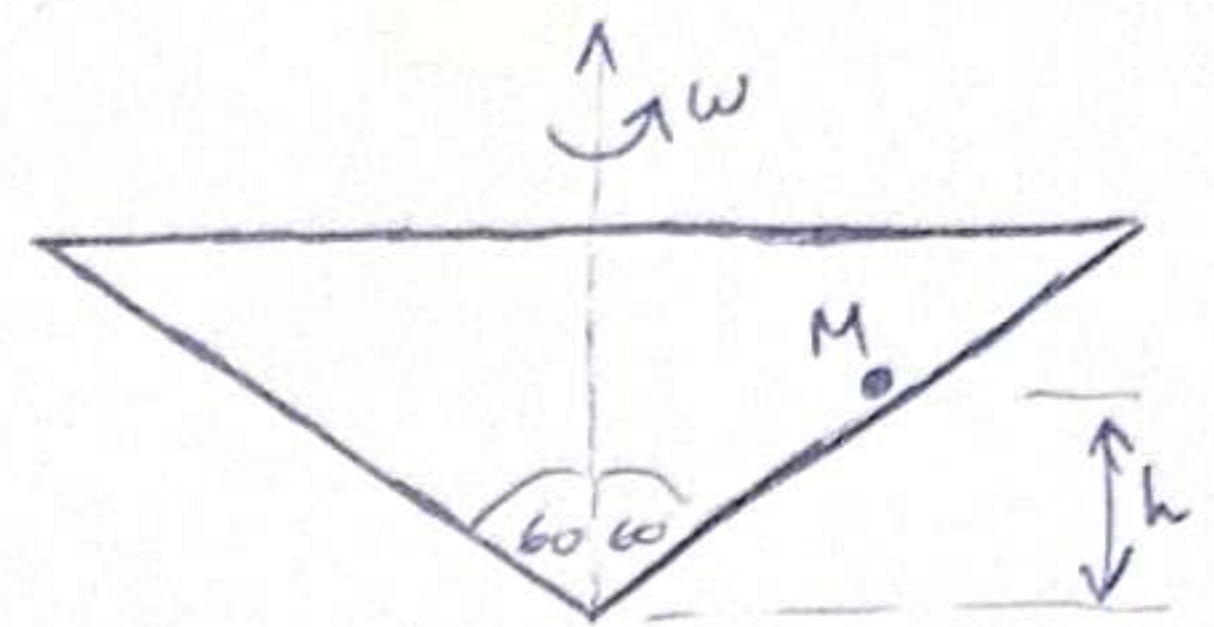


6.1. vertikalna ravna

$$w = \text{const.}$$

$w = ?$ - da bi točka bila
v relativno ravnoteži na višini h



$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_b$$

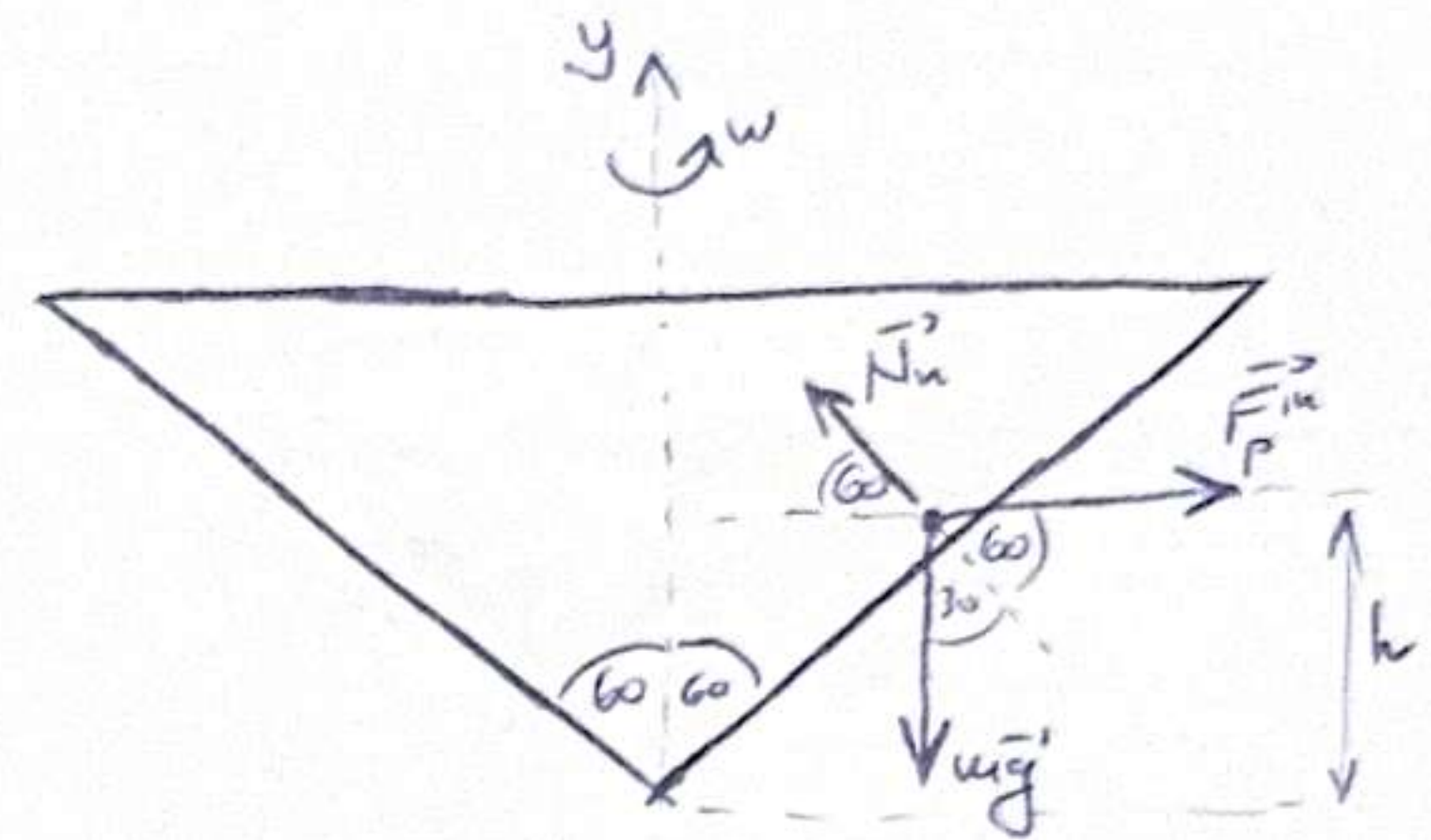
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\begin{aligned} \vec{F}_p^{in} &= -m(\vec{a}_n + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 \cdot x \cdot (\vec{i}') = \\ &= m\omega^2 \frac{h}{\sin 60} \sin 60 \cdot (\vec{i}') = m\omega^2 \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} (\vec{i}') = m\omega^2 \cdot h \sqrt{3} \cdot (\vec{i}') \end{aligned}$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 0$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_b = N_b \cdot (\vec{b})$$



$$\sum: 0 = -mg \sin 30 + F_p^{in} \cos 30 + 0 + 0 + 0$$

$$0 = -\frac{1}{2}mg + m\omega^2 h \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$m\omega^2 h \cdot \frac{3}{2} = \frac{1}{2}mg$$

$$\boxed{\omega = \sqrt{\frac{g}{3h}}}$$

- 6.3. problem u
 $xy = a^2$
 $\omega = \text{const}$
 vertikalna osa, uzmeću uvažiti
 položaj relativne ramolice?
 reakcija zice u tom položaju?

$$y = \frac{1}{x} \cdot a^2$$

$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_p^{\text{in}} + \vec{F}_{\text{cor}} + \vec{H}_n + \vec{H}_g$$

$$\vec{m}\vec{g} = mg \cdot (\vec{j})$$

$$\vec{F}_p^{\text{in}} = -m(\vec{a}_{\text{rel}} + \vec{E} \times \vec{J} + \vec{\omega} \times (\vec{\omega} \times \vec{J})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v}_r^{\text{rel}} = 0$$

$$\vec{H}_n = H_n \cdot (-\vec{n})$$

$$\vec{H}_g = H_g \cdot (\vec{k})$$

$$t: 0 = -mg \sin \varphi + F_p^{\text{in}} \cos \varphi + 0 + 0$$

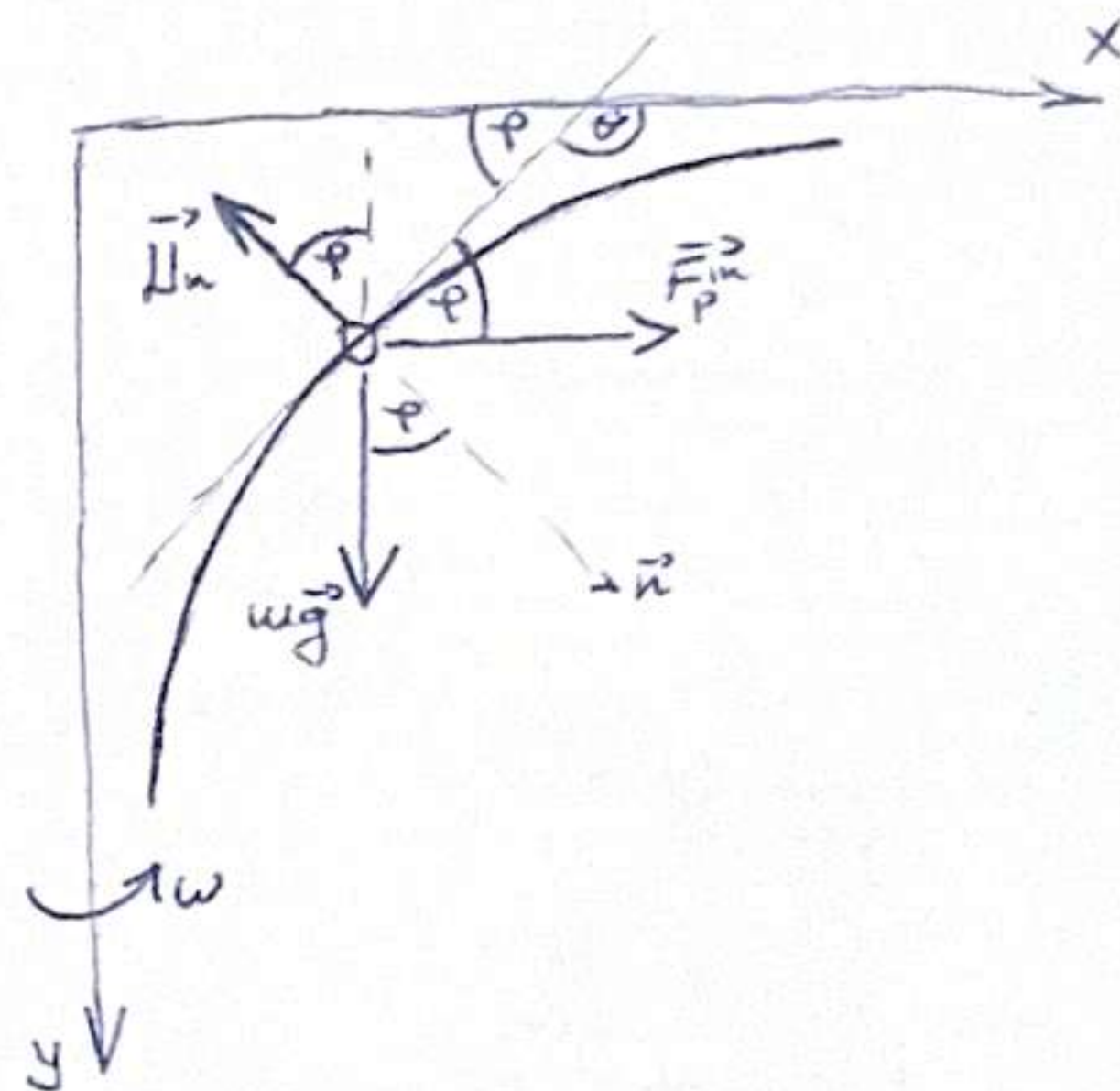
$$0 = -mg \frac{a^2}{\sqrt{x^4 + a^4}} + m\omega^2 x \cdot \frac{x^2}{\sqrt{x^4 + a^4}}$$

$$0 = -ga^2 + \omega^2 x^3$$

$$x = \sqrt[3]{\frac{ga^2}{\omega^2}}$$

$$y = \frac{1}{x} a^2 = \sqrt[3]{\frac{\omega^2}{ga^2}} \cdot a^2 = a \sqrt[3]{\frac{a\omega^2}{g}}$$

$$y = a \sqrt[3]{\frac{a\omega^2}{g}}$$



$$\tan \theta = y' = -1 \cdot x^{-2} \cdot a^2 = -\frac{a^2}{x^2}$$

$$\tan \varphi = \tan(180 - \theta) = -\tan \theta = \frac{a^2}{x^2}$$



$$\sin \varphi = \frac{\tan \varphi}{\sqrt{1 + \tan^2 \varphi}} = \frac{\frac{a^2}{x^2}}{\sqrt{1 + \frac{a^4}{x^4}}} = \frac{\frac{a^2}{x^2}}{\sqrt{\frac{x^4 + a^4}{x^4}}} = \frac{a^2}{\sqrt{x^4 + a^4}}$$

$$\cos \varphi = \frac{1}{\sqrt{1 + \tan^2 \varphi}} = \frac{1}{\sqrt{1 + \frac{a^4}{x^4}}} = \frac{1}{\sqrt{\frac{x^4 + a^4}{x^4}}} = \frac{x^2}{\sqrt{x^4 + a^4}}$$

$$n: 0 = mg \cos \varphi + F_p^{\text{in}} \sin \varphi - H_n + 0$$

$$H_n = mg \cos \varphi + F_p^{\text{in}} \sin \varphi = mg \frac{x^2}{\sqrt{x^4 + a^4}} + m\omega^2 x \frac{a^2}{\sqrt{x^4 + a^4}} = 0 \quad (\text{komplicirano})$$

$$y: 0 = mg + 0 - H_n \cos \varphi + 0$$

$$H_n = \frac{mg}{\cos \varphi} = mg \frac{\sqrt{x^4 + a^4}}{x^2} = mg \cdot \frac{\sqrt{\frac{a^4 g^4}{\omega^8} + a^4}}{\sqrt[3]{\frac{a^4 g^4}{\omega^4}}} = mg \sqrt[3]{\frac{(a^4 g^4)^{4/3}}{(\frac{a^4 g^4}{\omega^4})^{4/3}}} \Rightarrow H_n = mg \sqrt[3]{1 + \left(\frac{\omega^2 a}{g}\right)^{4/3}}$$

$$z: 0 = 0 + 0 + H_g \Rightarrow H_g = 0$$

6.4. prsten: m

b: $\vec{v}_0 = v_0 \cdot \vec{i}$

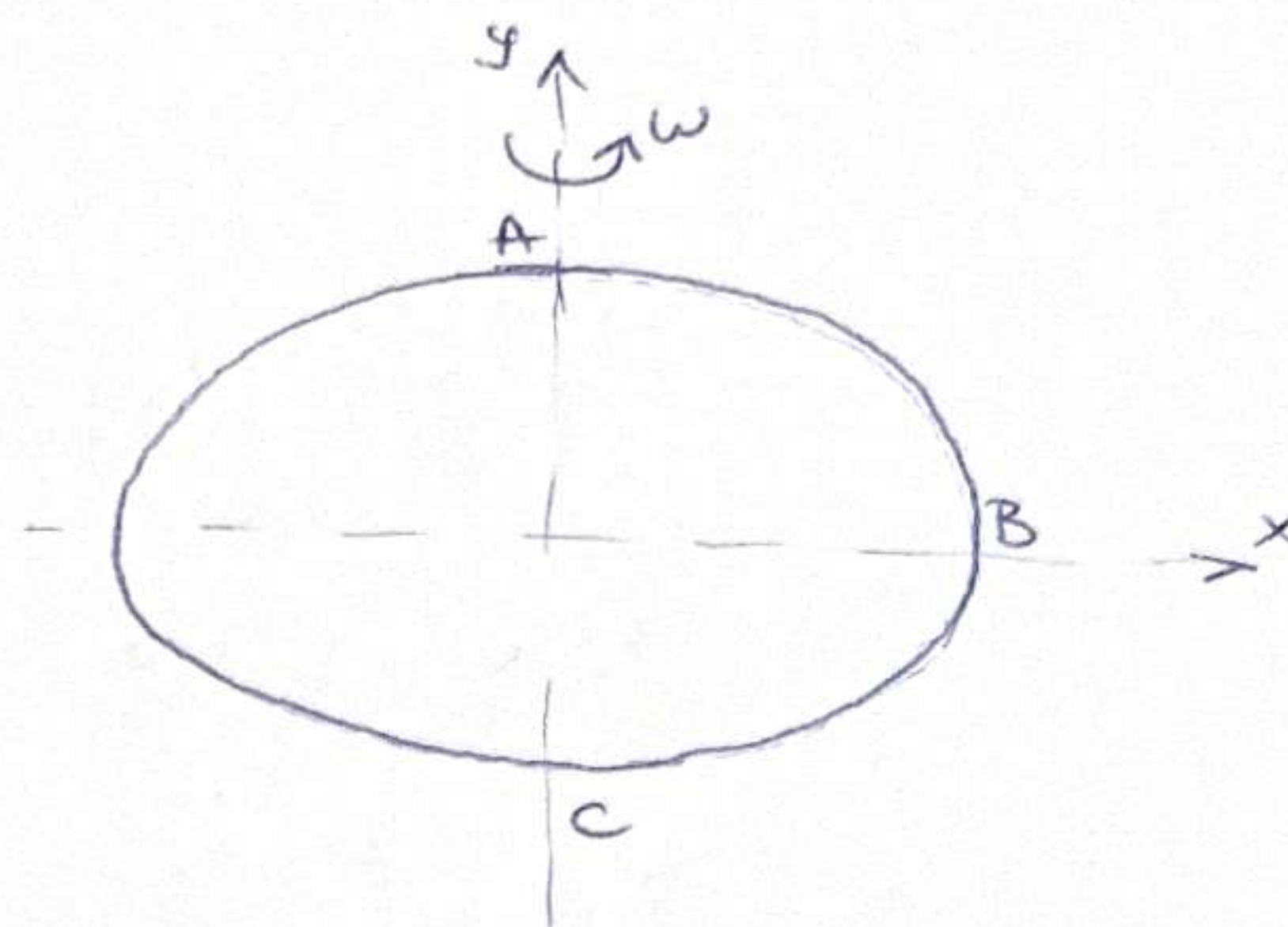
polotaj $A(0, b)$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\omega = \text{const}$ - obrtna fre

$N_B = ?$, $N_C = ?$

$B(a, 0)$, $C(0, -b)$



$$m\vec{a} = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_b$$

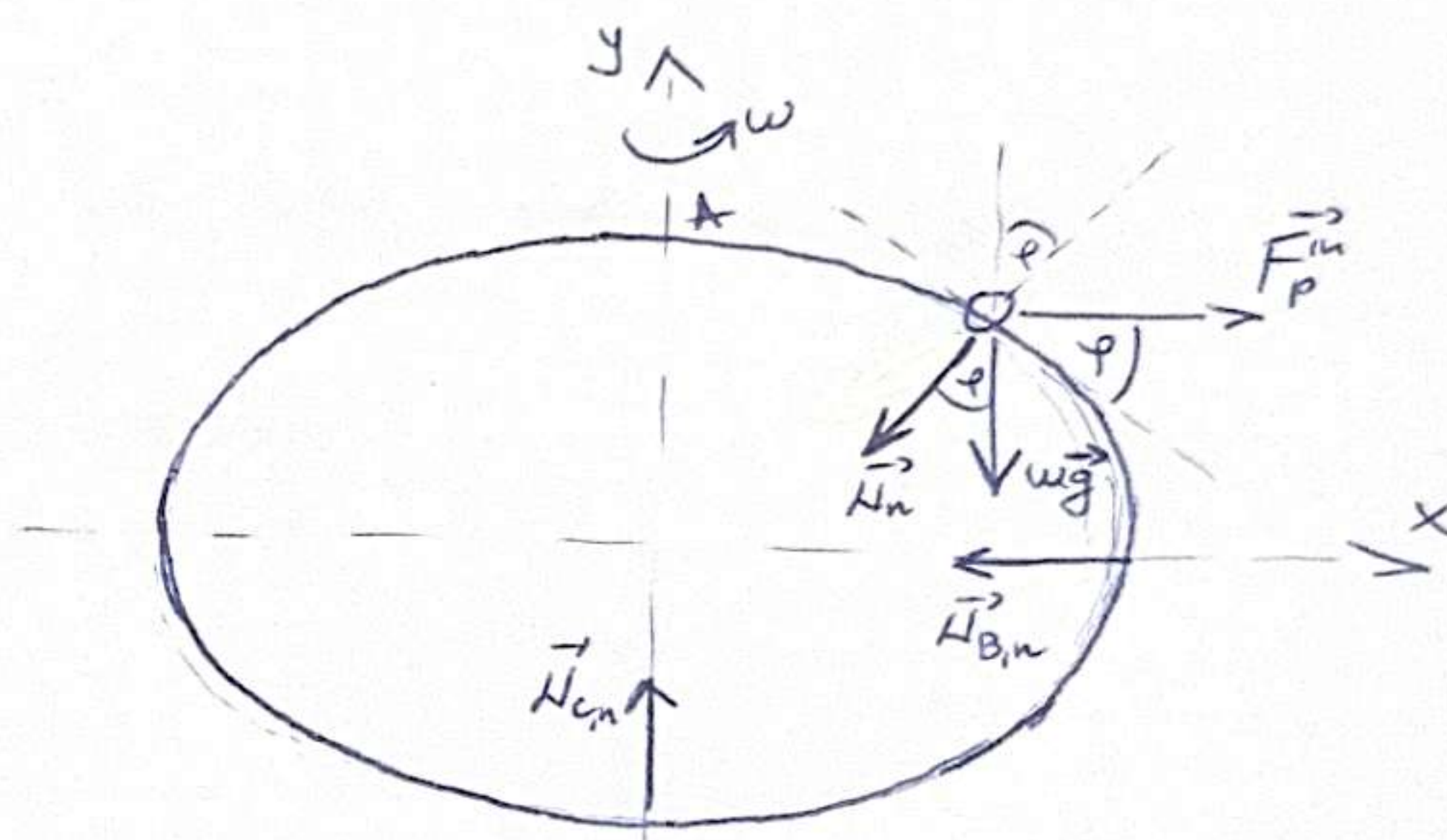
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{cm} + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_{rel} = 2m\omega \cdot v_{rel} \cdot \cos\varphi \cdot (\vec{k})$$

$$\vec{N}_n = N_n \cdot (+\vec{n})$$

$$\vec{N}_b = N_b \cdot (-\vec{k})$$



n: $m \cdot a_{rn} = mg \cos\varphi - F_p^{in} \sin\varphi + N_n$

$m \cdot \frac{v_{rel}^2}{R_k} = mg \cos\varphi - m\omega^2 x + N_n$

$N_n = -mg \cos\varphi + m\omega^2 x + m \cdot \frac{v_{rel}^2}{R_k}$, $v_{rel} = ?$
 $R_k = ?$

b: $0 = F_{cor}^{in} - N_b \rightarrow N_b = F_{cor}^{in} = 2m\omega \cdot v_{rel} \cdot \cos\varphi$

$T - T_0 = A$ (rel)

$T_0 = \frac{1}{2} m v_0^2$

$T = \frac{1}{2} m v_{rel}^2$

$A = \int_{y_0=b}^y -mg dy + \int_{x_0=0}^x F_p^{in} dx = mg(b-y) + \int_0^x m\omega^2 x dx = mg(b-y) + \frac{1}{2} m\omega^2 x^2$

$\frac{1}{2} m (v_{rel}^2 - v_0^2) = mg(b-y) + \frac{1}{2} m\omega^2 x^2$

$v_{rel}^2 = v_0^2 + 2g(b-y) + \omega^2 x^2$

$$R_k = \frac{(1+y'^2)^{3/2}}{|y''|}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \Rightarrow y = \left(b^2 - x^2 \frac{b^2}{a^2}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2} \left(b^2 - x^2 \frac{b^2}{a^2}\right)^{-\frac{1}{2}} \cdot (-2x \frac{b^2}{a^2}) = \frac{1}{2} y^{-1} \cdot (-2x \frac{b^2}{a^2}) = -\frac{x}{y} \frac{b^2}{a^2}$$

$$y'' = -\frac{1}{4} \left(b^2 - x^2 \frac{b^2}{a^2}\right)^{-\frac{3}{2}} \cdot (-2x \frac{b^2}{a^2})^2 + \frac{1}{2} \left(b^2 - x^2 \frac{b^2}{a^2}\right)^{-\frac{1}{2}} \cdot (-2 \frac{b^2}{a^2}) =$$

$$y'' = \frac{1}{2} y^{-3} \cdot \left(x \frac{b^2}{a^2}\right)^2 - y^{-1} \frac{b^2}{a^2} = \frac{1}{2y^3} \left(x \frac{b^2}{a^2}\right)^2 - \frac{1}{y} \frac{b^2}{a^2}$$

položaj B ($x=a$, $y=0$, $\varphi=\frac{\pi}{2}$)

$R_k(y=0)$: $y'(y=0) = \infty$
 $y''(y=0) = \infty - \infty \Rightarrow$ ne more ovako, probajmo drugače

$$\begin{aligned} x &= a \sin t & \dot{x} &= a \cos t & \ddot{x} &= -a \sin t \\ y &= b \cos t & \dot{y} &= -b \sin t & \ddot{y} &= -b \cos t \end{aligned}$$

$$R_k = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{|\ddot{x}\dot{y} - \dot{x}\ddot{y}|}$$

položaj B

$$\dot{x}_B = 0, \quad \dot{y}_B = -b, \quad \ddot{x}_B = -a, \quad \ddot{y}_B = 0$$

$$R_{k,B} = \frac{(0+b^2)^{3/2}}{|0+ab|} = \frac{b^3}{ab} = \frac{b^2}{a}$$

$$U_{rel,B} = U_0^2 + 2gb + \omega^2 a^2$$

$$H_{n,B} = 0 + m\omega^2 a + m \frac{(U_0^2 + 2gb + \omega^2 a^2) a}{b^2}$$

$$\boxed{H_{n,B} = m \frac{a}{b^2} (U_0^2 + 2gb + (a^2 + b^2)\omega^2)}, \quad \boxed{H_{e,B} = 0}, \quad \boxed{H_B = H_{n,B}}$$

položaj C ($x=0$, $y=-b$, $\varphi=\pi$)

$$\dot{x}_C = -a, \quad \dot{y}_C = 0, \quad \ddot{x}_C = 0, \quad \ddot{y}_C = b$$

$$R_{k,C} = \frac{(a^2+0)^{3/2}}{|-ab-0|} = \frac{a^3}{ab} = \frac{a^2}{b}$$

$$U_{rel,C}^2 = U_0^2 + 2g(b - (-b)) + \omega^2 \cdot 0 = U_0^2 + 4gb$$

$$H_{n,C} = mg + m\omega^2 \cdot 0 + m \cdot \frac{(U_0^2 + 4gb)b}{a^2} = m\left(g + \frac{b}{a^2}(U_0^2 + 4gb)\right)$$

$$\boxed{H_{n,C} = m\left(g + \frac{b}{a^2}(U_0^2 + 4gb)\right)}, \quad \boxed{H_{e,C} = -2m\omega\sqrt{U_0^2 + 4gb}}, \quad \boxed{H_C = \sqrt{H_{n,C}^2 + H_{e,C}^2}}$$

6.5) ploča: $l, \vec{a}$ ($a = \text{const}$)

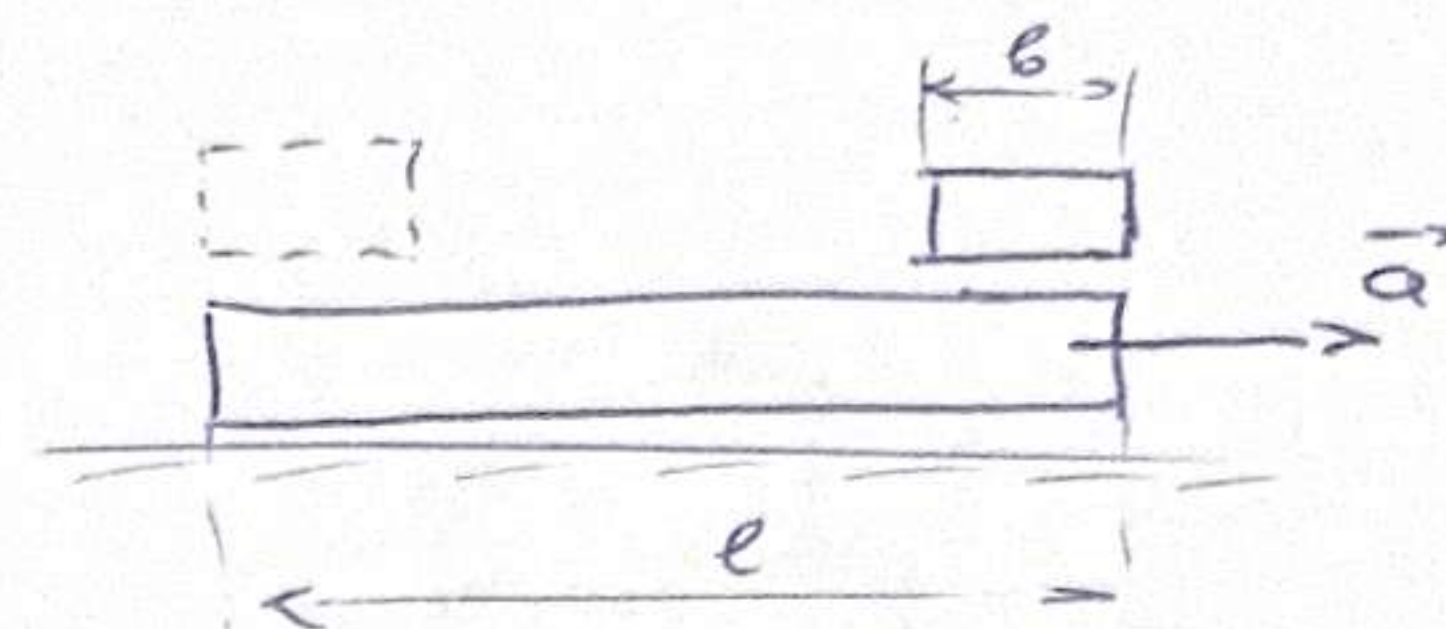
teło A: b

t_0 : $v_{A,rel} = 0$

A je na pólnoim krawu płoče

t_1 : A je na drugom krawu płoče

$t_1 = ?$



zadatok: uaci $x(t)$, a onda reši da je $x(t_1) = l - b$

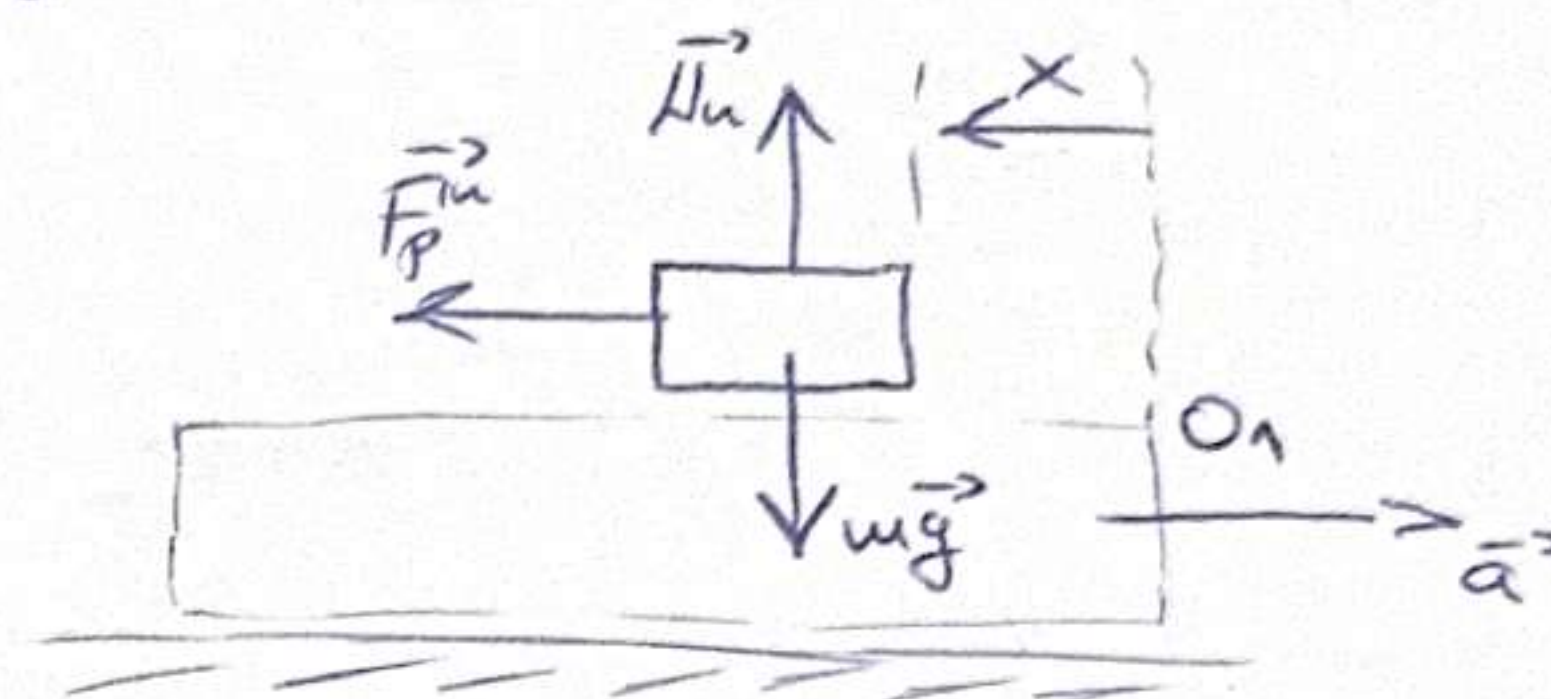
$$m\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{cor} + \vec{H}_u + \vec{H}_G \rightarrow 0$$

$$m\vec{g} = mg \cdot (-\vec{j}')$$

$$\vec{F}_p = -m(\vec{a}_{cm} + \vec{r} \times \vec{\omega}' + \vec{\omega}' \times (\vec{\omega}' \times \vec{r})) = ma \cdot (\vec{i}')$$

$$\vec{F}_{cor} = -2m\vec{\omega}' \times \vec{v}_r = 0$$

$$\vec{H}_u = H_u \cdot (\vec{j}')$$



$$x_0: m\ddot{x} = 0 + F_p + 0$$

$$m\ddot{x} = ma \quad \int dt$$

$$\dot{x} = \int a dt = at + c_1 \quad \int dt$$

$$x = \frac{1}{2}at^2 + c_2$$

$$x = \frac{1}{2}at^2$$

$$x(t_1) = \frac{1}{2}at_1^2 = l - b$$

$$t_1 = \sqrt{\frac{2(l-b)}{a}}$$

6.6. nastavak na prethodni zadatak, samo što između tela i ploče postoji koeficijent μ .

$$x: m\ddot{x} = F_p^{\text{in}} - F_{\text{fr}} = ma - \mu \cdot N = ma - \mu \cdot mg$$

$$\ddot{x} = a - \mu g \quad | \int dt$$

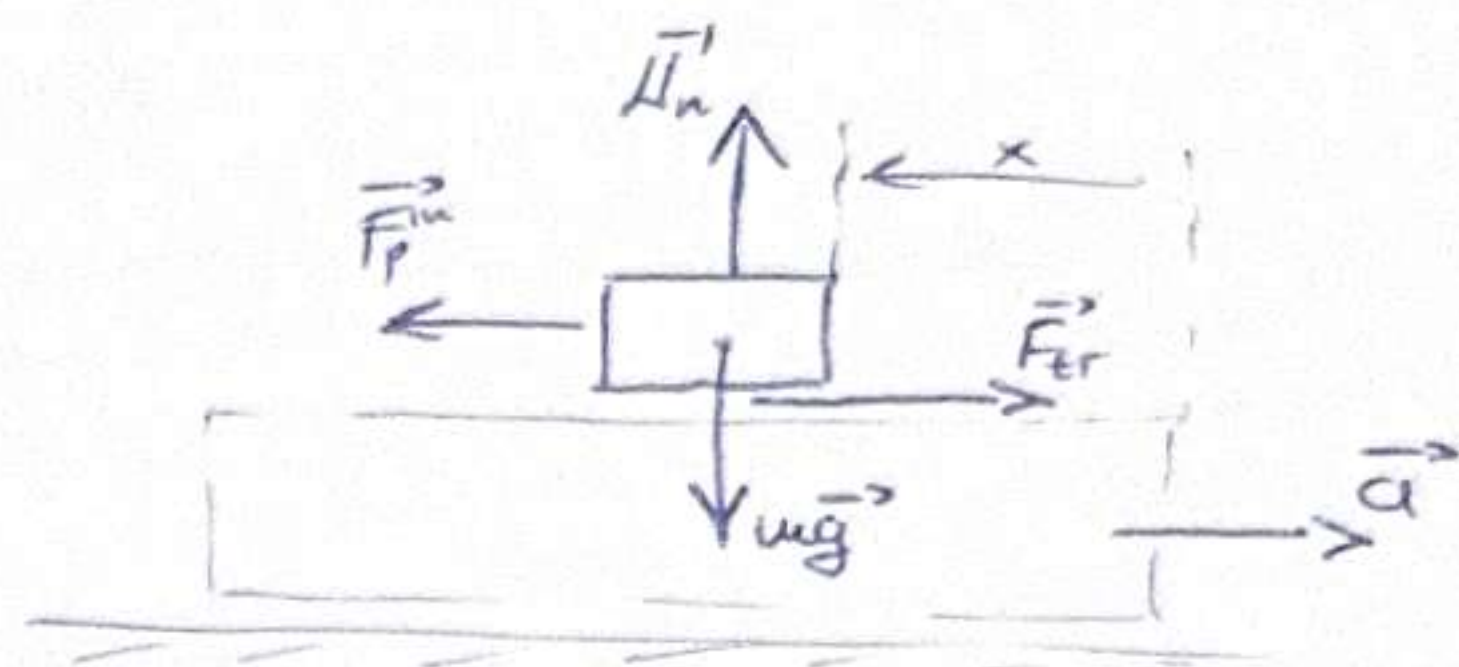
$$\dot{x} = (a - \mu g)t + \cancel{c_1^{\text{to}}} \quad | \int dt$$

$$x = \frac{1}{2}(a - \mu g)t^2 + \cancel{c_2^{\text{to}}}$$

$$x(t_1) = \frac{1}{2}(a - \mu g)t_1^2 = l - b$$

$$\Rightarrow \boxed{t_1 = \sqrt{\frac{2(l-b)}{a - \mu g}}}$$

$$\text{uslov: } a - \mu g > 0$$

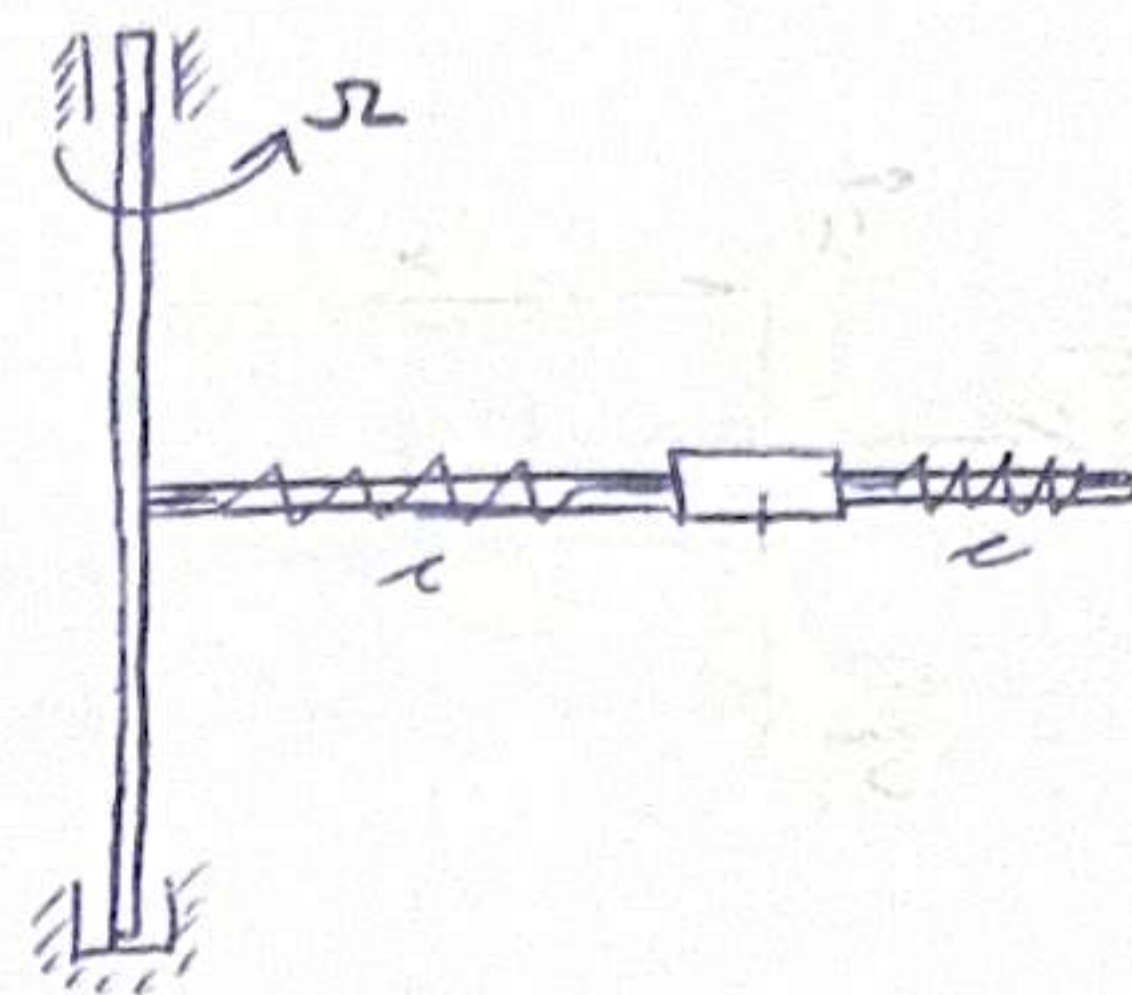


6.7. OA: ℓ , $\Omega = \text{const.}$

ključ: m

opruge: c , $\ell_0 = \frac{\ell}{2}$

j-ua rel. krećuća ključa po stepu?



$$\ell_e = \ell + \ell = 2\ell$$

$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_c + \vec{F}_p + \vec{F}_{\text{cor}} + \vec{N}_u + \vec{N}_e$$

$$\vec{m}\vec{g} = mg \cdot (-\vec{j})$$

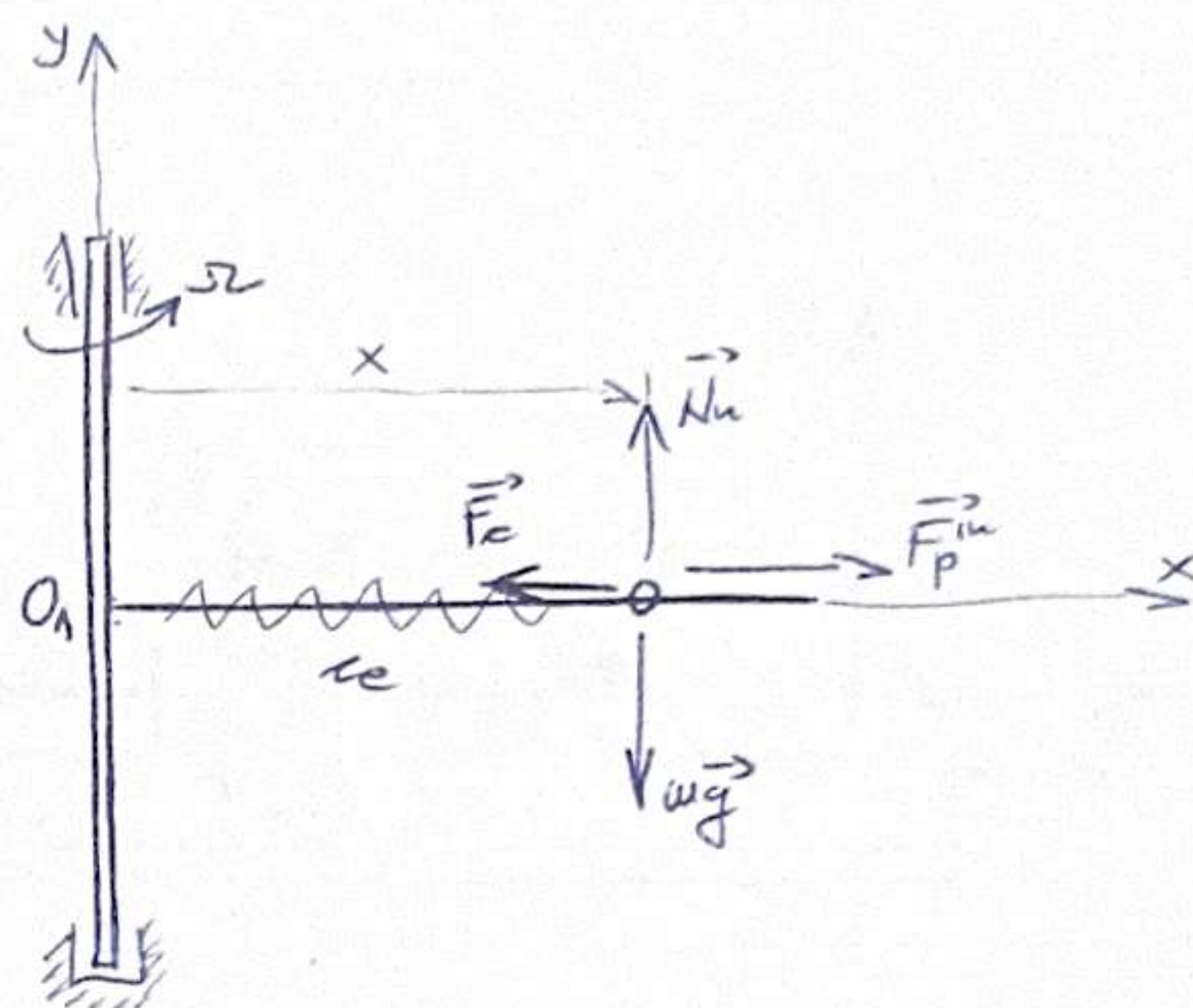
$$\vec{F}_c = c \cdot \Delta \ell \cdot (-\vec{i}) = 2c \cdot (x - \frac{\ell}{2}) \cdot (-\vec{i}) = c \cdot (2x - \ell) \cdot (-\vec{i})$$

$$\vec{F}_p = -m(\vec{\omega} \times \vec{r} + \vec{r} \times \vec{\omega} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\lambda^2 \cdot x \cdot (\vec{i})$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v}_r = 2m \cdot \Omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{N}_u = N_u \cdot (\vec{j})$$

$$\vec{N}_e = N_e \cdot (-\vec{k})$$



$$x: m\ddot{x} = -c(2x - \ell) + m\Omega^2 x \quad | : m$$

$$\ddot{x} + \frac{c}{m} \cdot 2x - \Omega^2 x = \frac{c\ell}{m}$$

$$\ddot{x} + \frac{2c - \Omega^2 m}{m} x = \frac{c\ell}{m}$$

$$1) \quad 2c > \Omega^2 m \Rightarrow \omega^2 = \frac{2c - \Omega^2 m}{m}, \quad h = \frac{c\ell}{m}$$

$$H: \lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm i\omega \rightarrow x_h = A_1 \cos\left(\sqrt{\frac{2c - \Omega^2 m}{m}} t\right) + A_2 \sin\left(\sqrt{\frac{2c - \Omega^2 m}{m}} t\right)$$

$$P: G(t) = h = e^{0t} (h \cos \omega t + 0 \sin \omega t)$$

$$x_p = t^0 e^{0t} (A \cos \omega t + B \sin \omega t) = A, \quad \ddot{x}_p = 0$$

$$0 + \omega^2 A = h \rightarrow A = \frac{h}{\omega^2} = \frac{\frac{c\ell}{m}}{\frac{2c - \Omega^2 m}{m}} = \frac{c\ell}{2c - \Omega^2 m}$$

$$x = A_1 \cos\left(\sqrt{\frac{2c - \Omega^2 m}{m}} t\right) + A_2 \sin\left(\sqrt{\frac{2c - \Omega^2 m}{m}} t\right) + \frac{c\ell}{2c - \Omega^2 m}$$

$$2) \quad 2c = \Omega^2 m \Rightarrow \ddot{x} = \frac{c\ell}{m} \Rightarrow \dot{x} = \frac{c\ell}{m} t + B_1 \Rightarrow x = \frac{c\ell}{2m} t^2 + B_1 t + B_2$$

$$3) \quad 2c < \Omega^2 m \Rightarrow \omega^2 = \frac{\Omega^2 m - 2c}{m}, \quad h = \frac{c\ell}{m}$$

$$H: \lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega \pm 0i \Rightarrow x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$P: x_p = A, \quad \ddot{x}_p = 0 \Rightarrow 0 - \omega^2 A = h \Rightarrow A = -\frac{h}{\omega^2} = -\frac{c\ell}{2c - \Omega^2 m}$$

$$x = C_1 e^{\sqrt{\frac{\Omega^2 m - 2c}{m}} t} + C_2 e^{-\sqrt{\frac{\Omega^2 m - 2c}{m}} t} + \frac{c\ell}{2c - \Omega^2 m}$$

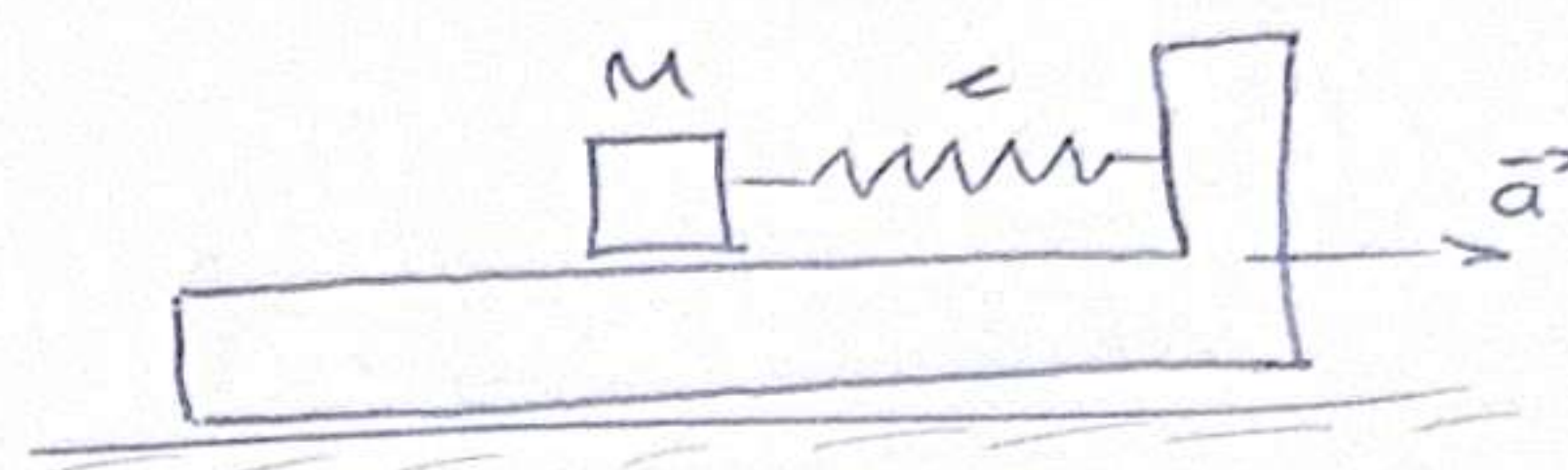
6.8) M: $m = 10 \text{ kg}$

opruga: $c = \frac{4 \text{ kN}}{\text{m}} = 4000 \frac{\text{N}}{\text{m}}$

$\Delta l_{\max} = ?$

$t_0 = 0$: A miruje, M miruje

$a_A = a = 3 \frac{\text{m}}{\text{s}^2}$



$$m \vec{a}_r = m \vec{g} + \vec{F}_c + \vec{F}_p + \vec{F}_{\text{Cor}} + \vec{N}_n + \vec{H}_c$$

$m \vec{g} = mg \cdot (-\vec{j})$

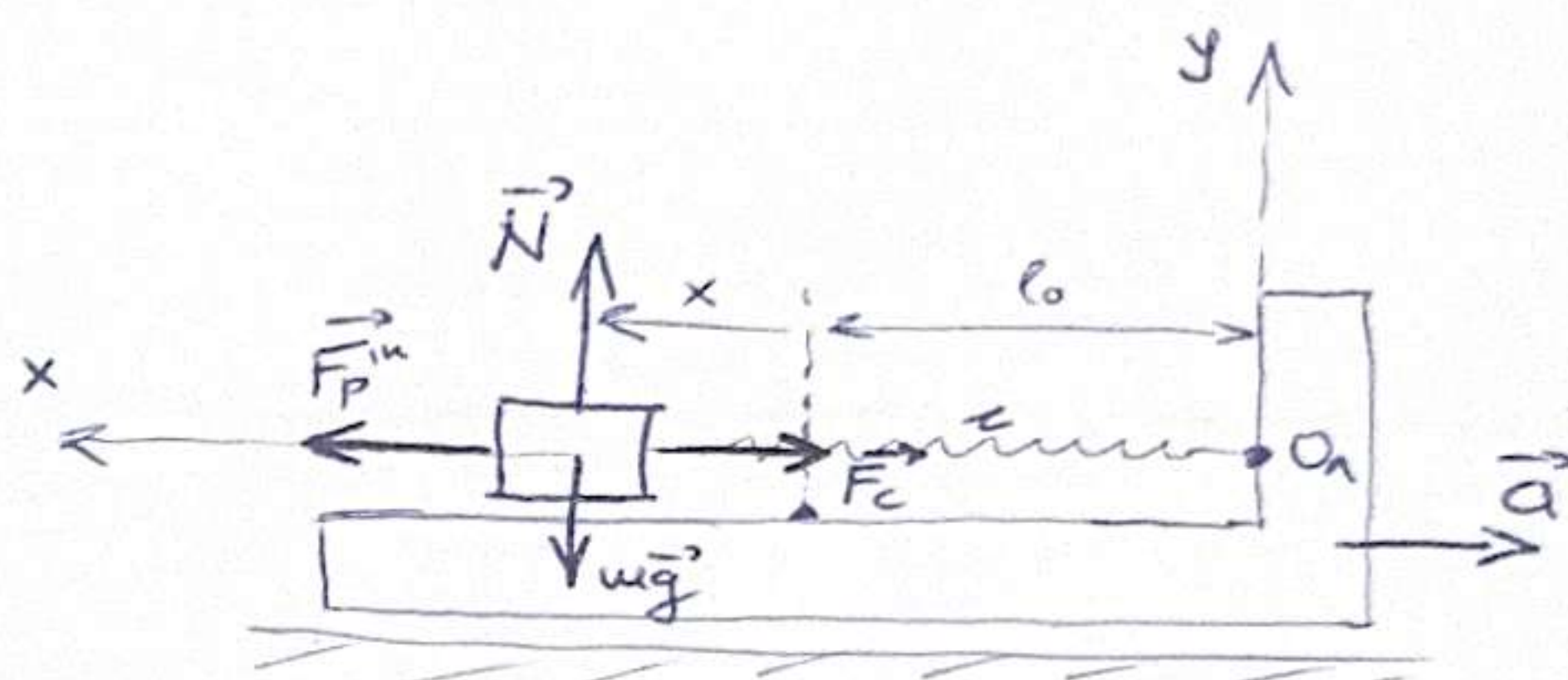
$\vec{F}_c = c \cdot \Delta l \cdot (-\vec{i}) = c \cdot x \cdot (-\vec{i})$

$\vec{F}_p = -m(\vec{a}_n + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m \cdot a \cdot (\vec{i})$

$\vec{F}_{\text{Cor}} = -2m \vec{\omega} \times \vec{v}_r = 0$

$\vec{N}_n = N \cdot (\vec{j}) = \vec{N}$

$\vec{H}_c = 0$



x : $m \ddot{x} = -cx + ma$ $| : m$
 $\ddot{x} + \frac{c}{m} x = a$, $\omega^2 = \frac{c}{m}$, $h = a$
 $\ddot{x} + \omega^2 x = h$

H : $\lambda^2 + \omega^2 \cdot 1 = 0 \rightarrow \lambda_{1/2} = 0 \pm i \cdot \omega \rightarrow x_h = C_1 \cos(\sqrt{\frac{c}{m}} t) + C_2 \sin(\sqrt{\frac{c}{m}} t)$

P : $\phi(t) = h = e^{0t} (h \cos \omega t + 0 \sin \omega t)$

$x_p = t^0 e^{0t} (A \cos \omega t + B \sin \omega t) = A$, $x_p'' = 0$

$0 + \omega^2 A = h \rightarrow A = \frac{h}{\omega^2} = \frac{a}{\frac{c}{m}} = \frac{ma}{c}$

$x = C_1 \cos(\sqrt{\frac{c}{m}} t) + C_2 \sin(\sqrt{\frac{c}{m}} t) + \frac{ma}{c}$

$\dot{x} = -\sqrt{\frac{c}{m}} C_1 \sin(\sqrt{\frac{c}{m}} t) + \sqrt{\frac{c}{m}} C_2 \cos(\sqrt{\frac{c}{m}} t)$

$x(t_0) = C_1 \cdot 1 + 0 + \frac{ma}{c} = 0 \rightarrow C_1 = -\frac{ma}{c}$

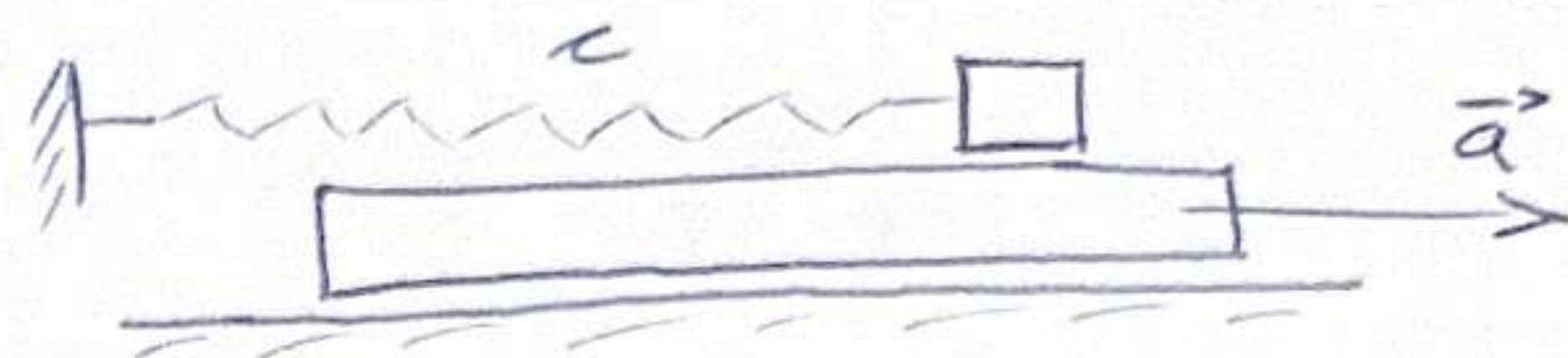
$\dot{x}(t_0) = 0 + \sqrt{\frac{c}{m}} C_2 \cdot 1 = 0 \rightarrow C_2 = 0$

$x = -\frac{ma}{c} \cos(\sqrt{\frac{c}{m}} t) + \frac{ma}{c}$

$x_{\max} = -\frac{ma}{c} \cdot (-1) + \frac{ma}{c} = \frac{2ma}{c} = \frac{2 \cdot 10 \cdot 3}{4000} = 0,015 \text{ m} = 1,5 \text{ cm}$

$\Delta l_{\max} = 1,5 \text{ cm}$

6.9. ploča: $\vec{a}$
 klasič: m
 opruga: $c = k^2 m$
 koef. tr: μ



$t_0 = 0$: ploča miruje
 opruga sebi sama $s_0 = \frac{\mu g}{k^2}$
 klasič v_0

$t_1 = \frac{\pi}{3k}$: klasič se pri t_1 zaustavi na ploči

$v_0 = ?$

pod kojim uslovima je to moguće?
 u konacnom intervalu vremena?

$$\vec{m}\ddot{\vec{r}} = \vec{m}\vec{g} + \vec{F}_{tr} + \vec{F}_c + \vec{F}_p + \vec{F}_{cor} + \vec{U}$$

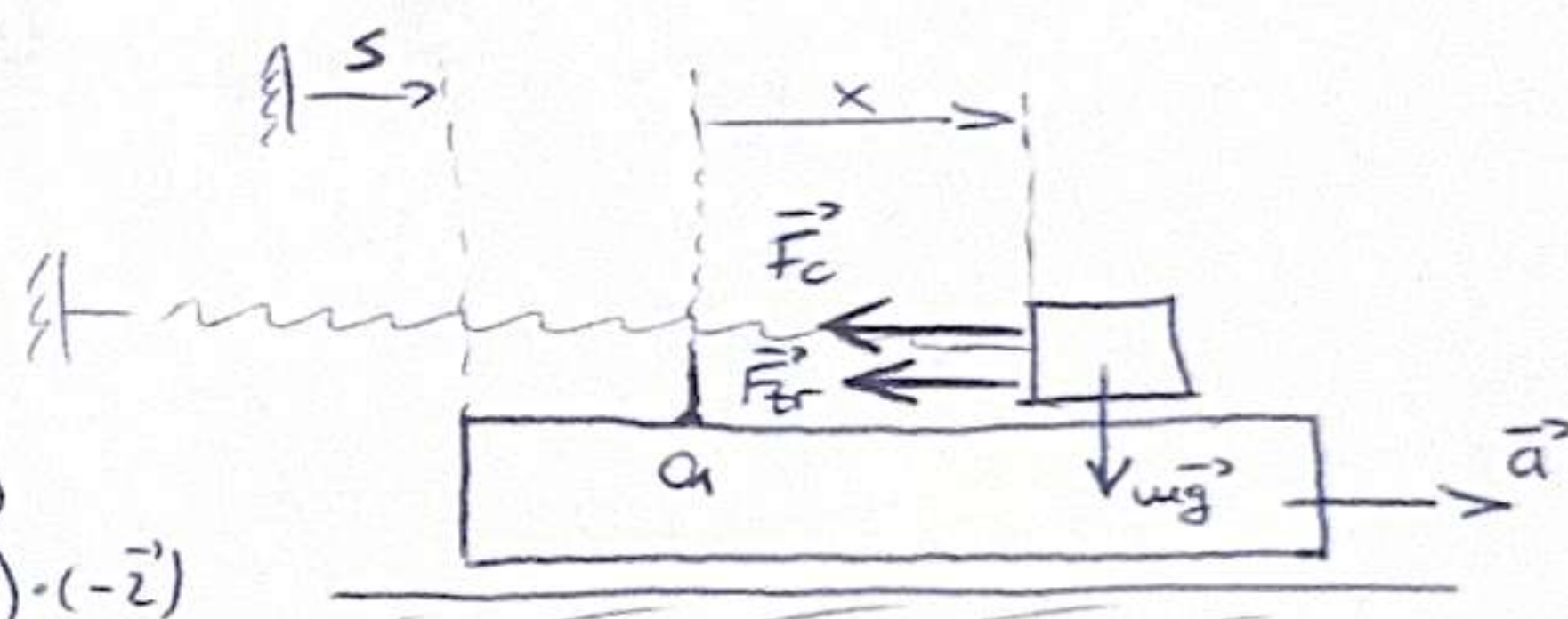
$$\vec{m}\vec{g} = m g \cdot (-\vec{j})$$

$$\vec{F}_{tr} = \mu \cdot N \cdot (-\vec{i}) = \mu \cdot m g \cdot (-\vec{i})$$

$$\vec{F}_c = c \cdot \Delta l \cdot (-\vec{i}) = c \cdot (-\Delta l_0 + s + x) \cdot (-\vec{i}) = k^2 m \left(-\frac{\mu g}{k^2} + s + x \right) \cdot (-\vec{i}) \quad (s = ?)$$

$$\vec{F}_p = -m(\ddot{\vec{a}}_{on} + \vec{\omega} \times \vec{r} + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m \cdot a \cdot (-\vec{i})$$

$$\vec{U} = U \cdot (\vec{j})$$



$$\ddot{s} = a \quad | \int dt$$

$$\dot{s} = at + C_1 \quad | \int dt$$

$$s = \frac{1}{2}at^2 + C_2$$

$$x: m\ddot{x} = -\mu \cdot m g - k^2 m \left(-\frac{\mu g}{k^2} + \frac{1}{2}at^2 + x \right) - ma$$

$$m\ddot{x} + k^2 m x = -\mu m g + m \mu g - \frac{1}{2}k^2 m a t^2 - ma$$

$$\ddot{x} + k^2 x = -\frac{1}{2}k^2 a t^2 - a$$

$$\# : \lambda^2 + k^2 = 0 \rightarrow \lambda_{in} = 0 \pm i \cdot k \rightarrow x_{in} = C_1 \cos kt + C_2 \sin kt$$

$$P: G(t) = -\frac{1}{2}k^2 a t^2 - a = e^{0t} (P_1(t) \cos 0t + P_2(t) \sin 0t)$$

$$x_p = t^0 e^{0t} (At^2 + Bt + C) \cos 0t + (Dt^2 + Et + F) \sin 0t = At^2 + Bt + C, \quad \ddot{x}_p = 2A$$

$$2A + k^2 A t^2 + k^2 B t + k^2 C = -\frac{1}{2}k^2 a t^2 - a$$

$$\rightarrow B = 0, \quad A = -\frac{1}{2}a, \quad C = \frac{-a - 2 \cdot (-\frac{1}{2}a)}{k^2} = 0 \Rightarrow x_p = -\frac{1}{2}at^2$$

$$x = C_1 \cos kt + C_2 \sin kt - \frac{1}{2}at^2$$

$$\dot{x} = -C_1 k \sin kt + C_2 k \cos kt - at$$

$$x(t_0) = 0 \rightarrow C_1 \cdot 1 + 0 - 0 = 0 \rightarrow C_1 = 0$$

$$\dot{x}(t_0) = v_0 \rightarrow -0 + C_2 \cdot k - 0 = v_0 \rightarrow C_2 = \frac{v_0}{k}$$

$$x = \frac{v_0}{k} \sin(kt) - \frac{1}{2} at^2$$

$$\dot{x}(t_1 = \frac{\pi}{3k}) = 0 \rightarrow \frac{v_0}{k} \cdot k \cos(k \cdot \frac{\pi}{3k}) - a \cdot \frac{\pi}{3k} = 0 \rightarrow v_0 \cdot \frac{1}{2} - \frac{a\pi}{3k} = 0$$

$$\hookrightarrow \boxed{v_0 = \frac{2}{3} \frac{a\pi}{k}}$$

uslovi relativne ravnoteže u trenutku t_1 :

$$m \vec{a}_r(t_1 = \frac{\pi}{3k}) = 0, \quad \text{stanje ravnoteže: } F_{tr} \leq \mu \cdot mg, \quad \text{; to suprotnog smjera!}$$

$$x: \quad 0 = + F_{tr} - k^2 m \left(- \frac{\mu g}{k^2} + \frac{1}{2} a t_1^2 + x(t_1) \right) - ma$$

$$0 = F_{tr} - k^2 m \left(- \frac{\mu g}{k^2} + \frac{1}{2} a \frac{\pi^2}{9k^2} + \frac{v_0}{k} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} a \frac{\pi^2}{9k^2} \right) - ma$$

$$0 = F_{tr} + m\mu g - k \cdot m \cdot \frac{v_0 \sqrt{3}}{2} - ma$$

$$0 = F_{tr} + m\mu g - k \cdot m \cdot \frac{2}{3} \frac{a\pi}{k} \frac{\sqrt{3}}{2} - ma$$

$$0 = F_{tr} + m\mu g - \frac{\sqrt{3}}{3} m a \pi - ma$$

$$F_{tr} = ma + \frac{\sqrt{3}}{3} a \pi \cdot m - m\mu g$$

$$\text{granični slučaj: } F_{tr} = F_{tr, \max} = m\mu g$$

$$m\mu g = ma + \frac{\sqrt{3}}{3} a \pi m - m\mu g$$

$$2\mu g = \frac{a}{3} (3 + \pi \sqrt{3})$$

$$\boxed{\mu \geq \frac{a}{6g} (3 + \pi \sqrt{3})}$$

6.10. $y = \frac{x^2}{2a}$

$w = \sqrt{\frac{g}{a}} = \text{const.}$

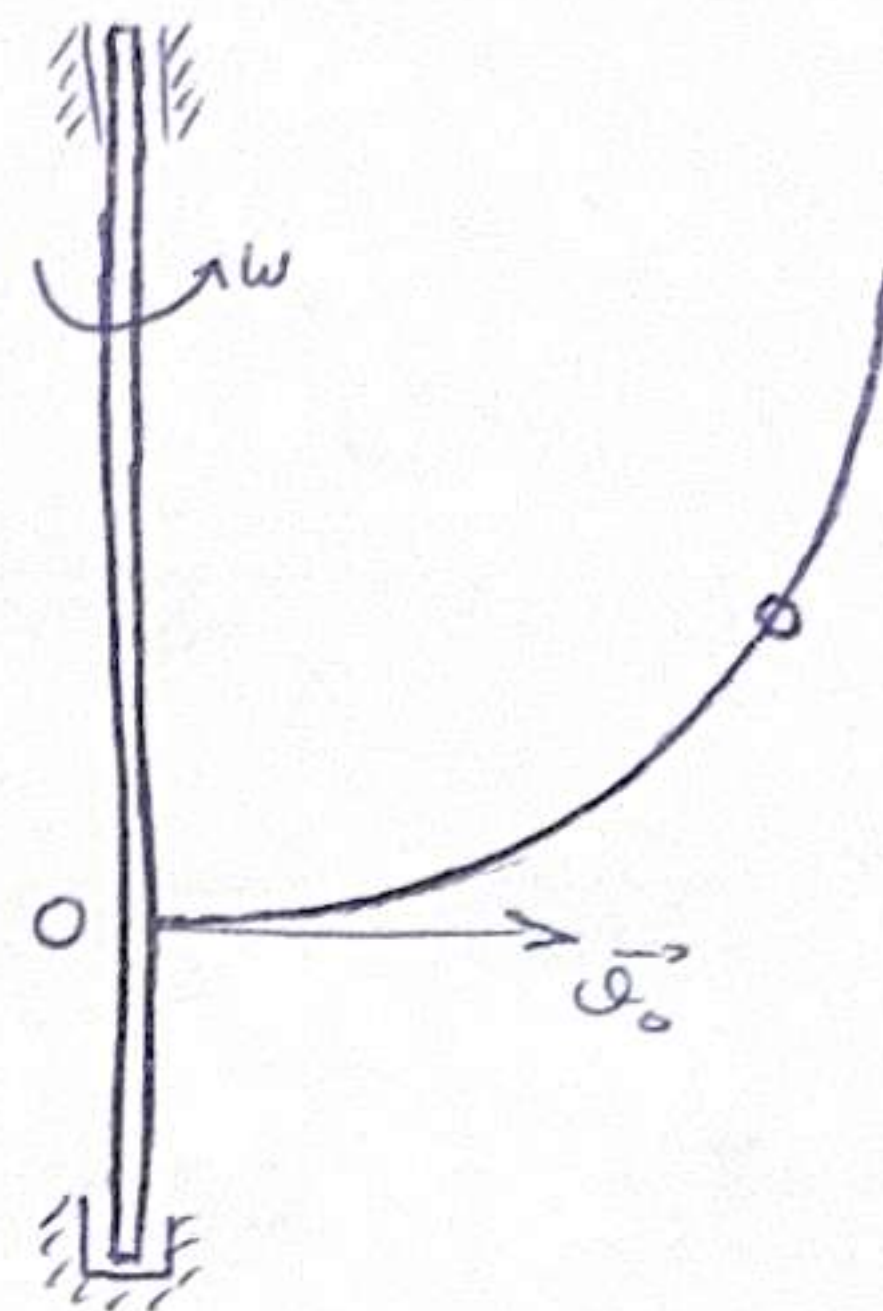
vertikalna osa O_y usmerena navzgor

prsten mase m

$t_0 = 0$: prsten je v temenu parabole

$v_0 = \sqrt{ag}$

U v prostotlinam položaj?



$m\vec{a} = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{U}_n + \vec{U}_e$

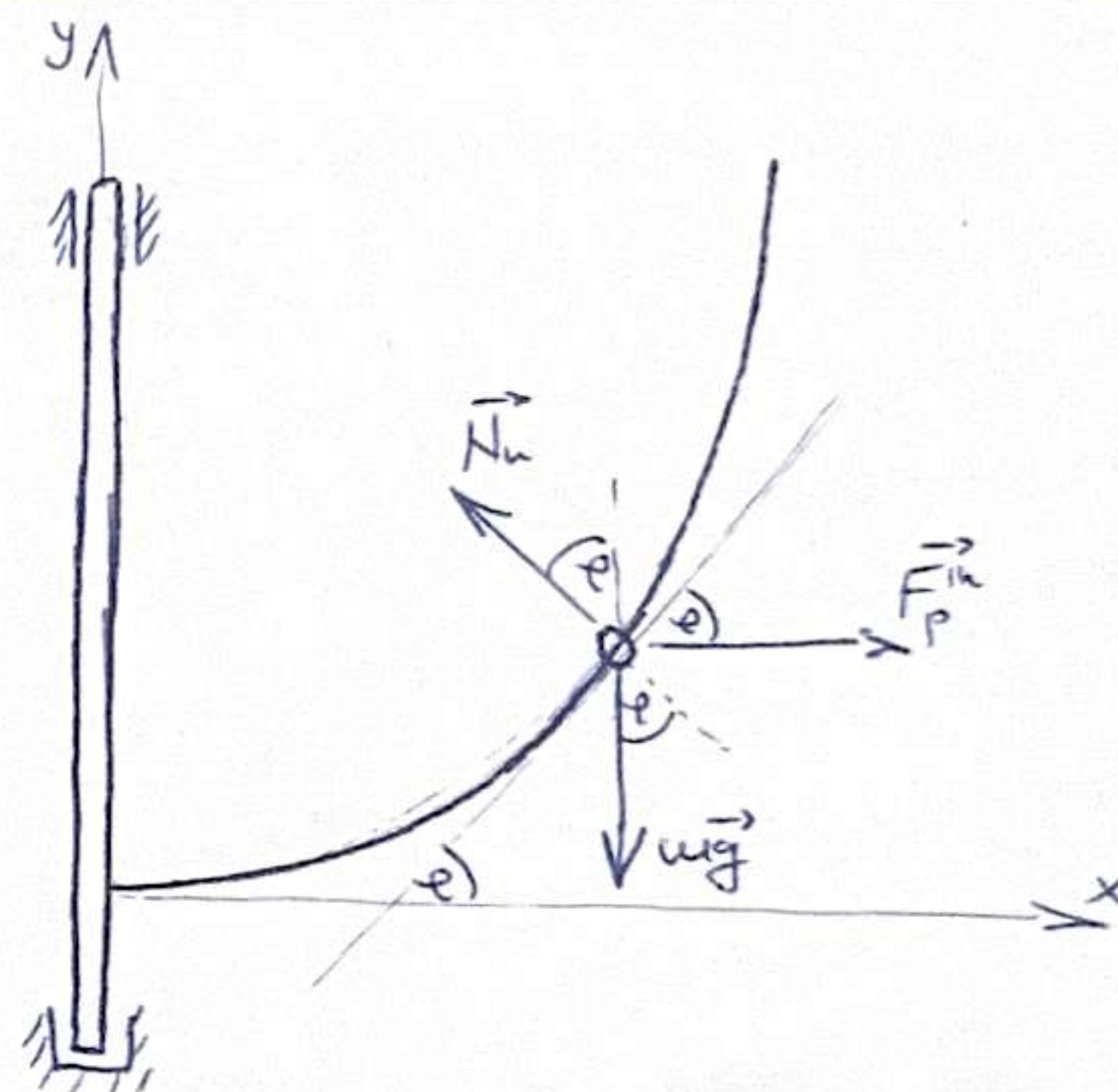
$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{on}^0 + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{U}_n = U_n \cdot (\vec{n})$

$\vec{U}_e = U_e \cdot (\vec{k})$



I način: razstavimo samo na x ali samo na y

II način: razstavimo na n (lahko) i dobijemo U_n (dosta komplikacija)

n : $ma_n = -mg \cos \varphi - F_p^{in} \sin \varphi + U_n$

$a_n = \frac{v_{rec}^2}{R_c}$, $v_{rec} = ?$, $R_c = ?$, $\sin \varphi = ?$, $\cos \varphi = ?$

$T - T_0 = A$ (rel)

$T = \frac{1}{2} m v_{rec}^2$

$T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} m ag$

$A = \int_0^x -mg dy + \int_0^x F_p^{in} dx = -\frac{1}{2} mg \frac{x^2}{a} + \int_0^x m\omega^2 x dx = -\frac{1}{2} \frac{mgx^2}{a} + \frac{1}{2} m \frac{g}{a} \cdot x^2 = 0$

$\frac{1}{2} m v_{rec}^2 - \frac{1}{2} m ag = 0 \Rightarrow v_{rec} = \sqrt{ag}$

$$R_c = \frac{(1+y'^2)^{3/2}}{|y''|}, \quad y' = \frac{x}{a}, \quad y'' = \frac{1}{a}$$

$$R_c = \frac{(1 + \frac{x^2}{a^2})^{3/2}}{\frac{1}{a}} = \frac{(a^2 + x^2)^{3/2}}{(a^2)^{3/2}} = \frac{(a^2 + x^2)^{3/2} \cdot a}{a^3} = \frac{(a^2 + x^2)^{3/2}}{a^2}$$

$$\begin{aligned} \tan \varphi = y' = \frac{x}{a} &\rightarrow \sin \varphi = \frac{\tan \varphi}{\sqrt{1 + \tan^2 \varphi}} = \frac{\frac{x}{a}}{\sqrt{1 + \frac{x^2}{a^2}}} = \frac{\frac{x}{a}}{\sqrt{\frac{a^2 + x^2}{a^2}}} = \frac{x}{\sqrt{a^2 + x^2}} \\ &\rightarrow \cos \varphi = \frac{1}{\sqrt{1 + \tan^2 \varphi}} = \frac{1}{\sqrt{1 + \frac{x^2}{a^2}}} = \frac{1}{\sqrt{\frac{a^2 + x^2}{a^2}}} = \frac{a}{\sqrt{a^2 + x^2}} \end{aligned}$$

$$U_n = m \cdot a_{n,rel} + mg \cos \varphi + F_p^m \sin \varphi$$

$$U_n = m \cdot \frac{ga}{(a^2 + x^2)^{3/2}} + mg \frac{a}{\sqrt{a^2 + x^2}} + m \omega^2 x \cdot \frac{x}{\sqrt{a^2 + x^2}}$$

$$U_n = m \cdot \frac{ga^3}{(a^2 + x^2)^{3/2}} + mg \frac{a}{\sqrt{a^2 + x^2}} + m \frac{g}{a} \frac{x^2}{\sqrt{a^2 + x^2}}$$

$$U_n = \frac{mg}{(a^2 + x^2)^{3/2}} \left(a^3 + a(a^2 + x^2) + \frac{x^2}{a}(a^2 + x^2) \right)$$

$$U_n = \frac{mg}{(a^2 + x^2)^{3/2}} \left(2a^3 + 2ax^2 + \frac{x^4}{a} \right)$$

- isto je rešenje i u zGrci, samo u drugom zapisu (malo sređeno)

$$z: 0 = F_{\omega r}^m - U_e$$

$$U_e = F_{\omega r}^m = 2m\omega \cdot \dot{x} = 2m \cdot \sqrt{\frac{g}{a}} \cdot \varphi_{rel} \cdot \omega r = 2m \sqrt{\frac{g}{a}} \cdot \sqrt{ag} \cdot \frac{a}{\sqrt{a^2 + x^2}}$$

$$U_e = \frac{2mga}{\sqrt{a^2 + x^2}}$$

$$U = \sqrt{U_n^2 + U_e^2}$$

6.11. lopta R

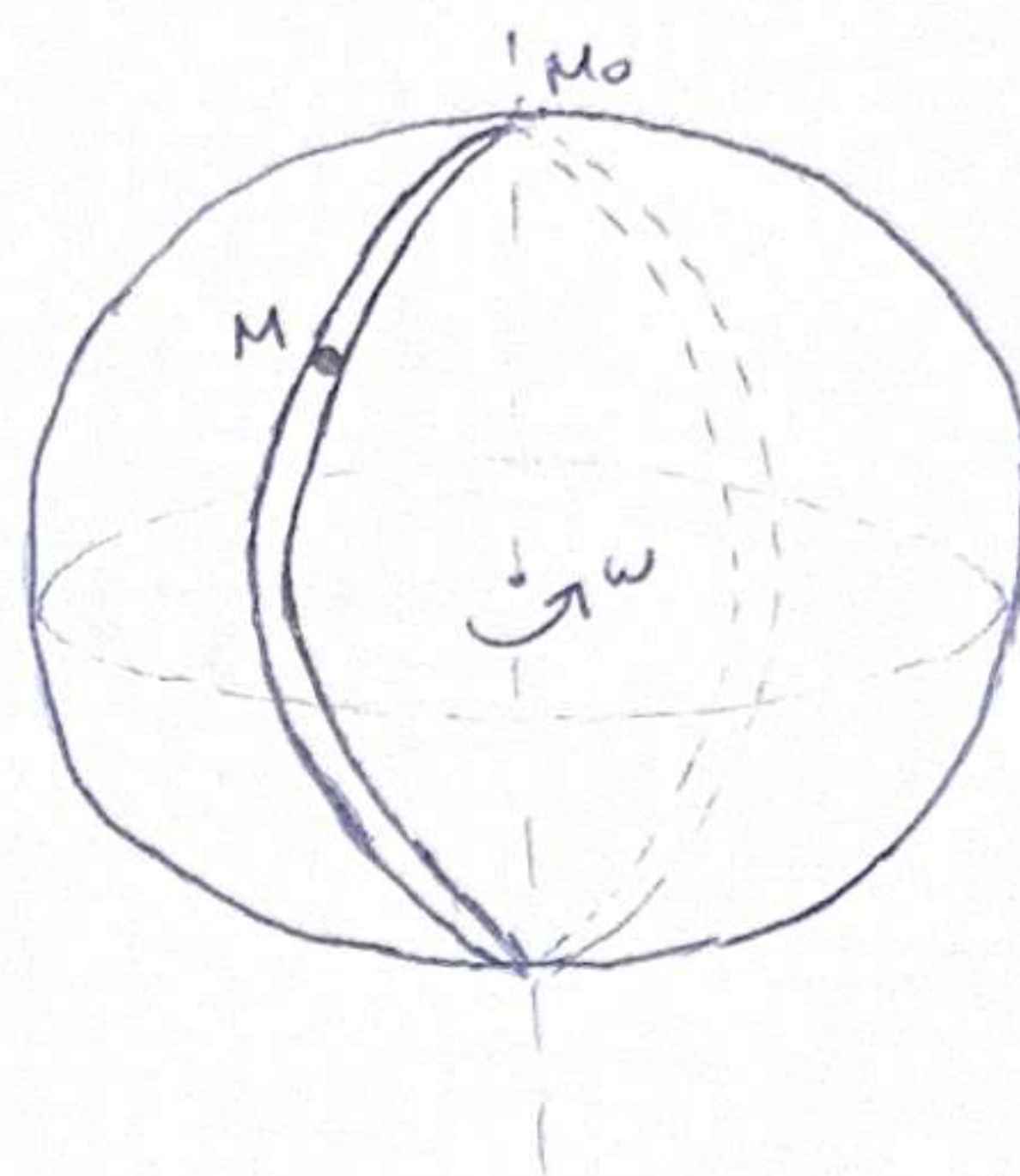
kuglica: m

$$\omega = \sqrt{\frac{g}{2R}} = \text{const}$$

vertikalna ravan

to: kuglica u M_0

$$U_0 = \frac{\sqrt{gR}}{5}$$



položaj u kojem kuglica upadne žleb?

horizontalna reakcija žleba u tom trenutku?

$$\vec{m}\vec{a} = \vec{m}\vec{g} + \vec{F}_p + \vec{F}_{\text{cor}} + \vec{N}_h + \vec{N}_c$$

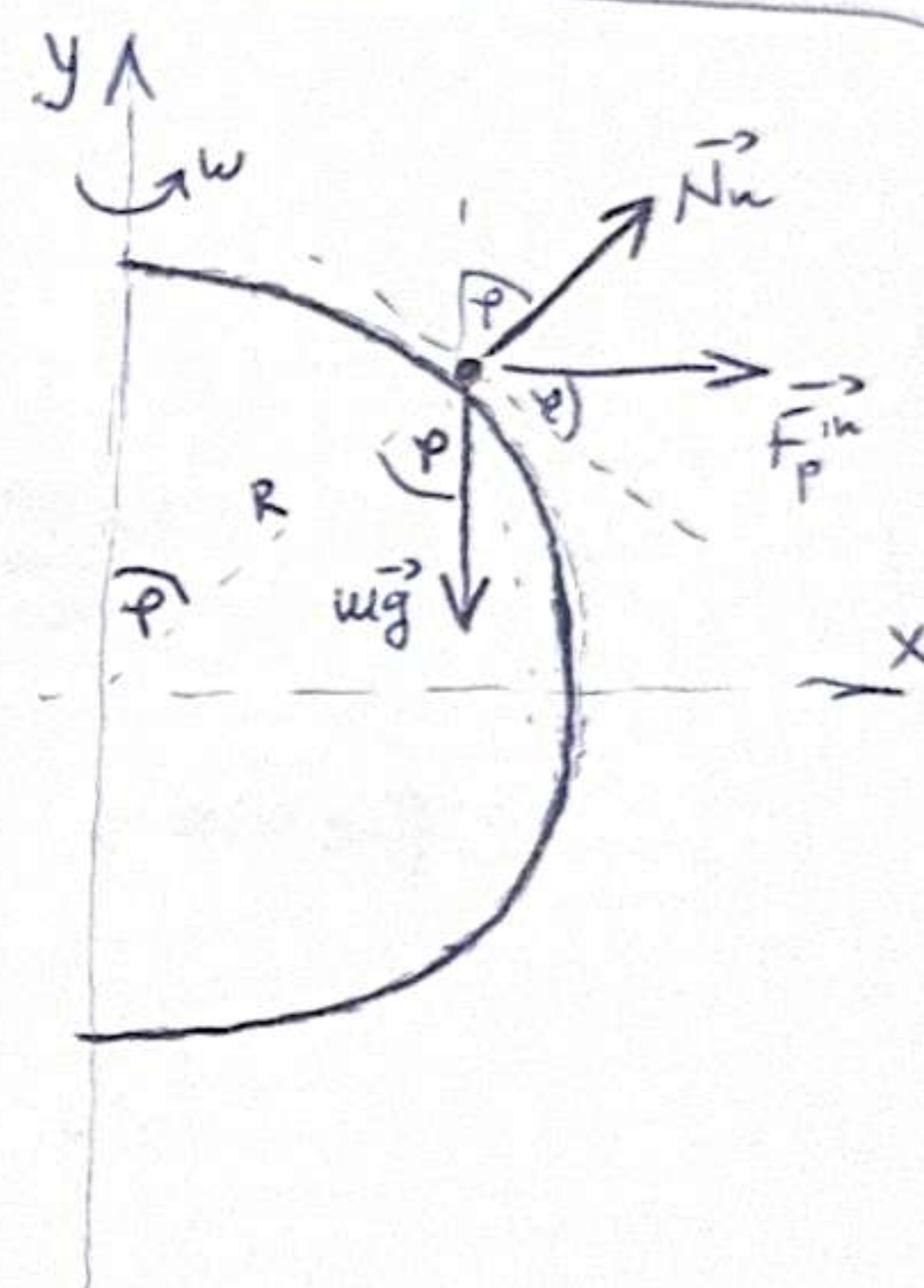
$$\vec{m}\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p = -m(\vec{a}_{\text{rot}} + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v} = +2m\omega \cdot \dot{x} \cdot (-\vec{k})$$

$$\vec{N}_h = N_h \cdot (-\vec{h})$$

$$\vec{N}_c = N_c \cdot (\vec{c})$$



cilj: naći $L(\varphi)$ i reći da je $L(\varphi_1) = 0 \Rightarrow$ odmah dobijemo φ_1

$$n: m \cdot a_n = mg \cos \varphi - F_p \sin \varphi - N_h$$

$$a_n = \frac{v_{\text{rel}}^2}{R_c}, \quad v_{\text{rel}} = ?, \quad R_c = R$$

$$T - T_0 = A \quad (\text{rel})$$

$$T_0 = \frac{1}{2} m U_0^2 = \frac{1}{2} m \frac{gR}{25} = \frac{1}{50} mgR$$

$$T = \frac{1}{2} m v_{\text{rel}}^2$$

$$A = \int_R^{R \cos \varphi} -mg dy + \int_0^{R \sin \varphi} F_p dx = mgR(1 - \cos \varphi) + \int_0^{R \sin \varphi} m\omega^2 x dx = mgR(1 - \cos \varphi) + \frac{1}{2} m \omega^2 R^2 \sin^2 \varphi =$$

$$A = mgR(1 - \cos \varphi) + \frac{1}{2} m \frac{g}{2R} R^2 \sin^2 \varphi = mgR(1 - \cos \varphi + \frac{1}{4} \sin^2 \varphi)$$

$$\frac{1}{2} m v_{\text{rel}}^2 - \frac{1}{50} mgR = mgR(1 - \cos \varphi + \frac{1}{4} \sin^2 \varphi)$$

$$v_{\text{rel}}^2 = \frac{1}{25} gR + 2gR(1 - \cos \varphi + \frac{1}{4} \sin^2 \varphi)$$

$$H_n = mg \cos \varphi - F_p^m \sin \varphi - m a_n$$

$$H_n = mg \cos \varphi - m \cdot \frac{g}{25} \left(R \sin \varphi \cdot \sin \varphi - m \cdot \frac{1}{25} g - m \cdot 2g (1 - \cos \varphi + \frac{1}{4} \sin^2 \varphi) \right)$$

$$H_n = mg \cos \varphi - \frac{1}{2} mg \sin^2 \varphi - \frac{1}{25} mg - 2mg + 2mg \cos \varphi - \frac{1}{2} mg \sin^2 \varphi$$

$$H_n = -\frac{51}{25} mg + 3 mg \cos \varphi - mg \sin^2 \varphi$$

$$H_n(\varphi_1) = 0 \rightarrow -\frac{51}{25} + 3 \cos \varphi - \sin^2 \varphi = 0$$

$$-\frac{51}{25} + 3 \cos \varphi - (1 - \cos^2 \varphi) = 0$$

$$-\frac{76}{25} + 3 \cos \varphi + \cos^2 \varphi = 0$$

$$(\cos \varphi_1)_{1/2} = \frac{-3 \pm \sqrt{9 + \frac{4 \cdot 76}{25}}}{2} = \frac{-3 \pm \sqrt{\frac{529}{25}}}{2} = \frac{-\frac{15}{5} \pm \frac{23}{5}}{2} = \frac{-15 \pm 23}{10}$$

$$(\cos \varphi_1)_1 = \frac{-15 - 23}{10} < 0 \quad - \text{otpada rešenja (jer je } \varphi \in (0, 30))$$

$$(\cos \varphi_1)_2 = \frac{-15 + 23}{10} = \frac{8}{10} = \frac{4}{5} \quad - \text{rešenje}$$

$$\boxed{\varphi_1 = \arccos\left(\frac{4}{5}\right)}$$

$$\Sigma: 0 = -F_{\text{cor}}^m + H_0 \Rightarrow H_0 = F_{\text{cor}}^m = 2m\omega \cdot \dot{x}, \quad \dot{x}(\varphi_1) = ?$$

$$\dot{x}(\varphi_1) = v_{\text{rel}} \cdot \cos \varphi_1 = \sqrt{\frac{1}{25} g R + 2gR(1 - \cos \varphi_1 + \frac{1}{4} \sin^2 \varphi_1)} \cdot \cos \varphi_1, \quad \cos \varphi_1 = \frac{4}{5}, \quad \sin^2 \varphi_1 = 1 - \cos^2 \varphi_1 = \frac{9}{25}$$

$$\dot{x}(\varphi_1) = \sqrt{\frac{1}{25} g R + 2gR(1 - \frac{4}{5} + \frac{1}{4} \cdot \frac{9}{25})} \cdot \frac{4}{5} = \sqrt{\frac{1}{25} g R + 2gR \cdot \frac{29}{100}} \cdot \frac{4}{5} = \sqrt{\frac{31}{50} g R} \cdot \frac{4}{5} = \frac{4\sqrt{31}}{25\sqrt{2}} \sqrt{gR}$$

$$H_0 = 2m \cdot \sqrt{\frac{g}{25}} \cdot \frac{4\sqrt{31}}{25\sqrt{2}} \sqrt{gR} = \frac{4\sqrt{31}}{25} mg$$

$$\boxed{H_0 = \frac{4\sqrt{31}}{25} mg}$$

6.12. vertikalna ravan

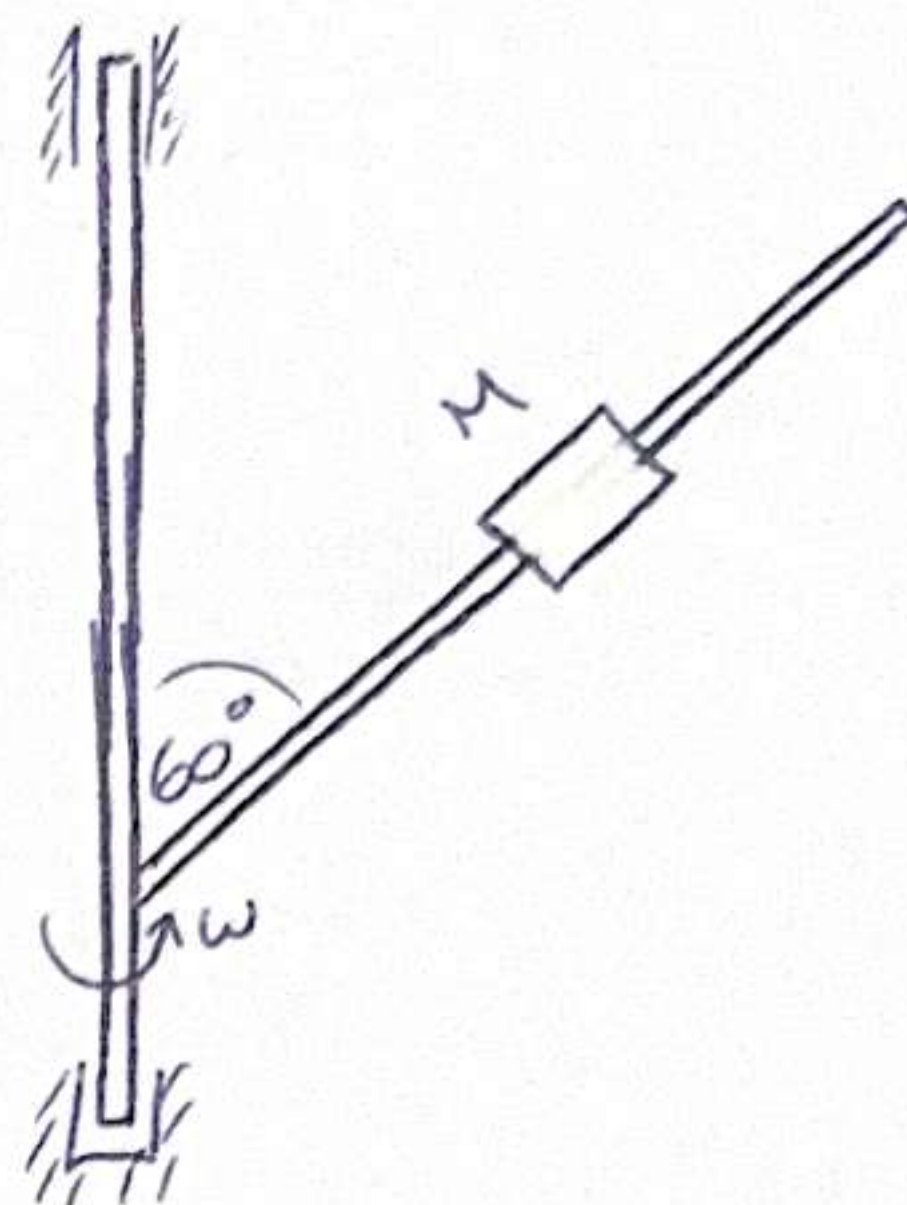
ugaona brzina ω

klizac: m

to: klizac je u položaju $OM_1 = a$

$\omega(t) = ?$ - da li se klizac kreće $v_{rel} = v_0 = \text{const.}$

$L = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{L}_u + \vec{L}_G$$

$$m\vec{g} = mg \cdot (-\vec{j})$$

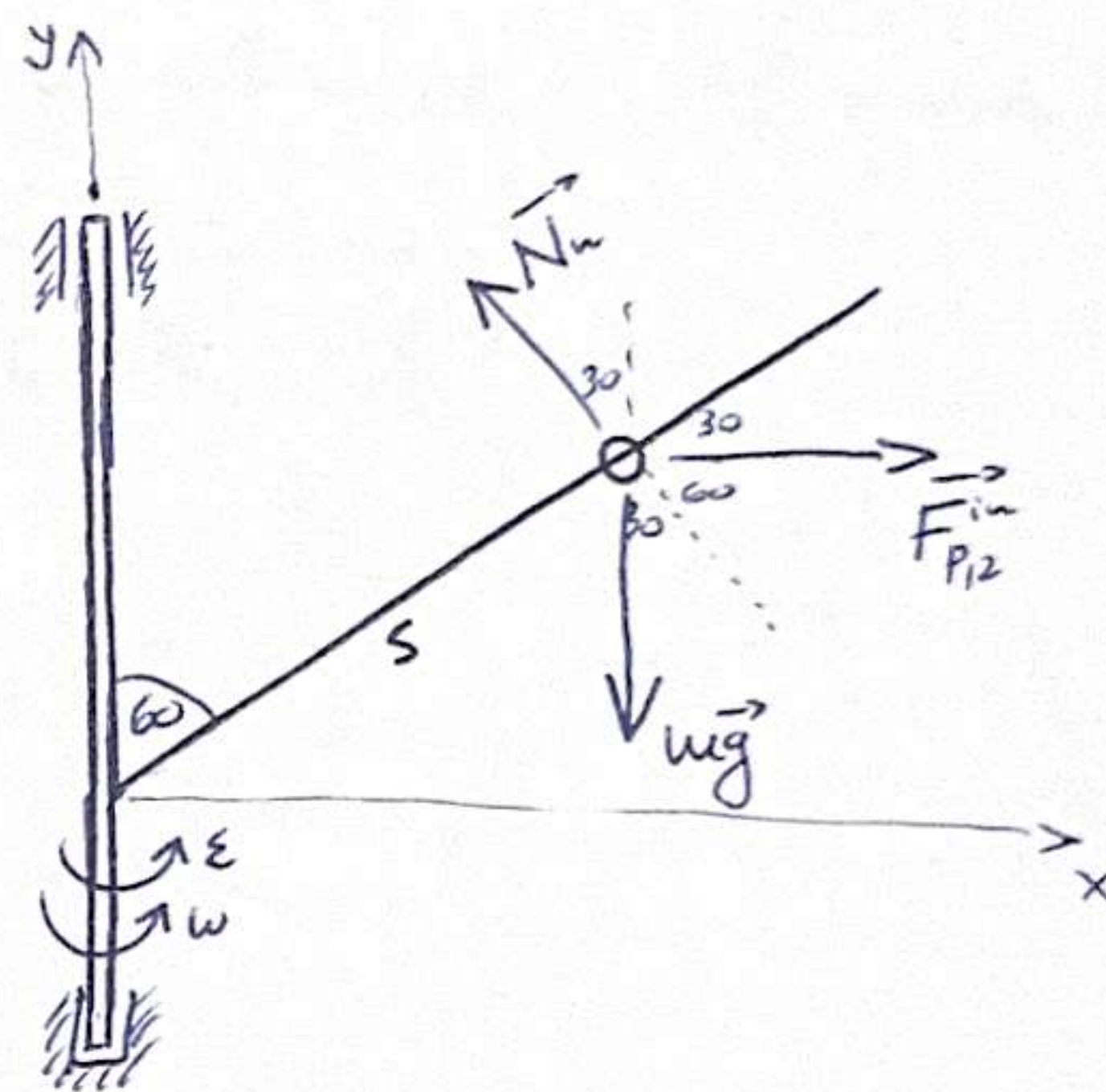
$$\vec{F}_p^{in} = -m(\vec{a}_{rel} + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m \cdot \epsilon \cdot x \cdot (\vec{k}) + m\omega^2 x \cdot (-\vec{i}) = \vec{F}_{p1}^{in} + \vec{F}_{p2}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{L}_u = L_u \cdot (\vec{u})$$

$$\vec{L}_G = L_G \cdot (-\vec{k})$$



$$t: m \cdot a_{r,t} = -mg \sin 30 + F_{p2}^{in} \cos 30$$

$$0 = -\frac{1}{2}mg + m\omega^2 x \frac{\sqrt{3}}{2} \quad | :m$$

$$\omega^2 = \frac{1}{2} \frac{g}{x} \frac{2}{\sqrt{3}} = \frac{g}{x\sqrt{3}}, \quad x = ?$$

$$\dot{s} = 0 \rightarrow \ddot{s} = \text{const.} = v_0 \rightarrow s = v_0 t + \text{const.} = v_0 t + a$$

$$x = s \cdot \sin 60 = (v_0 t + a) \frac{\sqrt{3}}{2} \Rightarrow \omega^2 = \frac{g}{(v_0 t + a) \frac{\sqrt{3}}{2} \sqrt{3}} = \frac{2g}{3(v_0 t + a)}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{2g}{3(v_0 t + a)}}}$$

$$y: 0 = -mg + L_u \cos 30$$

$$L_u = \frac{mg}{\cos 30} = \frac{mg}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} mg$$

$$z: 0 = F_{p1}^{in} + F_{cor}^{in} - L_G$$

$$L_G = F_{p1}^{in} + F_{cor}^{in} = m \cdot \epsilon \cdot x + 2m \cdot \omega \cdot \dot{x}$$

$$\epsilon = ? \quad \dot{x} = v_0 \cos 30 = v_0 \frac{\sqrt{3}}{2}$$

$$\omega^2 = \frac{2}{3} \frac{g}{v_0 t + a} \quad \left| \frac{d}{dt} \right.$$

$$2 \cdot \omega \cdot \varepsilon = \frac{2}{3} \frac{0 - g \cdot v_0}{(v_0 t + a)^2} = \frac{2}{3} \frac{g}{(v_0 t + a)} \cdot \left(- \frac{v_0}{(v_0 t + a)} \right) = \omega^2 \cdot \left(- \frac{v_0}{v_0 t + a} \right) \quad / : 2\omega$$

$$\varepsilon = - \frac{1}{2} \omega \cdot \frac{v_0}{v_0 t + a}$$

$$H_6 = m \cdot \left(- \frac{1}{2} \omega \frac{v_0}{v_0 t + a} \right) \cdot (v_0 t + a) \frac{\sqrt{3}}{2} + 2m\omega \cdot v_0 \frac{\sqrt{3}}{2} = - \frac{\sqrt{3}}{4} m\omega v_0 + \sqrt{3} m\omega v_0 = \frac{3\sqrt{3}}{4} m\omega v_0$$

$$H = \sqrt{H_u^2 + H_6^2} = \sqrt{\frac{4}{3} \omega^2 g^2 + m\omega^2 v_0^2 \cdot \frac{27}{16}} = m \cdot \sqrt{\frac{4}{3} g^2 + \frac{2}{3} \frac{g}{v_0 t + a} \cdot v_0^2 - \frac{27}{16}}$$

$$H = m \sqrt{\frac{4}{3} g^2 + \frac{g}{8} \frac{v_0^2}{v_0 t + a}}$$

6.13. $O_1A, AB, BO_2: \phi, e$

vertikalna ravan

$O_1A, BO_2: \omega = \text{const.}$

klasec: m

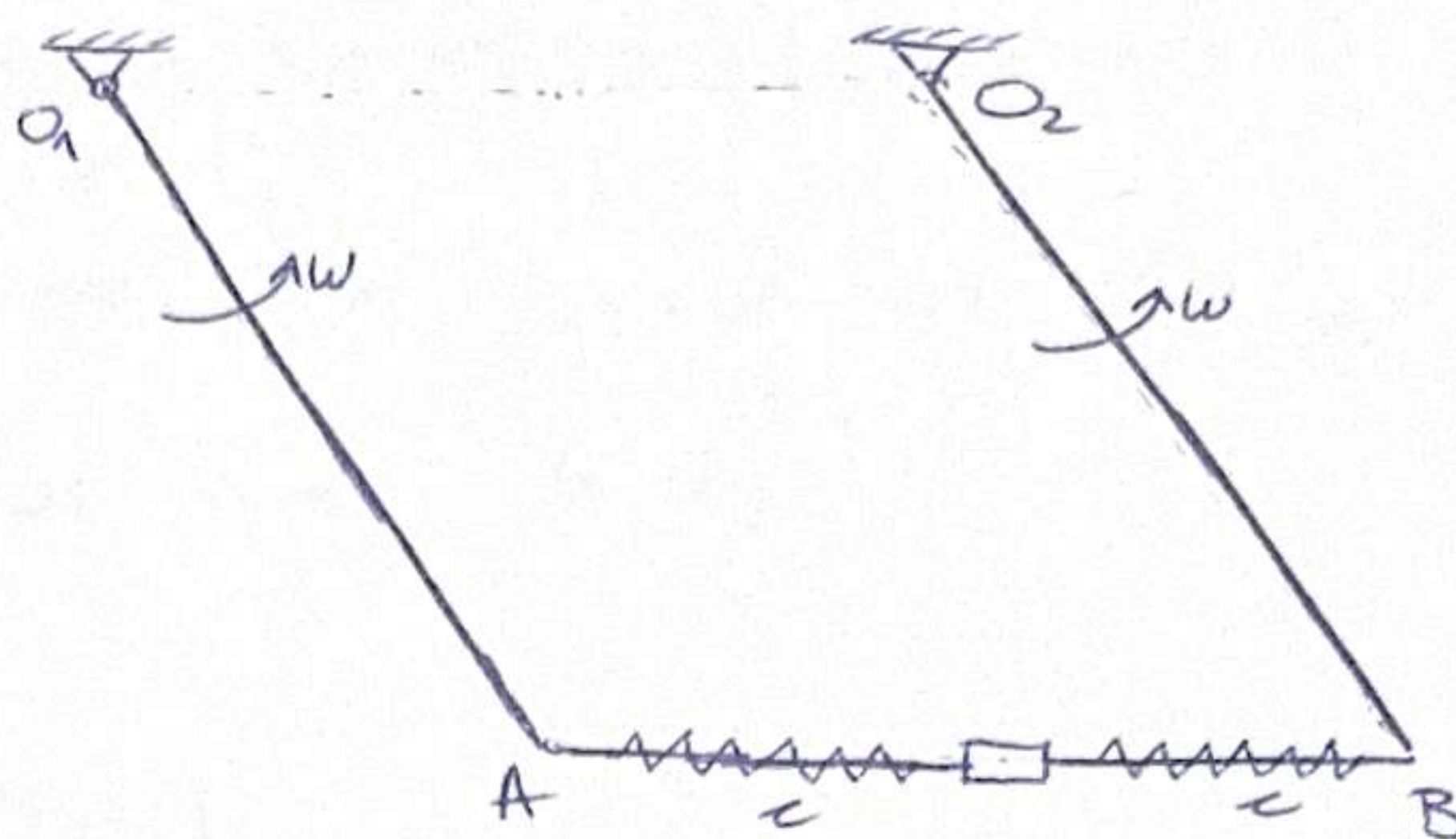
opruge: $c = \frac{m\omega^2}{2}$

$t_0 = 0: O_1A, O_2B: \text{vertikalni}$
klasec: na sredini AB
relativno u mirovanju

opruge: neopregnute

$x(t) = ?$ - j -na relativnog kretanja
klaseca po stepu AB

$H(t) = ?$ - reakcija između stepa i klaseca



$$c_e = c_1 + c_2 = 2c = m\omega^2$$

$$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{H}_n + \vec{H}_t$$

$$m\vec{g} = mg \cdot (-\vec{j}')$$

$$\vec{F}_c = c_e \cdot x \cdot (-\vec{i}') = m\omega^2 x \cdot (-\vec{i}')$$

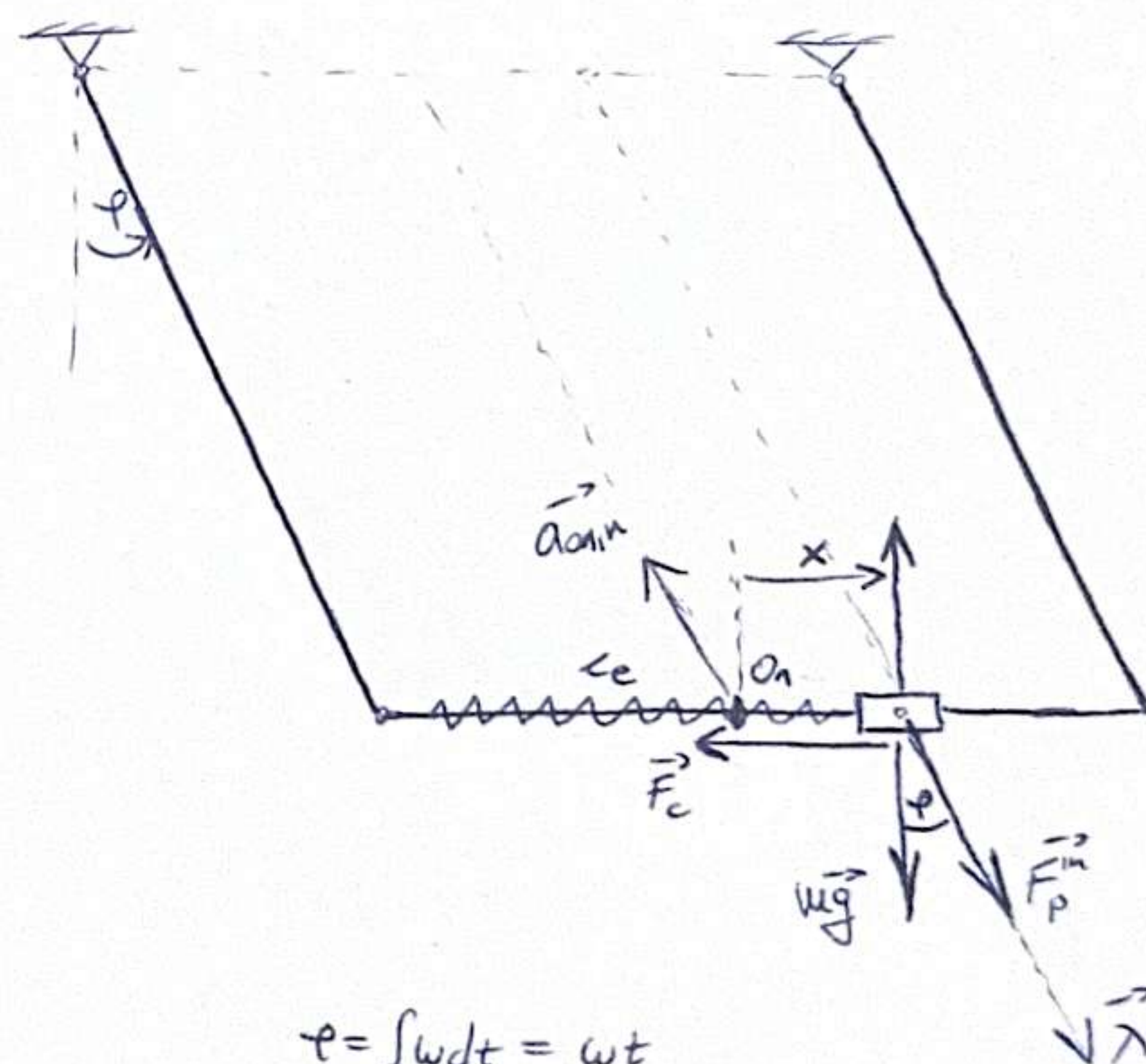
$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= -m(\vec{a}_{on,n} + \vec{a}_{on,t}) = m \cdot \omega^2 \cdot e \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m \vec{\omega} \times \vec{v}_r = 0$$

$$\vec{H}_n = H_n \cdot (\vec{j}')$$

$$\vec{H}_t = 0$$



$$\phi = \int \omega dt = \omega t$$

X: $m\ddot{x} = -m\omega^2 x + m\omega^2 e \sin(\omega t)$

$$\ddot{x} + \omega^2 x = \omega^2 e \sin(\omega t)$$

H: $\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i, \quad (\alpha = 0, \beta = \omega)$

$$x_h = C_1 \cos \omega t + C_2 \sin \omega t$$

P: $\phi(t) = e^{0t} (0 \cos(\omega t) + \omega^2 e \sin(\omega t)), \quad (\alpha = 0, \beta = \omega)$

$$x_p = t^1 e^{0t} (A \cos \omega t + B \sin \omega t)$$

$$\dot{x}_p = A \cos \omega t + B \sin \omega t + \omega t (-A \sin \omega t + B \cos \omega t)$$

$$\ddot{x}_p = (-A \sin \omega t + B \cos \omega t) \cdot \omega + \omega \cdot 2 + \omega^2 t (-A \cos \omega t - B \sin \omega t)$$

$$2W(-A \sin \omega t + B \cos \omega t) + \omega^2 t(-A \cos \omega t - B \sin \omega t) + \omega^2 t(A \cos \omega t + B \sin \omega t) = \omega^2 e \sin \omega t$$

$$\sin \omega t(-2AW - \cancel{\omega^2 t B} + \cancel{\omega^2 t B}) = \omega^2 e \sin \omega t \Rightarrow -2A = \omega e \Rightarrow A = -\frac{1}{2} \omega e$$

$$\cos \omega t(2BW - \cancel{\omega^2 t A} + \cancel{\omega^2 t A}) = 0 \cos \omega t \Rightarrow B = 0$$

$$X = C_1 \cos \omega t + C_2 \sin \omega t - \frac{1}{2} \omega e \cos \omega t$$

$$\dot{X} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t - \frac{1}{2} \omega e \sin \omega t + \frac{1}{2} \omega^2 e t \sin \omega t$$

$$X(t_0) = C_1 + 0 - 0 = 0 \rightarrow C_1 = 0$$

$$\dot{X}(t_0) = 0 + C_2 \omega - \frac{1}{2} \omega e + 0 = 0 \rightarrow C_2 = \frac{1}{2} e$$

$$\boxed{X = \frac{1}{2} e (\sin \omega t - \omega t \cos \omega t)}$$

$$y: 0 = -mg - m \omega^2 e \cos \omega t + U_n$$

$$\boxed{U_n = mg + m \omega^2 e \cos \omega t}$$

6.14. Obratuje: $\omega = \text{const.}$

AB i OC leže u horizontalnoj ravni

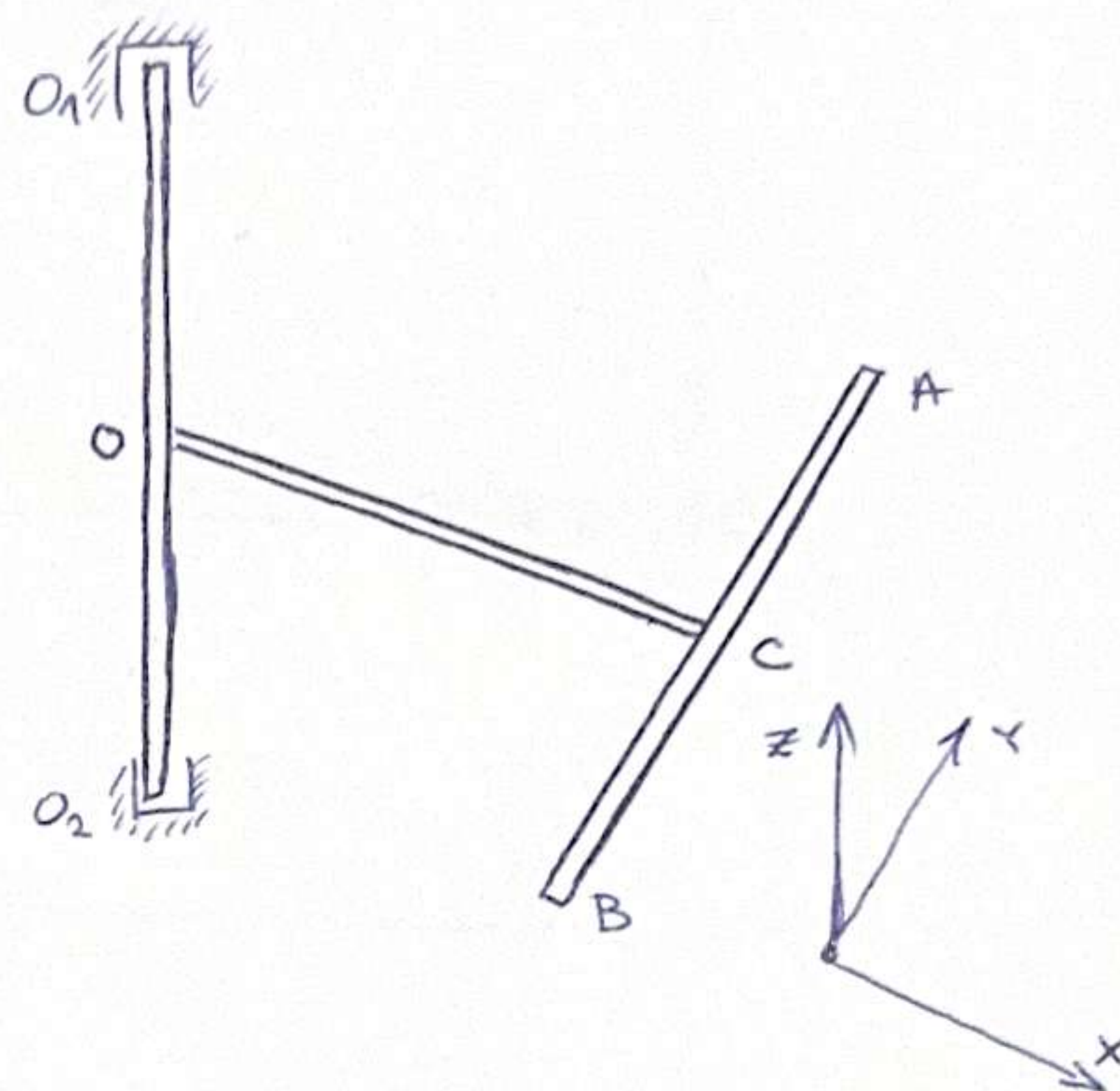
$$\overline{AC} = \overline{CB} = e, \quad \overline{OC} = e$$

tačka mase m

to: tačka je u A
relativna brzina v_0

$v_0 = ?$ - da li tačka dospela
u položaj B

$$H_c = ?$$



$$\vec{m}\vec{a} = \vec{m}\vec{g} + \vec{F}_p^{\text{in}} + \vec{F}_{\text{cor}}^{\text{in}} + \vec{H}_n + \vec{H}_c$$

$$\vec{m}\vec{g} = mg \cdot (-\vec{k}')$$

$$\vec{F}_p^{\text{in}} = -m(\vec{a}_{\text{rel}} + \vec{\varepsilon} \times \vec{r}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}')) =$$

$$= m\omega^2 x \cdot (\vec{i}') + m\omega^2 y \cdot (\vec{j}')$$

$$\vec{F}_{\text{cor}}^{\text{in}} = -2m\vec{\omega} \times \vec{v}' = +2m\omega \cdot v_r \cdot (-\vec{i}')$$

$$\vec{H}_n = H_n \cdot (\vec{i}')$$

$$\vec{H}_c = H_c \cdot (\vec{k}')$$

Da li tačka dođe do položaja B, treba proveriti
da li će doći do položaja C, jer do C sila $\vec{F}_{p,y}^{\text{in}}$
deluje nepovoljno, a posle toga deluje u pravcu kretanja
ka B. Dakle, ako stigne do C, sigurno će i do B.

$$T_c - T_A = A_{AC}$$

$$\frac{1}{2}mv_c^2 - \frac{1}{2}mv_0^2 = \int_e^0 F_{p,y}^{\text{in}} dy = \int_e^0 m\omega^2 y dy = \frac{1}{2}m\omega^2(0 - e^2) = -\frac{1}{2}m\omega^2 e^2$$

$$v_0^2 - v_c^2 = \omega^2 e^2, \quad \text{u granicama slučaja } v_c \approx 0 \rightarrow \boxed{v_0 > \omega e}$$

$$z: 0 = -mg + H_c \Rightarrow H_c = mg$$

$$x: 0 = m\omega^2 x - 2m\omega \cdot v_{r,c} + H_n, \quad v_c = \sqrt{v_0^2 - \omega^2 e^2}$$

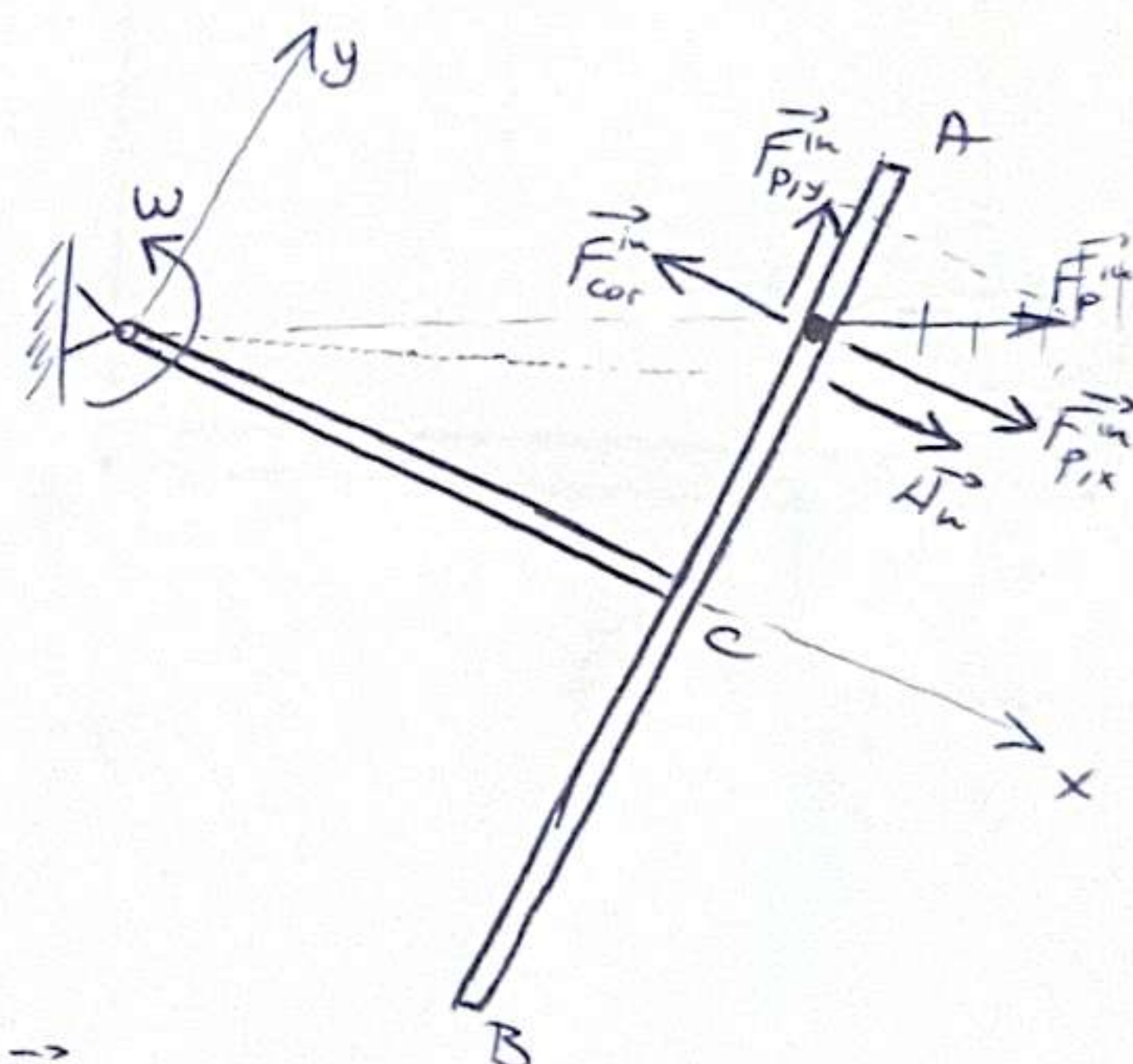
$$H_n = 2m\omega v_c - m\omega^2 x = 2m\omega \sqrt{v_0^2 - \omega^2 e^2} - m\omega^2 e$$

$$H = \sqrt{H_n^2 + H_c^2}$$

$$\Rightarrow$$

$$\boxed{H = m \sqrt{g^2 + (2\omega \sqrt{v_0^2 - \omega^2 e^2} - \omega^2 e)^2}}$$

pregled odboja
(horizontalna ravan)



6.15) horizontalna ravan
kvadratna ploča ABCD

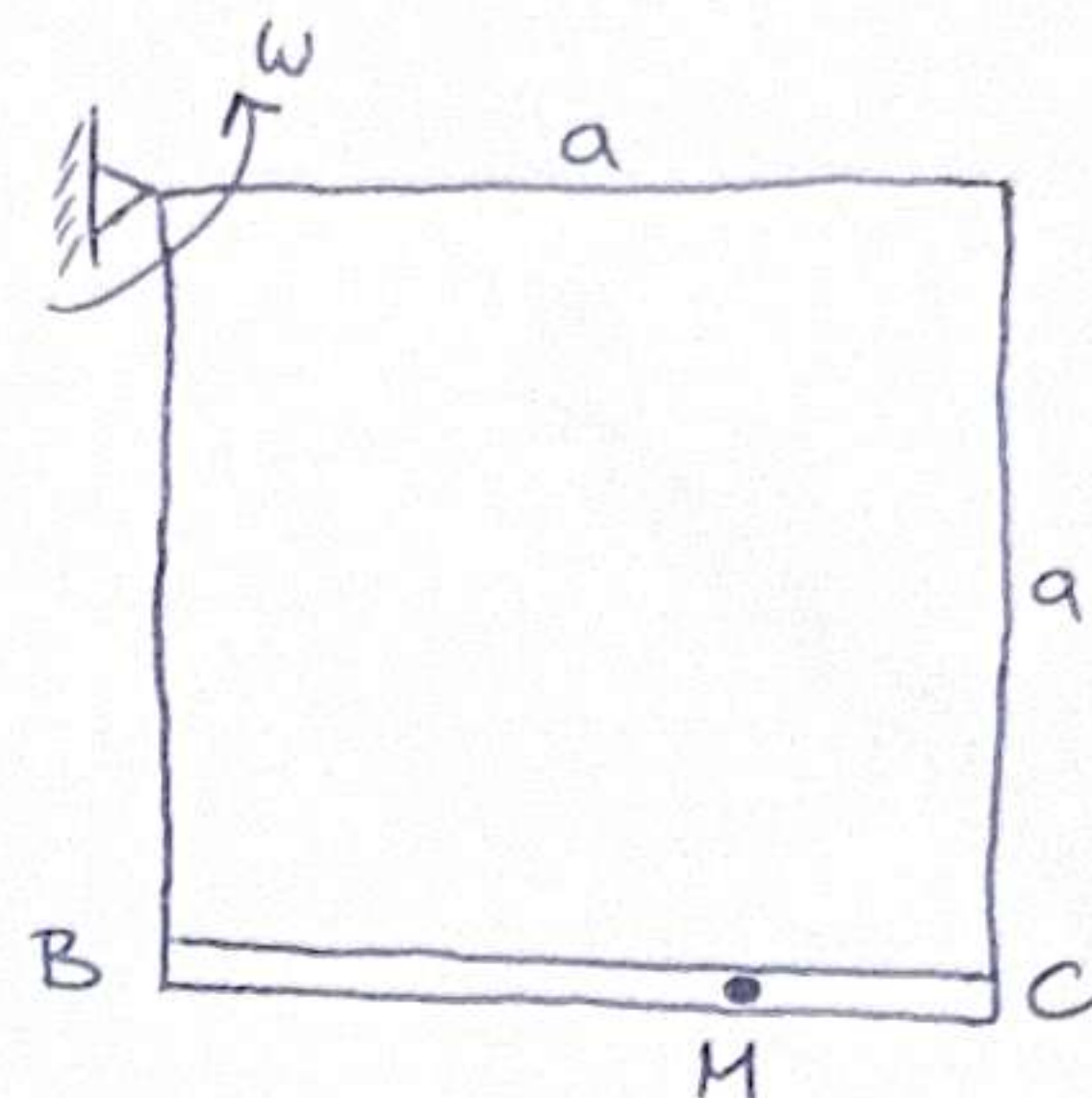
$\omega = \text{const.}$

stranica ploče a

$\omega = ?$

t_0 : tačka M je u B
 $\omega_0 = \omega_0$

t_1 : tačka M uprta kandel (položaj C)
 $\omega_1 = \omega_1 = 2\omega_0$



$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{H}_n + \vec{H}_G$$

$$\vec{\omega}\vec{g} = \omega\vec{g} \cdot (-\vec{k})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m\omega^2 \cdot x \cdot (\vec{i}) + m\omega^2 \cdot y \cdot (\vec{j}) = \vec{F}_{p,x}^{in} + \vec{F}_{p,y}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \omega_n \cdot (\vec{j})$$

$$\vec{H}_n = H_n \cdot (-\vec{j})$$

$$\vec{H}_G = H_G \cdot (\vec{k})$$

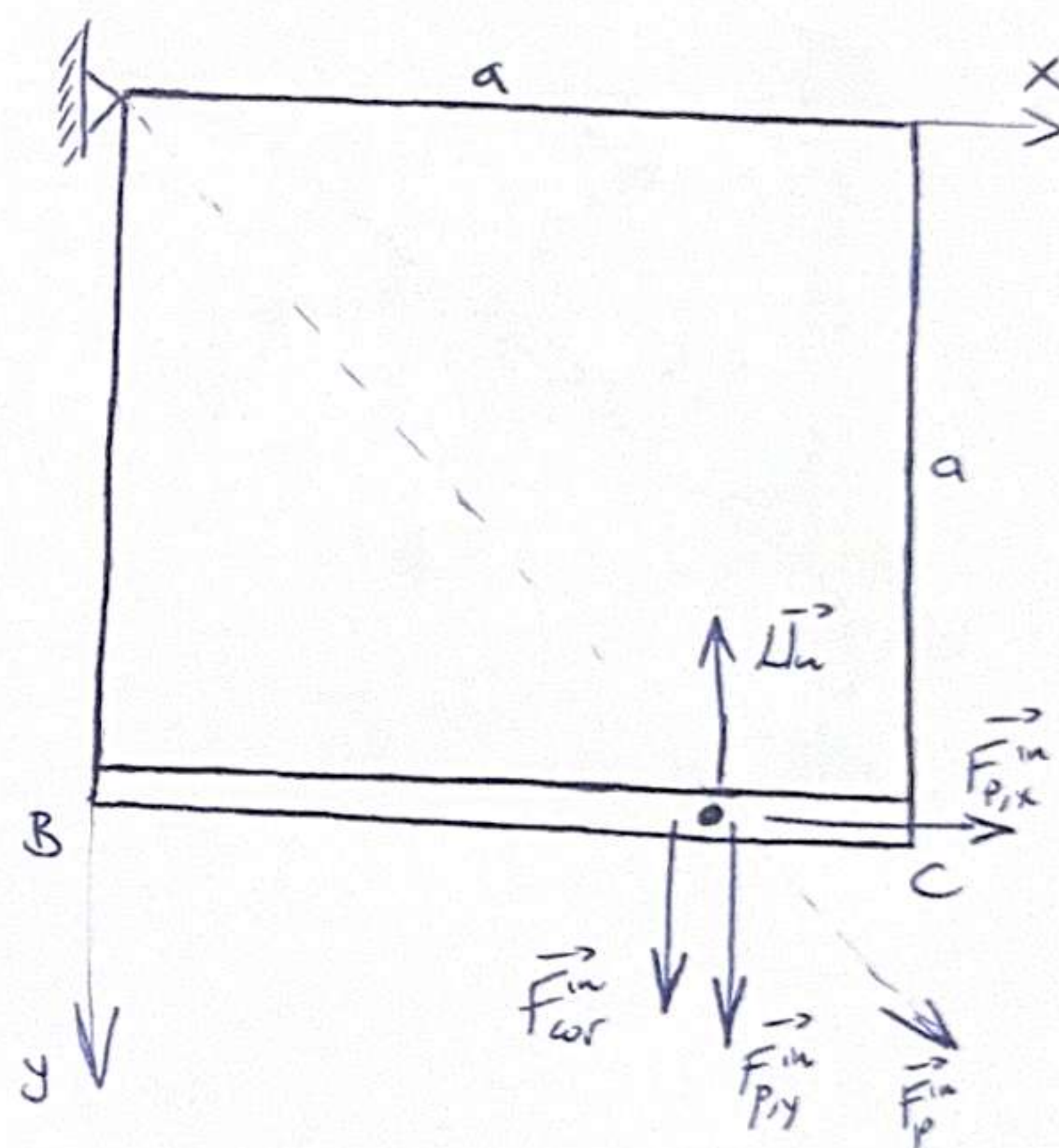
$$T_C - T_B = \Delta E_c$$

$$\frac{1}{2} m \omega_c^2 - \frac{1}{2} m \omega_B^2 = \int_0^a \vec{F}_{p,x}^{in} dx$$

$$\frac{1}{2} m \cdot 4\omega_0^2 - \frac{1}{2} m \omega_0^2 = \int_0^a m\omega^2 x dx$$

$$\frac{3}{2} m \omega_0^2 = \frac{1}{2} m \omega^2 a^2$$

$$\boxed{\omega = \frac{\omega_0 \sqrt{3}}{a}}$$



6.16. vertikalni ravan

tačka masu m

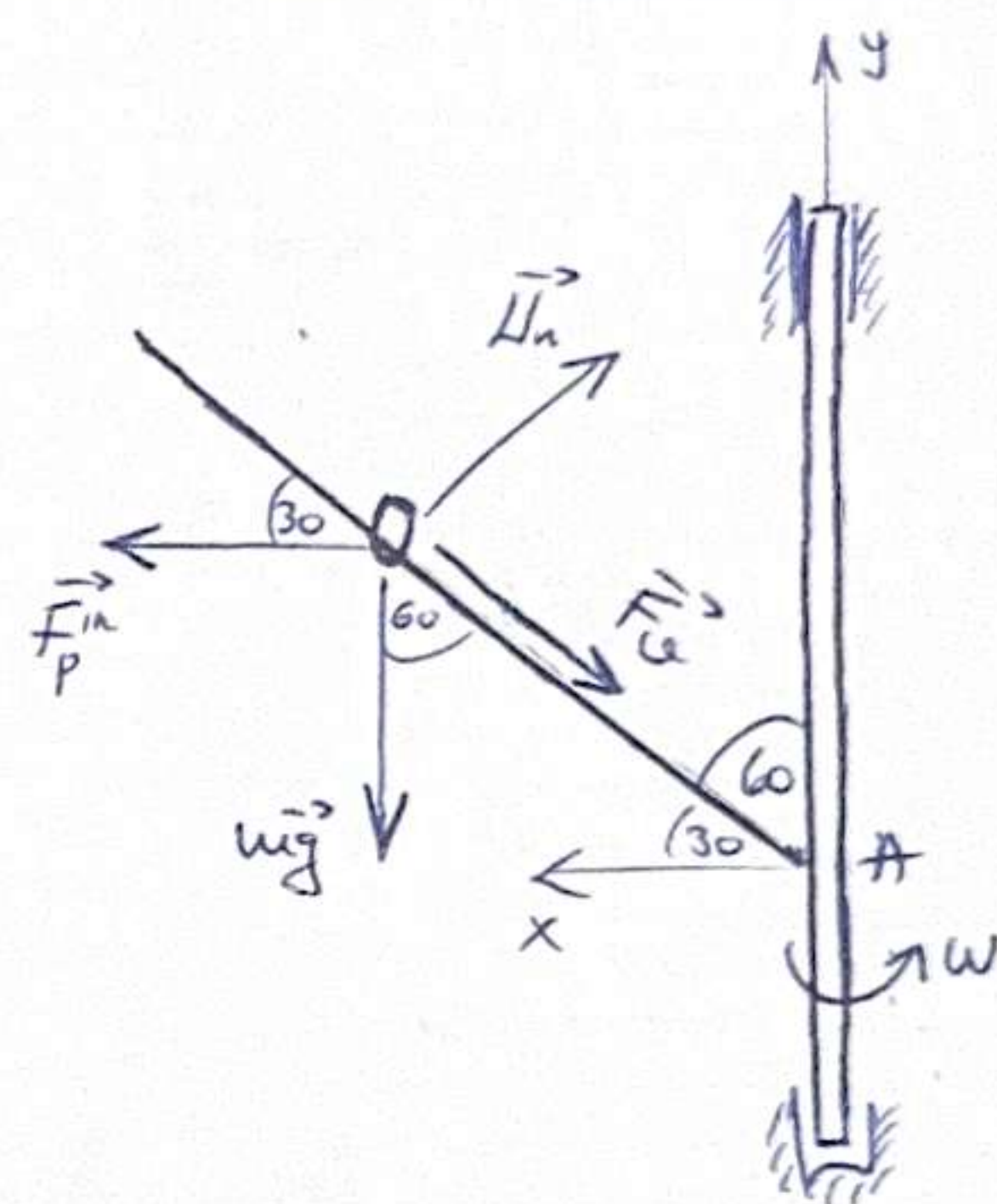
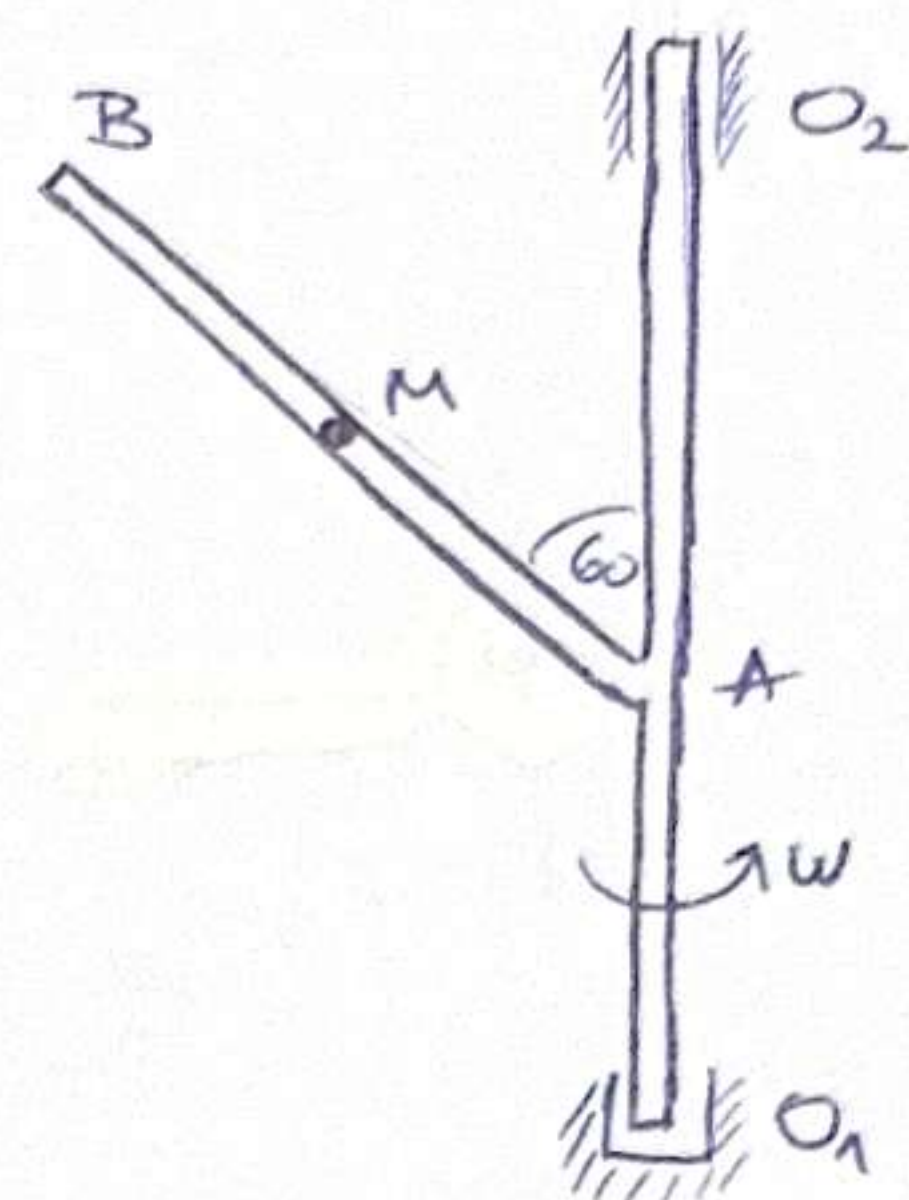
$\omega = \text{const.}$

sila otpora sračunat proporcionalna prvom stepenu brzine
koef. proporcionalnosti je $m\omega$

$t_0 = 0$: $\varphi = \frac{g}{\omega}$, položaj A

t_1 : $\varphi_r = \frac{1}{2} \varphi$

$U(t_1) = ?$



$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_d + \vec{F}_p + \vec{F}_{cor} + \vec{L}_n + \vec{L}_e$$

$$\vec{m}\vec{g} = m\vec{g} \cdot (-\vec{j})$$

$$\vec{F}_d = m \cdot \omega \cdot \varphi_r \cdot (-\vec{e})$$

$$\vec{F}_p = -m(\vec{a}_{O_1} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 \cdot x \cdot (\vec{i})$$

$$\vec{F}_{cor} = -2m\vec{\omega} \times \vec{v}_r = 2m \cdot \omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{L}_n = L_n \cdot (\vec{n})$$

$$\vec{L}_e = L_e \cdot (-\vec{k})$$

n: $0 = -mg \sin 60 - F_p \sin 30 + L_n = 0$

$$L_n = \frac{\sqrt{3}}{2} mg + \frac{1}{2} m\omega^2 x, \quad x(t_1) = ?$$

z: $0 = 2m\omega \cdot \dot{x} - L_e$

$$L_e = 2m\omega \cdot \dot{x} = 2m\omega \cdot \varphi \omega \sin 30 = 2m\omega \cdot \frac{1}{2} \frac{g}{\omega} \frac{\sqrt{3}}{2} = mg \frac{\sqrt{3}}{2}$$

$$\underline{T_1 - T_0 = A_{01}} \quad (\text{rel})$$

$$T_1 = \frac{1}{2} m \varphi_1^2 = \frac{1}{2} m \left(\frac{1}{2} \varphi \right)^2 = \frac{1}{8} m \varphi^2 = \frac{1}{8} \frac{mg^2}{\omega^2}$$

$$T_0 = \frac{1}{2} m \varphi_0^2 = \frac{1}{2} m \varphi^2 = \frac{1}{2} \frac{mg^2}{\omega^2}$$

$$A_{01} = \int_0^x F_p dx + \left(\int_0^{s_1} -F_d ds \right) = \text{ne more da se resi!}$$

$$t: \ddot{ms} = -F_c + F_p^m \cos 30^\circ - mg \cos 60^\circ$$

$$\ddot{ms} = -m\omega \cdot \omega_r + m\omega^2 x \cdot \frac{\sqrt{3}}{2} - mg \cdot \frac{1}{2}$$

$$\ddot{s} = -\omega \cdot \dot{s} + \omega^2 s \cos 30^\circ \frac{\sqrt{3}}{2} - \frac{1}{2}g$$

$$\ddot{s} + \omega \dot{s} - \omega^2 \cdot \frac{3}{4}s = -\frac{1}{2}g$$

$$\# : \lambda^2 + \omega \lambda - \frac{3}{4}\omega^2 = 0 \rightarrow \lambda_{1/2} = \frac{-\omega \pm \sqrt{\omega^2 + 3\omega^2}}{2} = \frac{-\omega \pm 2\omega}{2}$$

$$s_h = C_1 e^{\frac{1}{2}\omega t} + C_2 e^{-\frac{1}{2}\omega t}$$

$$p: b(t) = e^{\omega t} (-\frac{1}{2}g \cos \omega t + 0 \sin \omega t)$$

$$s_p = t^0 e^{\omega t} (A \cos \omega t + B \sin \omega t) = A, \quad \dot{s}_p = 0, \quad \ddot{s}_p = 0$$

$$-\frac{3}{4}\omega^2 A = -\frac{1}{2}g \rightarrow A = \frac{2}{3} \frac{g}{\omega^2}$$

$$s = C_1 e^{\frac{1}{2}\omega t} + C_2 e^{-\frac{3}{2}\omega t} + \frac{2}{3} \frac{g}{\omega^2}$$

$$\dot{s} = C_1 \cdot \frac{1}{2}\omega e^{\frac{1}{2}\omega t} - C_2 \cdot \frac{3}{2}\omega e^{-\frac{3}{2}\omega t}$$

$$\dot{s}(t_0) = \frac{g}{\omega} \rightarrow \frac{1}{2}\omega C_1 - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$$

$$s(t_0) = 0 \rightarrow C_1 + C_2 + \frac{2}{3} \frac{g}{\omega^2} = 0 \rightarrow C_1 = -\frac{2}{3} \frac{g}{\omega^2} - C_2$$

$$\frac{1}{2}\omega \left(-\frac{2}{3} \frac{g}{\omega^2} - C_2\right) - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$$

$$-\frac{1}{3} \frac{g}{\omega} - \frac{1}{2}\omega C_2 - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$$

$$-2\omega C_2 = \frac{4}{3} \frac{g}{\omega} \rightarrow C_2 = -\frac{2}{3} \frac{g}{\omega^2} \rightarrow C_1 = 0$$

$$\boxed{s = \frac{2}{3} \frac{g}{\omega^2} (1 - e^{-\frac{3}{2}\omega t})}$$

$$\dot{s} = \frac{2}{3} \frac{g}{\omega^2} \left(\frac{3}{2}\omega e^{-\frac{3}{2}\omega t}\right) = \frac{g}{\omega} e^{-\frac{3}{2}\omega t}$$

$$\dot{s}(t_1) = \frac{1}{2} \frac{g}{\omega} \rightarrow \frac{g}{\omega} e^{-\frac{3}{2}\omega t_1} = \frac{1}{2} \frac{g}{\omega} \rightarrow e^{-\frac{3}{2}\omega t_1} = \frac{1}{2}$$

$$s(t_1) = \frac{2}{3} \frac{g}{\omega^2} \left(1 - \frac{1}{2}\right) = \frac{1}{3} \frac{g}{\omega^2}$$

$$x(t_1) = s(t_1) \cdot \cos 30^\circ = \frac{\sqrt{3}}{2} \frac{1}{3} \frac{g}{\omega^2} = \frac{\sqrt{3}}{6} \frac{g}{\omega^2}$$

$$H_u = \frac{\sqrt{3}}{2} \omega g + \frac{1}{2} m \omega^2 \frac{\sqrt{3}}{6} \frac{g}{\omega^2} = \frac{7\sqrt{3}}{12} \omega g$$

$$H = \sqrt{H_u^2 + H_c^2} = \omega g \sqrt{\frac{49 \cdot 3}{144} + \frac{3}{4}} = \omega g \sqrt{\frac{255}{144}} = \omega g \sqrt{\frac{85}{48}} = \frac{1}{4} \omega g \sqrt{\frac{85}{3}}$$

$$\boxed{H(t_1) = \frac{1}{4} \omega g \sqrt{\frac{85}{3}}}$$

6.17. kuglica mase m
horizontalna ravan

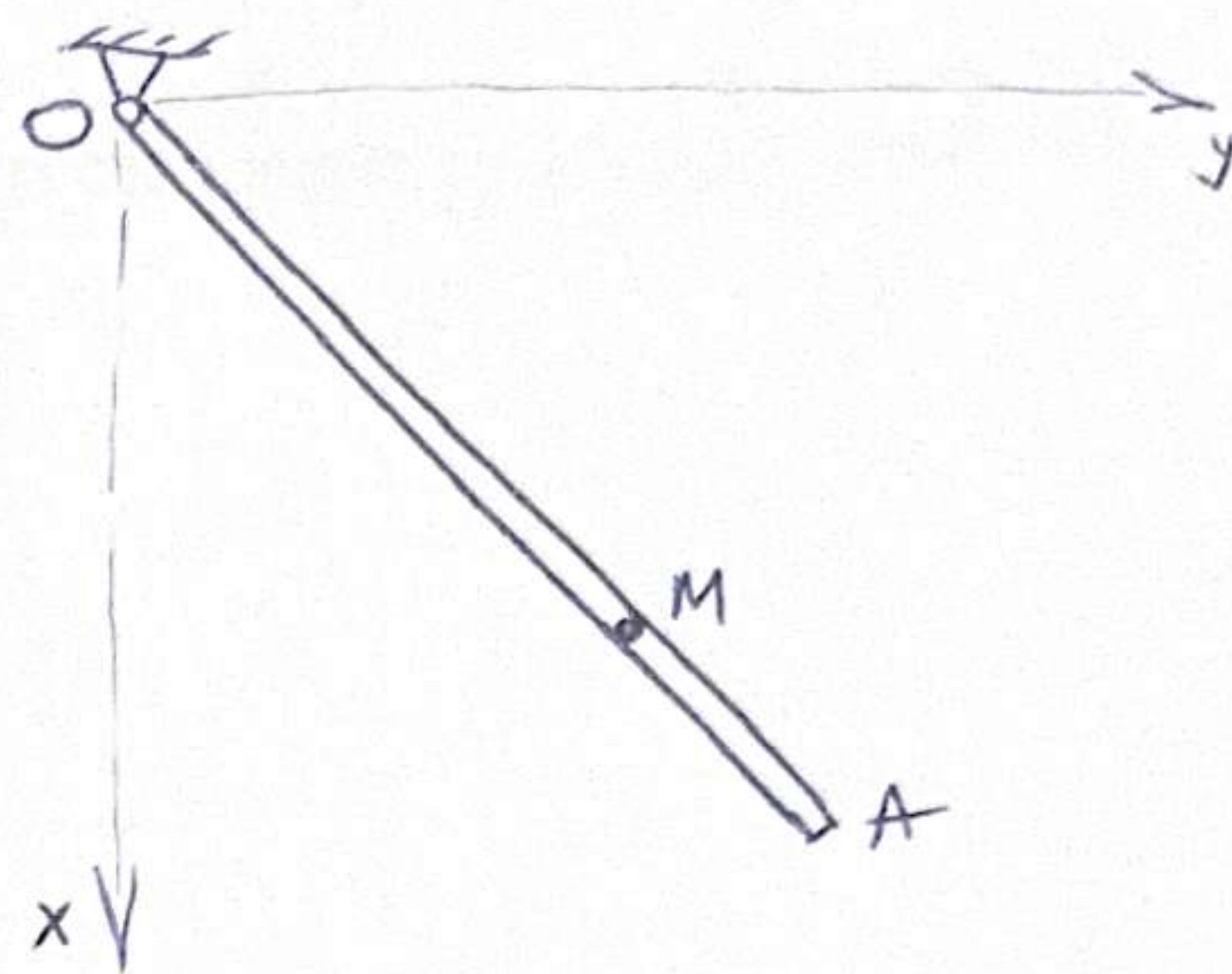
$$\omega = \text{const.}$$

$$\overline{OA} = \ell$$

do: M je na sredini cevi
 $\varphi = 0$

ta: tacka uprta cev

$$t_n = ?$$



$$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{cor} + \vec{L}_n + \vec{L}_c}$$

$$m\vec{g} = mg \cdot (-\vec{e}_1)$$

$$\vec{F}_p = -m(\ddot{\vec{r}}_{cm} + \vec{e} \times \vec{r} \times \ddot{\vec{e}} + \vec{\omega}' \times (\vec{\omega}' \times \vec{r})) = m\omega^2 \cdot x \cdot (\vec{e}_2)$$

$$\vec{F}_{cor} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (-\vec{e}_1)$$

$$\vec{L}_n = L_n \cdot (\vec{e}_1)$$

$$\vec{L}_c = L_c \cdot (\vec{e}_1)$$

$$x: \quad m\ddot{x} = m\omega^2 x$$

$$\ddot{x} - \omega^2 x = 0$$

$$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1,2} = \pm \omega \pm i$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$$

$$x(t_0) = C_1 + C_2 = \frac{\ell}{2}$$

$$\dot{x}(t_0) = C_1 \omega - C_2 \omega = 0 \rightarrow C_1 = C_2$$

$$\Rightarrow C_1 = C_2 = \frac{1}{4}\ell$$

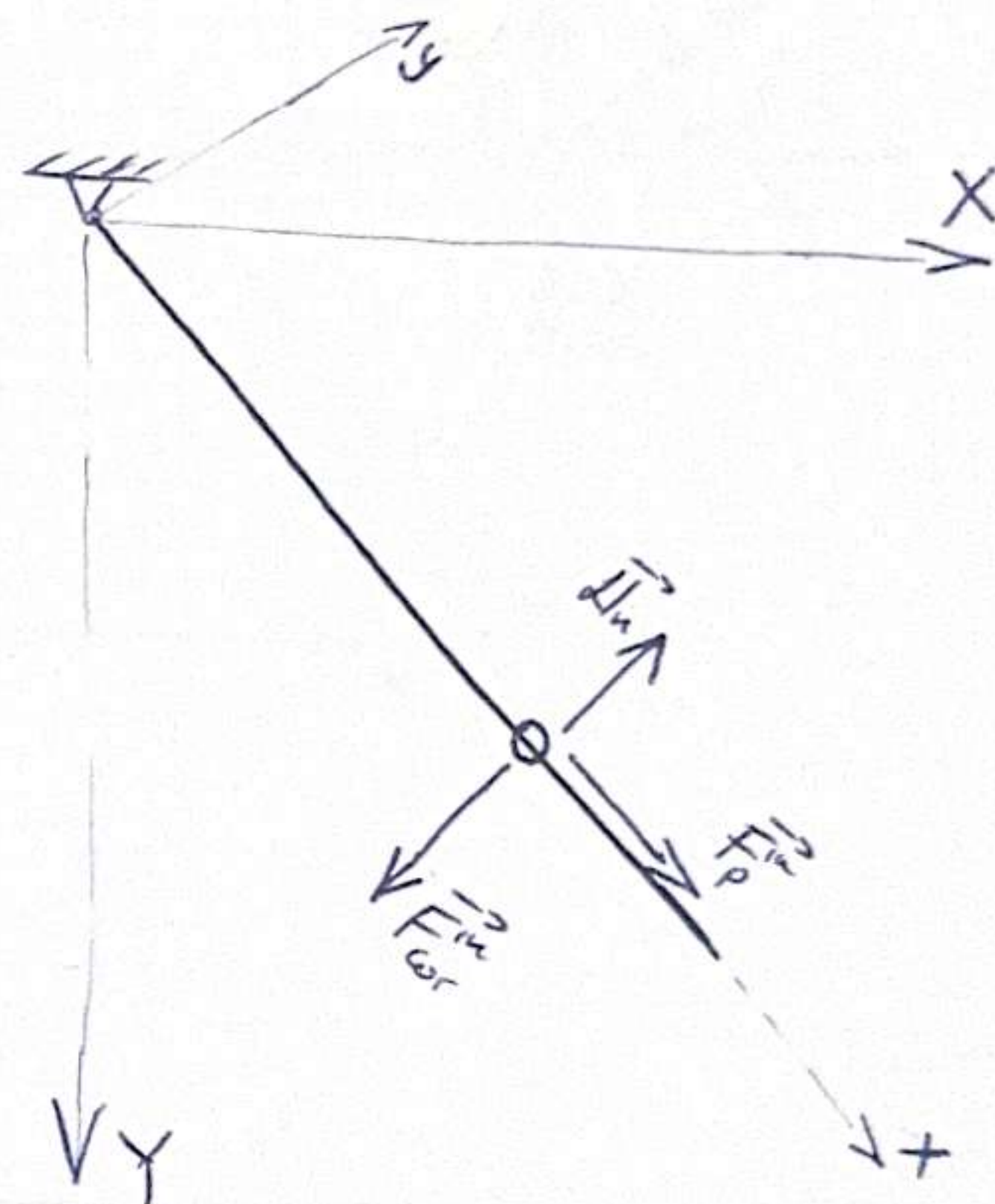
$$x = \frac{1}{4}\ell(e^{\omega t} + e^{-\omega t}) = \frac{1}{4}\ell \cdot 2 \cdot \frac{e^{\omega t} + e^{-\omega t}}{2} = \frac{1}{2}\ell \cosh(\omega t)$$

$$x(t_n) = \frac{1}{2}\ell \cosh(\omega t_n) = \ell$$

$$\cosh(\omega t_n) = 2$$

$$\boxed{t_n = \frac{1}{\omega} \operatorname{arccosh}(2)}$$

- U rešenju u zbirci je izračunato preko $\ln$. Rezultat je isti, jer C_1 preko rešenja uglo matematičkim operacijama da se transformiše u drugo.



6.18. horizontalna ravan

$$w = \text{const.}$$

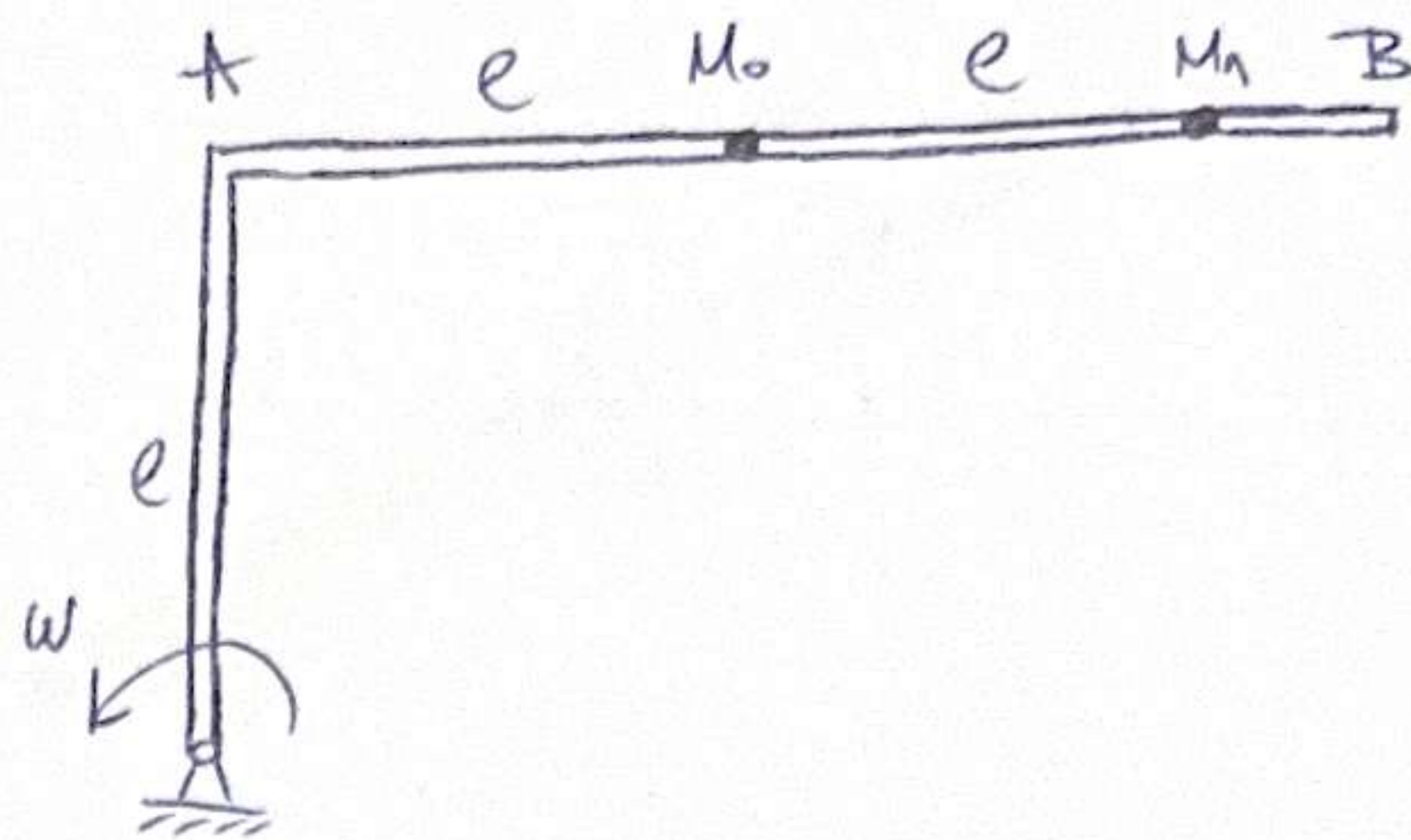
tacka mase m

to: tacka u M_0

$$v_{ro} = ew$$

1) $t_n = ?$ - trenutak kada je tacka u M_n

2) $N_n(t_n) = ?$



$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_n + \vec{N}_G$$

$$m\vec{g} = mg \cdot (-\vec{k})$$

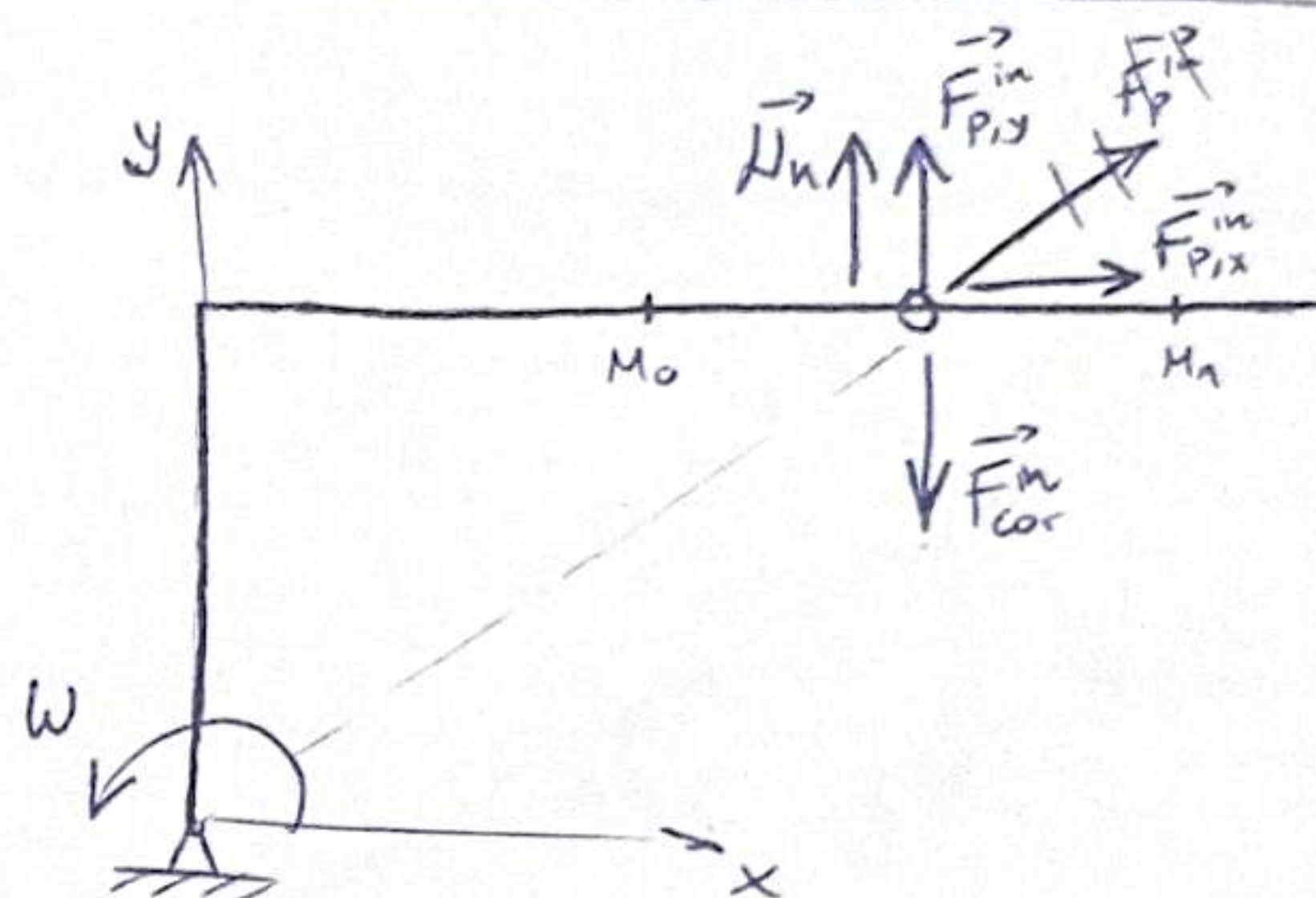
$$\vec{F}_p^m = -m(\vec{a}_m + \vec{v} \times \vec{v} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m\omega^2 x \cdot (\vec{i}) + m\omega^2 y \cdot (\vec{j}) = \vec{F}_{p,x}^m + \vec{F}_{p,y}^m$$

$$\vec{F}_{cor}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (-\vec{j})$$

$$\vec{N}_n = N_n \cdot \vec{j}$$

$$\vec{N}_G = N_G \cdot \vec{k}$$



$$x: m\ddot{x} = m\omega^2 x$$

$$\ddot{x} - \omega^2 x = 0$$

$$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega \pm i0$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$$

$$x(t_0) = C_1 + C_2 = e$$

$$\dot{x}(t_0) = C_1 \omega - C_2 \omega = ew$$

$$C_1 = e + C_2 \rightarrow e + C_2 + C_2 = e$$

$$\Rightarrow C_2 = 0, C_1 = e$$

$$x = e \cdot e^{\omega t}$$

$$x(t_n) = e \cdot e^{\omega t_n} = 2e$$

$$e^{\omega t_n} = 2 \quad | \ln$$

$$\omega t_n = \ln 2$$

$$t_n = \frac{1}{\omega} \ln 2$$

$$y: 0 = m\omega^2 y - 2m\omega \dot{x} + N_n$$

$$\dot{x}(t_n) = e \cdot \omega \cdot e^{\omega t_n} = e \cdot \omega \cdot 2$$

$$N_n = 2m\omega \cdot 2e\omega - m\omega^2 \cdot e$$

$$N_n(t_n) = 3m e \omega^2$$

6.13 Vertikalna ravan

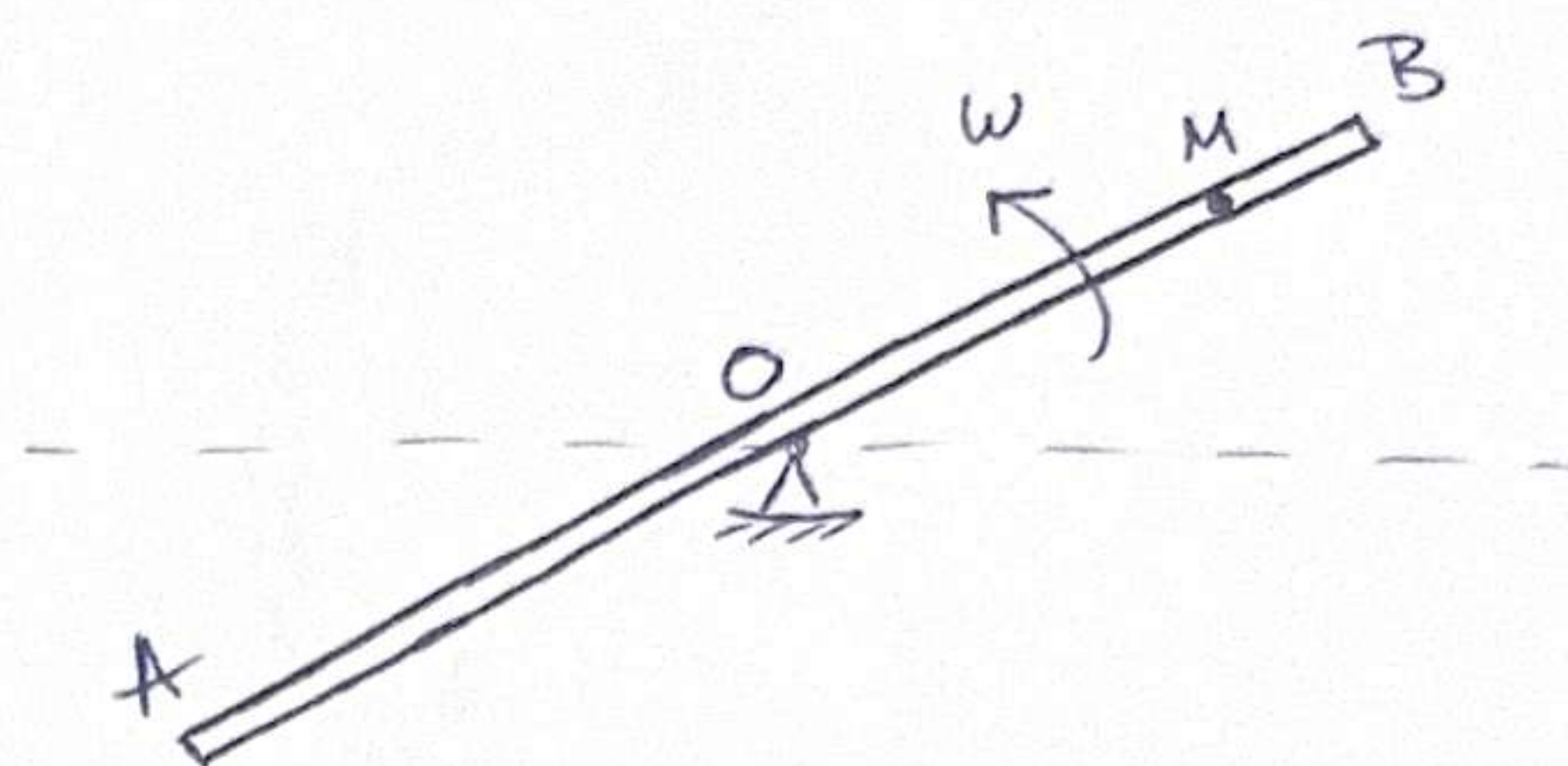
AB: $\omega = \text{const.}$

M: m

l_0 : cev horizontalna

M je u O

$$\omega_0 = \frac{g}{2\omega}$$



$x(t) = ?$ - j-na rel kretanja?
tacke M

$$H_{\text{max}} = ?$$

$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p^m + \vec{F}_{\text{cor}}^m + \vec{N}_h + \vec{H}_c^m$$

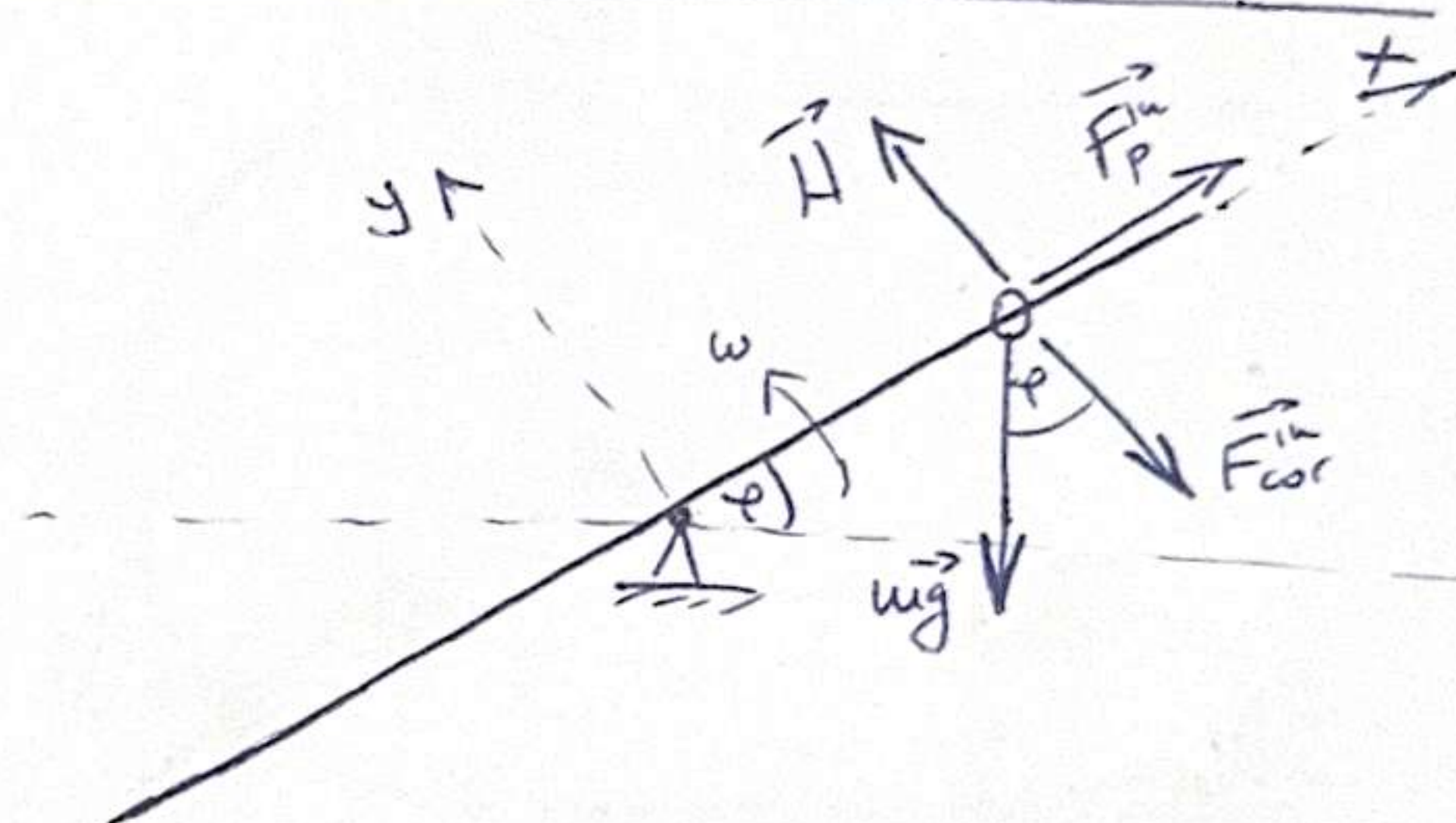
$$m\vec{g} = mg \cdot (-\vec{e})$$

$$\vec{F}_p^m = -m(\vec{a}_{\text{rot}} + \vec{\varepsilon}' \times \vec{r}' + \vec{\omega}' \times (\vec{\omega}' \times \vec{r}')) = m\omega^2 x \cdot (\vec{z}')^0$$

$$\vec{F}_{\text{cor}}^m = -2m\vec{\omega}' \times \vec{v}_r' = +2m\omega \cdot \dot{x} \cdot (-\vec{j}')^0$$

$$\vec{N}_h = N \cdot (\vec{j}') = \vec{H}$$

$$\vec{H}_c^m = 0$$



$$\varphi = \int \omega dt = \omega t$$

$$x: \quad m\ddot{x} = -mg \sin \varphi + m\omega^2 x$$

$$\ddot{x} - \omega^2 x = -g \sin \omega t$$

$$H: \quad \lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1,2} = \pm i\omega$$

$$x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$P: \quad G(t) = e^{0t} (0 \cos(\omega t) - g \sin(\omega t))$$

$$x_p = t^0 e^{0t} (A \cos \omega t + B \sin \omega t)$$

$$\dot{x}_p = -A\omega \sin \omega t + B\omega \cos \omega t$$

$$\ddot{x}_p = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t$$

$$\cos(\omega t)(-A\omega^2 - \omega^2 A) + \sin(\omega t)(-B\omega^2 - \omega^2 B) =$$

$$= 0 \cdot \cos(\omega t) + (-g) \cdot \sin(\omega t)$$

$$\Rightarrow A=0, \quad 2B\omega^2 = g \rightarrow B = \frac{1}{2} \frac{g}{\omega^2}$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t} + \frac{1}{2} \frac{g}{\omega^2} \sin \omega t$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t} + \frac{1}{2} \frac{g}{\omega} \cos \omega t$$

$$x(t_0) = C_1 + C_2 + 0 = 0 \rightarrow C_1 = -C_2$$

$$\dot{x}(t_0) = C_1 \omega - C_2 \omega + \frac{1}{2} \frac{g}{\omega} = \frac{1}{2} \frac{g}{\omega} \rightarrow C_1 = -C_2 = 0$$

$$x = \frac{1}{2} \frac{g}{\omega^2} \sin(\omega t)$$

$$y: \quad 0 = -mg \cos \varphi - 2m\omega \dot{x} + H$$

$$H = mg \cos \omega t + 2m\omega \cdot \frac{1}{2} \frac{g}{\omega} \cos \omega t$$

$$H = 2mg \cos \omega t$$

$$H_{\text{max}} = 2mg$$

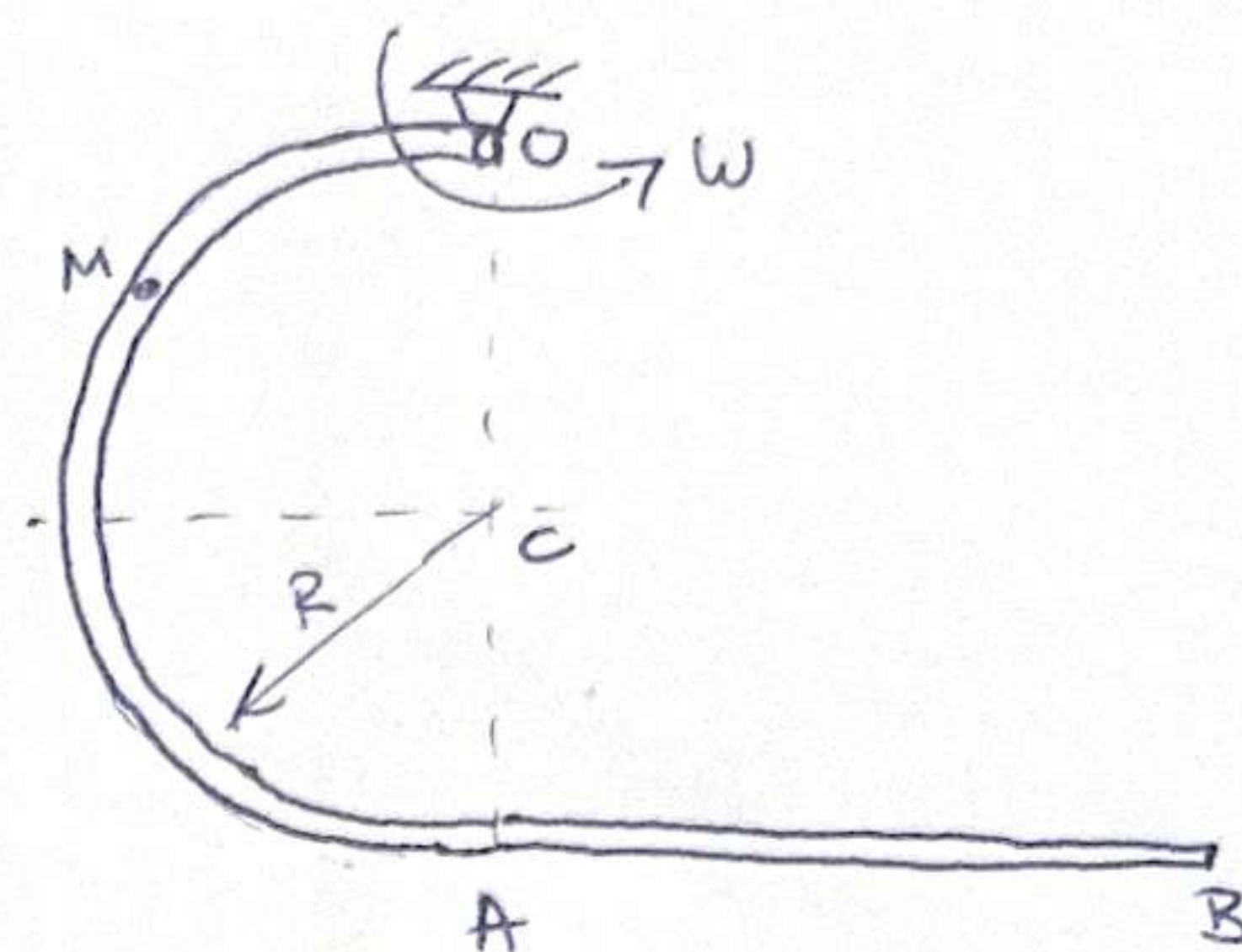
6.20) horizontalna ravnina

$\omega = \text{const.}$

tačka mase m

to: M je u O , $0 \leq 20$

konačna j -na rel. kretanja na delu AB ?



deonika OA

$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_P^{\text{in}} + \vec{F}_{\text{cor}}^{\text{in}} + \vec{N}_n + \vec{N}_G$$

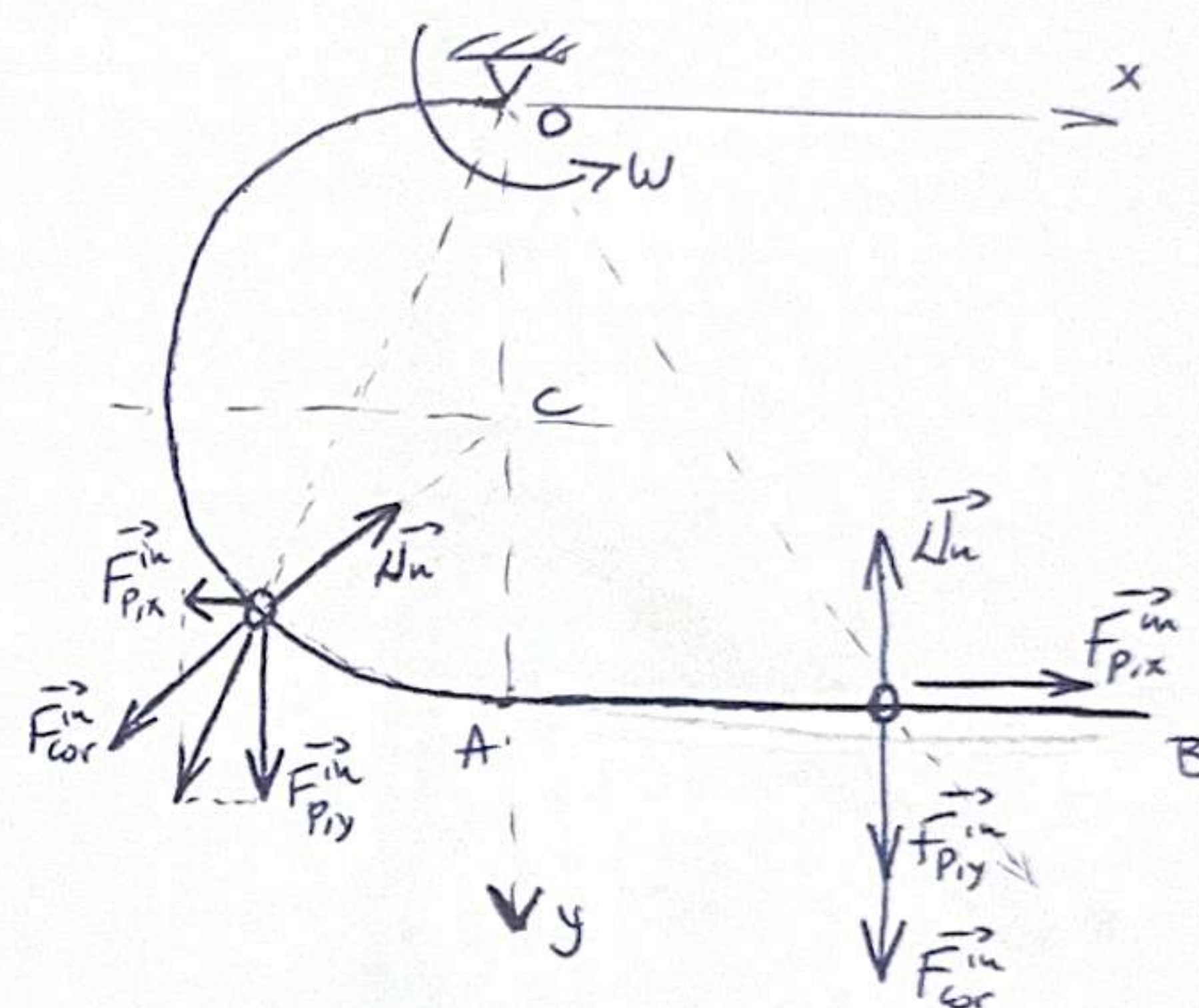
$$\vec{m}\vec{g} = m\vec{g} \cdot (\vec{k})$$

$$\vec{F}_P^{\text{in}} = -m(\vec{a}_{\text{cor}} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 \cdot |\vec{r}| \cdot (-\vec{r}) + m\omega^2 y \cdot (\vec{j}) = \vec{F}_{P,x}^{\text{in}} + \vec{F}_{P,y}^{\text{in}}$$

$$\vec{F}_{\text{cor}}^{\text{in}} = -2m\vec{\omega} \times \vec{v}_r = +2m \cdot \omega \cdot v_r \cdot (-\vec{n})$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_G = N_G \cdot (-\vec{k})$$



$$\vec{T}_A - \vec{T}_O = A \vec{O}A, \quad (\text{rel})$$

$$\frac{1}{2} m v_{Ar}^2 - 0 = \int_0^0 \vec{F}_{P,x}^{\text{in}} dx + \int_0^{2R} \vec{F}_{P,y}^{\text{in}} dy = \int_0^{2R} m\omega^2 y dy = \frac{1}{2} m\omega^2 (4R^2) = 2m\omega^2 R^2$$

$$v_{Ar}^2 = 4\omega^2 R^2 \rightarrow v_{Ar} = 2\omega R$$

deonika AB

$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_P^{\text{in}} + \vec{F}_{\text{cor}}^{\text{in}} + \vec{N}_n + \vec{N}_G$$

$$\vec{m}\vec{g} = m\vec{g} \cdot (\vec{k})$$

$$\vec{F}_P^{\text{in}} = -m(\vec{a}_{\text{cor}} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (-\vec{i}) + m\omega^2 y \cdot (\vec{j}) = \vec{F}_{P,x}^{\text{in}} + \vec{F}_{P,y}^{\text{in}}$$

$$\vec{F}_{\text{cor}}^{\text{in}} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$$

$$\vec{N}_n = N_n \cdot (-\vec{j})$$

$$\vec{N}_G = N_G \cdot (-\vec{k})$$

$$x: m\ddot{x} = m\omega^2 x$$

$$\ddot{x} - \omega^2 x = 0$$

$$\# : x^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega \pm 0i$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$$

$$x(t_0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$$

$$\dot{x}(t_0) = C_1 \omega - C_2 \omega = 2\omega R \Rightarrow 2C_1 = 2R$$

$$\Rightarrow C_1 = R = -C_2$$

$$x = R(e^{\omega t} - e^{-\omega t})$$

6.21.

α - ugao

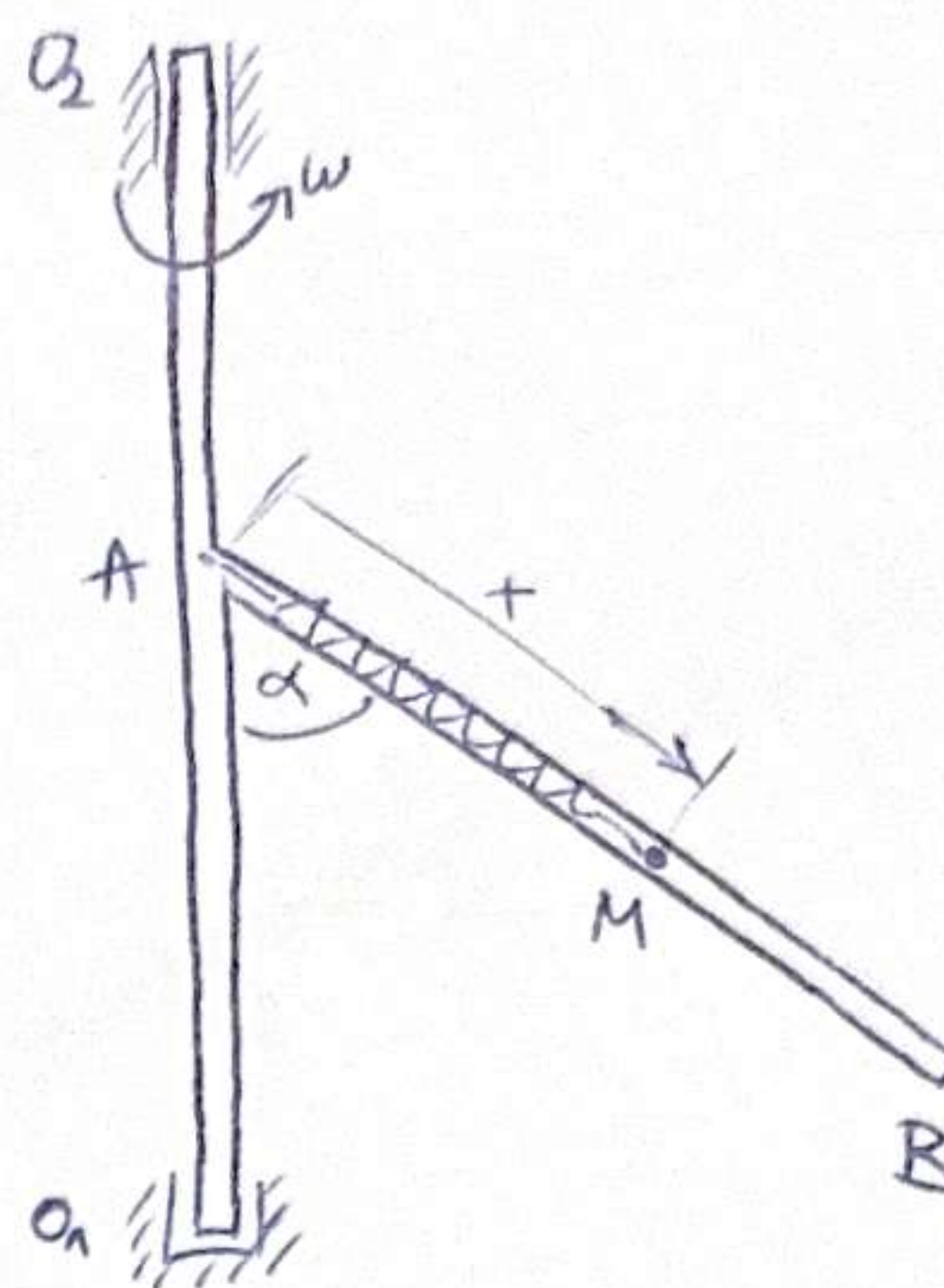
$\omega = \text{const}$

kuglica: m

opruga: c, l_0

1) d.f. j-na rel. kretanja kuglice?
uslov za α da li kretanje bilo oscilatorno?

2) u slučaju da je ispunjen uslov za oscilatorno kretanje, j-na rel. kretanja?
to: $x_0 = l_0, v_{x0} = 0$



$$1) \quad m \vec{a}_r = m \vec{g} + \vec{F}_c + \vec{F}_p^m + \vec{F}_{cor} + \vec{L}_n + \vec{L}_c$$

$$m \vec{g} = m g \cdot (-\vec{j})$$

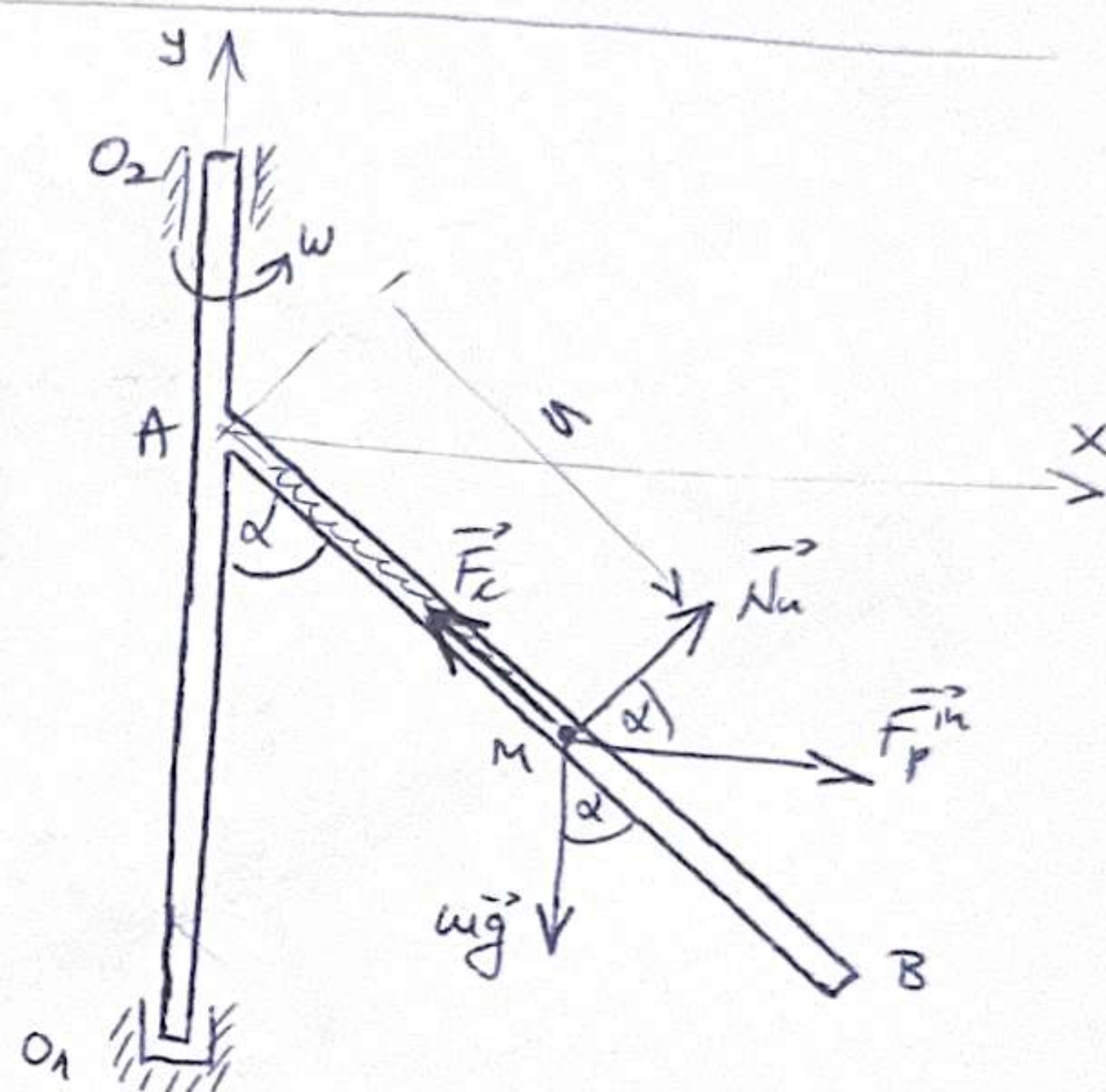
$$\vec{F}_c = c \cdot (s - l_0) \cdot (-\vec{t})$$

$$\vec{F}_p^m = -m(\ddot{\vec{r}}_M + \vec{\omega} \times \vec{r}_M + \vec{\omega} \times (\vec{\omega} \times \vec{r}_M)) = m \omega^2 x \cdot (-\vec{t})$$

$$\vec{F}_{cor} = -2m \vec{\omega} \times \vec{v}_r = 2m \omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{L}_n = L_n \cdot (\vec{n})$$

$$\vec{L}_c = L_c \cdot (-\vec{k})$$



$$t: \quad m \ddot{s} = m g \cos \alpha - c(s - l_0) + m \omega^2 x \sin \alpha$$

$$\ddot{s} = g \cos \alpha - \frac{c}{m}(s - l_0) + \omega^2 \cdot s \cdot \sin^2 \alpha$$

$$\ddot{s} + \left(\frac{c}{m} - \omega^2 \sin^2 \alpha \right) \cdot s = g \cos \alpha + \frac{c}{m} l_0$$

$$\ddot{s} + \omega_1^2 s = h_1$$

uslov za

osc. kretanje:

$\omega_1 > 0$

$\rightarrow$

$\frac{c}{m} - \omega^2 \sin^2 \alpha > 0$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$\rightarrow$

$\alpha < \arcsin(\sqrt{\frac{c}{m \omega^2}})$

$$2) \quad \lambda^2 + \omega_1^2 = 0 \rightarrow x_h = C_1 \cos(\omega_1 t) + C_2 \sin(\omega_1 t)$$

$$P: \quad \phi(t) = e^{0t} (h_1 \cos(\omega_1 t) + 0 \sin(\omega_1 t)) \rightarrow x_p = t^0 e^{0t} (A \cos(\omega_1 t) + B \sin(\omega_1 t)) = A, \quad \ddot{x}_p = 0$$

$$\omega_1^2 A = h_1 \rightarrow A = \frac{h_1}{\omega_1^2} \rightarrow x_p = \frac{h_1}{\omega_1^2}$$

$$x = C_1 \cos(\omega_1 t) + C_2 \sin(\omega_1 t) + \frac{h_1}{\omega_1^2}$$

$$\dot{x} = -C_1 \omega_1 \sin(\omega_1 t) + C_2 \omega_1 \cos(\omega_1 t)$$

$$x(t_0) = C_1 + 0 + \frac{h_1}{\omega_1^2} = l_0 \rightarrow C_1 = l_0 - \frac{h_1}{\omega_1^2}$$

$$\dot{x}(t_0) = 0 + C_2 \omega_1 = 0 \rightarrow C_2 = 0$$

$$x = \left(l_0 - \frac{h_1}{\omega_1^2} \right) \cos(\omega_1 t) + \frac{h_1}{\omega_1^2}$$

$$x = \left(l_0 - \frac{g \cos \alpha + \frac{c}{m} l_0}{\frac{c}{m} - \omega^2 \sin^2 \alpha} \right) \cos(\sqrt{\frac{c}{m} - \omega^2 \sin^2 \alpha} \cdot t) + \frac{g \cos \alpha + \frac{c}{m} l_0}{\frac{c}{m} - \omega^2 \sin^2 \alpha}$$

6.22. horizontalna ravan
jednakosmerni kretanje stanice a

$$\omega = \text{const.}$$

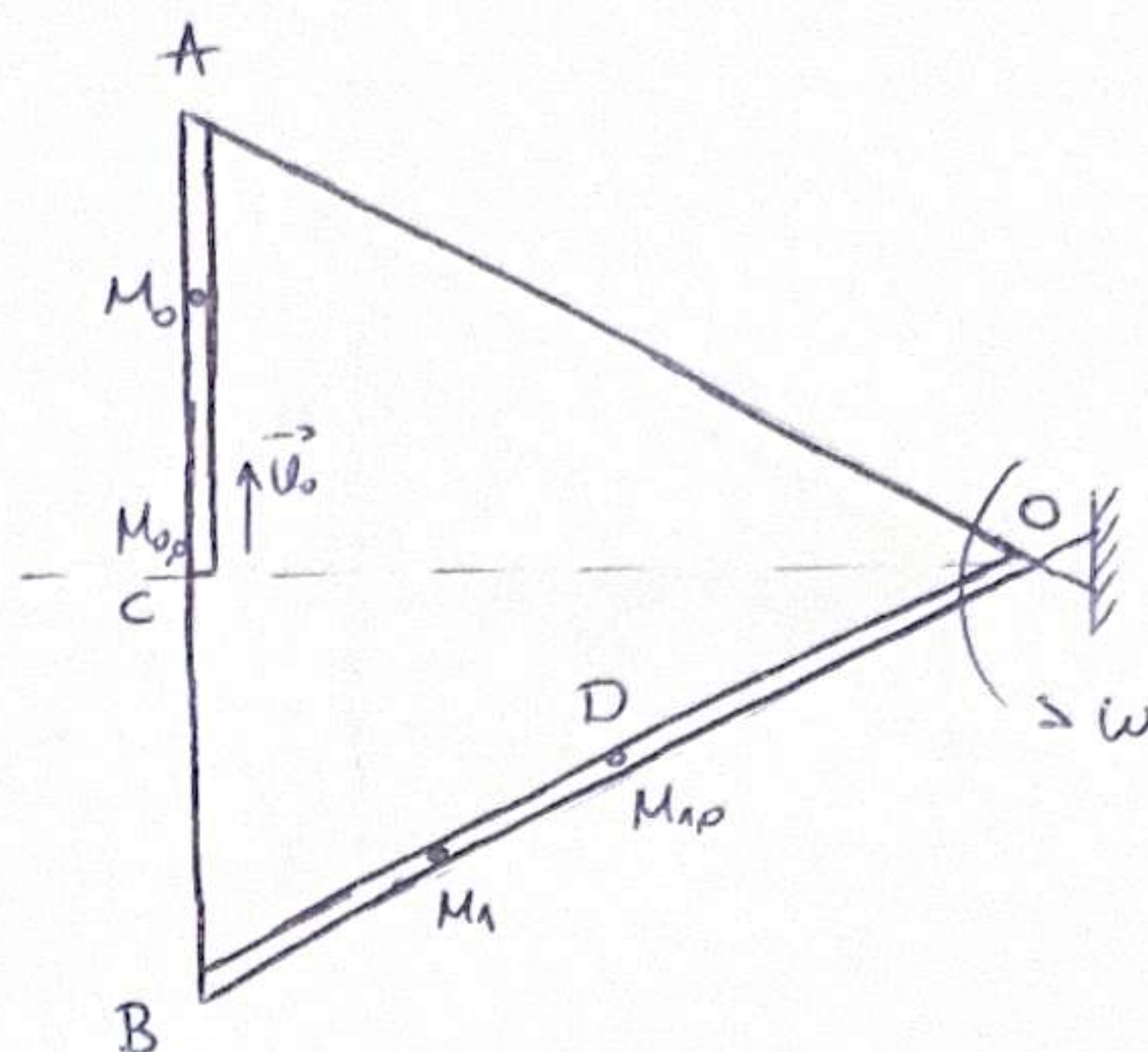
to: M_0 u C (C na sredini)

$$v_{0,0} = v_0$$

M_1 u D (D na sredini)

$$v_{0,0} = 0$$

$v_0 = ?$ - da li obe tačke
istovremeno napuste kanal



tačka M_1 :

$$m \vec{a}_r = m \vec{g} + \vec{F}_{p1} + \vec{F}_{cor1} + \vec{N}_{1n} + \vec{N}_{1c}$$

$$m \vec{g} = m g \cdot (-\vec{k}_1)$$

$$\vec{F}_{p1} = -m(\vec{a}_{on} + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m \omega^2 x_1 \cdot (\vec{i}_1)$$

$$\vec{F}_{cor1} = -2m \vec{\omega} \times \vec{v}_r = 2m \cdot \omega \cdot \dot{x} \cdot (\vec{j}_1)$$

$$\vec{N}_{1n} = N_{1n} \cdot (-\vec{j}_1)$$

$$\vec{N}_{1c} = N_{1c} \cdot (\vec{k}_1)$$

$$x_1: m \ddot{x}_1 = m \omega^2 x_1$$

$$\ddot{x}_1 - \omega^2 x_1 = 0$$

$$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega \pm i0$$

$$x = c_1 e^{\omega t} + c_2 e^{-\omega t}$$

$$\dot{x} = c_1 \omega e^{\omega t} - c_2 \omega e^{-\omega t}$$

$$\dot{x}(0) = c_1 \omega - c_2 \omega = 0 \rightarrow c_1 = c_2$$

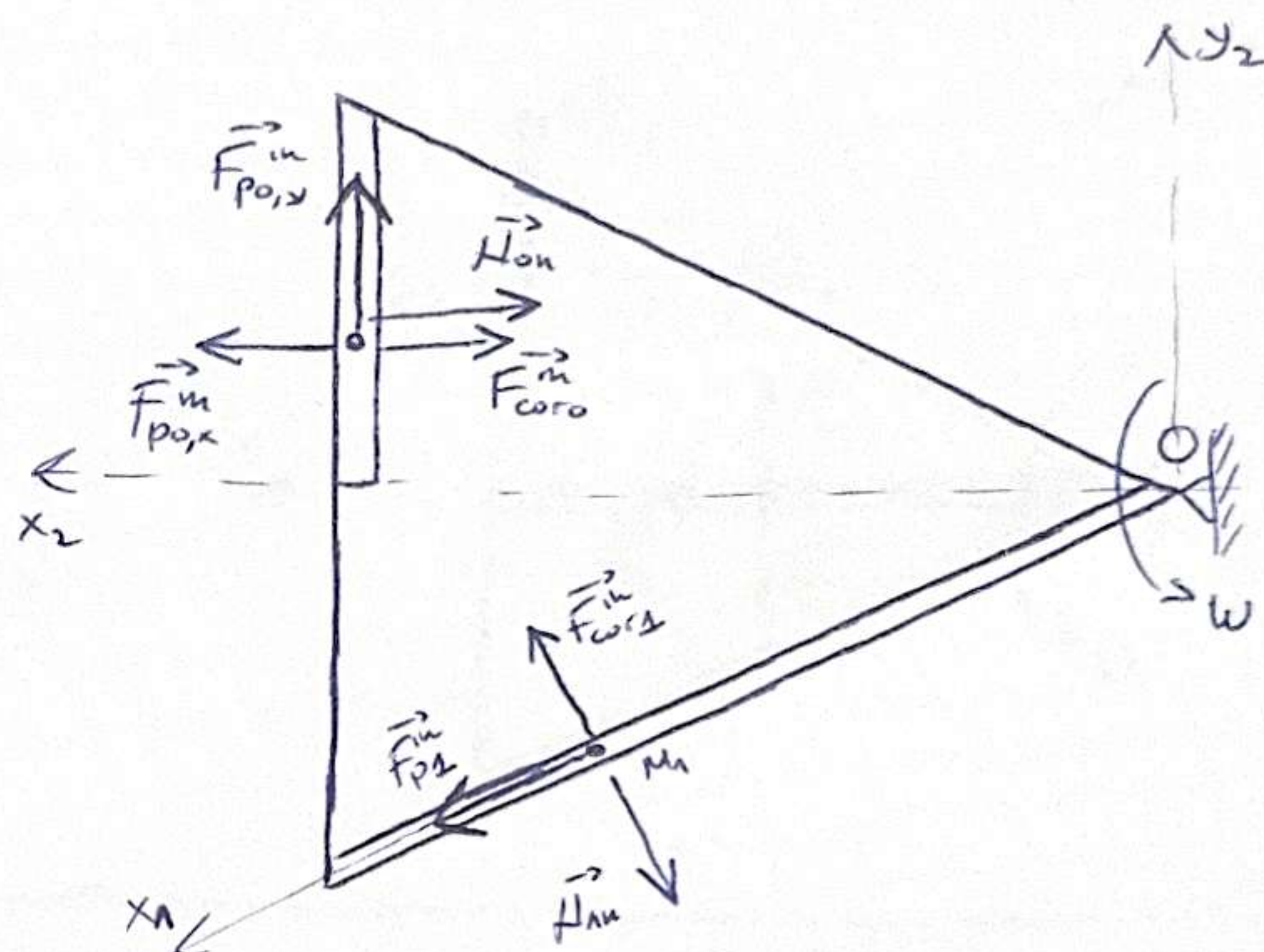
$$x(0) = c_1 + c_2 = \frac{a}{2} \rightarrow c_1 = c_2 = \frac{a}{4}$$

$$x = \frac{1}{4} a (e^{\omega t} + e^{-\omega t}) = \frac{1}{4} a \cdot 2 \frac{e^{\omega t} + e^{-\omega t}}{2}$$

$$x = \frac{1}{2} a \cosh(\omega t)$$

$$x(t_n) = \frac{1}{2} a \cosh(\omega t_n) = a$$

$$\cosh(\omega t_n) = 2$$



tačka M_0

$$m \vec{a}_{or} = m \vec{g} + \vec{F}_{p0} + \vec{F}_{cor0} + \vec{N}_{0n} + \vec{N}_{0c}$$

$$m \vec{g} = m g \cdot (-\vec{k}_2)$$

$$\vec{F}_{p0} = -m(\vec{a}_{on} + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m \omega^2 x_2 \cdot (\vec{i}_2) + m \omega^2 y_2 \cdot (\vec{j}_2)$$

$$\vec{F}_{cor0} = -2m \vec{\omega} \times \vec{v}_r = 2m \omega y_2 \cdot (-\vec{i}_2)$$

$$\vec{N}_{0n} = N_{0n} \cdot (-\vec{i}_2)$$

$$\vec{N}_{0c} = N_{0c} \cdot (\vec{k}_2)$$

$$y_2: m \ddot{y}_2 = m \omega^2 y_2$$

$$\ddot{y}_2 - \omega^2 y_2 = 0$$

$$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega \pm i0$$

$$y = c_1 e^{\omega t} + c_2 e^{-\omega t}, \quad \dot{y} = c_1 \omega e^{\omega t} - c_2 \omega e^{-\omega t}$$

$$y(0) = c_1 + c_2 = 0, \quad \dot{y}(0) = c_1 \omega - c_2 \omega = v_0$$

$$\Rightarrow c_1 = \frac{v_0}{2\omega}, \quad c_2 = -\frac{v_0}{2\omega}$$

$$y = \frac{v_0}{2\omega} (e^{\omega t} - e^{-\omega t}) = \frac{v_0}{\omega} \sinh(\omega t)$$

$$y(t_n) = \frac{v_0}{\omega} \sinh(\omega t_n) = \frac{a}{2}, \quad \cosh^2(\omega t_n) - \sinh^2(\omega t_n) = 1$$

$$4 - \sinh^2(\omega t_n) = 1 \rightarrow \sinh(\omega t_n) = \sqrt{3}$$

$$y(t_n) = \frac{v_0}{\omega} \sqrt{3} = \frac{a}{2} \Rightarrow v_0 = \frac{a\omega}{2\sqrt{3}} \Rightarrow \boxed{v_0 = \frac{a\omega\sqrt{3}}{6}}$$

6.23) horizontalna ravan
jednolično kružno kretanje

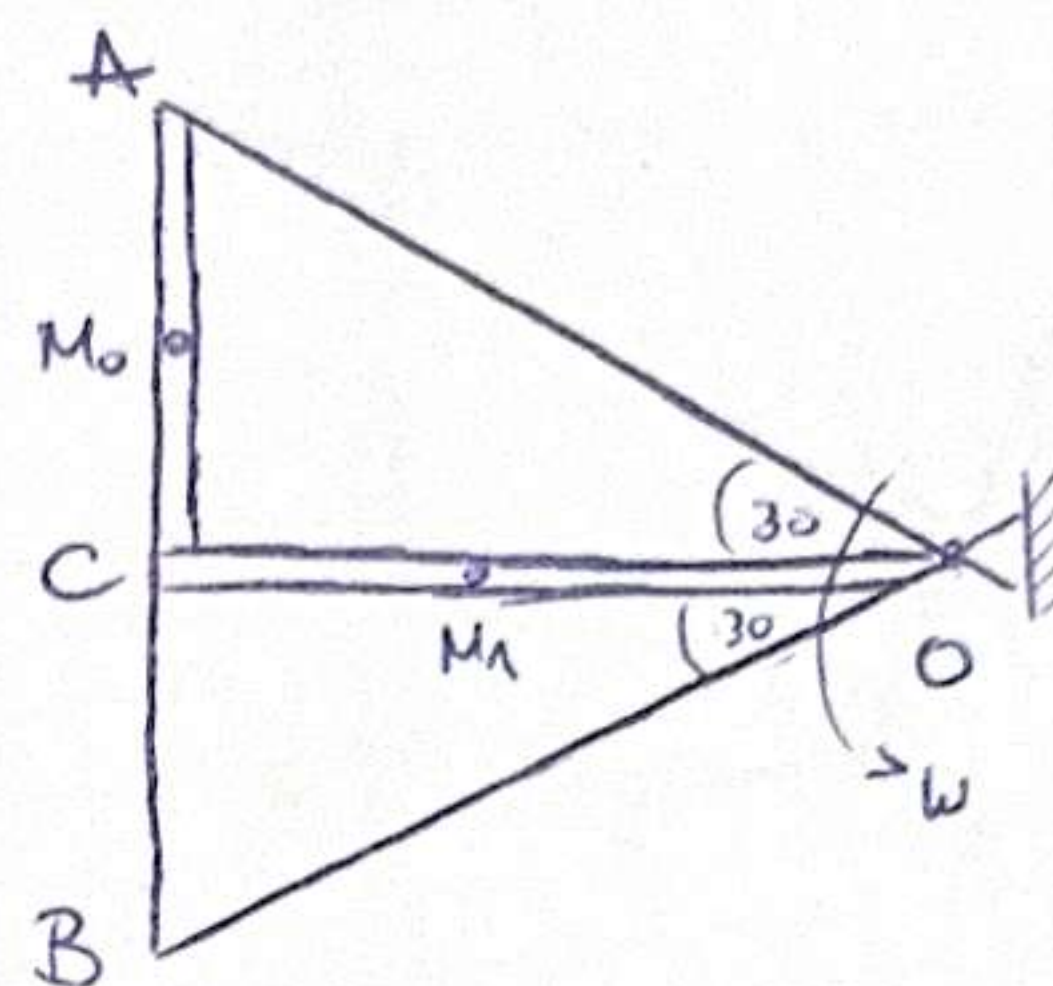
$$\omega = \text{const.}$$

$$t_0: M_0 \text{ u } C$$

$$\varphi_{C,0} = \varphi_0$$

$$M_1 \text{ u } O$$

$$\varphi_{O,1} = \varphi_1$$



$$\frac{\varphi_0}{\varphi_1} = ? \quad - \text{ da bi istovremeno } \text{upustile ploču}$$

tacka M1:

$$m \vec{a}_{1r} = m \vec{g} + \vec{F}_{P1}^m + \vec{F}_{Cor1}^m + \vec{L}_{M1} + \vec{L}_{G1}$$

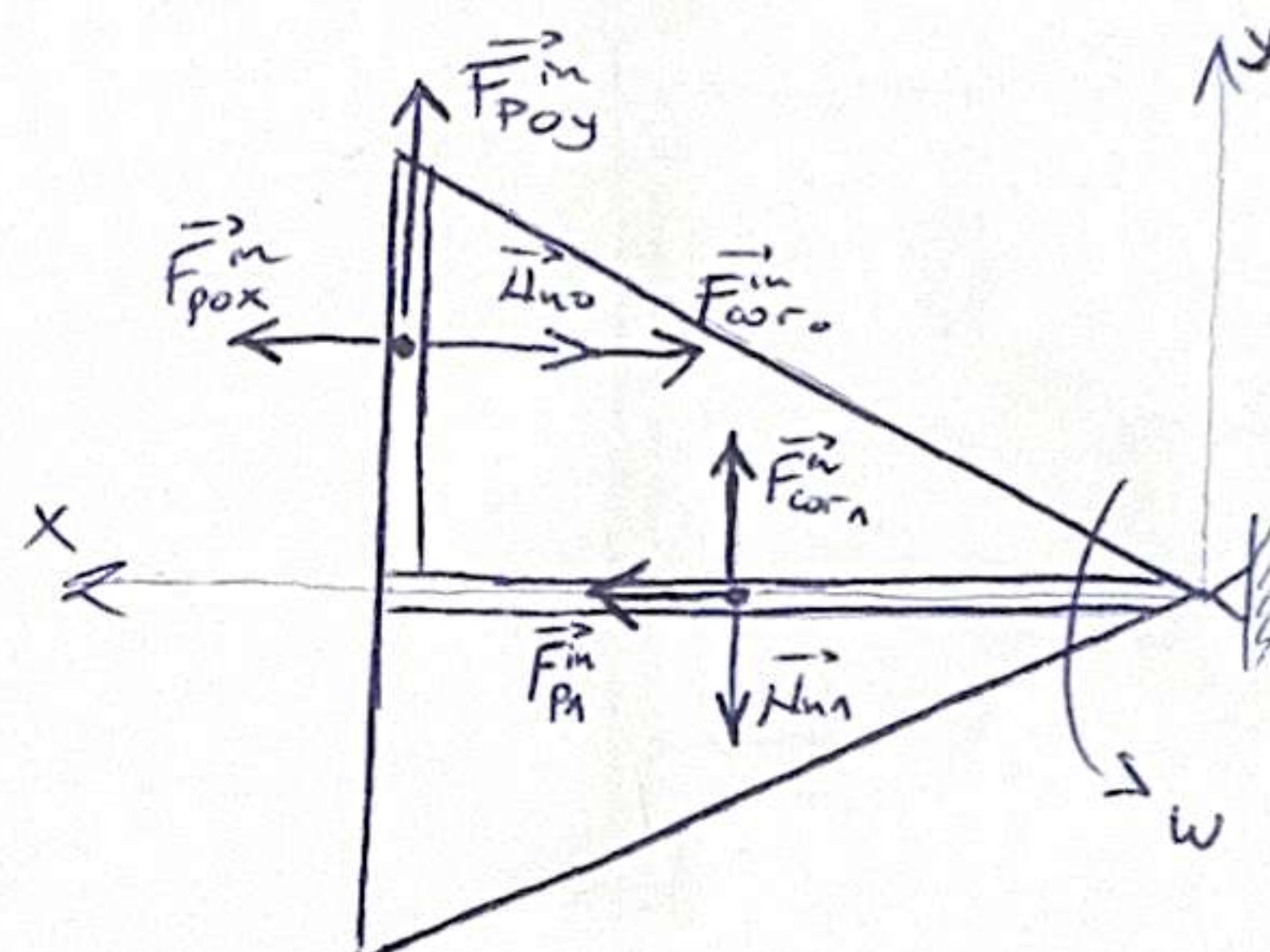
$$m \vec{g}' = m \vec{g} \cdot (-\vec{e})$$

$$\vec{F}_{P1}^m = -m(\vec{a}_{on}^i + \vec{\varepsilon} \times \vec{r}^i + \vec{\omega} \times (\vec{\omega} \times \vec{r}^i)) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{Cor1}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$$

$$\vec{L}_{M1} = L_{M1} \cdot (-\vec{j})$$

$$\vec{L}_{G1} = L_{G1} \cdot (\vec{e})$$



$$x: \ddot{x} = \omega^2 x$$

$$\ddot{x} - \omega^2 x = 0$$

$$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1,2} = \pm \omega \pm i0$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$$

$$x(t_0) = 0 \rightarrow C_1 + C_2 = 0 \rightarrow C_1 = -C_2$$

$$\dot{x}(t_0) = C_1 \omega - C_2 \omega = \varphi_1$$

$$2C_1 \omega = \varphi_1$$

$$C_1 = \frac{\varphi_1}{2\omega} = -C_2$$

$$x = \frac{\varphi_1}{2\omega} (e^{\omega t} - e^{-\omega t}) = \frac{\varphi_1}{\omega} \sinh(\omega t)$$

$$x(t_1) = \frac{\varphi_1}{\omega} \sinh(\omega t_1) = a \cdot \omega \cdot 30 = \frac{a\sqrt{3}}{2}$$

tacka M0

$$m \vec{a}_{r0} = m \vec{g} + \vec{F}_{P0}^m + \vec{F}_{Cor}^m + \vec{L}_{M0} + \vec{L}_{G0}$$

$$m \vec{g}' = m \vec{g} \cdot (-\vec{e})$$

$$\vec{F}_{P0}^m = -m(\vec{a}_{on}^i + \vec{\varepsilon} \times \vec{r}^i + \vec{\omega} \times (\vec{\omega} \times \vec{r}^i)) = m\omega^2 x \cdot (\vec{i}) + m\omega^2 y \cdot (\vec{j}) = \vec{F}_{Pox}^m + \vec{F}_{Poy}^m$$

$$\vec{F}_{Cor0}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{y} \cdot (-\vec{i})$$

$$\vec{L}_{M0} = L_{M0} \cdot (-\vec{i})$$

$$\vec{L}_{G0} = L_{G0} \cdot (\vec{e})$$

$$y: \ddot{y} = \omega^2 y$$

$$\ddot{y} - \omega^2 y = 0$$

...

$$y = \frac{\varphi_0}{\omega} \sinh(\omega t_1)$$

$$y(t_1) = \frac{\varphi_0}{\omega} \sinh(\omega t_1) = \frac{a}{2}$$

$$x(t_1) = y(t_1) \sqrt{3}$$

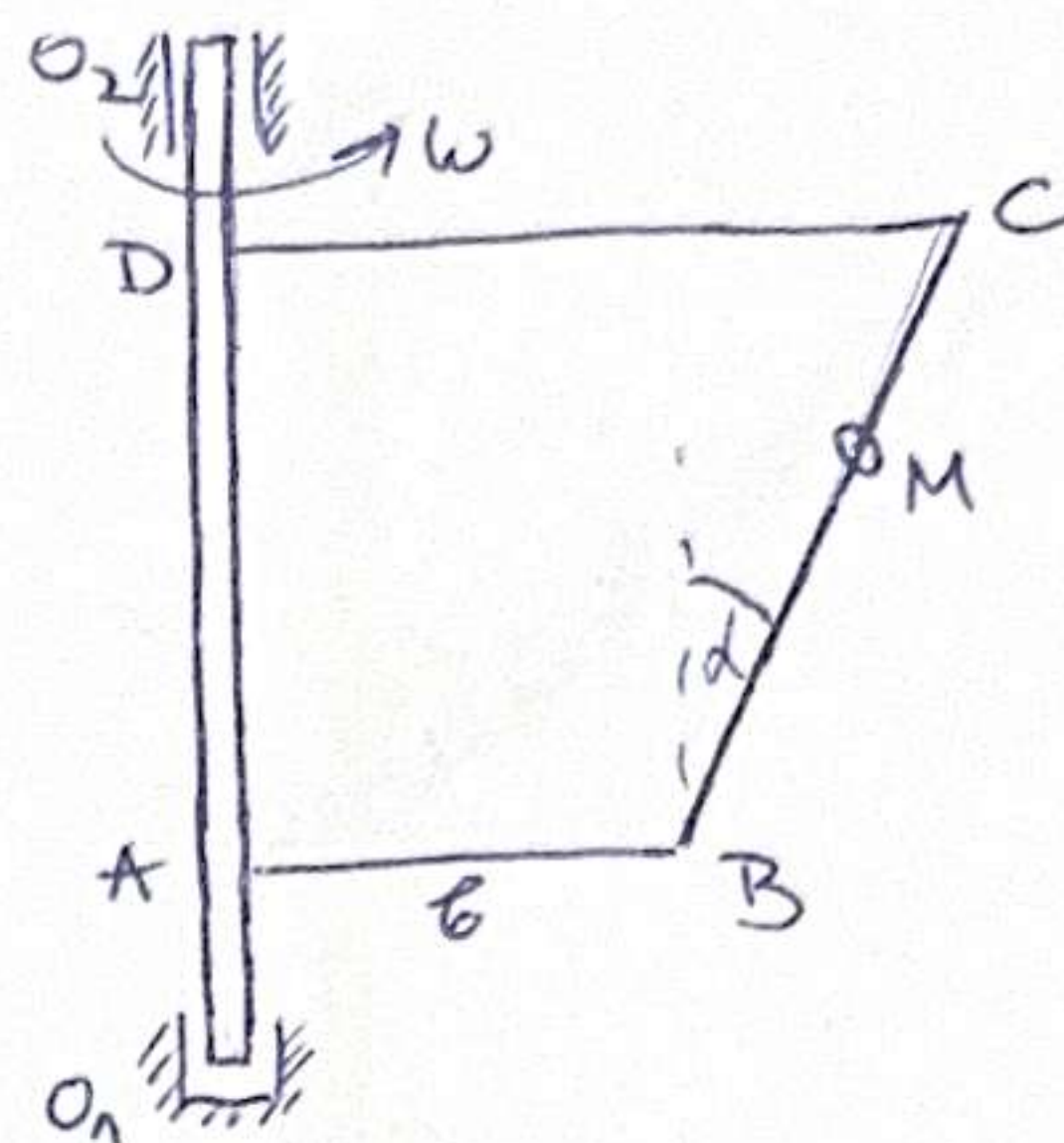
$$\frac{\varphi_1}{\omega} \sinh(\omega t_1) = \frac{\varphi_0}{\omega} \sinh(\omega t_1) \sqrt{3}$$

$$\boxed{\frac{\varphi_0}{\varphi_1} = \frac{\sqrt{3}}{3}}$$

prosten: m

b: $\text{prosten } \cup B, \quad U_{ro} = 0$

2) $w_{\max} = ?$ - da li postoji ostao u B



$$1) \vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{cor} + \vec{N}_n + \vec{N}_B$$

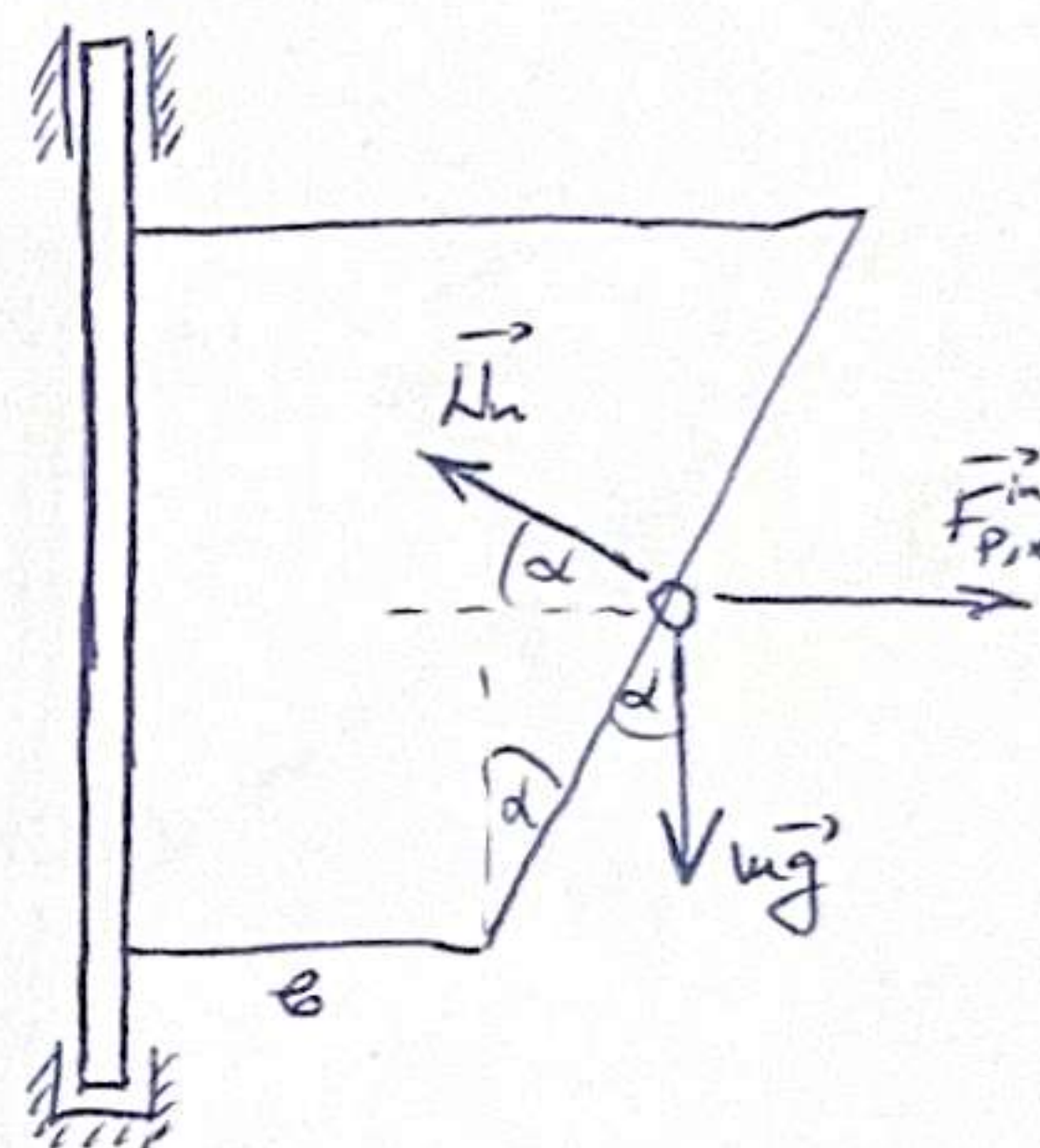
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\begin{aligned}\vec{F}_p^m &= -m(\vec{a}_{\text{cm}} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = \\ &= m \cdot \varepsilon \cdot x \cdot (\vec{k}) + m \omega^2 x \cdot (\vec{z}) = \vec{F}_{p,z} + \vec{F}_{p,x}\end{aligned}$$

$$\vec{F}_{\text{cor}} = -2m \vec{\omega} \times \vec{v}_r = 2m \omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{L}_n = L_n \cdot (\vec{h})$$

$$\vec{U}_6' = U_6 \cdot (-\vec{e}_1)$$



$$t: \quad \vec{m}_S = -m_g \cos \theta + \vec{F}_{p,x}^{\text{in}} \sin \theta$$

$$\cancel{y_a} = -\cancel{y_g} \cos \alpha + \cancel{y_w} \omega^2 \times \sin \alpha$$

$$w = \sqrt{\frac{a + g \cos \theta}{x \sin \theta}} \quad x = ?$$

$$\ddot{s} = a \rightarrow \dot{s} = at + \cancel{C_1}^0 \rightarrow s = \frac{1}{2}at^2 + \cancel{C_2}^0 \rightarrow x = 6 + s \cdot \sin \alpha = 6 + \frac{1}{2}at^2 \sin \alpha$$

$$w = \sqrt{\frac{a + g \cos \theta}{(6 + \frac{1}{2}at^2 \sin \theta) \sin \theta}}$$

2) t: $0 = -mg \cos \theta + F_{p,x} \sin \theta$

$$\underline{0 = -mg \cos \alpha + m \omega^2 r \sin \alpha}$$

$$W_{\max} = \sqrt{\frac{g_{\text{old}}}{65 \text{ m/s}}}$$

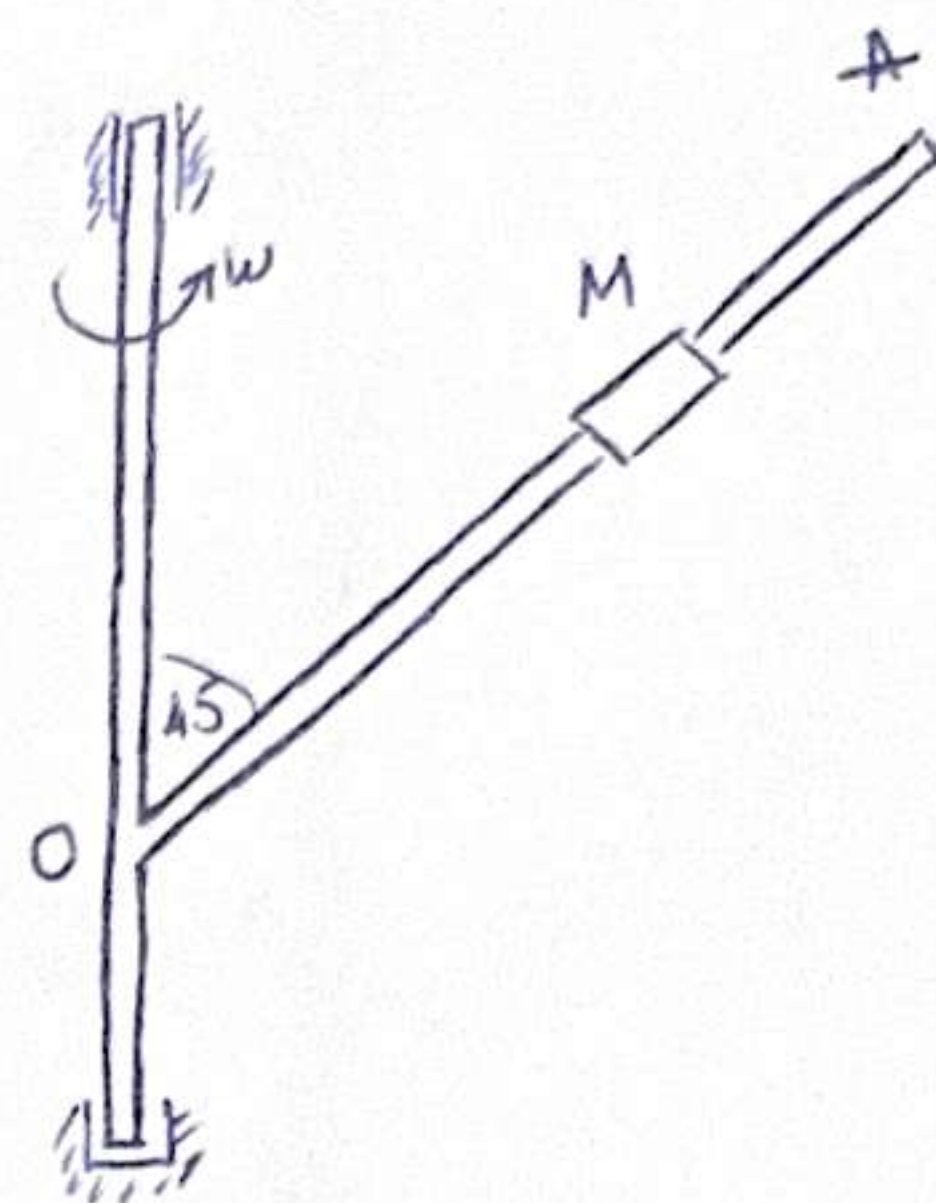
6.25.

OA: 6

vertikalna ravan

 $\omega = \text{const.}$

klizač: m

to: klizač relativno miruje
na sredini OA $t_1 = ?$ - trenutak kada klizač
napušta štap

$$\vec{a}_r = \omega g \cdot (-\vec{j})$$

$$\vec{F}_p = -m(\vec{a}_r + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (\vec{z})$$

$$\vec{F}_{cor} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_g = N_g \cdot (-\vec{k})$$

$$t: m\ddot{s} = -mg \frac{\sqrt{2}}{2} + m\omega^2 x \frac{\sqrt{2}}{2}$$

$$\ddot{s} = -g \frac{\sqrt{2}}{2} + \omega^2 \cdot s \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2}$$

$$\ddot{s} - \frac{1}{2}\omega^2 s = -g \frac{\sqrt{2}}{2}$$

$$A: \lambda^2 - \frac{1}{2}\omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \frac{\sqrt{2}}{2}\omega \pm 0i$$

$$x_n = C_1 e^{\frac{\sqrt{2}}{2}\omega t} + C_2 e^{-\frac{\sqrt{2}}{2}\omega t}$$

$$P: \phi(t) = e^{\omega t} (-g \frac{\sqrt{2}}{2} \omega t + 0 \sin \omega t)$$

$$x_p = t e^{\omega t} (A \omega t + B \sin \omega t), \quad \ddot{x}_p = 0$$

$$-\frac{1}{2}\omega^2 \cdot A = -g \frac{\sqrt{2}}{2} \rightarrow A = \frac{g\sqrt{2}}{\omega^2}$$

$$x = C_1 e^{\frac{\sqrt{2}}{2}\omega t} + C_2 e^{-\frac{\sqrt{2}}{2}\omega t} + \frac{g\sqrt{2}}{\omega^2}$$

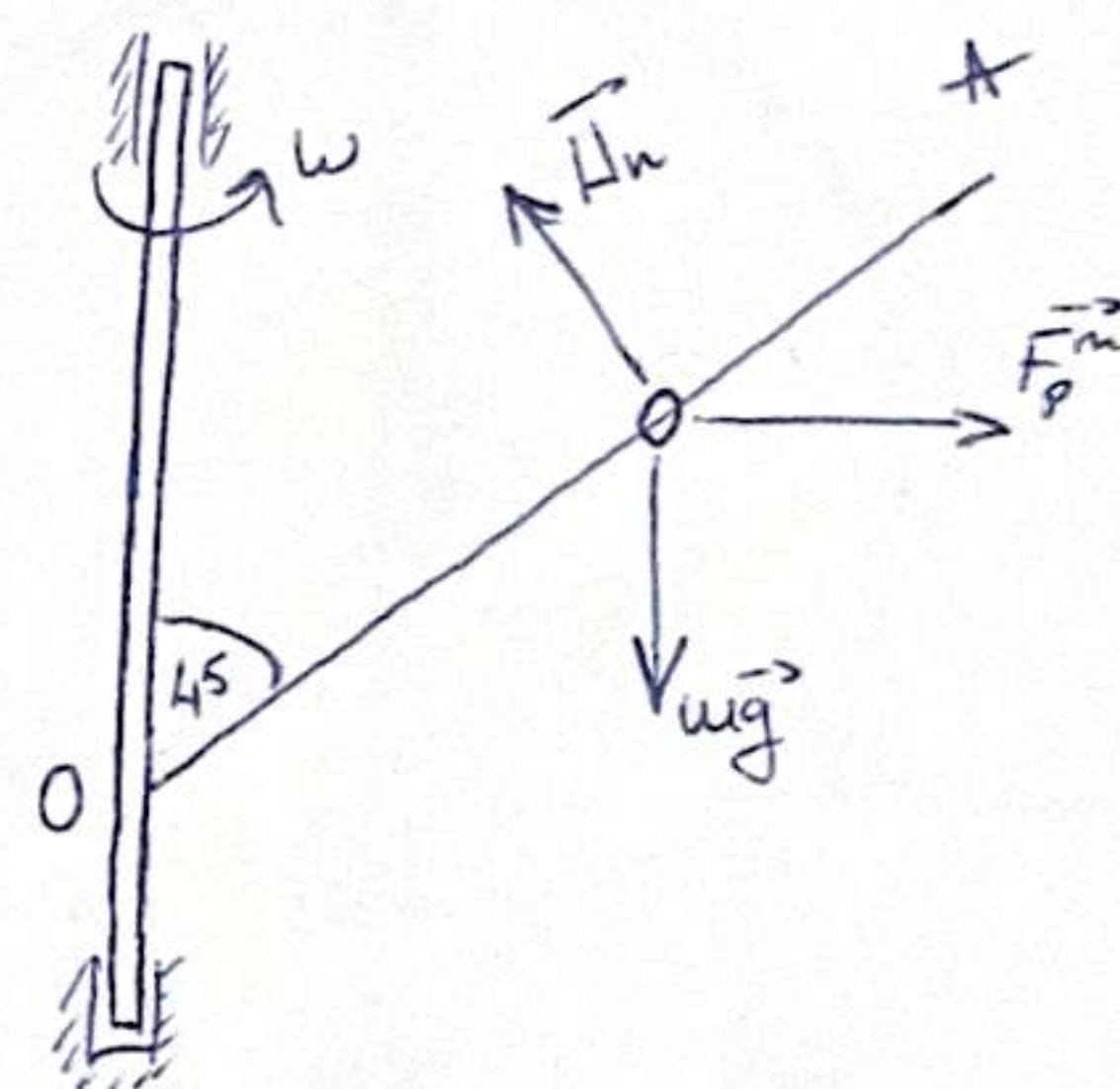
$$\dot{x} = \frac{\sqrt{2}}{2}\omega C_1 e^{\frac{\sqrt{2}}{2}\omega t} - \frac{\sqrt{2}}{2}\omega C_2 e^{-\frac{\sqrt{2}}{2}\omega t}$$

$$\dot{x}(t_0) = \frac{\sqrt{2}}{2}\omega C_1 - \frac{\sqrt{2}}{2}\omega C_2 = 0 \rightarrow C_1 = C_2$$

$$x(t_0) = C_1 \cdot 1 + C_2 \cdot 1 + \frac{g\sqrt{2}}{\omega^2} = \frac{6}{2}$$

$$2C_1 = \frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}$$

$$C_1 = C_2 = \frac{6}{4} - \frac{g\sqrt{2}}{\omega^2}$$



$$x = \left(\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}\right) \cosh\left(\frac{\sqrt{2}}{2}\omega t\right) + \frac{g\sqrt{2}}{\omega^2}$$

$$x(t_1) = \left(\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}\right) \cosh\left(\frac{\sqrt{2}}{2}\omega t_1\right) + \frac{g\sqrt{2}}{\omega^2} = 6$$

$$\cosh\left(\frac{\sqrt{2}}{2}\omega t_1\right) = \frac{6 - \frac{g\sqrt{2}}{\omega^2}}{\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}}$$

$$t_1 = \frac{\sqrt{2}}{\omega} \operatorname{arccosh}\left(\frac{6 - \frac{g\sqrt{2}}{\omega^2}}{\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}}\right)$$

napomena:

Rešenje u zbirci je izračunato preko logaritamske f-ije, a ne preko hiperboličke. Ta dva rešenja su potpuno jednaka, jednu rešenje bi se matematičkim operacijama moglo transformirati u drugo.

6.26.

knjma cev: r

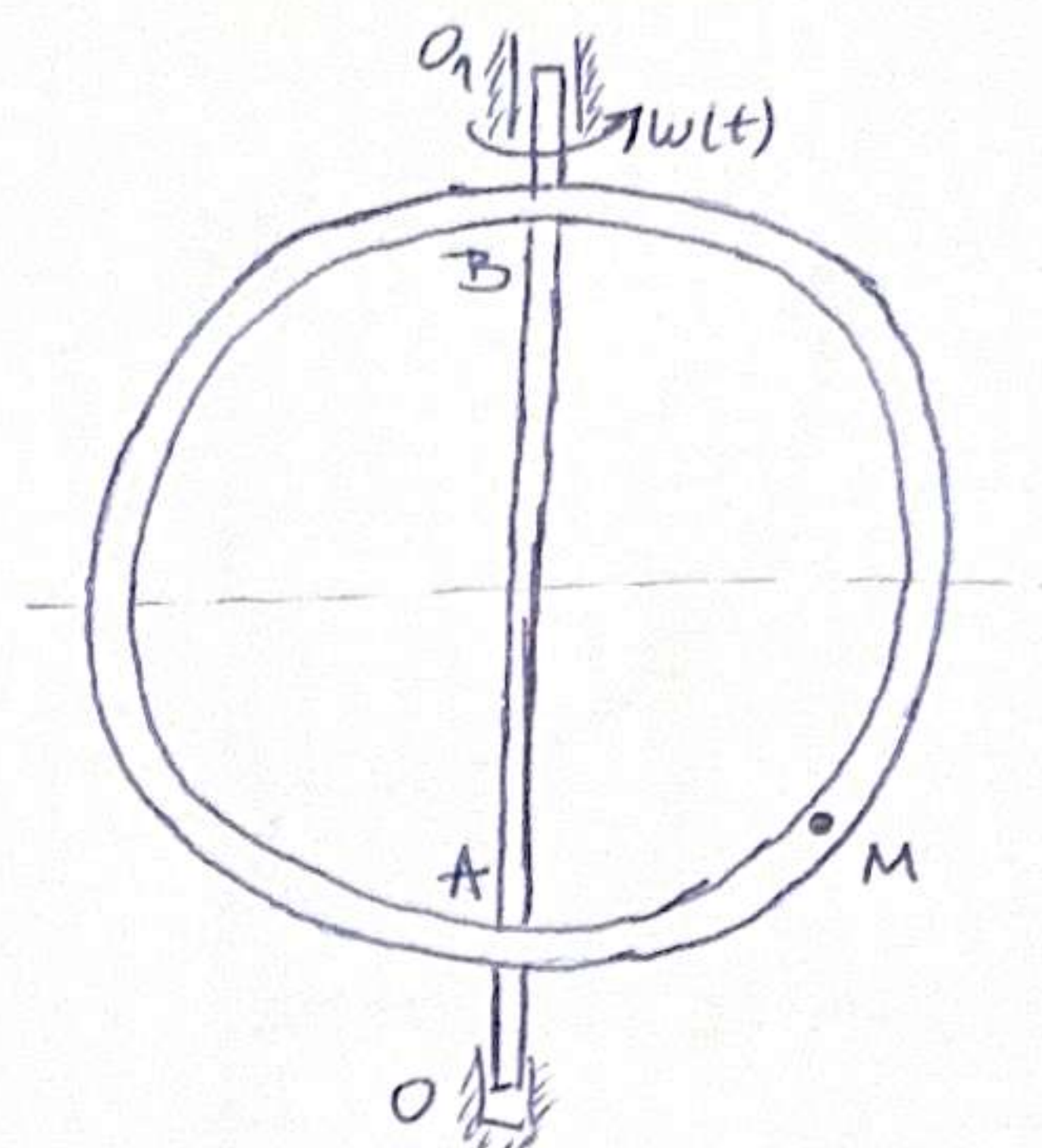
vertikalna ravnina

kuglica: m

to: kuglica v A , v_0

$\omega(t) = ?$

$a_{\text{rt}} = a = \text{const.}$ - relativno tang. ubrzanje kuglice



$$\vec{a}_r = \vec{a}_g + \vec{F}_p^{\text{in}} + \vec{F}_{\text{cor}} + \vec{N}_n + \vec{N}_c$$

$$\vec{a}_g = g \cdot (-\vec{j})$$

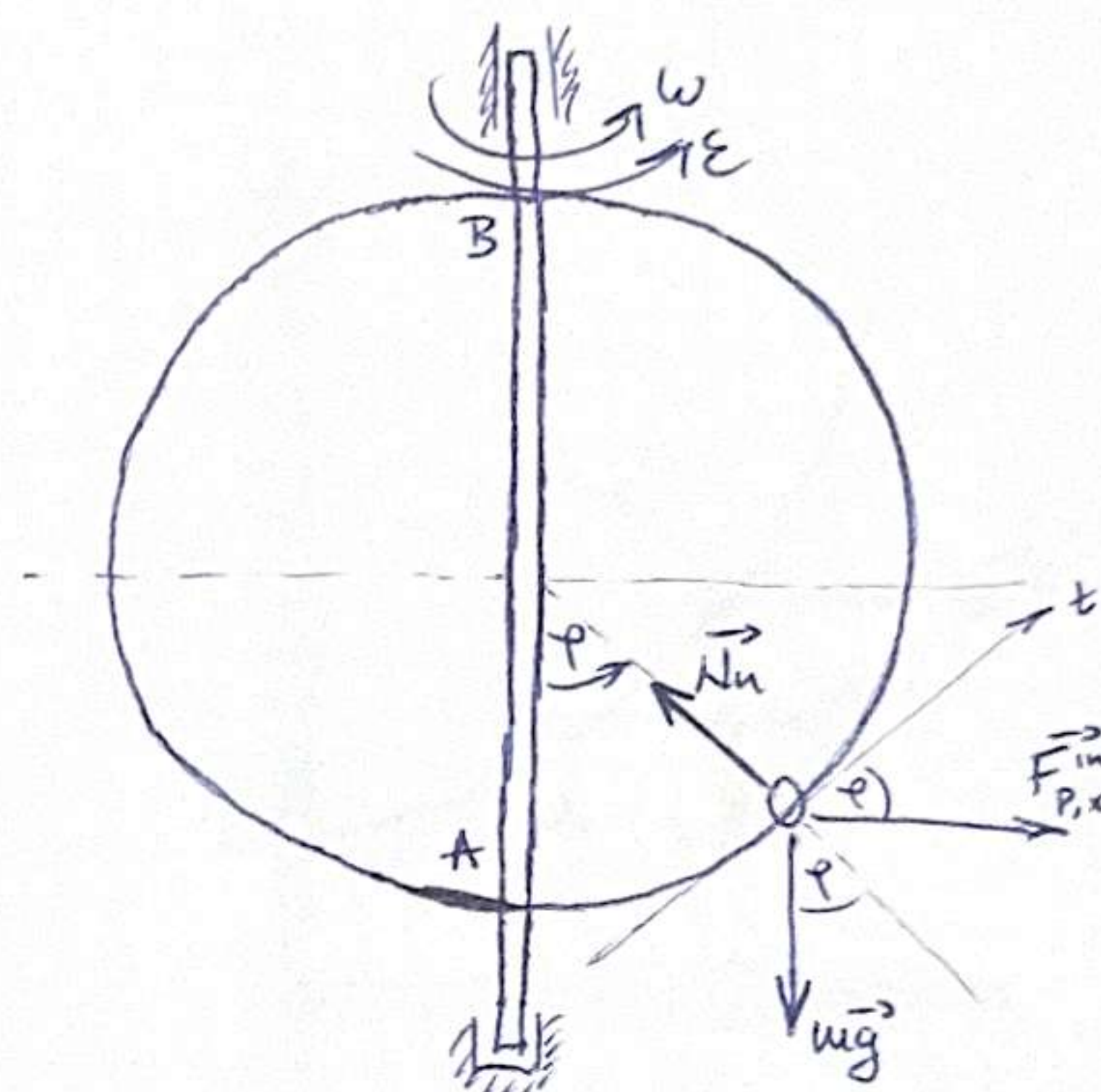
$$\vec{F}_p^{\text{in}} = -m(\vec{a}_{\text{rel}} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m \cdot \varepsilon \cdot x \cdot (\vec{e}_1) + m \cdot \omega^2 \cdot x \cdot (\vec{e}_2) = \vec{F}_{p,z}^{\text{in}} + \vec{F}_{p,x}^{\text{in}}$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{e}_1)$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_c = N_c \cdot (-\vec{e}_2)$$



$$t: m a_{\text{rt}} = -mg \sin \varphi + m \omega^2 x \cos \varphi, \quad \varphi = ?$$

$$\varepsilon = \frac{a_{\text{rt}}}{r} = \frac{a}{r} \Rightarrow \omega = \frac{a}{r} t + \omega_0 = \frac{a}{r} t + \frac{v_0}{r} \Rightarrow \varphi = \frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t$$

$$a = -g \sin \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right) + \omega^2 \cdot r \cdot \sin \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right) \cos \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right)$$

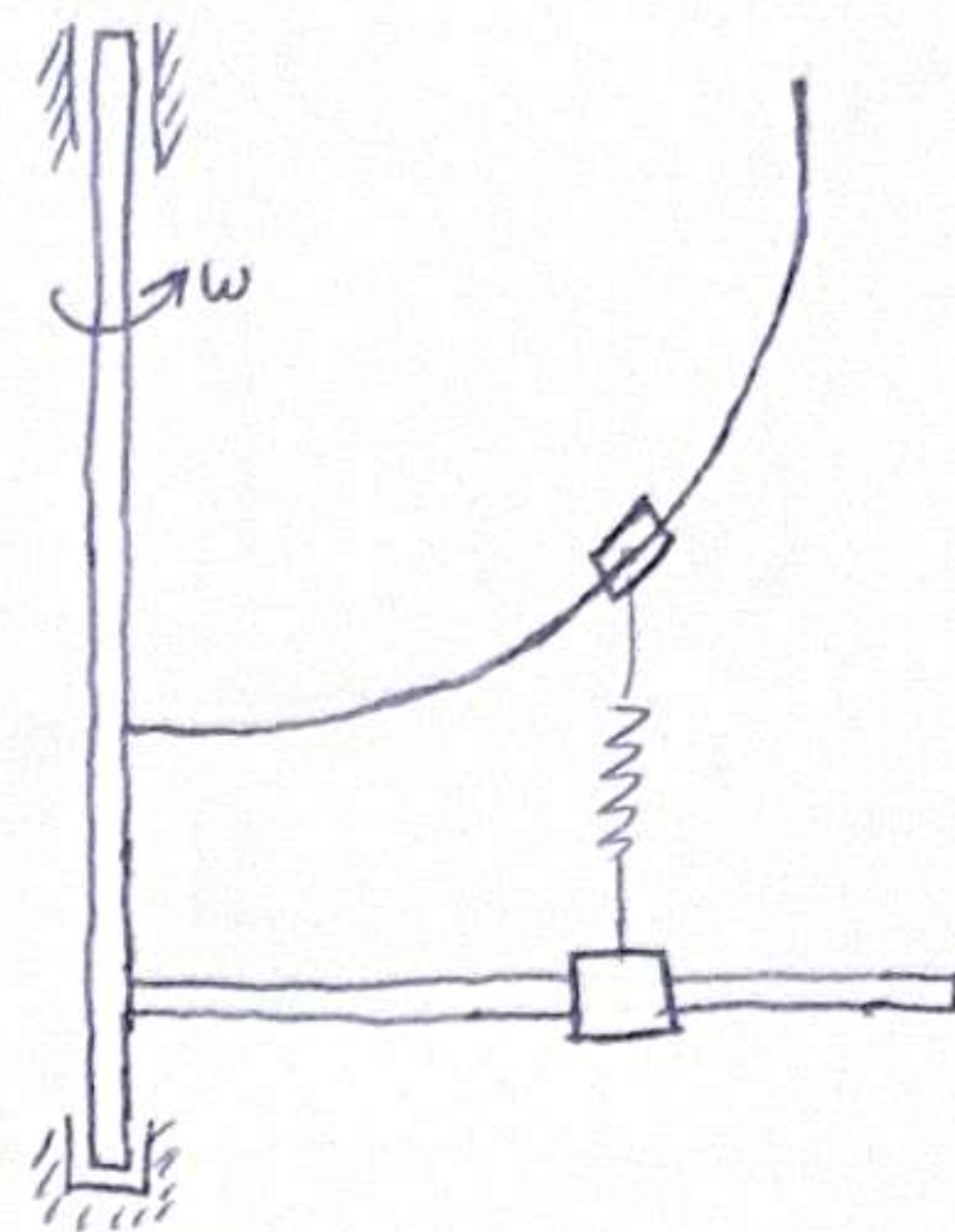
$$\omega^2 = \frac{a + g \sin \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right)}{r \sin \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right) \cos \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right)}$$

$$\boxed{\omega^2 = \frac{a + g \sin \left(\frac{1}{2} \frac{a}{r} t^2 + \frac{v_0}{r} t \right)}{\frac{1}{2} r \sin \left(\frac{a}{r} t^2 + \frac{2v_0}{r} t \right)}}$$

6.27.

klizac: m opruga: κ

vertikalna ravan

 $\omega = \text{const.}$ to: klizac u koord. položaj, φ_0
opruga neupregnuta $y(x) = ?$ - tako da $\varphi = \text{const.}$ 

$$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^m + \vec{F}_{\text{cor}}^m + \vec{H}_n + \vec{H}_g$$

$$m\vec{g} = mg \cdot (-\vec{j}')$$

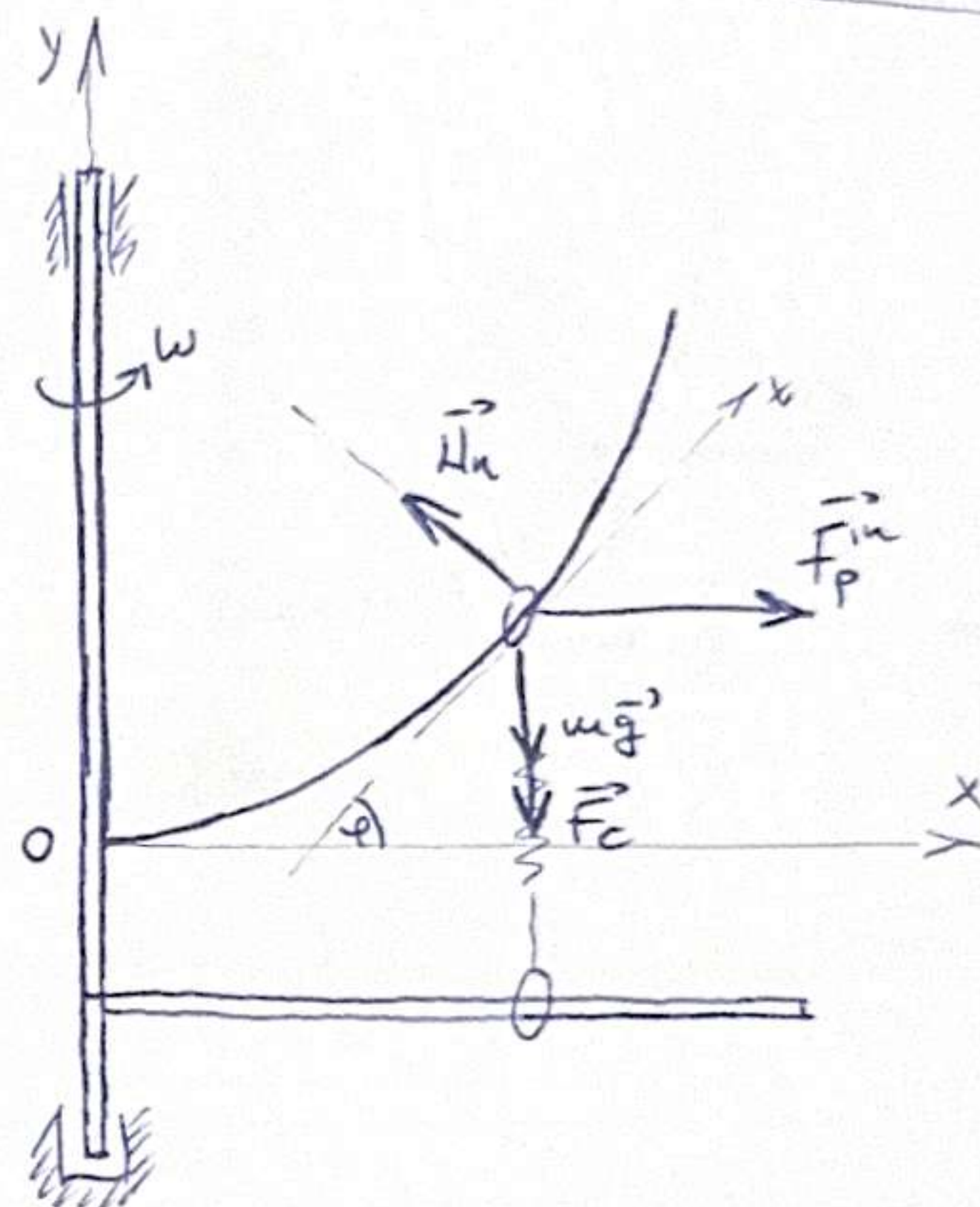
$$\vec{F}_c = \kappa \cdot y \cdot (-\vec{j}')$$

$$\vec{F}_p^m = -m(\vec{a}_n + \vec{E} \times \vec{j}' + \vec{\omega} \times (\vec{\omega} \times \vec{j}')) = m\omega^2 x \cdot (\vec{z})$$

$$\vec{F}_{\text{cor}}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{H}_n = H_n \cdot (\vec{n})$$

$$\vec{H}_g = H_g \cdot (-\vec{k})$$



$$\pm: 0 = -mg \sin \varphi - F_c \sin \varphi + F_p^m \cos \varphi$$

$$0 = -mg \sin \varphi - \kappa \cdot y \cdot \sin \varphi + m\omega^2 x \cos \varphi, \quad \varphi = ? \quad \rightarrow \quad \tan \varphi = y' = \frac{dy}{dx}$$

$$0 = -mg \tan \varphi - \kappa y \tan \varphi + m\omega^2 x$$

$$0 = -(mg + \kappa y) \cdot y' + m\omega^2 x$$

$$(mg + \kappa y) \frac{dy}{dx} = m\omega^2 x$$

$$mg dy + \kappa y dy = m\omega^2 x dx \quad | \int dt$$

$$mg \cdot y + \frac{1}{2} \kappa y^2 = \frac{1}{2} m\omega^2 x^2 \quad | \cdot 2$$

$$\boxed{\kappa y^2 + 2mgy - m\omega^2 x^2 = 0}$$

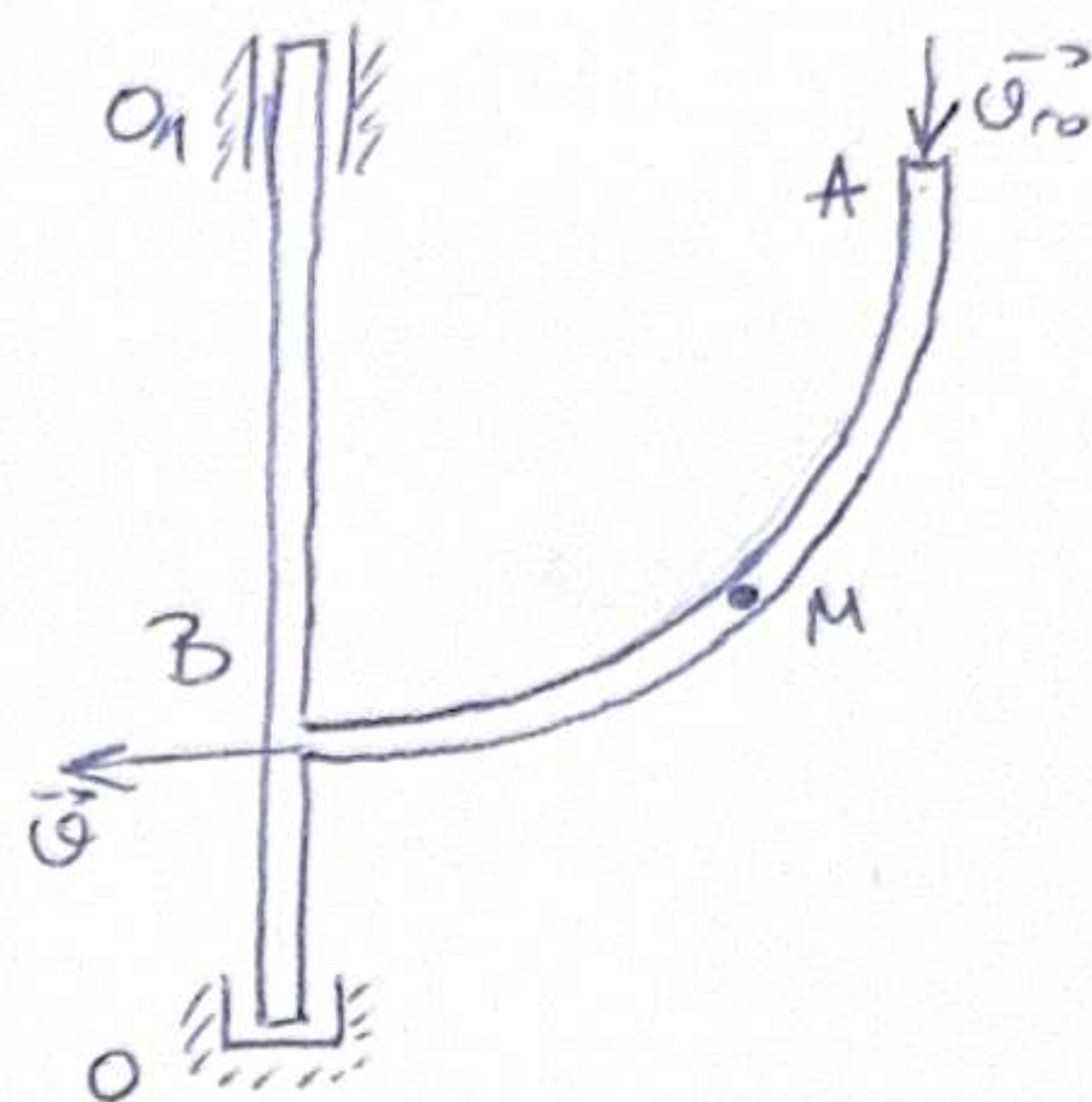
6.28) knjazi luk: R

vertikalna ravan

$w = \text{const.}$

$v_0 = ?$ - da li na otvoru B
brzina bita je 0?

težina: m



$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{cor} + \vec{L}_n + \vec{L}_G$$

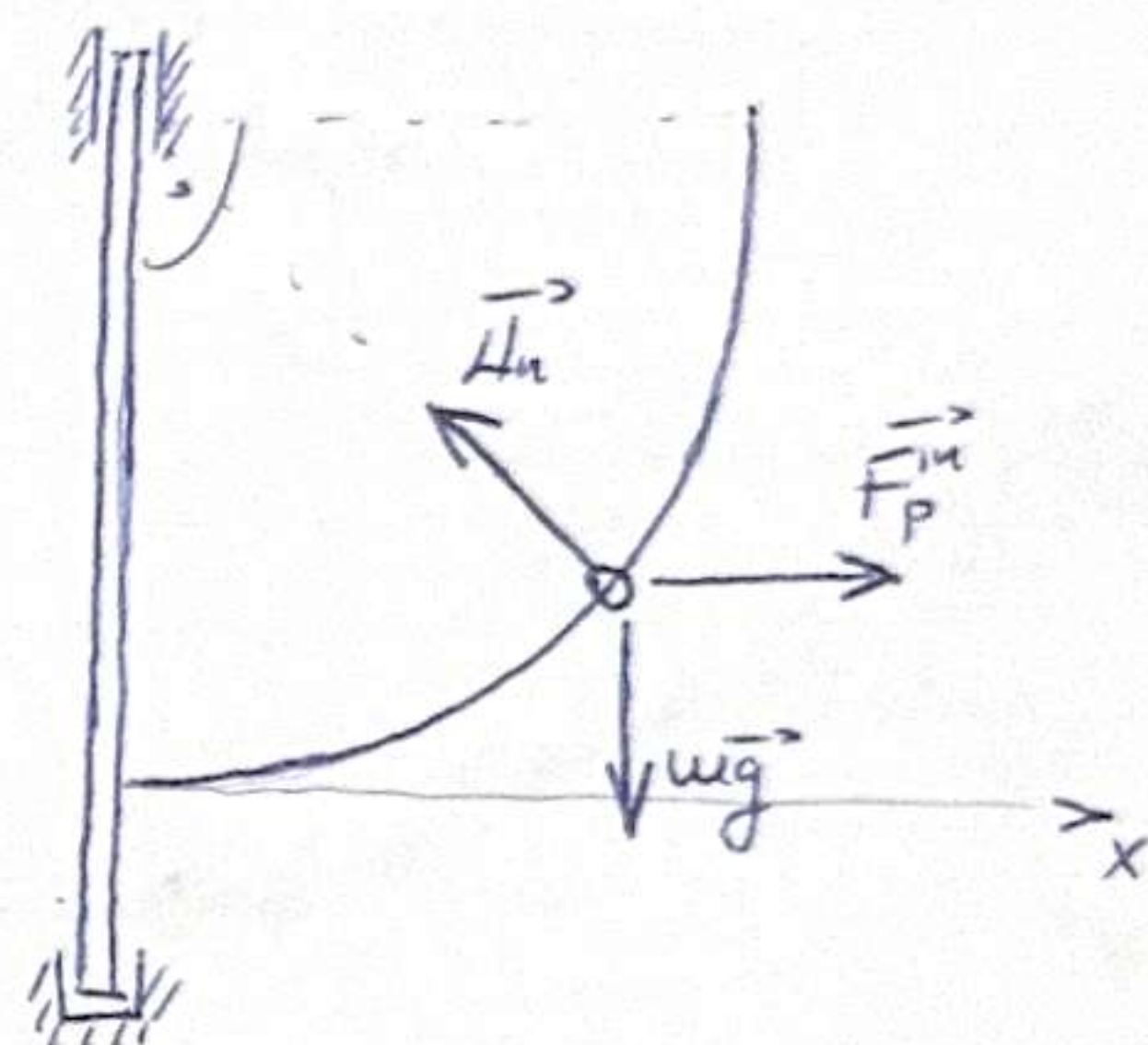
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p = -m(\vec{\omega} \times \vec{r} + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{L}_n = L_n \cdot (\vec{n})$$

$$\vec{L}_G = L_G \cdot (-\vec{k})$$



$$T_B - T_A = A_{AB}$$

$$A_{AB} = \int_R^0 -mg dy + \int_R^0 F_p dx = mgR + \int_0^R -m\omega^2 x dx = mgR - \frac{1}{2}m\omega^2 R^2$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgR - \frac{1}{2}m\omega^2 R^2$$

$$v_0^2 = v^2 - 2gR + \omega^2 R^2$$

6.29. $\omega = \text{const}$, $\overline{OA} = a$

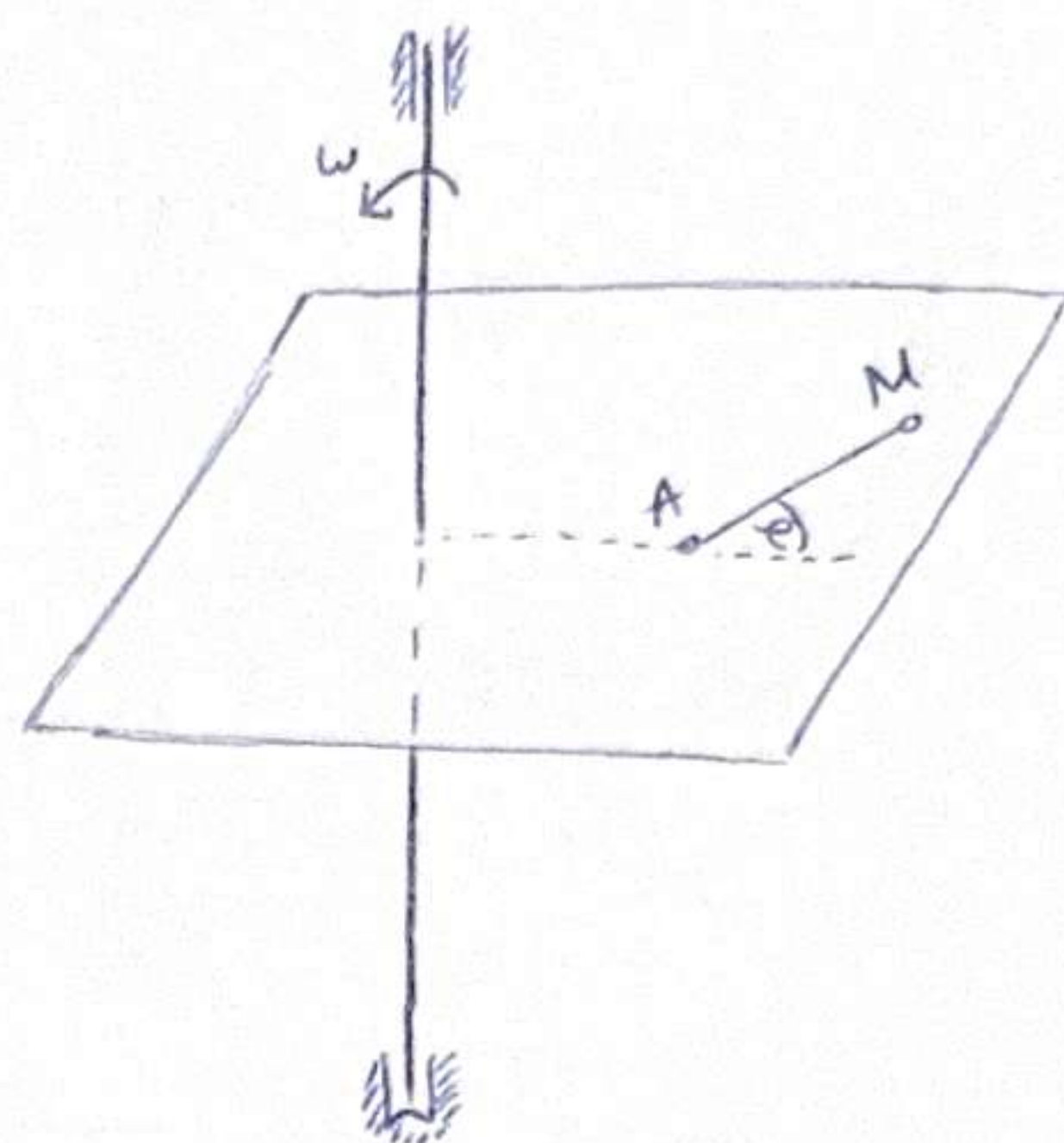
AB: ϕ, b

to: $\phi_0 = 0$, $\dot{\phi}_0 = \omega \sqrt{\frac{a}{b}}$

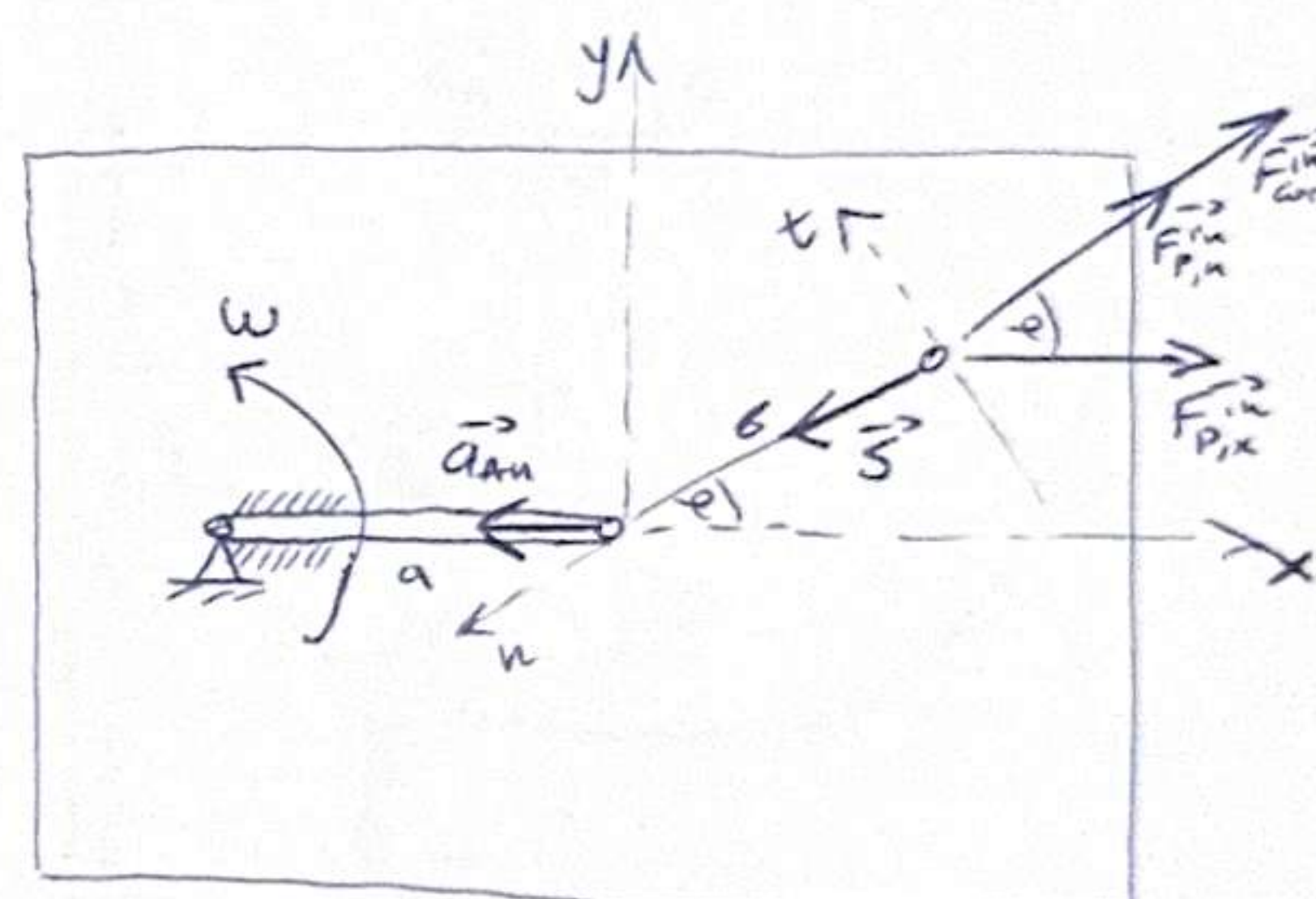
M: m

$\mathcal{U}_r(\phi) = ?$

$S_{\max} = ?$



početak odtočka 2



$a_{An} = \omega^2 \cdot a$

$$m\vec{a}_r = m\vec{g} + \vec{S} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}$$

$m\vec{g} = mg \cdot (-\vec{k}')$

$\vec{S} = S \cdot (\vec{n}')$

$$\begin{aligned} \vec{F}_p^{in} &= -m(\vec{a}_A + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = \\ &= m \cdot a_{An} \cdot (\vec{i}') + m\omega^2 \cdot b \cdot (-\vec{n}') = \vec{F}_{p,x}^{in} + \vec{F}_{p,n}^{in} \end{aligned}$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m \cdot \omega \cdot \mathcal{U}_r \cdot (-\vec{n}')$$

$\vec{N} = N \cdot (\vec{k}')$

$$\underbrace{T - T_0 = A}_{\text{Graf}} \quad (\text{rel})$$

$$A = \int_b^{6\omega\sqrt{a}} \vec{F}_{p,x}^{in} \cdot d\vec{x} = \int_b^{6\omega\sqrt{a}} m \cdot \omega^2 a dx = m\omega^2 a (6\omega\sqrt{a} - b) = -m\omega^2 a \cdot b (1 - \cos\phi)$$

$$\frac{1}{2} m \cdot \mathcal{U}_r^2 - \frac{1}{2} m \cdot (b \cdot \dot{\phi}_0)^2 = -m\omega^2 a \cdot b \cdot (1 - \cos\phi) \quad | \cdot \frac{2}{m}$$

$$\mathcal{U}_r^2 - b^2 \cdot \omega^2 \cdot \frac{a}{b} = -2\omega^2 a b (1 - \cos\phi)$$

$$\mathcal{U}_r^2 = \omega^2 \cdot a b - 2\omega^2 a b + 2\omega^2 a b \cos\phi \Rightarrow \boxed{\mathcal{U}_r = \omega \sqrt{a b (2\cos\phi - 1)}}$$

n: $m \cdot a_{An} = S - m \cdot a_{An} \cos\phi - m\omega^2 b - 2m\omega \cdot \mathcal{U}_r$

$$S = m \cdot \frac{\mathcal{U}_r^2}{R_k} + m\omega^2 a \cdot \cos\phi + m\omega^2 b + 2m\omega \mathcal{U}_r$$

$$S = m \cdot \frac{\omega^2 \cdot a b (2\cos\phi - 1)}{b} + m\omega^2 a \cos\phi + m\omega^2 b + 2m\omega^2 \sqrt{a b (2\cos\phi - 1)}$$

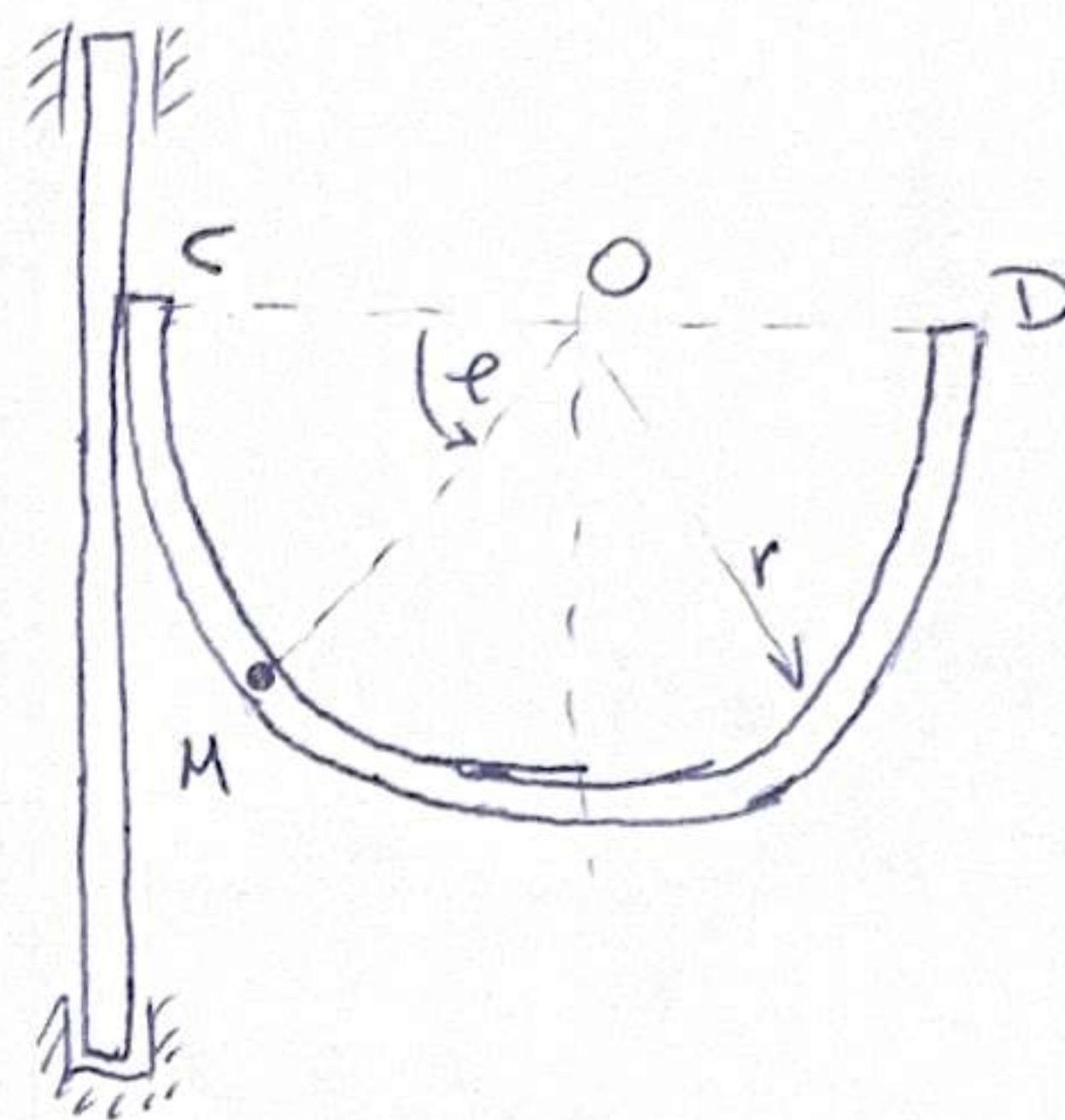
$$S_{\max} = S(\cos\phi = 1) = m\omega^2 a + m\omega^2 a + m\omega^2 b + 2m\omega^2 \sqrt{a b}$$

$$\boxed{S_{\max} = m\omega^2 (2a + b + 2\sqrt{a b})}$$

6.30

cevi: r

vertikalna ravan

 $\omega = \text{const}$ $t_0 = 0$: M je u C $\mathcal{Q}_0 = 0$ $\mathcal{Q}_r(\varphi) = ?$ $\omega = ?$ - da bi apsolutna brzina tačke M pri nepuštanju cevi bila $\mathcal{Q}_0$ 

$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_n + \vec{L}_G$$

$$\vec{m}\vec{g} = mg \cdot (-\vec{j})$$

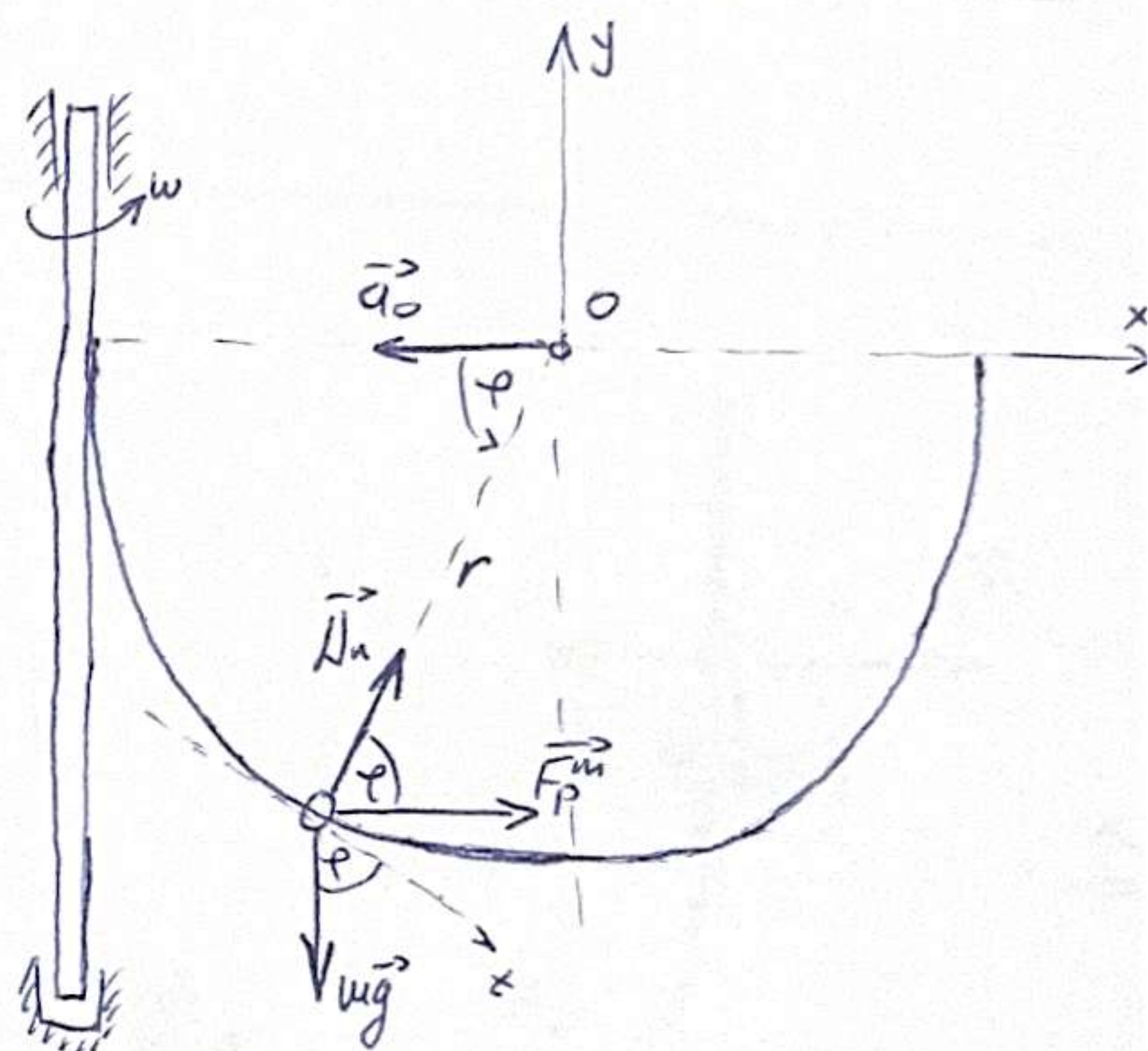
$$\vec{F}_p^m = -m(\vec{a}_0 + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m \cdot a_0 \cdot (\vec{i}) + m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^m = -2m\vec{\omega} \times \vec{Q}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{L}_G = L_G \cdot (-\vec{k})$$

$$T - T_0 = A \quad (\text{rel})$$



$$a_0 = r\omega^2$$

$$A = \int_0^{r \sin \varphi} -mg dy + \int_{-r}^{-r \cos \varphi} F_p^m dx = mg \cdot r \sin \varphi + \int_{-r}^{-r \cos \varphi} (m \cdot r\omega^2 + m\omega^2 x) dx =$$

$$A = mgr \sin \varphi + mr\omega^2(-r \cos \varphi + r) + \frac{1}{2}m\omega^2((-r \cos \varphi)^2 - (-r)^2) =$$

$$A = mgr \sin \varphi + mr^2\omega^2 - mr^2\omega^2 \cos \varphi + \frac{1}{2}m\omega^2 r^2 \cos^2 \varphi - \frac{1}{2}m\omega^2 r^2$$

$$\frac{1}{2}m\mathcal{Q}_r^2 - 0 = mgr \sin \varphi + \frac{1}{2}mr^2\omega^2 - mr^2\omega^2 \cos \varphi + \frac{1}{2}m\omega^2 r^2 \cos^2 \varphi$$

$$\mathcal{Q}_r^2 = 2gr \sin \varphi + r^2\omega^2(1 - 2 \cos \varphi + \cos^2 \varphi)$$

$$\mathcal{Q}_r = \sqrt{2gr \sin \varphi + r^2\omega^2(1 - \cos \varphi)^2}$$

$$\mathcal{Q}_{r,D} = \mathcal{Q}_r(\varphi=180) = \sqrt{2gr \cdot 0 + r^2\omega^2(1 - (-1))^2} = \sqrt{4r^2\omega^2} = 2r\omega$$

$$\mathcal{Q}_{p,D} = \omega \cdot 2r$$

$$\mathcal{Q}_D^2 = \mathcal{Q}_{r,D}^2 + \mathcal{Q}_{p,D}^2 = 4r^2\omega^2 + 4\omega^2 r^2 = 8r^2\omega^2$$

$$\omega = \frac{\mathcal{Q}_D}{r\sqrt{8}} = \frac{\mathcal{Q}_D}{r \cdot 2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\mathcal{Q}_D \sqrt{2}}{4r}$$

$$\omega = \frac{\mathcal{Q}_D \sqrt{2}}{4r}$$

6.31. vertikalna ravan
 $\omega = \sqrt{\frac{g}{R}} = \text{const.}$

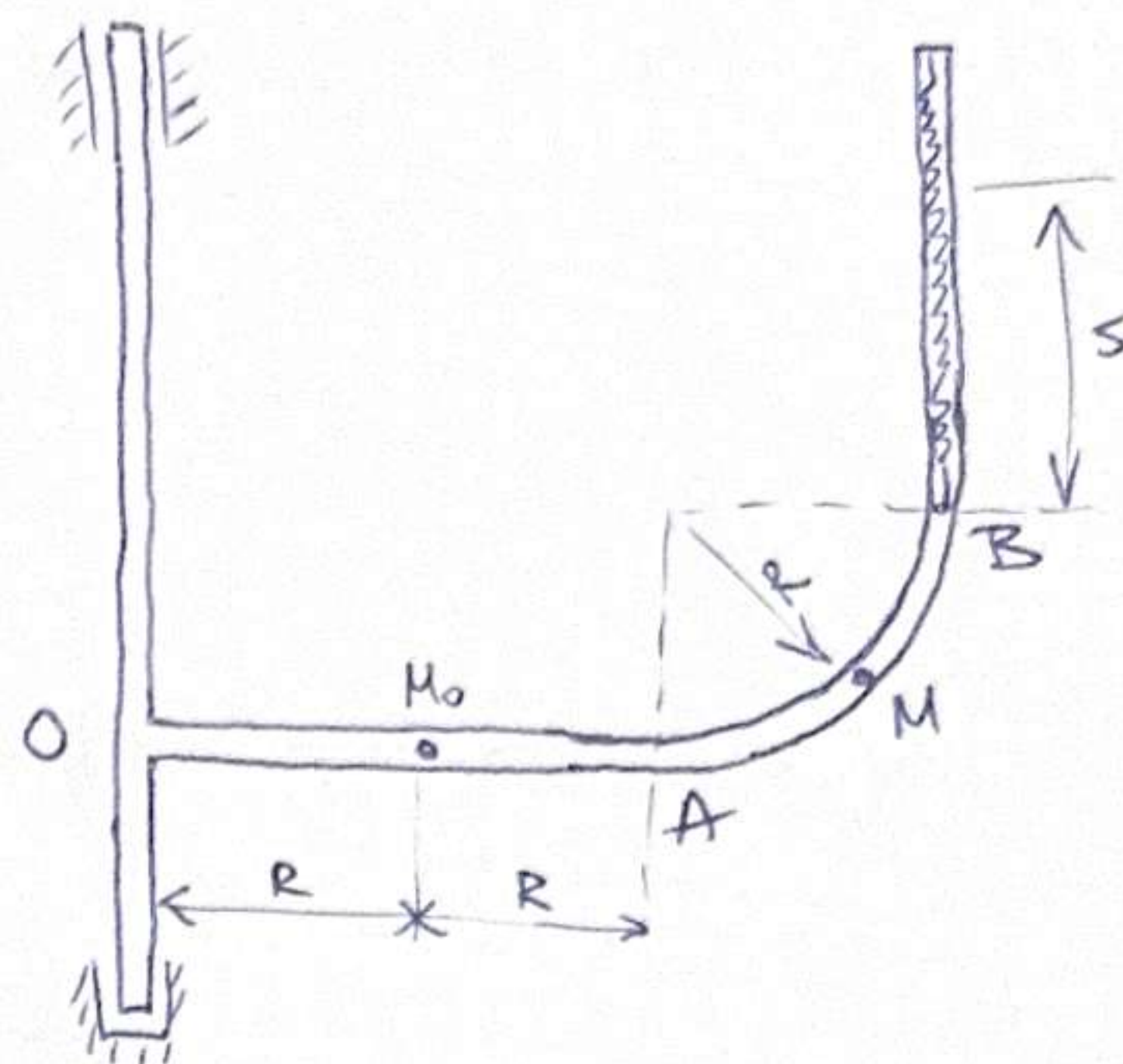
$M: m$

$t_0 = 0: M_1 \text{ je u } M_0$
 relativno mirovanje

$t_1: M_1 \text{ je u } M_1$

1) $N_A = ?$

2) $\tau = ?$ - da li $S_{\text{max}} = R$



deonice $M_0 - A$

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^m + \vec{F}_{\text{cor}}^m + \vec{N}_n + \vec{N}_c$$

$$m\vec{g} = m\vec{g} \cdot (-\vec{j})$$

$$\vec{F}_p^m = -m(\vec{g}_{\text{rel}} + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x \cdot (-\vec{i})$$

$$\vec{F}_{\text{cor}}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (-\vec{k})$$

$$\vec{N}_n = N_n \cdot (-\vec{j})$$

$$\vec{N}_c = N_c \cdot (-\vec{k})$$

polozaj A

$$x = x_A - \varepsilon \Rightarrow 0 = -mg + N_{An} \Rightarrow N_{An} = mg$$

$$x = x_A + \varepsilon \Rightarrow m a_{\text{Ar},n} = -mg + N_{An} \Rightarrow N_{An} = mg + m a_{\text{Ar},n}, \quad a_{\text{Ar},n} = \frac{v_{\text{Ar}}^2}{R_c}, \quad v_{\text{Ar}} = ? \quad R_c = R$$

$$T_A - T_{M_0} = A_{M_0M} \quad (\text{rel})$$

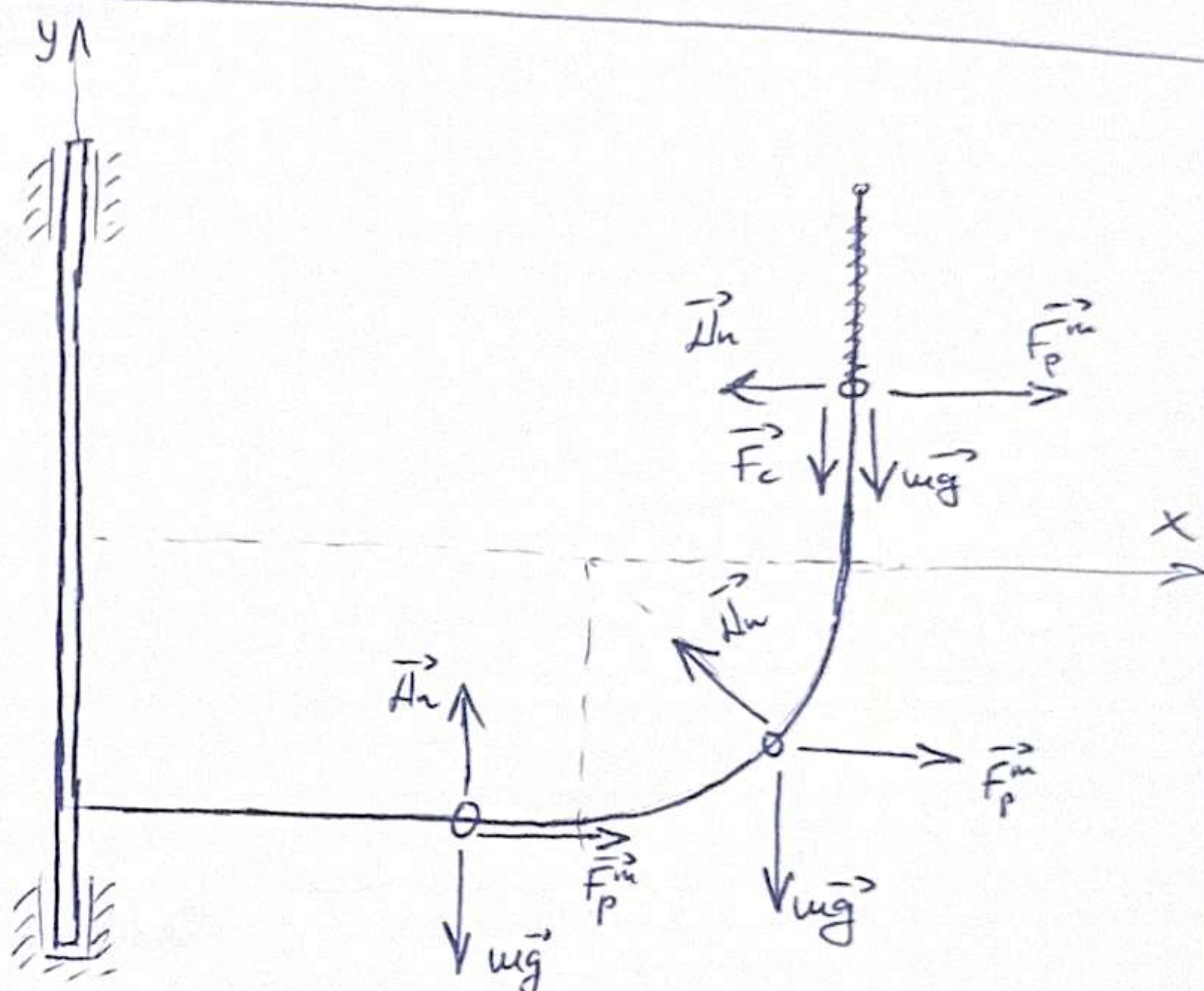
$$A_{M_0M} = \int_R^{2R} F_p^m dx = \int_R^{2R} m\omega^2 x dx = \frac{1}{2} m\omega^2 (4R^2 - R^2) = \frac{3}{2} mR^2\omega^2$$

$$\frac{1}{2} m v_{\text{Ar}}^2 - 0 = \frac{3}{2} m\omega^2 R^2 \Rightarrow v_{\text{Ar}}^2 = 3\omega^2 R^2$$

$$N_{An} = mg + m \frac{3\omega^2 R^2}{R} = mg + m \cdot 3 \frac{g}{R} R = mg + 3mg = 4mg \Rightarrow N_{An} = 4mg$$

$$Z: 0 = 2m\omega \dot{x} - N_{Ac} \Rightarrow N_{Ac} = 2m\omega \cdot v_r = 2m\omega \cdot \sqrt{3}\omega R = 2m \frac{g}{R} \sqrt{3} R = 2\sqrt{3} mg$$

$$N_A = \sqrt{N_{An}^2 + N_{Ac}^2} = \begin{cases} mg\sqrt{1+12} = mg\sqrt{13}, & x = x_A - \varepsilon \\ mg\sqrt{16+12} = mg \cdot 2\sqrt{5}, & x = x_A + \varepsilon \end{cases}$$



deonica A-B: ista kao Mo-A, samo što je $\vec{A}_n = A_n \cdot (\vec{n})$
(izračun za ostale sile ružič)

$$\underline{T_B - T_{Mo} = A_{MoB}} \quad (\text{rel})$$

$$A_{MoB} = \int_R^{3R} F_p^m dx + \int_0^R -mg dy = \int_R^{3R} m\omega^2 x dx - mgR = \frac{1}{2} m\omega^2 (9R^2 - R^2) - mgR =$$

$$A_{MoB} = 4m\omega^2 R^2 - mgR = 4m \frac{g}{R} R^2 - mgR = 3mgR$$

$$\frac{1}{2} m v_B^2 - 0 = 3mgR \quad \Rightarrow \quad \underline{v_B^2 = 6gR}$$

deonica B-Mn: sve sile su iste osim:

$$\vec{A}_n = +A_n \cdot (-\vec{i})$$

$$\vec{F}_c = c \cdot y \cdot (-\vec{j})$$

$$\underline{T_{Mn} - T_B = A_{BMn}} \quad (\text{rel})$$

$$A_{BMn} = \int_0^R -mg dy + \int_0^R -c y dy = -mgR - \frac{1}{2} cR^2$$

$$0 - \frac{1}{2} m v_B^2 = -mgR - \frac{1}{2} cR^2$$

$$-\frac{1}{2} m \cdot 6gR = -mgR - \frac{1}{2} cR^2$$

$$\frac{1}{2} cR^2 = 2mgR$$

$$\boxed{c = \frac{4mg}{R}}$$

6.33. vertikalna ravan

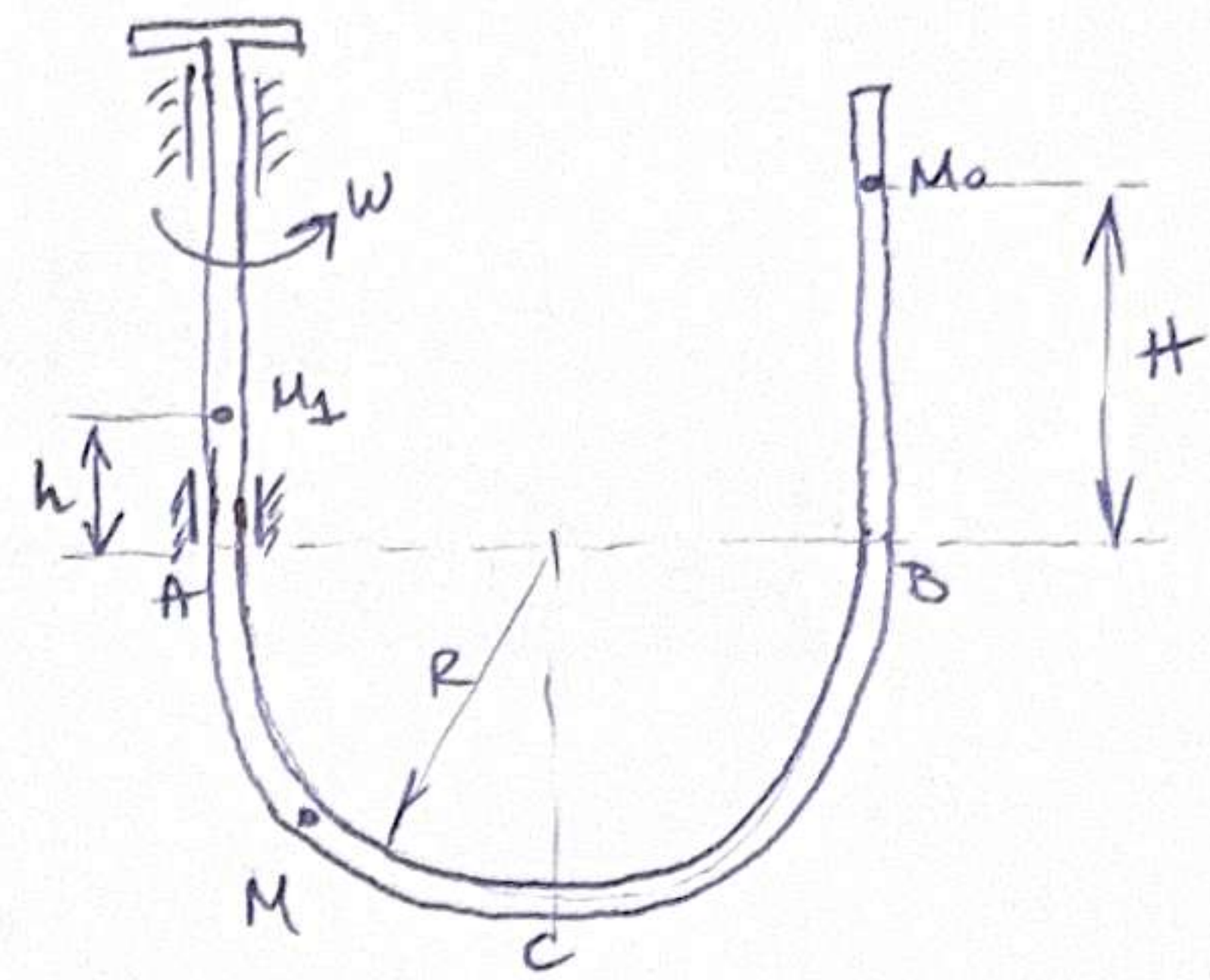
$$\omega = \sqrt{\frac{g}{R}}$$

tačka mase m

$$\text{do: } H = 3R, \quad \omega_0 = 0$$

1) $h = ?$ - max. položaj

2) $\omega_c = ?$



deonika $M_0 - B$:

$$m\vec{a}_T = m\vec{g} + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{A}_n + \vec{A}_b$$

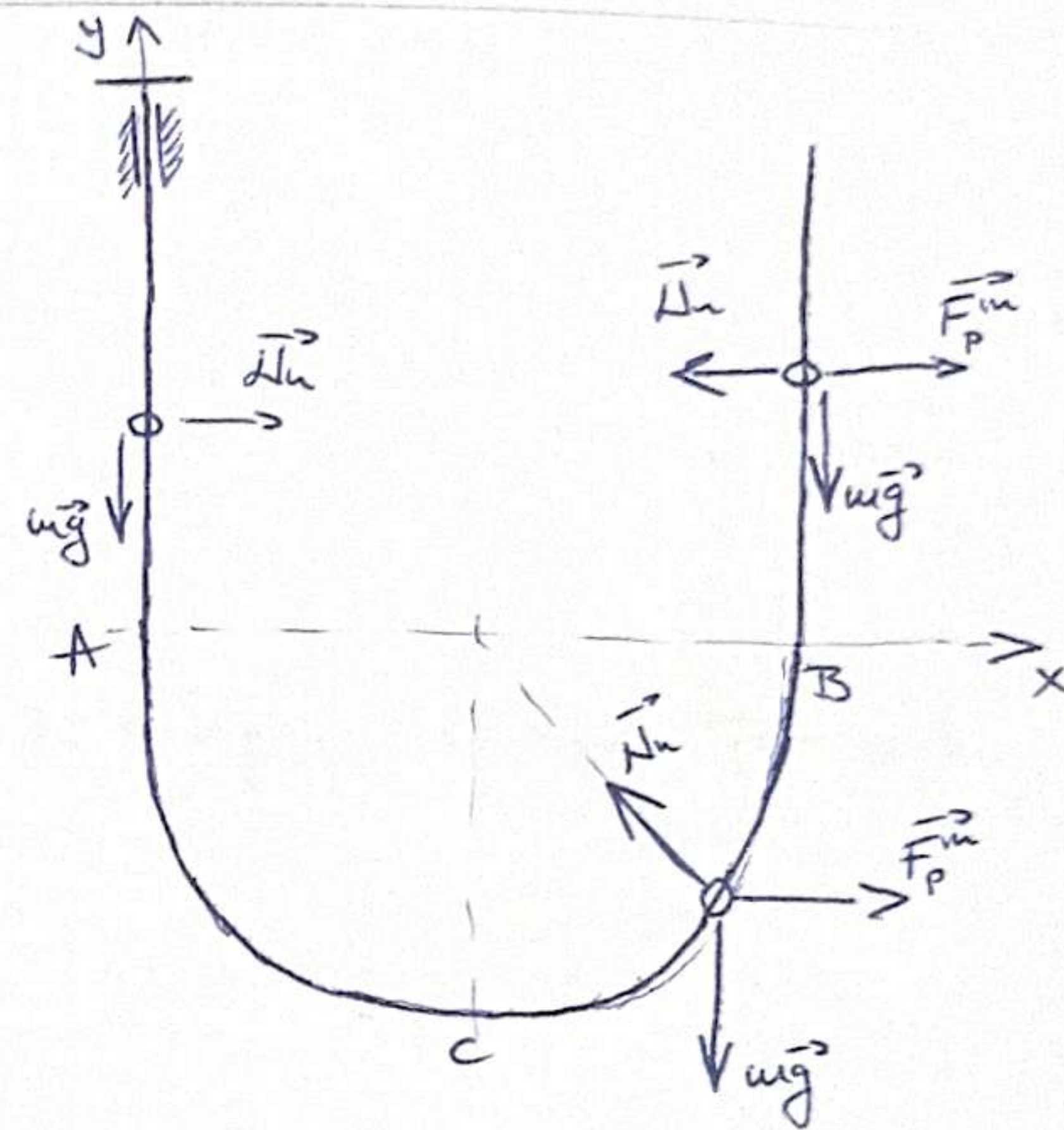
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^m = -m(\vec{\omega} \times \vec{r} + \vec{\omega}' \times \vec{r} + \vec{\omega} \times (\vec{\omega}' \times \vec{r})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^m = -2m\vec{\omega} \times \vec{v}_r = 0$$

$$\vec{A}_n = A_n \cdot \vec{n}$$

$$\vec{A}_b = A_b \cdot \vec{k}$$



deonika $B - A$: sve sile su iste, osim:

$$\vec{F}_{cor}^m = +2m\omega \cdot \dot{x} \cdot (\vec{k})$$

položaj C:

$$n: m \cdot a_{n,c} = -mg + A_{n,c}$$

$$A_{n,c} = m(g + a_{n,c}), \quad a_{n,c} = \frac{v_{c,c}^2}{R_c}, \quad \omega_c = ?, \quad R_c = R$$

$$T_c - T_{M_0} = A_{M_0c} \quad (\text{rel})$$

$$A_{M_0c} = \int_{-R}^{-R+H} -mg dy + \int_{2R}^R F_p^m dx = -mg(-R+H) + \int_{2R}^R m\omega^2 x dx = mg(R+H) + \frac{1}{2} m\omega^2 (R^2 - 4R^2) =$$

$$A_{M_0c} = 4mgR - \frac{3}{2} m \frac{g}{R} R^2 = \frac{5}{2} mgR$$

$$\frac{1}{2} m \omega_c^2 - 0 = \frac{5}{2} mgR$$

$$\omega_c^2 = 5gR$$

$$A_{n,c} = m(g + \frac{5gR}{R}) = 6mg$$

$$z: 0 = +2m\omega \dot{x}_c + U_{G,c}$$

$$U_{G,c} = -2m\omega \cdot \dot{x}_c = -2m \sqrt{\frac{g}{R}} \cdot (-\mathcal{O}_{r,c}) = 2m \sqrt{\frac{g}{R}} \cdot \sqrt{5gR} = 2\sqrt{5} mg$$

$$U_c = \sqrt{U_{H,c}^2 + U_{G,c}^2} = mg \sqrt{36 + 20} = \sqrt{56} mg \Rightarrow \boxed{U_c = \sqrt{56} mg}$$

deonika A - M₁: ista kao prethodna, samo gde važi:

$$\vec{F}_p^m = 0$$

$$\vec{F}_{\text{or}}^m = 0$$

$$\boxed{T_{M_1} - T_{M_0} = A_{M_0 M_1}}$$

$$A_{M_0 M_1} = \int_H^h -mg dy + \int_{2R}^0 \vec{F}_p^m dx = mg(H - h) + \int_{2R}^0 m\omega^2 x dx = mg(3R - h) + \frac{1}{2}m\omega^2(0 - 4R^2)$$

$$A_{M_0 M_1} = 3mgR - mgh - 2m \frac{g}{R} R^2 = mgR - mgh$$

$$0 - 0 = mgR - mgh \Rightarrow \boxed{h = R}$$

6.34. ABCD: a

$\omega = \text{const.}$

horizontalna ravan

M: m

t_0 : M je u B

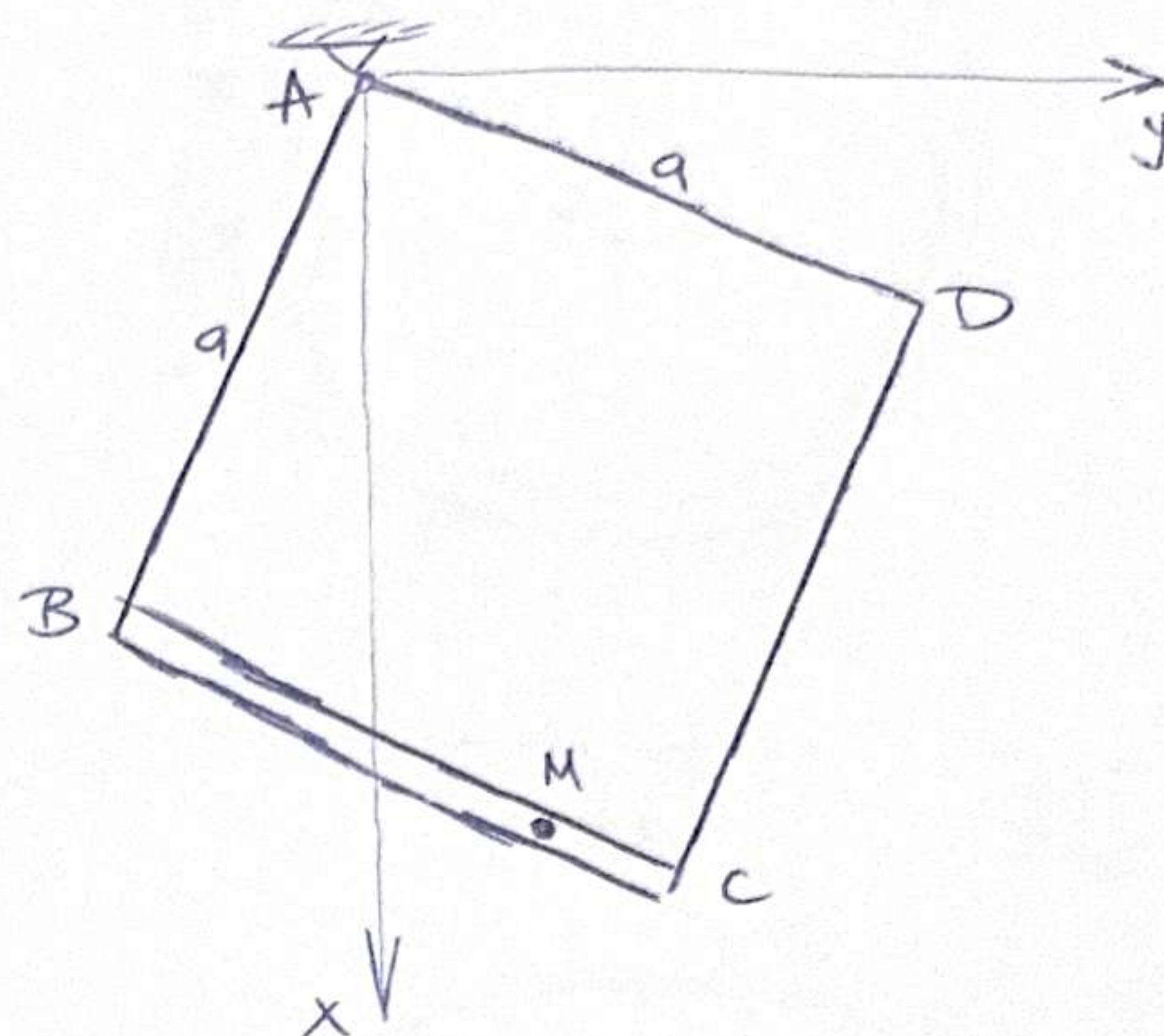
$$v_{r0} = v_0$$

t_1 : M je u C

$$v_{r1} = 2v_0$$

$AB \parallel A_x$ (A_x usmerena od A ka B)

Kada i-ue kreću se tačka M
u odnosu na debljinu ploče u pokretu: A_{xyz} ?



$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_u + \vec{N}_G$$

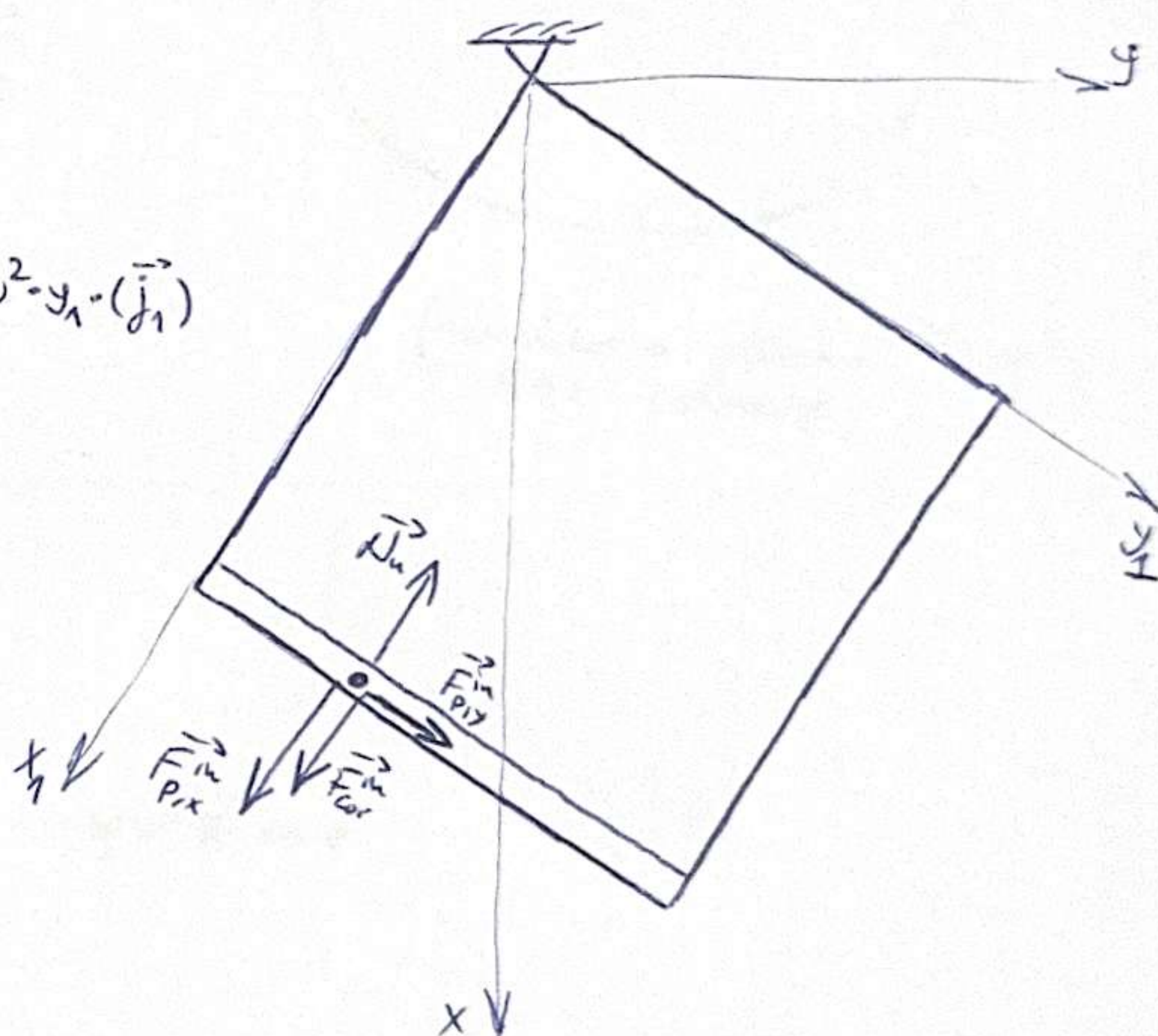
$$\vec{m}\vec{g} = m\vec{g}(-\vec{k}_1)$$

$$\vec{F}_p^m = -m(\vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m\omega^2 x_1 \cdot (\vec{i}_1) + m\omega^2 y_1 \cdot (\vec{j}_1)$$

$$\vec{F}_{cor}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot v_r \cdot (\vec{i}_1)$$

$$\vec{N}_u = +N_u \cdot (-\vec{j}_1)$$

$$\vec{N}_G = N_G \cdot (\vec{k}_1)$$



$$T_C - T_B = A_{BC} \quad (\text{rel})$$

$$A_{BC} = \int_0^a F_{p,y}^m dy_1 = \int_0^a m\omega^2 y_1 dy_1 = \frac{1}{2} m\omega^2 a^2$$

$$\frac{1}{2} m (2v_0)^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} m\omega^2 a^2$$

$$\frac{3}{2} m v_0^2 = \frac{1}{2} m\omega^2 a^2 \Rightarrow \underline{\underline{\omega = \frac{v_0}{a} \sqrt{3}}}$$

reschetamo vreme u trenutak kada tačka M napusti ploču:

$$t_0 = 0: \quad x_0 = a, \quad y_0 = a, \quad \dot{y}_0 = \omega a + 2v_0 = \frac{v_0}{a} \sqrt{3} a + 2v_0, \quad \dot{x}_0 = -\omega a = -v_0 \sqrt{3}$$

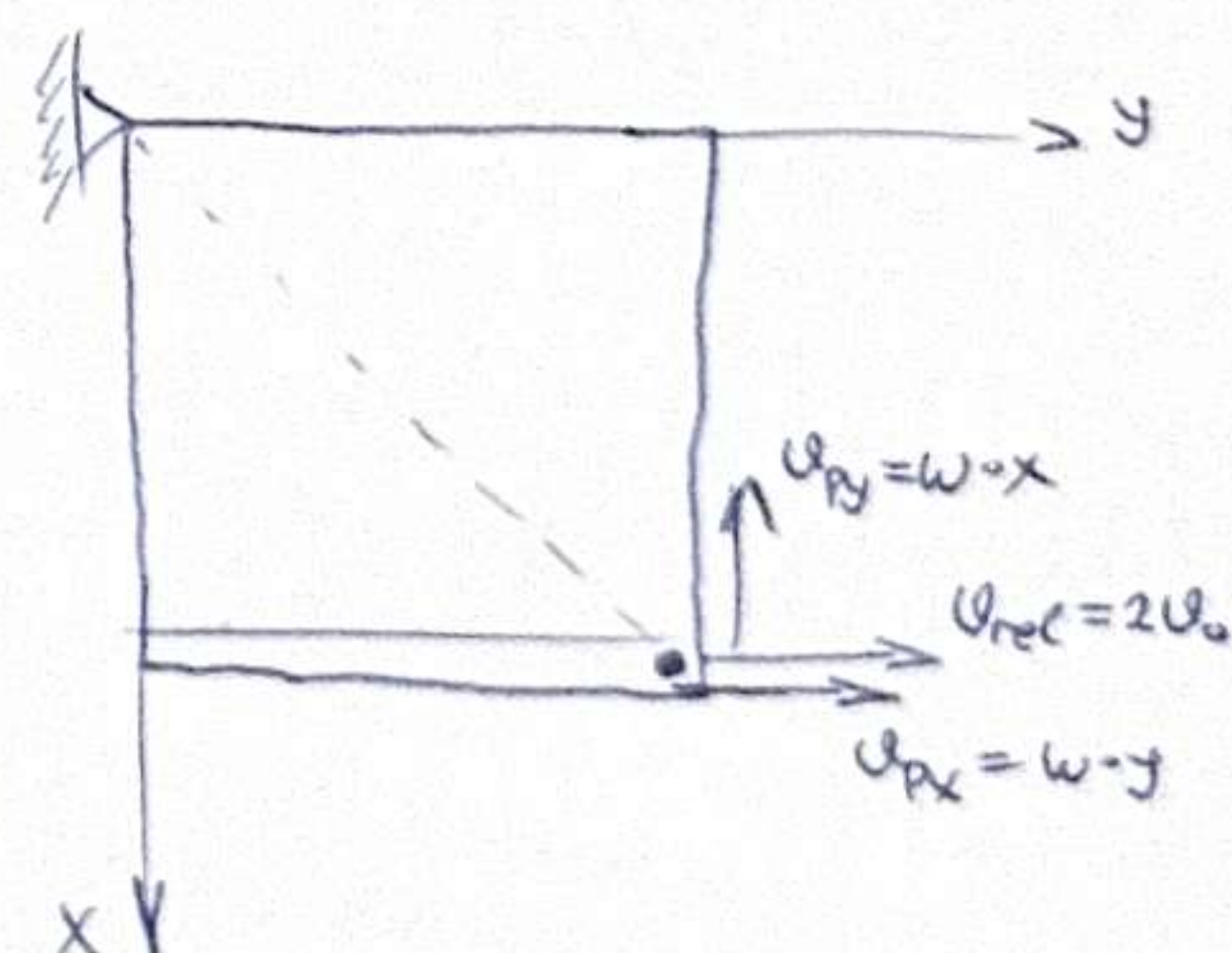
$$\vec{m}\vec{a} = \vec{m}\vec{g}$$

$$y: \quad m\ddot{y} = 0 \rightarrow \dot{y} = \text{const} = v_0(2 + \sqrt{3}) \rightarrow$$

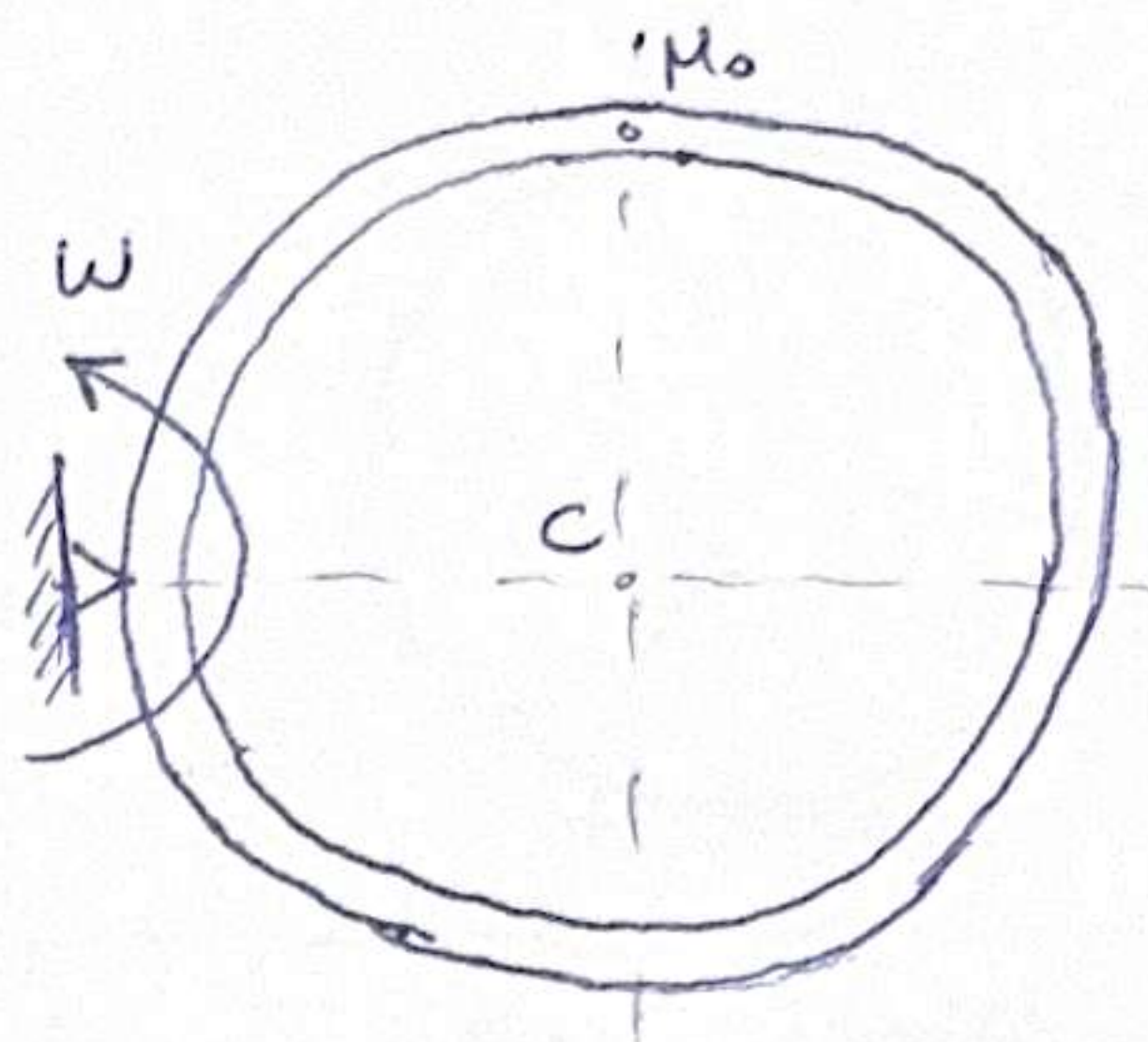
$$x: \quad m\ddot{x} = 0 \rightarrow \dot{x} = \text{const} = -v_0 \sqrt{3} \rightarrow$$

$$z: \quad m\ddot{z} = -mg \rightarrow \dot{z} = -gt \rightarrow$$

$$\begin{aligned} y &= v_0(2 + \sqrt{3})t + a \\ x &= -v_0 \sqrt{3}t + a \\ z &= -\frac{1}{2}gt^2 \end{aligned}$$



- 6.35. horizontalna ravnina
 $\omega = \text{const.}$
 kuglica: m
 to: M i M_0 , uistye
 $\mathcal{O}_r(\varphi) = ?$, $\mathcal{L}(\varphi) = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{cor} + \vec{N}_c + \vec{N}_G$$

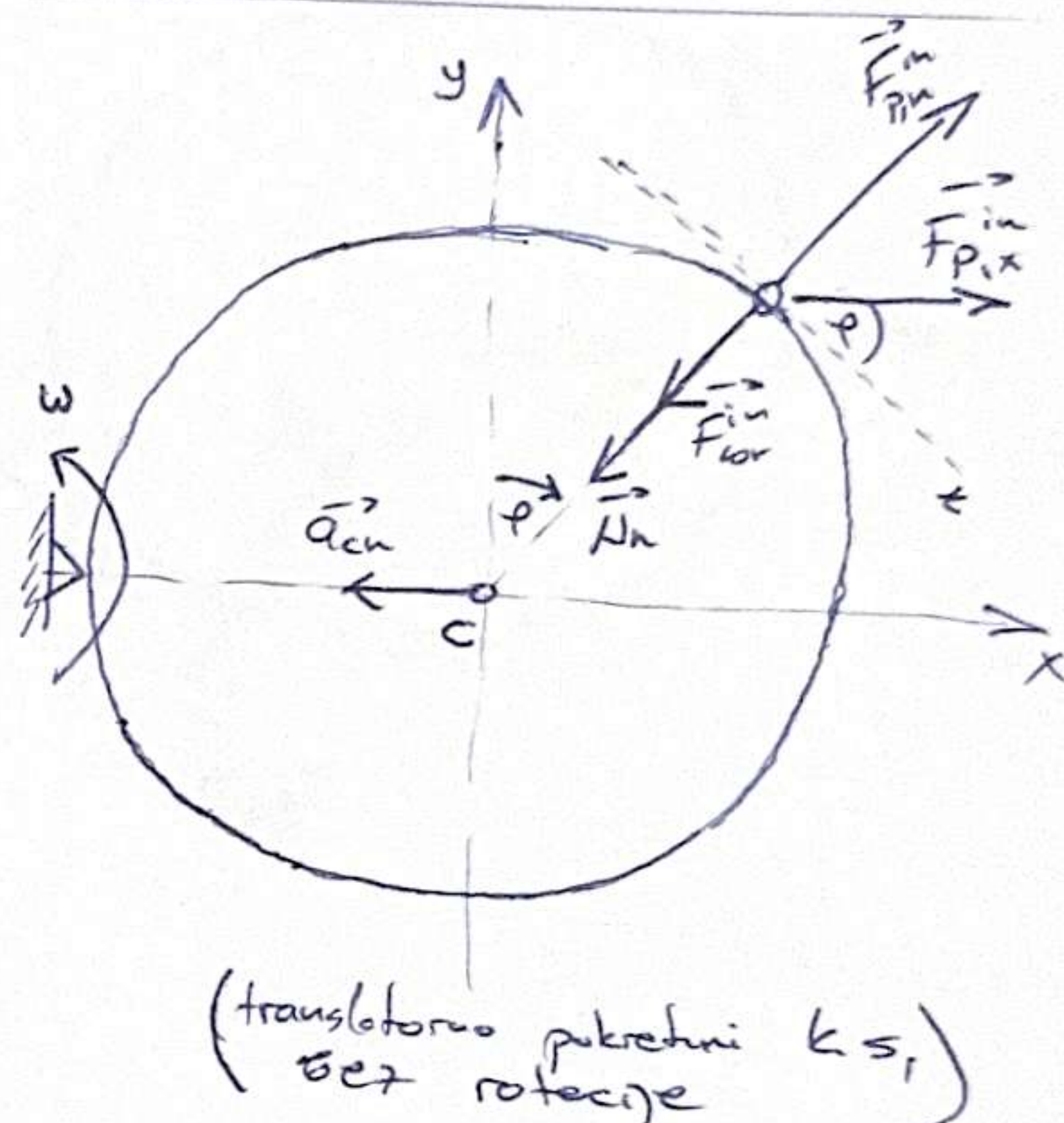
$$m\vec{g} = +mg \cdot (-\vec{k})$$

$$\begin{aligned} \vec{F}_p &= -m(\vec{a}_c + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = \\ &= m a_{cn}(\vec{i}') + m\omega^2 R \cdot (-\vec{n}') = \\ &= m\omega^2 R \cdot (\vec{i}') + m\omega^2 R \cdot (-\vec{n}') = \vec{F}_{p,x} + \vec{F}_{p,n} \end{aligned}$$

$$\vec{F}_{cor} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \mathcal{O}_r \cdot (\vec{n}')$$

$$\vec{N}_n = N_n \cdot (\vec{n}')$$

$$\vec{N}_G = N_G \cdot (\vec{k})$$



$$T_M - T_{M0} = A_{MOM} \quad (\text{rel})$$

$$A_{MOM} = \int_{M_0}^M \vec{F}_{p,x} d\vec{s} = \int_0^{R \sin \varphi} \vec{F}_{p,x} dx = \int_0^{R \sin \varphi} m\omega^2 R dx = m\omega^2 R^2 \sin \varphi$$

$$\frac{1}{2} m \mathcal{O}_r^2 - 0 = m\omega^2 R^2 \sin \varphi$$

$$\boxed{\mathcal{O}_r = \omega R \sqrt{2 \sin \varphi}}$$

$$n: m \cdot a_{r,n} = -F_{p,x} \sin \varphi - F_{p,n} + F_{cor} + N_n$$

$$N_n = m a_{r,n} + F_{p,x} \sin \varphi + F_{p,n} - F_{cor}$$

$$N_n = m \frac{\mathcal{O}_{rel}^2}{R} + m\omega^2 R \sin \varphi + m\omega^2 R - 2m\omega \mathcal{O}_{rel}$$

$$N_n = m \frac{\omega^2 R^2 \cdot 2 \sin \varphi}{R} + m\omega^2 R \sin \varphi + m\omega^2 R - 2m\omega^2 R \sqrt{2 \sin \varphi}$$

$$\underline{N_n = m\omega^2 R (1 + 3 \sin \varphi - 2 \sqrt{2 \sin \varphi})}$$

$$z: 0 = -mg + N_G \Rightarrow \underline{N_G = mg}$$

$$\boxed{N = \sqrt{N_n^2 + N_G^2}}$$

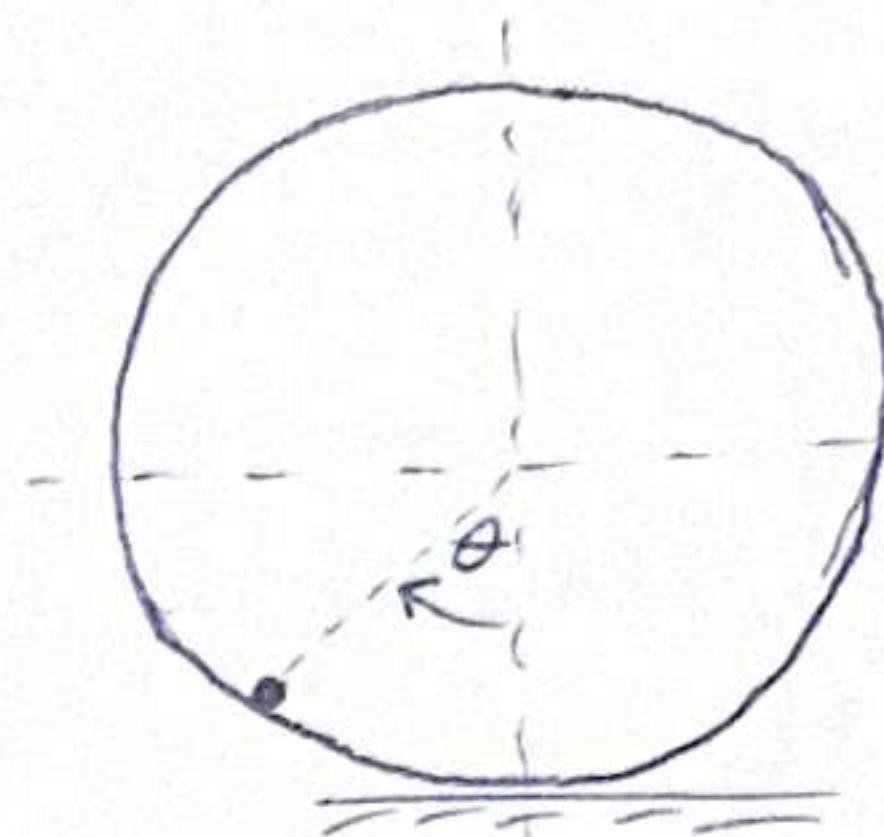
6.35.

μ - koef. trenja između kamara i cilindra

$\Phi = ?$ - konstantan ugo otklona usled trejja

1) $\varphi_c = \text{const.} = \varphi$

$$2) \quad q_c = \text{const.} = 2g$$



$$m \vec{a}_r = m \vec{g} + \vec{F}_{tr} + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_u + \vec{N}_G^{10}$$

$$m\vec{g} = mg - (-\vec{j})$$

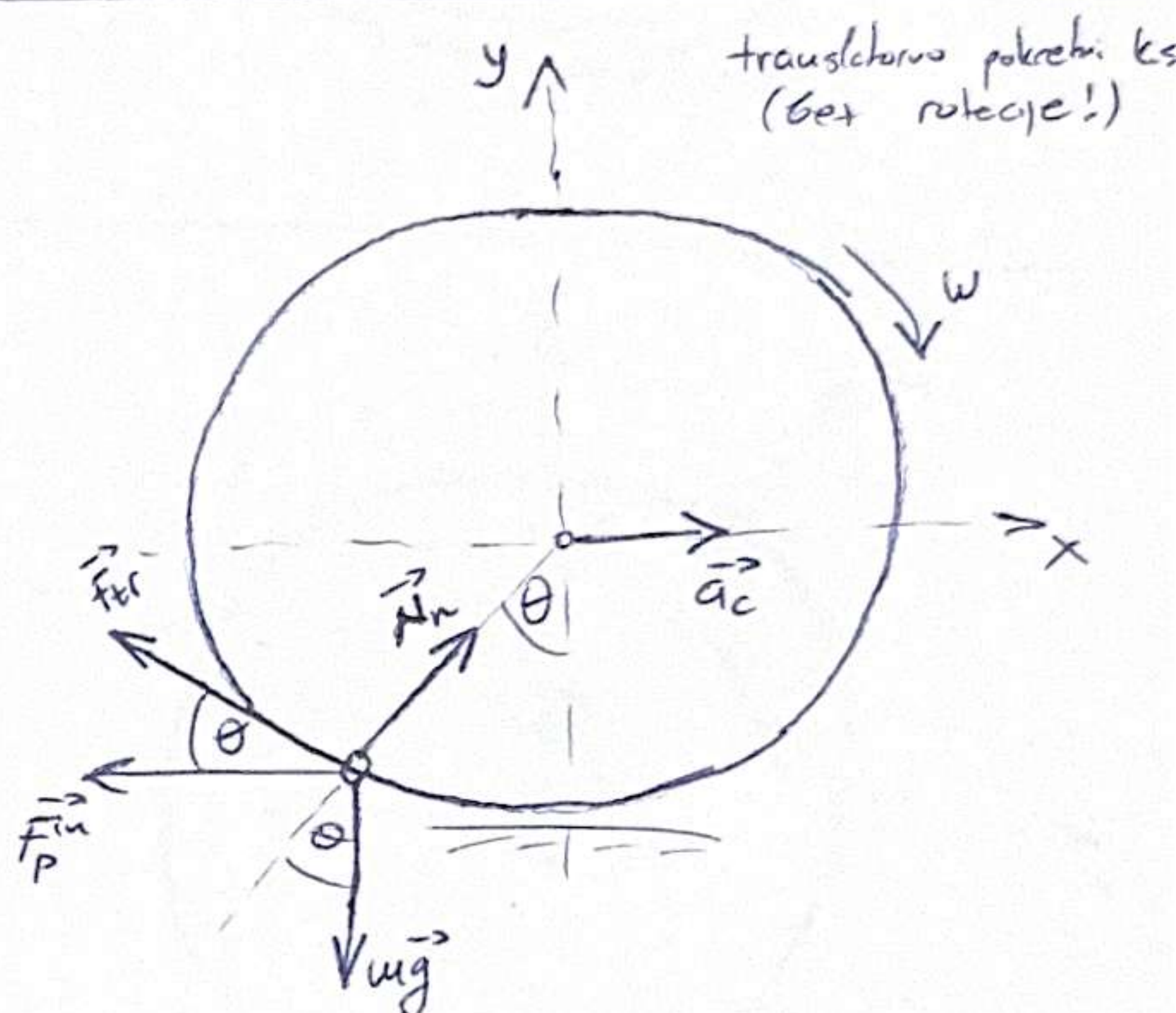
$$\vec{F}_{tr} = \mu \cdot M_n \cdot (-\vec{t})$$

$$\vec{F}_p^{\text{in}} = -m(\vec{a}_c + \cancel{\vec{\xi} \times \vec{p}} + \cancel{\vec{\omega} \times (\vec{\omega} \times \vec{p})}) = m \cdot a_c \cdot (-\vec{z})$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega} \times \vec{v}_r = 0$$

$$\vec{L}_n = \Delta n \cdot \vec{u}$$

$$\vec{L}_x = 0$$



$$t: 0 = mg \sin \theta - F_{tr} - F_p^m \cos \theta$$

$$0 = mg \sin \theta - \mu \cdot N_n - m a_c \cdot \cos \theta, \quad N_n = ?$$

$$\text{H: } 0 = -mg \cos \theta - F_p^{\text{in}} \sin \theta + N_n$$

$$N_n = m g \cos \theta + m a_c \sin \theta$$

$$t: 0 = mg \sin \theta - \mu \cdot m (g \cos \theta + a_c \sin \theta) - m a_c \cos \theta$$

1) $U_c = \text{const.} \rightarrow a_c = 0$

$$0 = f \sin \theta - \mu f \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \mu = \tan \theta$$

$$\boxed{\vartheta = \arctg \mu}$$

2) $a_c = 2g$

$$0 = g \sin \theta - \mu g \cos \theta - \mu \cdot 2g \sin \theta - 2g \cos \theta$$

$$0 = \sin \theta (g - 2\mu g) - \cos \theta (2g + \mu g)$$

$$\frac{\sin \theta}{\cos \theta} = \frac{2g + \mu g}{g - 2\mu g} = \frac{2 + \mu}{1 - 2\mu} = \tan \theta$$

$$\theta = \arctg \frac{2+\mu}{1-\mu}$$

6.38) vertikalni ravan

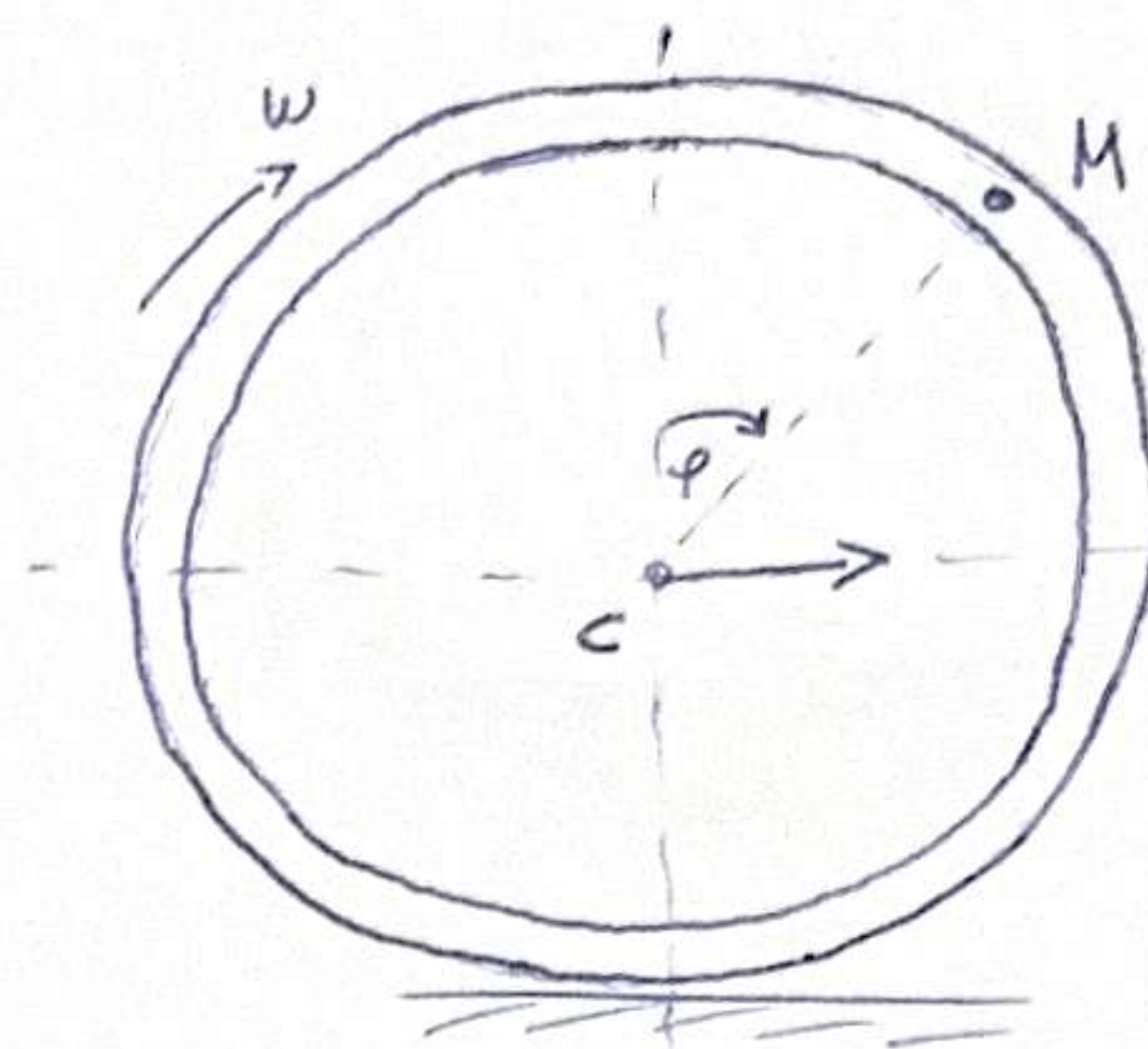
$M: m$

to: M je v relativnom miru
v najvišem položaju

$\mathcal{H}_n(\varphi) = ?$

1) $\omega = \text{const}$

2) $\varepsilon = \text{const}$, $\omega(t_0) = \omega_0$



$$\vec{m}\vec{a}_r = \vec{m}\vec{g} + \vec{F}_p^{\text{in}} + \vec{F}_{\text{cor}}^{\text{in}} + \vec{N}_n + \vec{N}_G$$

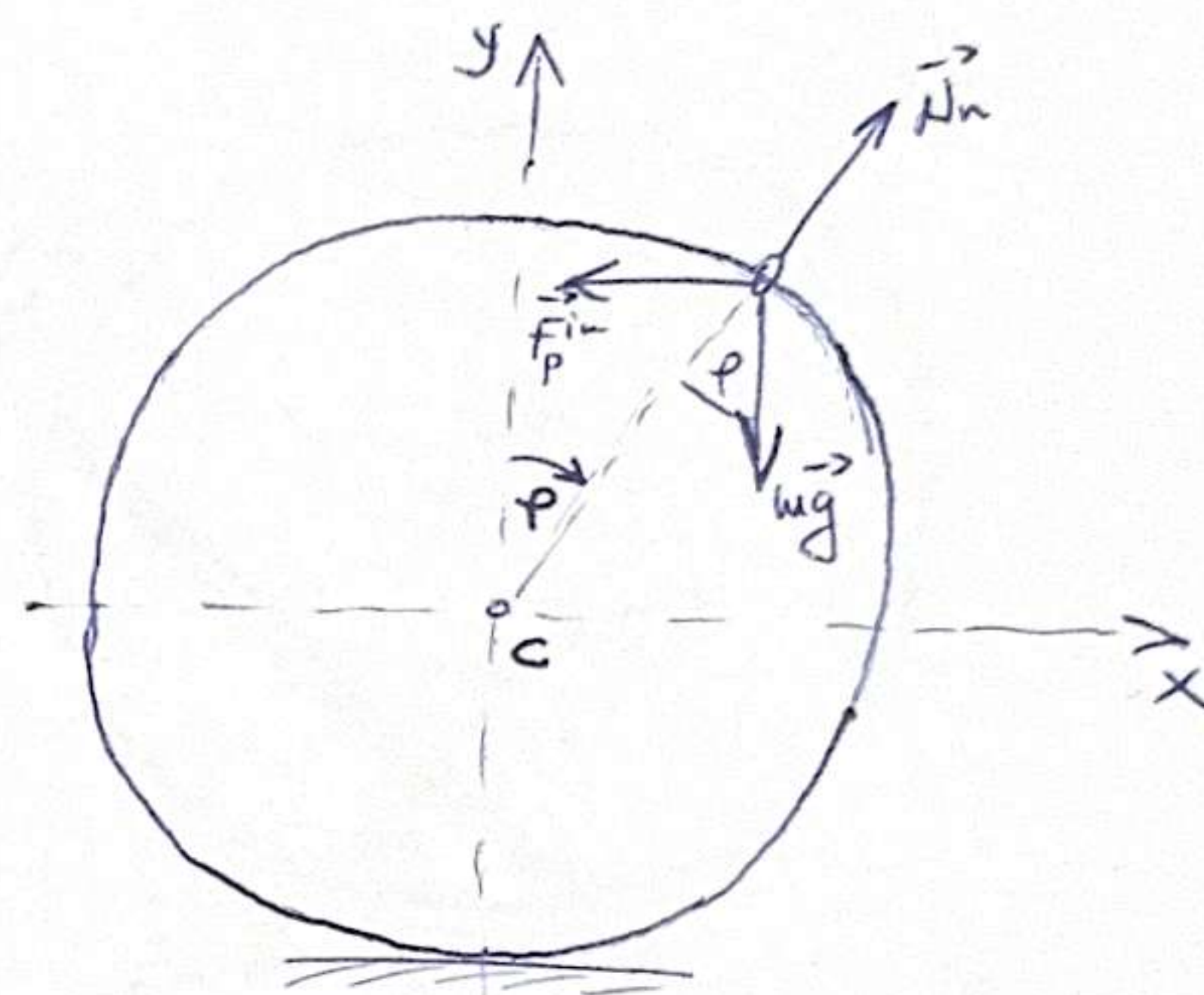
$$\vec{m}\vec{g} = +mg \cdot (-\vec{j})$$

$$\vec{F}_p^{\text{in}} = -m(\vec{a}_c + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = m \cdot a_c \cdot (-\vec{i})$$

$$\vec{F}_{\text{cor}}^{\text{in}} = -2m \cdot \vec{\omega} \times \vec{v}_r = 0$$

$$\vec{N}_n = +N_n \cdot (-\vec{n})$$

$$\vec{N}_G = 0$$



n: $m \cdot \frac{v_r^2}{R} = mg \cos \varphi + F_p^{\text{in}} \sin \varphi - N_n$

$$N_n = mg \cos \varphi + m a_c \sin \varphi - m \frac{v_r^2}{R}, \quad v_r = ?$$

$T - T_0 = A$ (rel)

$$A_{\text{rel}} = \int_R^{R \cos \varphi} -mg dy + \int_0^{\varphi} -F_p^{\text{in}} dx = mgR(1 - \cos \varphi) - m \cdot a_c \cdot R \sin \varphi$$

$$\frac{1}{2} m v_{\text{rel}}^2 - \frac{1}{2} m v_{r0}^2 = mgR(1 - \cos \varphi) - m \cdot a_c \cdot R \sin \varphi$$

$$\frac{1}{2} m v_{\text{rel}}^2 - \frac{1}{2} m R^2 \omega_0^2 = mgR(1 - \cos \varphi) - m a_c \cdot R \sin \varphi$$

$$v_{\text{rel}}^2 = R^2 \omega_0^2 + 2gR(1 - \cos \varphi) - 2a_c R \sin \varphi$$

$$N_n = mg \cos \varphi + m a_c \sin \varphi - m R \omega_0^2 - 2mg(1 - \cos \varphi) + 2m a_c \sin \varphi$$

$$N_n = 3mg \cos \varphi + 3m a_c \sin \varphi - m R \omega_0^2 - 2mg$$

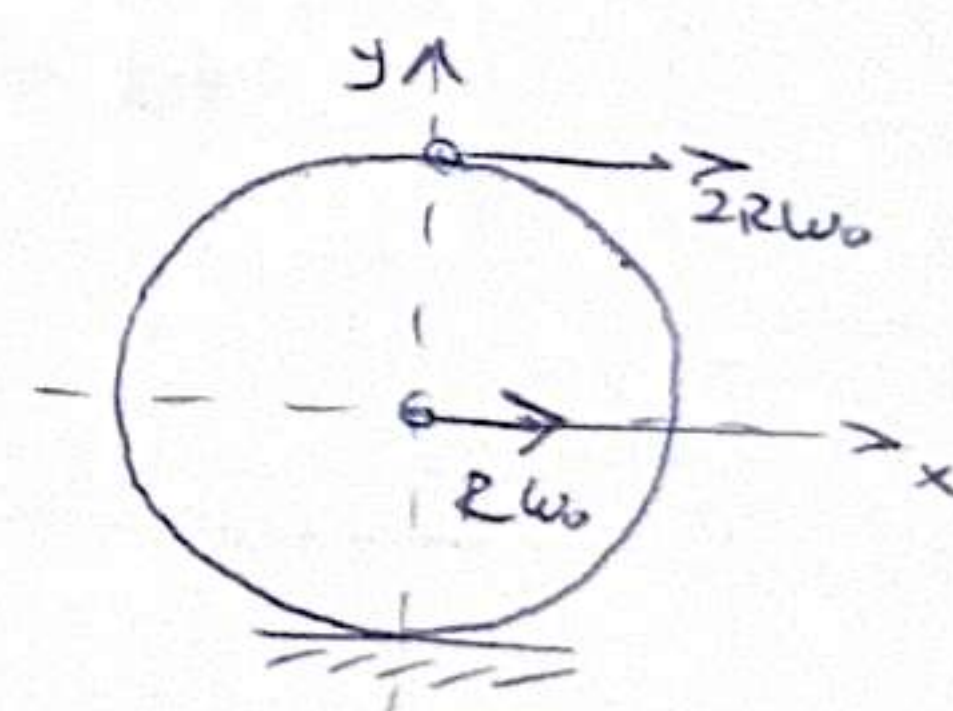
1) $\omega = \text{const} \Rightarrow a_c = R\varepsilon = 0 \Rightarrow$

$$N_n = 3mg \cos \varphi - m R \omega_0^2 - 2mg$$

2) $\varepsilon = \text{const} \Rightarrow a_c = R\varepsilon \Rightarrow$

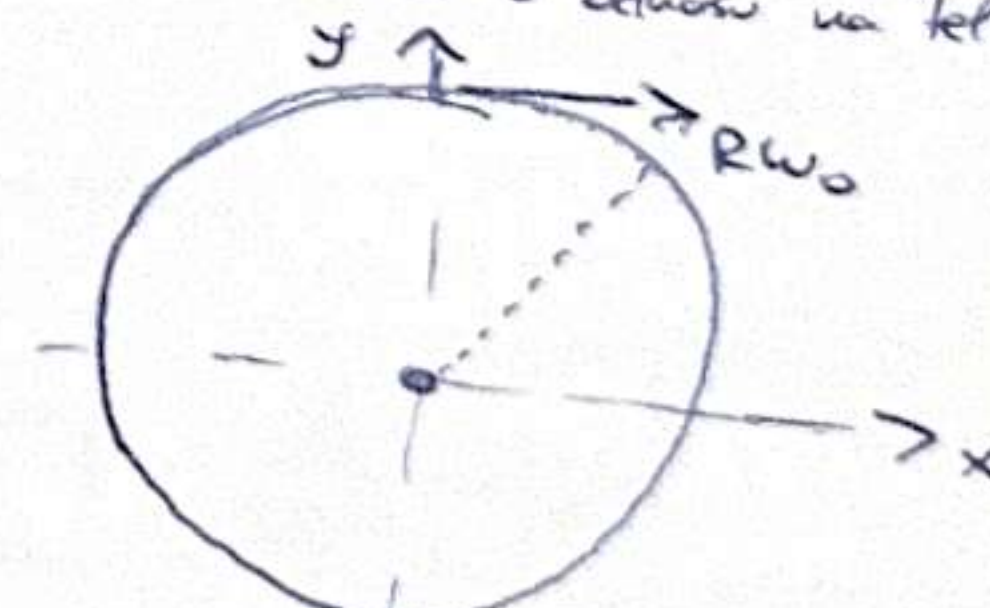
$$N_n = 3mg \cos \varphi + 3m\varepsilon \cdot R \sin \varphi - m R \omega_0^2 - 2mg$$

(to) - apsolutno



(to) - relativno

(v odnosu na Cxy
a ne v odnosu na telo)



(samo suv slobodni teži)
(određi Cmm R*omega)

6.39.

disk: R

$\omega = \text{const.}$

vertikalna ravan

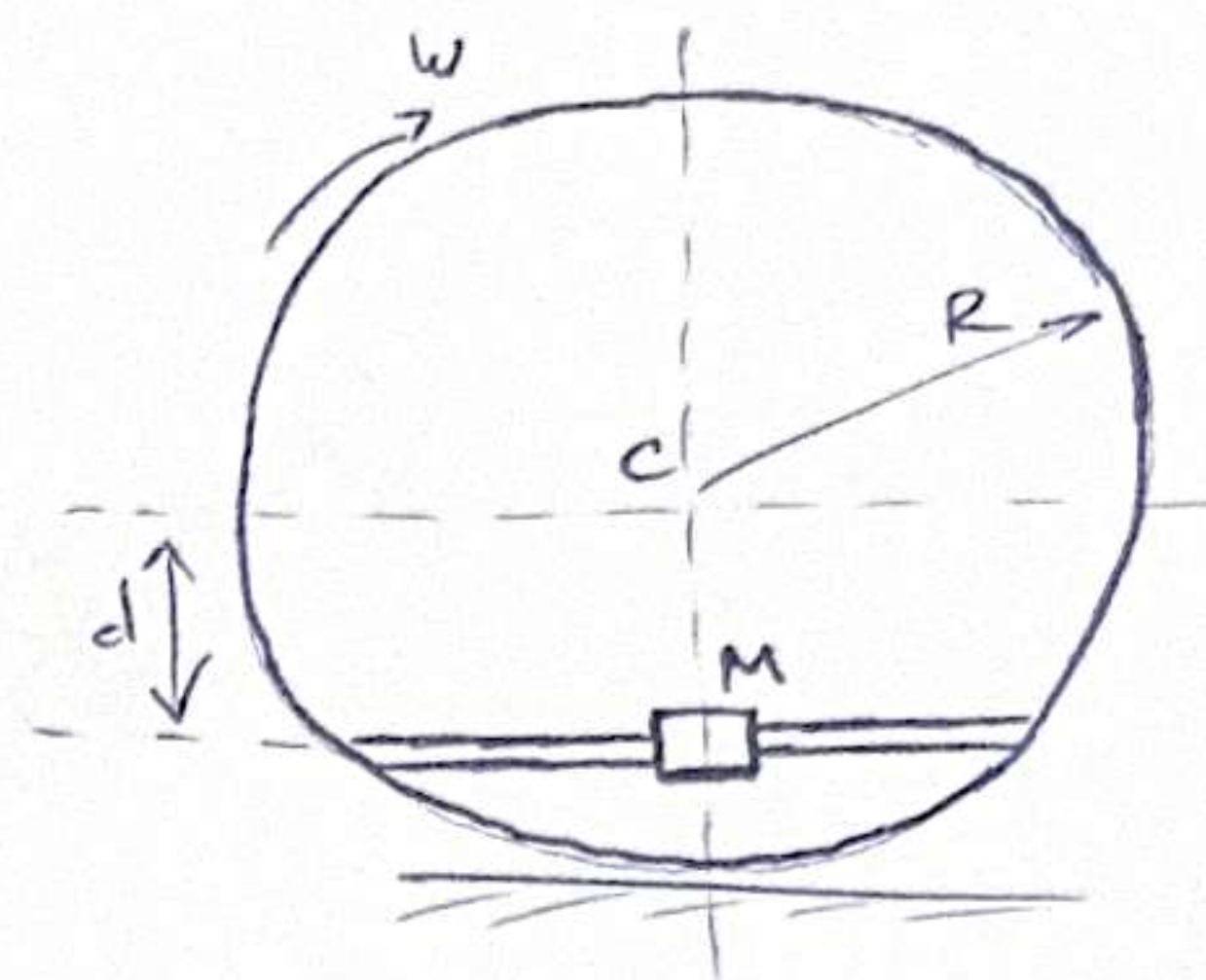
klasi: m

to: žleboj je horizontalan

$U_0 = U_0$

f-ua rel kretanja u odnosu na disk?

$N(t) = ?$



$$m\vec{a}_r = m\vec{g}' + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_n + \vec{N}_g$$

$$m\vec{g}' = mg \cdot (-\vec{j})$$

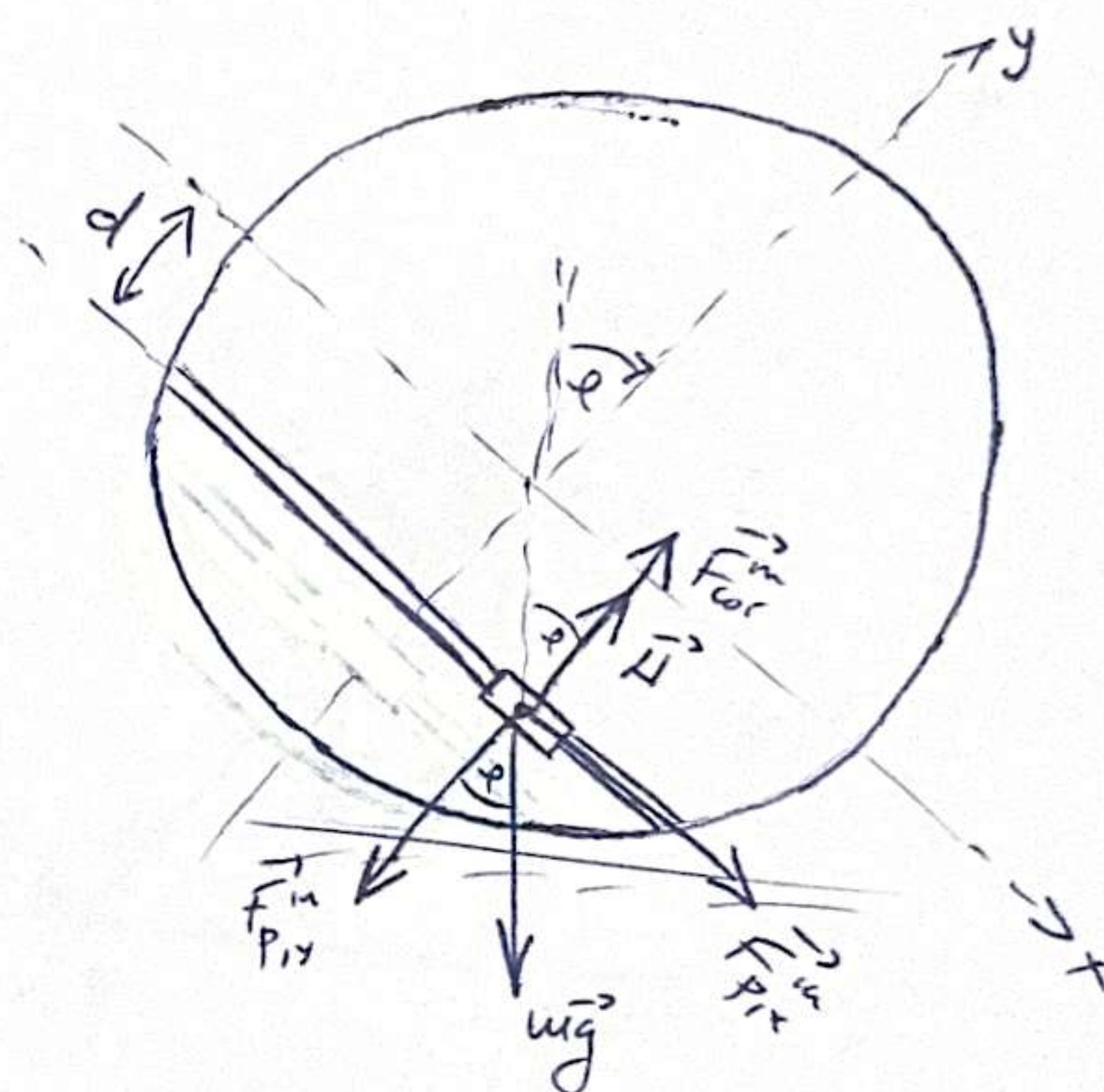
$$\vec{F}_p^m = -m(\vec{a}_c + \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m\omega^2 d \cdot (-\vec{j}) + m\omega^2 x \cdot (\vec{i}) = \vec{F}_{p,y}^m + \vec{F}_{p,x}^m$$

$$\vec{F}_{cor}^m = -2m\vec{\omega}' \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$$

$$\vec{N}_n = N \cdot \vec{n}' = \vec{N}'$$

$$\vec{N}_g = 0$$



$$\varphi = \int \omega dt = \omega t$$

$$x: \quad m\ddot{x} = mg \sin \varphi + m\omega^2 x$$

$$\ddot{x} - \omega^2 x = g \sin(\omega t)$$

$$\# : \quad \lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega + 0i$$

$$x_h = c_1 e^{\omega t} + c_2 e^{-\omega t}$$

$$P: \quad \phi(t) = e^{\omega t} (A \cos(\omega t) + B \sin(\omega t))$$

$$x_p = t^0 e^{\omega t} (A \cos(\omega t) + B \sin(\omega t))$$

$$\ddot{x}_p = -\omega^2 (A \cos(\omega t) + B \sin(\omega t))$$

$$\cos(\omega t) (-A\omega^2 - \omega^2 A) + \sin(\omega t) (-B\omega^2 - \omega^2 B) =$$

$$= \cos(\omega t) \cdot 0 + \sin(\omega t) \cdot g$$

$$\Rightarrow A = 0, -2B\omega^2 = g \rightarrow B = -\frac{g}{2\omega^2}$$

$$x = c_1 e^{\omega t} + c_2 e^{-\omega t} - \frac{g}{2\omega^2} \sin(\omega t)$$

$$\dot{x} = c_1 \omega e^{\omega t} - c_2 \omega e^{-\omega t} - \frac{g}{2\omega} \cos(\omega t)$$

$$x(t_0) = c_1 + c_2 - 0 = 0 \rightarrow c_1 = -c_2$$

$$\dot{x}(t_0) = c_1 \omega - c_2 \omega - \frac{g}{2\omega} = U_0$$

$$c_1 \omega + c_1 \omega = U_0 + \frac{g}{2\omega}$$

$$\Rightarrow c_1 = \frac{U_0}{2\omega} + \frac{g}{4\omega^2} = -c_2$$

$$x = \left(\frac{U_0}{2\omega} + \frac{g}{4\omega^2} \right) (e^{\omega t} - e^{-\omega t}) - \frac{g}{2\omega^2} \sin(\omega t)$$

$$y: \quad 0 = -mg \cos \varphi - m\omega^2 d + 2m\omega \dot{x} + N$$

$$N = mg \cos(\omega t) + m\omega^2 d - 2m\omega \left(\left(\frac{U_0}{2\omega} + \frac{g}{4\omega^2} \right) \cdot \right.$$

$$\left. \cdot (\omega e^{\omega t} + \omega e^{-\omega t}) - \frac{g}{2\omega} \cos(\omega t) \right)$$

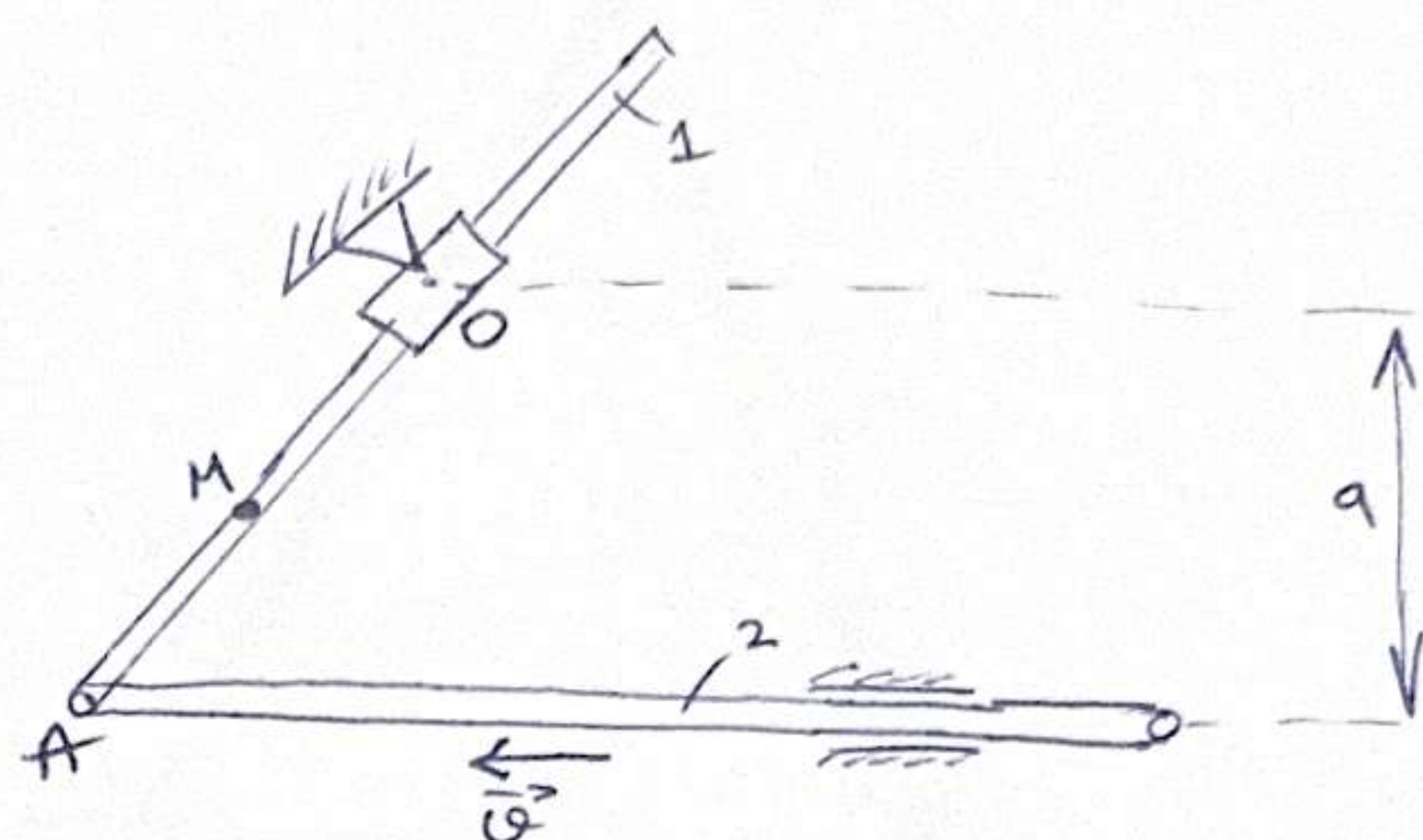
$$N = m\omega^2 d + 2mg \cos(\omega t) - m \left(U_0 \omega + \frac{g}{2} \right) (e^{\omega t} + e^{-\omega t})$$

6.40. 2: $\varphi_0 = ak = \text{const.}$

kuglica: m

def. i-na rel. kočin kuglice?

$t_0 = 0$: 1: je vertikalna



$$\omega = \frac{\varphi \omega_1 \varphi}{\overline{OA}} = \frac{\varphi \omega_1 \varphi}{\frac{a}{\omega_1 \varphi}} = \frac{\varphi_0}{a} \omega_1^2 \varphi$$

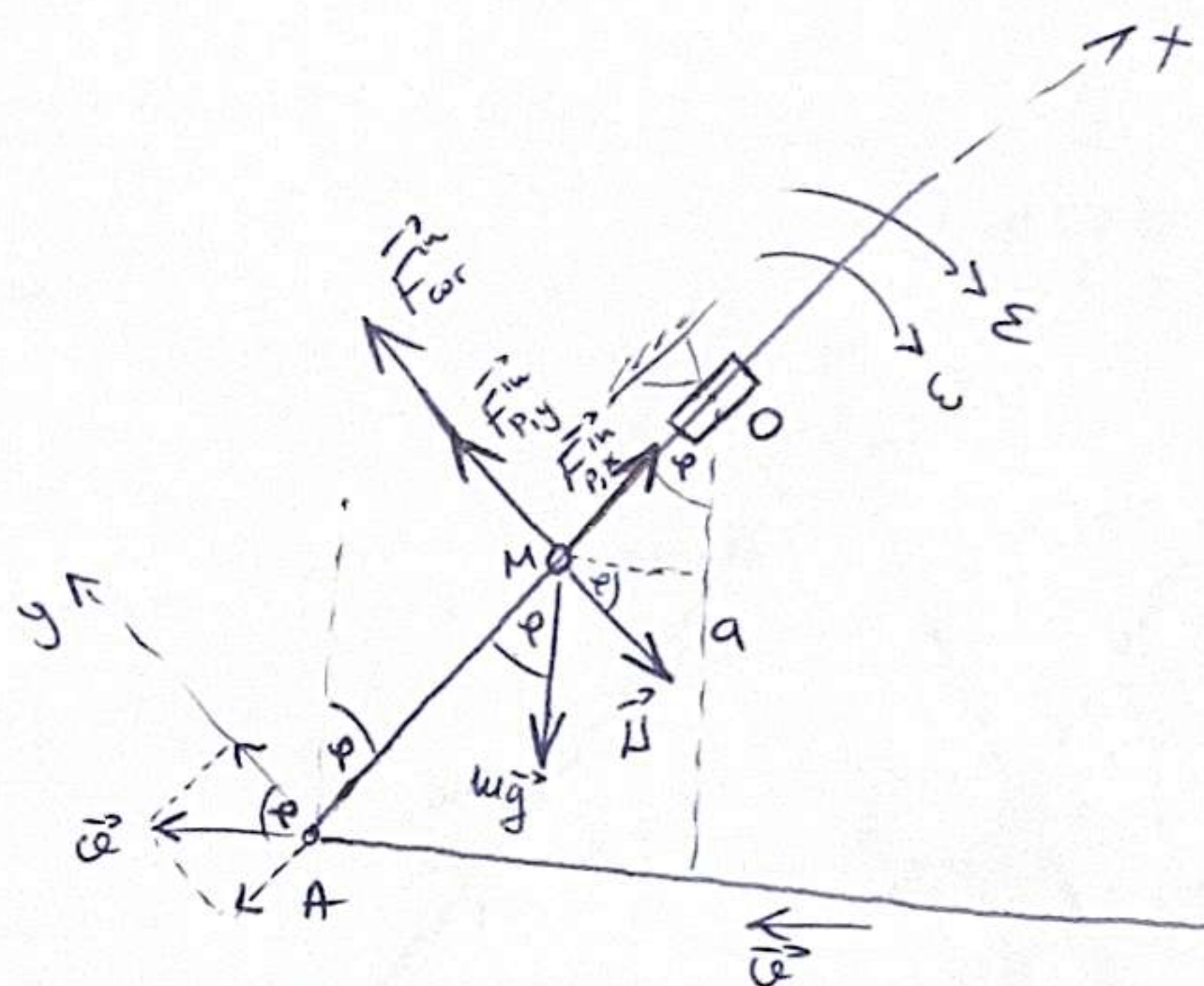
$$\frac{d\varphi}{dt} = \frac{\varphi_0}{a} \omega_1^2 \varphi$$

$$\frac{d\varphi}{\omega_1^2 \varphi} = \frac{\varphi_0}{a} dt \quad || \int$$

$$\lg \varphi = \frac{\varphi_0}{a} t + C_1 \rightarrow 0$$

$$\varphi = \arctg\left(\frac{\varphi_0}{a}\right)$$

$$\omega = \frac{1}{1 + \left(\frac{\varphi_0}{a} t\right)^2} \cdot \frac{\varphi_0}{a}$$



$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_p + \vec{F}_{\text{cor}} + \vec{N}_n + \vec{N}_c$$

$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p = -m(\ddot{\vec{r}} + \vec{\varepsilon}' \times \vec{r} + \vec{\omega}' \times (\vec{\omega}' \times \vec{r})) = m \cdot \varepsilon \cdot x \cdot (\vec{j}') + m \omega^2 x \cdot (\vec{i}')$$

$$\vec{F}_{\text{cor}} = -2m\vec{\omega}' \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j}')$$

$$\vec{N}_n = N \cdot (-\vec{j}) = \vec{N}'$$

$$\vec{N}_c = 0$$

$$x: m\ddot{x} = -mg \cos \varphi + m \omega^2 x$$

$$\cos \varphi = \frac{1}{\sqrt{1 + \tan^2 \varphi}} = \frac{1}{\sqrt{1 + \left(\frac{\varphi_0}{a} t\right)^2}}, \quad \varphi_0 = ak$$

$$\ddot{x} = -g \cdot \frac{1}{\sqrt{1 + k^2 t^2}} + \frac{1}{(1 + (kt)^2)^2} k^2 x$$

$$\ddot{x} - \frac{k^2}{(1 + k^2 t^2)^2} x + \frac{g}{\sqrt{1 + k^2 t^2}} = 0$$

6.41.

OA, AB: R

vertikalna ravan

$$\omega = \sqrt{\frac{3g}{R}} = \text{const.}$$

M: m

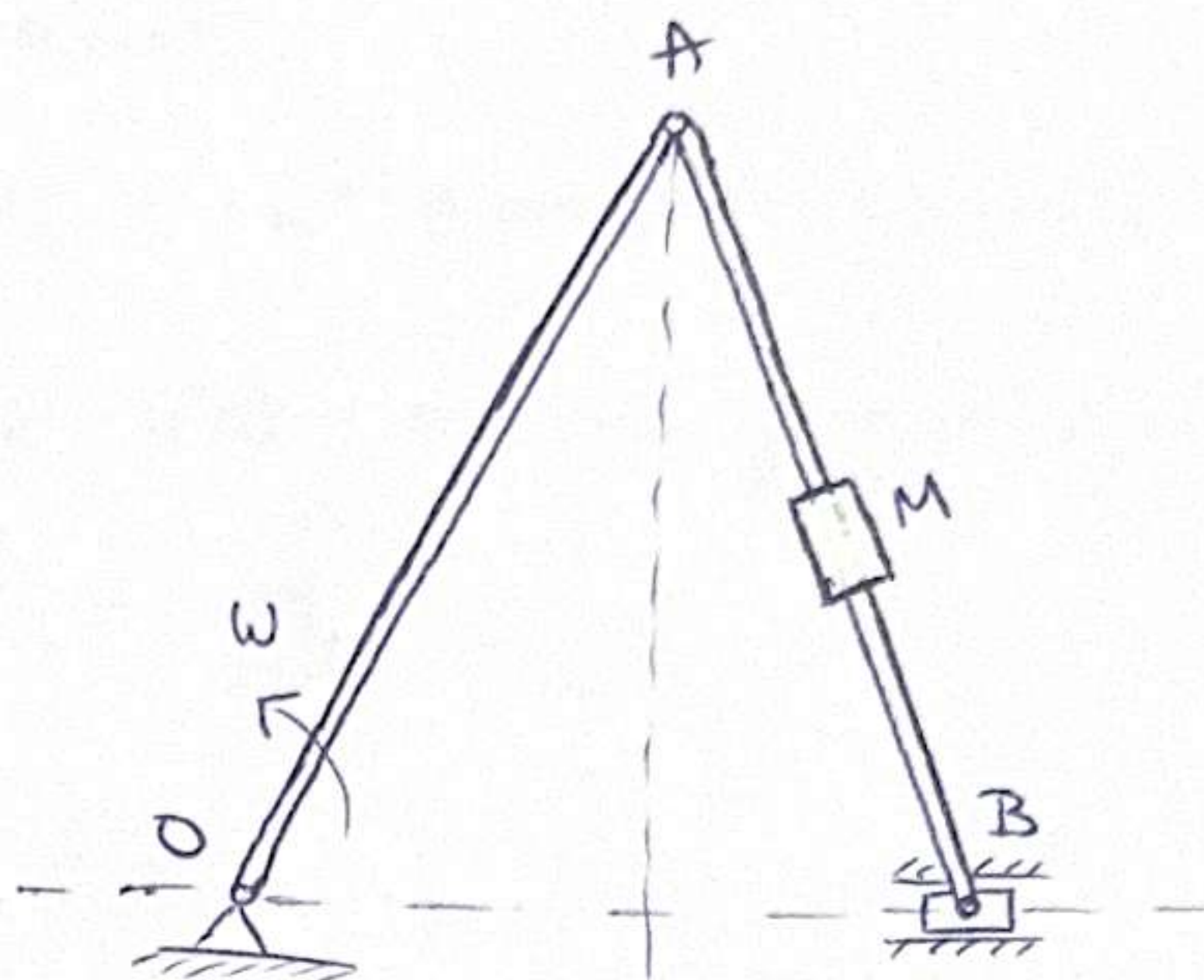
to: statovi horizontalni

M relativno mirno

položaj A

Konacna j-na rel. kretanja klase M?

$$N(t) = ?$$



oba stativa zaklapaju ugao phi sa horizontalom
=> iste su im y-ove brzine w

$$\vec{w}_A = \vec{w}_g + \vec{F}_p^m + \vec{F}_{cor}^m + \vec{N}_n + \vec{N}_g$$

$$\vec{w}_g = w_g \cdot (-\vec{j})$$

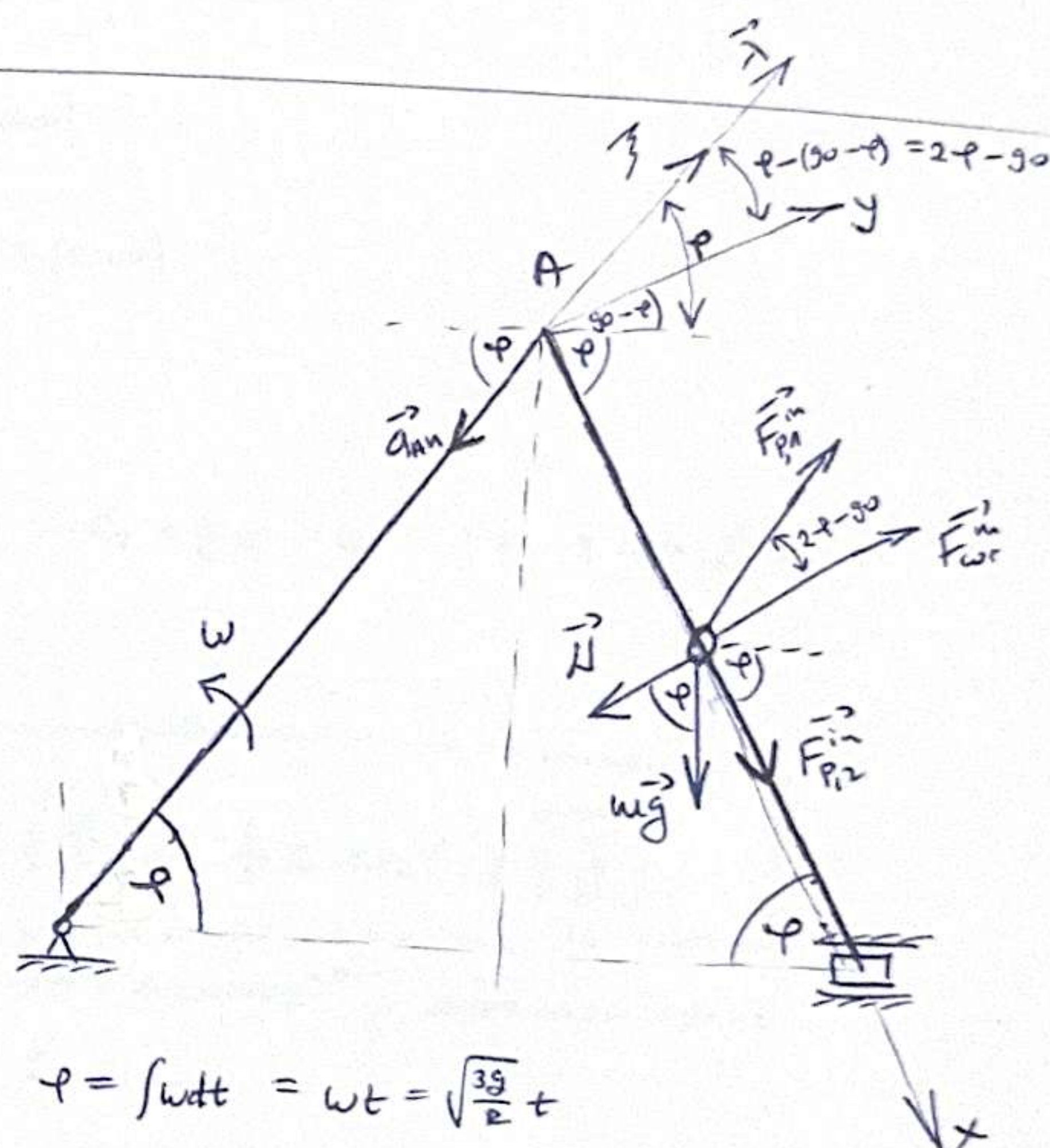
$$\vec{F}_p^m = -m(\vec{a}_A + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) =$$

$$= m \cdot a_{An} \cdot (\vec{\lambda}) + m\omega^2 x \cdot (\vec{z}) = \vec{F}_{p1}^m + \vec{F}_{p2}^m$$

$$\vec{F}_{cor}^m = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$$

$$\vec{N}_n = N \cdot (-\vec{j}) = \vec{N}$$

$$\vec{N}_g = 0$$



$$\varphi = \int \omega dt = \omega t = \sqrt{\frac{3g}{R}} t$$

$$a_{An} = \omega^2 \cdot R = \frac{3g}{R} \cdot R = 3g$$

$$x: m\ddot{x} = w_g \sin \varphi - m a_{An} \cdot \sin(2\varphi - 90^\circ) + m\omega^2 x$$

$$\ddot{x} - \frac{3g}{R} x = g \sin\left(\sqrt{\frac{3g}{R}} t\right) + 3g \cos\left(2\sqrt{\frac{3g}{R}} t\right)$$

$$\ddot{x} - \omega^2 x = g \sin(\omega t) + 3g \cos(2\omega t)$$

$$H: \lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm \omega + 0i \rightarrow x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

$$P_1: C_1(t) = g \sin(\omega t) = e^{\omega t} (0 \cos(\omega t) + g \sin(\omega t))$$

$$x_{p1} = t^0 e^{\omega t} (A \cos(\omega t) + B \sin(\omega t)), \quad \ddot{x}_{p1} = -\omega^2 (A \cos(\omega t) + B \sin(\omega t))$$

$$\cos(\omega t) (-\omega^2 A - \omega^2 A) + \sin(\omega t) (-\omega^2 B - \omega^2 B) = 0 \cos(\omega t) + g \sin(\omega t)$$

$$A = 0, \quad -2\omega^2 B = g \Rightarrow B = -\frac{g}{2\omega^2}$$

$$x_{p1} = -\frac{g}{2\omega^2} \sin(\omega t) = -\frac{g}{2 \cdot \frac{3g}{R}} = -\frac{1}{6} R$$

$$P_2: \quad b_2(t) = 3g \cos(2\omega t) = e^{0t} (3g \cos(2\omega t) + 0 \sin(2\omega t))$$

$$x_{p2} = t^0 e^{0t} (A \cos(2\omega t) + B \sin(2\omega t)), \quad \ddot{x}_{p2} = -4\omega^2 (A \cos(2\omega t) + B \sin(2\omega t))$$

$$\cos(2\omega t) (-4\omega^2 A - \omega^2 A) + \sin(2\omega t) (-4\omega^2 B - \omega^2 B) = \cos(2\omega t) \cdot 3g + \sin(2\omega t) \cdot 0$$

$$\Rightarrow B = 0, \quad -5\omega^2 = 3g \rightarrow B = -\frac{3g}{5\omega^2}$$

$$x_{p2} = -\frac{3g}{5\omega^2} \cos(2\omega t) = -\frac{3g}{5 \cdot \frac{3g}{R}} = -\frac{1}{5} R$$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t} - \frac{1}{6} R \sin(\omega t) - \frac{1}{5} R \cos(2\omega t)$$

$$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t} - \frac{1}{6} R \omega \cos(\omega t) + \frac{2}{5} R \omega \sin(2\omega t)$$

$$x(0) = C_1 + C_2 - \frac{1}{5} R = 0 \rightarrow C_2 = \frac{1}{5} R - C_1$$

$$\dot{x}(0) = C_1 \omega - C_2 \omega - \frac{1}{6} R \omega = 0 \rightarrow \frac{1}{5} R - C_1 - C_2 - \frac{1}{6} R = 0 \rightarrow 2C_2 = \frac{1}{30} R$$

$$\rightarrow C_2 = \frac{1}{60} R, \quad C_1 = \frac{11}{60} R$$

$$x = \frac{1}{60} R \left(11 e^{t\sqrt{\frac{3g}{R}}} + e^{-t\sqrt{\frac{3g}{R}}} \right) - \frac{1}{6} R \sin\left(t\sqrt{\frac{3g}{R}}\right) - \frac{1}{5} R \cos\left(2t\sqrt{\frac{3g}{R}}\right)$$

$$y: \quad 0 = -mg \cos \varphi + m \cdot g_{\text{An}} \cos(2\varphi - 90^\circ) + 2m\omega \cdot \dot{x} - U$$

$\cos(2\varphi - 90^\circ) = \cos 2\varphi \cos 90^\circ + \sin 2\varphi \sin 90^\circ = \sin 2\varphi$

$$U = -mg \cos\left(t\sqrt{\frac{3g}{R}}\right) + 3mg \sin\left(2t\sqrt{\frac{3g}{R}}\right) + 2m\sqrt{\frac{3g}{R}} \left(\frac{1}{60} R \cdot \sqrt{\frac{3g}{R}} \left(11 e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}} \right) - \frac{1}{6} R \sqrt{\frac{3g}{R}} \cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{2}{5} R \sqrt{\frac{3g}{R}} \sin\left(2t\sqrt{\frac{3g}{R}}\right) \right)$$

$$U = -mg \cos\left(t\sqrt{\frac{3g}{R}}\right) + 3mg \sin\left(2t\sqrt{\frac{3g}{R}}\right) + \frac{1}{10} mg \left(11 e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}} \right) - mg \cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{12}{5} mg \sin\left(2t\sqrt{\frac{3g}{R}}\right)$$

$$U = \frac{27}{5} mg \sin\left(2t\sqrt{\frac{3g}{R}}\right) - 2mg \cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{1}{10} mg \left(11 e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}} \right)$$

6.42.

vertikalna ravan

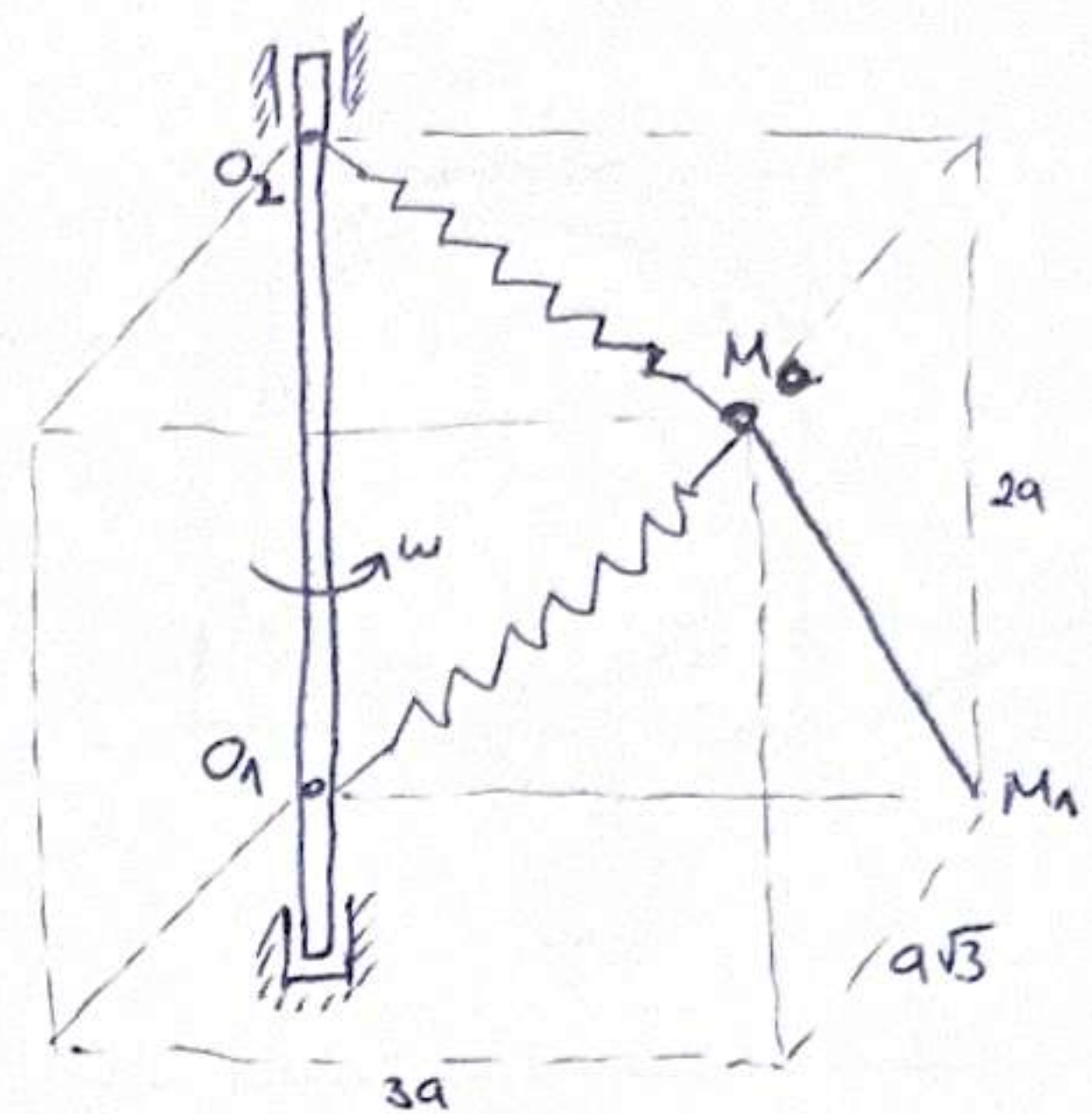
$\omega = \text{const.}$

klizac mase m

opruge: $c, l_0 = 3a$

$\omega = ?$ - da li klizac stignao u položaj M_1

to: klizac u M_0 , $U_{r0} = 0$



$$\vec{m}\vec{a}_r = m\vec{g} + \vec{F}_{c1} + \vec{F}_{c2} + \vec{F}_p^{in} + \vec{F}_{cor} + \vec{N}_n + \vec{N}_g$$

$$m\vec{g} = +mg \cdot (-\vec{k})$$

$$\vec{F}_{c1} = c \cdot \Delta l_1 \cdot (-\vec{\lambda}_1) = c \vec{r}_1 \cdot (-\vec{\lambda}_1)$$

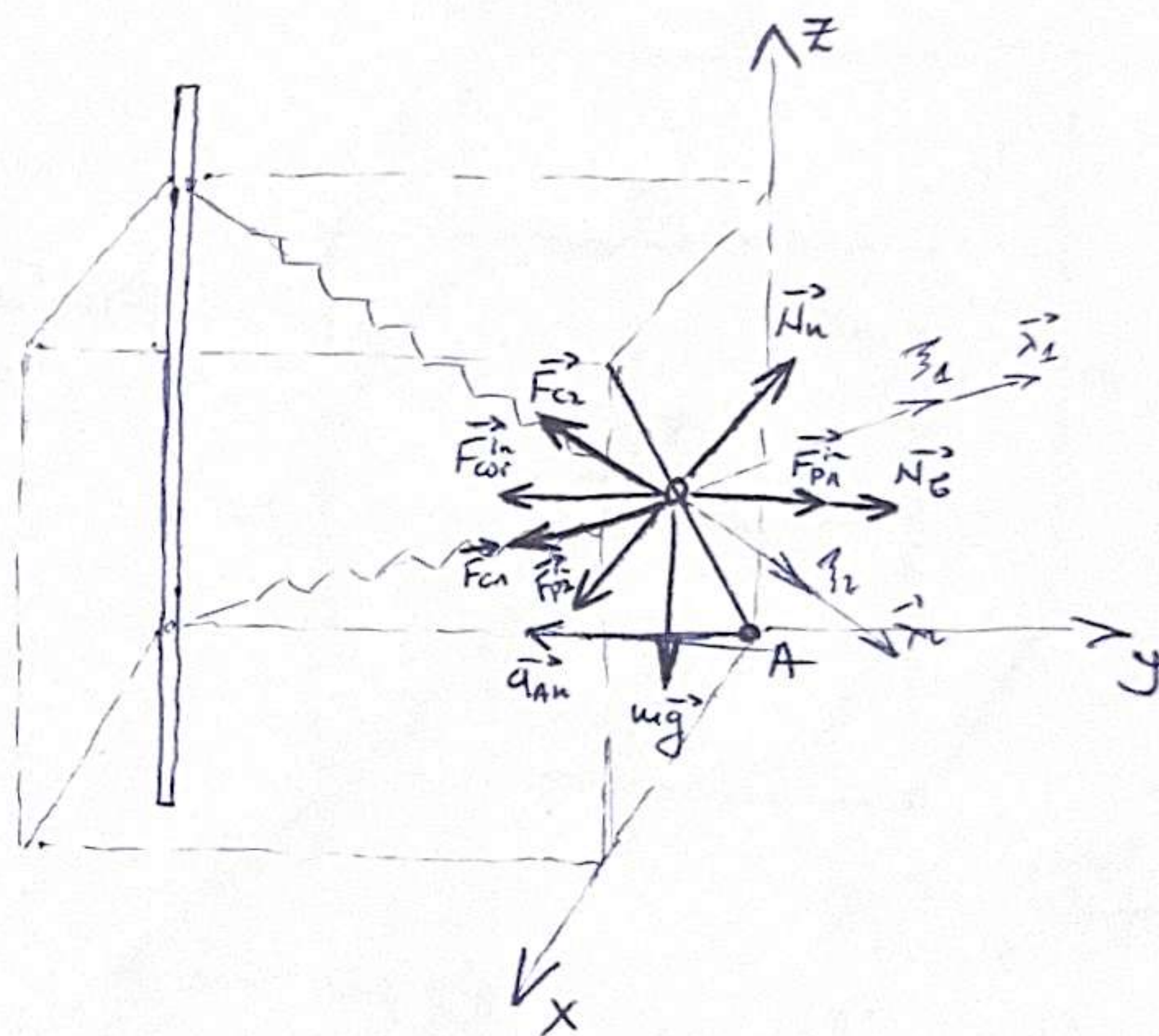
$$\vec{F}_{c2} = c \cdot \Delta l_2 \cdot (-\vec{\lambda}_2) = c \vec{r}_2 \cdot (-\vec{\lambda}_2)$$

$$\begin{aligned} \vec{F}_p^{in} &= -m(\vec{a}_A + \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = \\ &= m \cdot a_{An} \cdot (-\vec{j}) + m\omega^2 x \cdot (\vec{i}) = \vec{F}_{p1} + \vec{F}_{p2} \end{aligned}$$

$$\vec{F}_{cor} = -2m \vec{\omega} \times \vec{v}_r = +2m\omega \cdot \dot{x} \cdot (-\vec{j})$$

$$\vec{N}_n = N_n \cdot (-\vec{i})$$

$$\vec{N}_g = N_g \cdot (\vec{j})$$



$$T_{M1} - T_{M0} = A_{M0M1}$$

$$A_{M0M1} = \int_0^{2a} -mg dz + \int_{\vec{r}_{10}}^{\vec{r}_{11}} -c \vec{r}_1 d\vec{r}_1 + \int_{\vec{r}_{20}}^{\vec{r}_{21}} -c \vec{r}_2 d\vec{r}_2 + \int_0^{a\sqrt{3}} m\omega^2 x dx$$

$\vec{r}_{10} = 3a - 3a = 0, \vec{r}_{11} = \sqrt{(3a)^2 + (a\sqrt{3})^2} - 3a = a,$
 $\vec{r}_{20} = \sqrt{(a^2) + (2a)^2} = 2a = a(\sqrt{3} - 1), \vec{r}_{21} = \sqrt{(3a)^2 + (a\sqrt{3})^2} - 3a = a$

$$A_{M0M1} = -2mga + \frac{1}{2}c(\vec{r}_{10}^2 - \vec{r}_{11}^2) + \frac{1}{2}c(\vec{r}_{20}^2 - \vec{r}_{21}^2) + \frac{1}{2}m\omega^2 \cdot 3a^2 =$$

$$A_{M0M1} = -2mga + \frac{1}{2}c(0 - a^2) + \frac{1}{2}c(a^2(13 - 6\sqrt{3} + 9) - a^2(12 - 6\sqrt{12} + 9)) + \frac{3}{2}m\omega^2 a =$$

$$A_{M0M1} = -2mga - \frac{1}{2}ca^2 + \frac{1}{2}ca^2(4 - 6\sqrt{3} + 6\sqrt{12}) + \frac{3}{2}m\omega^2 a = -2mga - 3ca^2(\sqrt{3} - \sqrt{12}) + \frac{3}{2}m\omega^2 a$$

$$0 - 0 = -2mga - 3ca^2(\sqrt{3} - \sqrt{12}) + \frac{3}{2}m\omega^2 a$$

$$\frac{3}{2}m\omega^2 a = 2mg + 3ca(\sqrt{3} - \sqrt{12})$$

$$\boxed{\omega = \sqrt{\frac{\frac{4}{3}mg + 2ca(\sqrt{3} - \sqrt{12})}{ma}}}$$

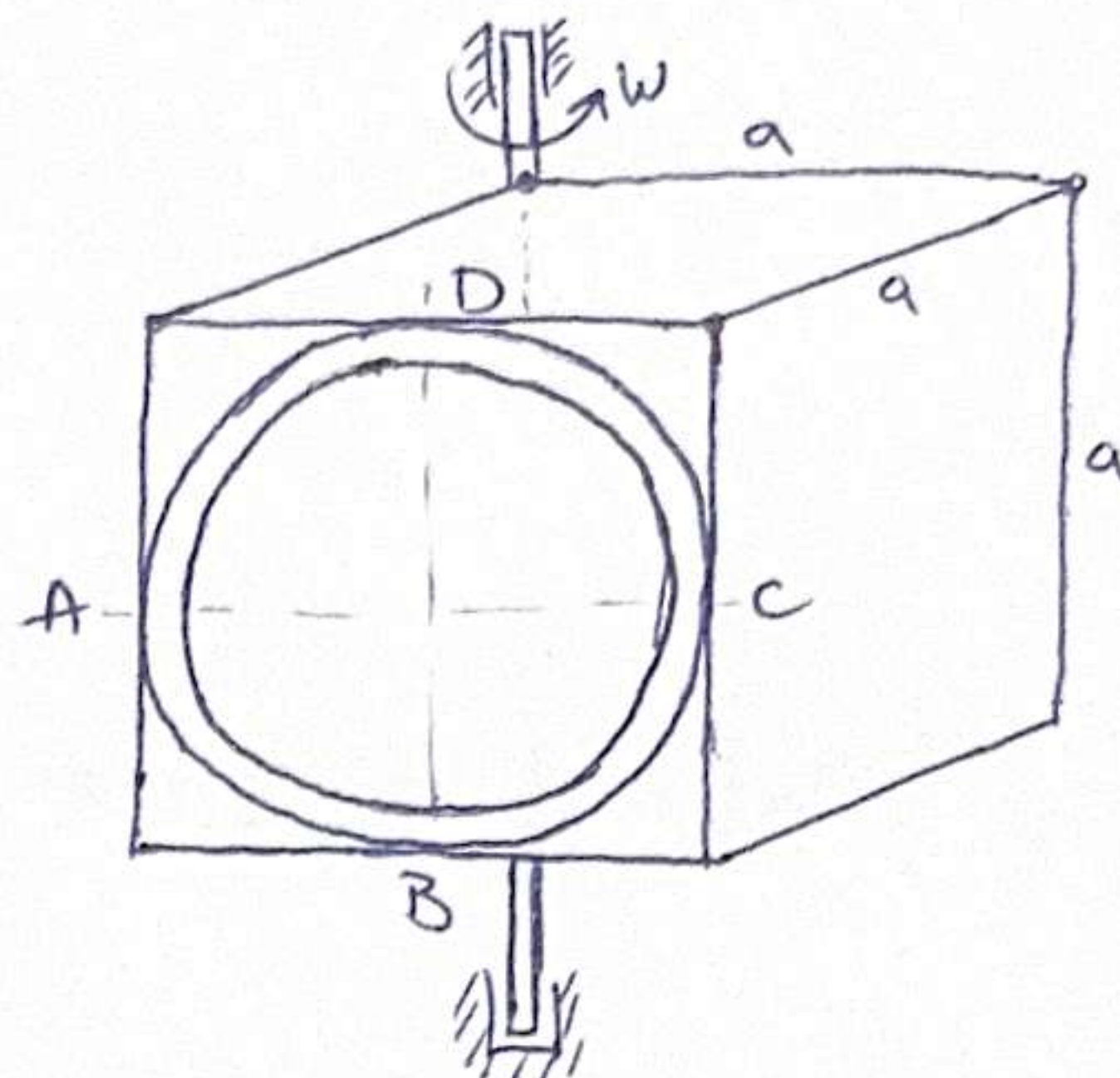
6.43 kuglica: ω

$\omega = \text{const.}$

to: kuglica u A, $v_{ro} = 0$

1) $\omega = ?$ - da li kuglica stigne u D

2) $U_B = ?$, $U_C = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{\text{in}} + \vec{F}_{\text{cor}}^{\text{in}} + \vec{U}_n + \vec{U}_G$$

$$m\vec{g} = +mg \cdot (-\vec{k})$$

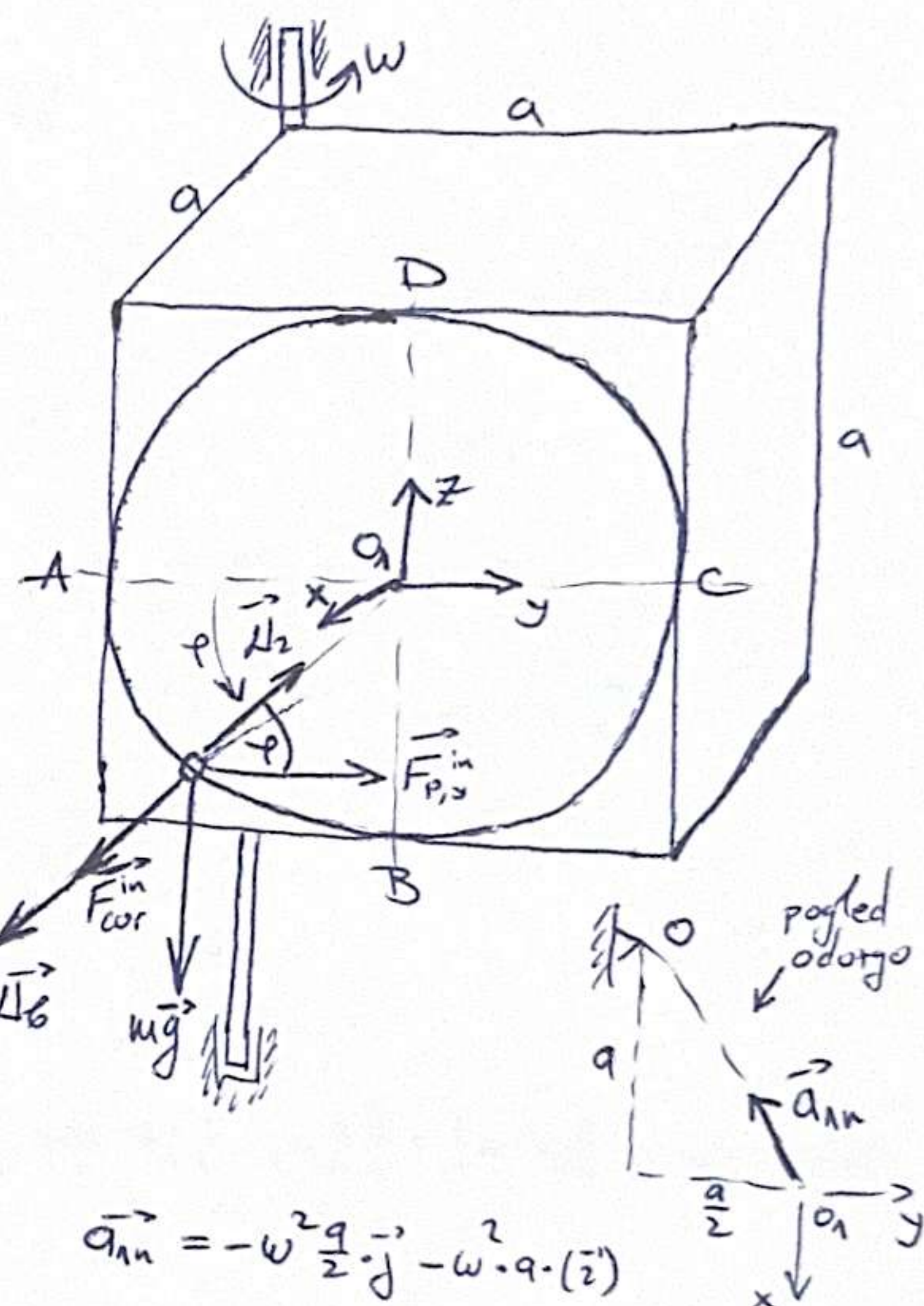
$$\vec{F}_p^{\text{in}} = -m(\vec{a}_{o1} + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})) = \frac{1}{2}m\omega^2 a \cdot (\vec{j}) + m\omega^2 a \cdot (\vec{i}) + m\omega^2 \cdot y \cdot \vec{j} =$$

$$= m\omega^2(y + \frac{1}{2}a)\vec{j} + m\omega^2 a \cdot (\vec{i}) = \vec{F}_{P,y}^{\text{in}} + \vec{F}_{P,x}^{\text{in}}$$

$$\vec{F}_{\text{cor}}^{\text{in}} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{y} \cdot (\vec{i})$$

$$\vec{U}_n = U_n \cdot (\vec{n})$$

$$\vec{U}_G = U_G \cdot (\vec{i})$$



$$\vec{a}_{nn} = -\omega^2 \frac{a}{2} \cdot \vec{j} - \omega^2 \cdot a \cdot (\vec{i})$$

$$T_D - T_A = A_{\text{AD}} \quad (\text{rec})$$

$$A_{\text{AD}} = \int_0^{\frac{a}{2}} -mg dz + \int_{-\frac{a}{2}}^0 m\omega^2(y + \frac{1}{2}a) dy =$$

$$= -\frac{1}{2}mga + \frac{1}{2}m\omega^2(0^2 - (-\frac{a}{2})^2) + \frac{1}{2}m\omega^2 a(0 - (-\frac{a}{2})) =$$

$$= -\frac{1}{2}mga - \frac{1}{8}m\omega^2 a^2 + \frac{1}{4}m\omega^2 a^2 = -\frac{1}{2}mga + \frac{1}{8}m\omega^2 a^2$$

$$0 - 0 = -\frac{1}{2}mga + \frac{1}{8}m\omega^2 a^2 \Rightarrow \omega = \sqrt{\frac{4g}{a}} \Rightarrow \boxed{\omega = 2\sqrt{\frac{g}{a}}}$$

$$n: m a_n = -mg \sin \varphi + m\omega^2(y - \frac{1}{2}a) \cos \varphi + U_n$$

$$U_n = m \cdot \frac{v_{\text{rec}}^2}{R_k} + mg \sin \varphi + m\omega^2(y - \frac{1}{2}a \cos \varphi)$$

$$x: 0 = m\omega^2 a + 2m\omega \dot{y} + U_G$$

$$U_G = -(m\omega^2 a + 2m\omega \dot{y})$$

tačka B

$$\underline{T_B - T_A = A_{AB}} \quad (\text{rec})$$

$$A_{AB} = \int_0^{-\frac{a}{2}} -mg dz + \int_{-\frac{a}{2}}^0 m\omega^2(y + \frac{1}{2}a) dy = \frac{1}{2}mga - \frac{1}{8}m\omega^2 a^2 + \frac{1}{4}m\omega^2 a = \frac{1}{2}mga + \frac{1}{8}m \cdot 4 \frac{g}{a} a^2 = mga$$

$$\frac{1}{2} m v_{Brec}^2 - 0 = mga \quad \Rightarrow \quad v_{Brec}^2 = 2ga$$

$$L_{n,B} = m \cdot \frac{2ga}{\frac{1}{2}a} + mg \cdot 1 + m\omega^2(0-0) = 4mg + mg = 5mg$$

$$L_{g,B} = -(m \cdot 4 \frac{g}{a} a + 2m \cdot 2\sqrt{\frac{g}{a}} \cdot \sqrt{2ga}) = -4mg(1+\sqrt{2})$$

$$L_B = \sqrt{L_{n,B}^2 + L_{g,B}^2} = mg \cdot \sqrt{25 + 16(1+2\sqrt{2}+2)}$$

$$\boxed{L_B = mg \sqrt{73 + 32\sqrt{2}}}$$

tačka C

$$\underline{T_C - T_A = A_{AC}} \quad (\text{rec})$$

$$A_{AC} = \int_0^0 -mg dz + \int_{-\frac{a}{2}}^{\frac{a}{2}} m\omega^2(y + \frac{1}{2}a) dy = \frac{1}{2}m\omega^2\left(\left(\frac{a}{2}\right)^2 - \left(-\frac{a}{2}\right)^2\right) + \frac{1}{2}m\omega^2 a \left(\frac{a}{2} - \left(-\frac{a}{2}\right)\right) = \frac{1}{2}m \cdot 4 \frac{g}{a} \cdot a \cdot a = 2mga$$

$$\frac{1}{2} m v_{Crec}^2 - 0 = 2mga \quad \Rightarrow \quad v_{Crec}^2 = 4ga$$

$$L_{n,C} = m \cdot \frac{4ga}{\frac{1}{2}a} + mg \cdot 0 + m\omega^2\left(\frac{1}{2}a - \frac{1}{2}a \cdot (-1)\right) = 8mg + m \cdot 4 \frac{g}{a} \cdot a = 12mg$$

$$L_{g,C} = -(m \cdot 4 \frac{g}{a} \cdot a + 0) = -4mg$$

$$L_C = \sqrt{L_{n,C}^2 + L_{g,C}^2} = mg \sqrt{144 + 16} = mg \cdot \sqrt{160}$$

$$\boxed{L_C = 4mg\sqrt{10}}$$