④.1.) $Q = 2\,N$

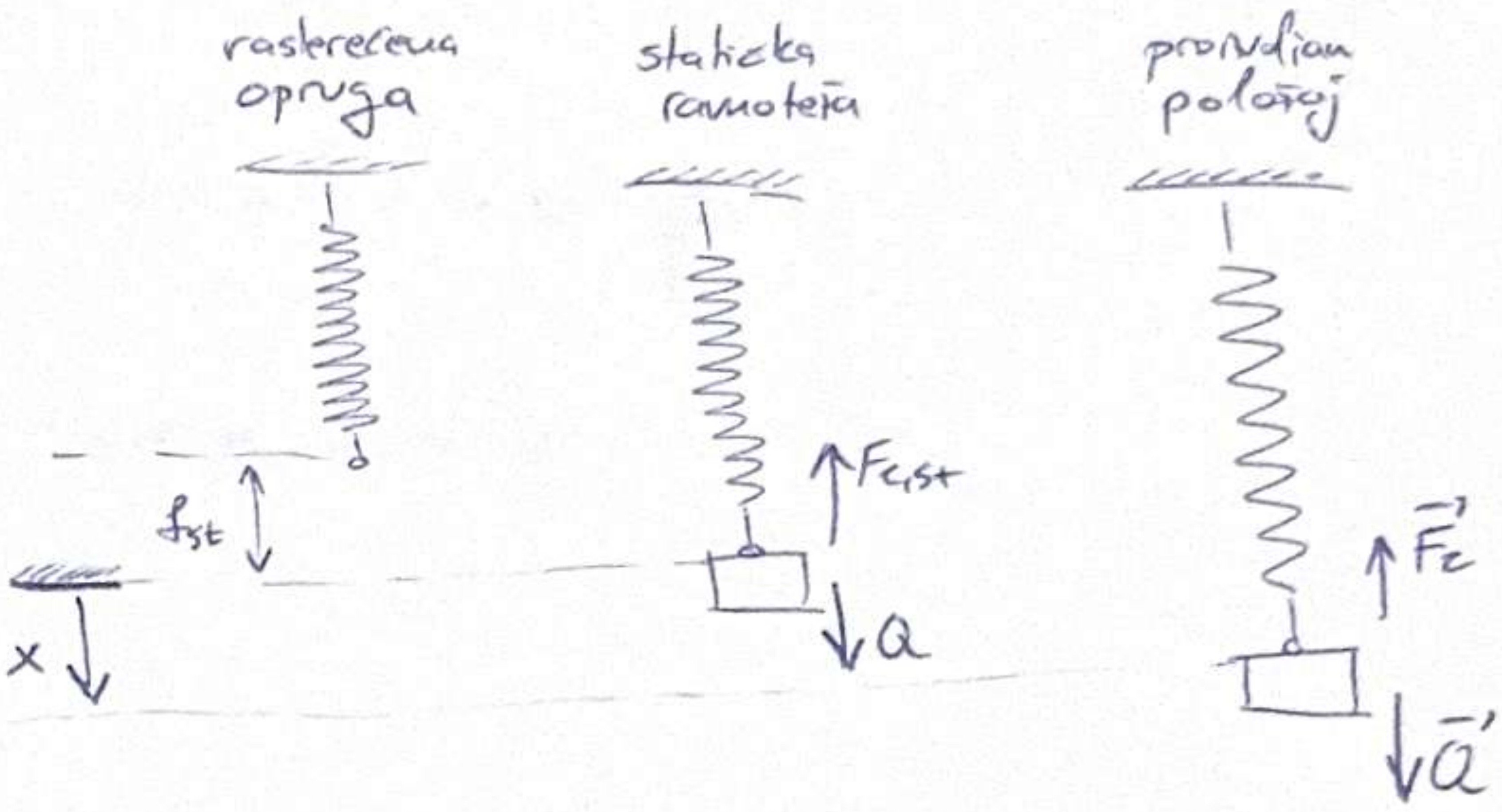
opruga: $F = 0{,}5\,N \rightarrow \Delta l = 1\,cm = 0{,}01\,m$

t_0: $v_0 = ?$

položaj ravnoteže

$a = 2\,cm = 0{,}02\,m$

$c = \frac{F}{\Delta l} = \frac{0{,}5\,N}{0{,}01\,m} = 50\,\frac{N}{m}$



položaj statičke ravnoteže:

$m\vec{a} = \vec{Q} + \vec{F}_{c,st}$

x: $0 = Q - F_{c,st}$

$0 = Q - c \cdot f_{st}$

proizvoljan položaj

$m\vec{a} = \vec{Q} + \vec{F_c}$

x: $m\ddot{x} = Q - F_c$

$\frac{Q}{g}\ddot{x} = Q - c(f_{st} + x)$

$\frac{Q}{g}\ddot{x} = \cancel{Q - cf_{st}} - cx \quad / \cdot \frac{g}{Q}$

$\ddot{x} + \frac{cg}{Q}x = 0 \quad , \quad \omega^2 = \frac{cg}{Q}$

$\ddot{x} + \omega^2 x = 0 \rightarrow \lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1,2} = 0 \pm \omega i = 0 \pm \sqrt{\frac{cg}{Q}} \cdot i$

$x = C_1 \cos\left(\sqrt{\frac{cg}{Q}}\,t\right) + C_2 \sin\left(\sqrt{\frac{cg}{Q}}\,t\right)$

$\dot{x} = -C_1 \cdot \sqrt{\frac{cg}{Q}} \sin\left(\sqrt{\frac{cg}{Q}}\,t\right) + C_2 \cdot \sqrt{\frac{cg}{Q}} \cdot \cos\left(\sqrt{\frac{cg}{Q}}\,t\right)$

$x(t_0) = 0 \rightarrow C_1 \cdot 1 + 0 = 0 \rightarrow C_1 = 0$

$\dot{x}(t_0) = 0 + C_2 \cdot \sqrt{\frac{cg}{Q}} \cdot 1 = v_0 \quad , \quad C_2 = ?$

$a^2 = C_1^2 + C_2^2 \rightarrow C_2^2 = a^2 - C_1^2 = 0{,}0004 - 0 \rightarrow C_2 = 0{,}02$

$v_0 = 0{,}02 \cdot \sqrt{\frac{50 \cdot 9{,}81}{2}} = \sqrt{0{,}0004 \cdot \frac{50 \cdot 9{,}81}{2}} = \sqrt{0{,}0981}\ \frac{m}{s}$

$$\boxed{v_0 = \sqrt{981}\ \frac{cm}{s}}$$

(4.2.) $Q = 4{,}9\ N$

$T_\omega = \frac{\pi}{4}\ s$

$F = ?$ – da li $\Delta l = 1 cm = 0{,}01 m$

$F = c \cdot \Delta l \quad , \quad c = ?$

$T_\omega = \frac{2\pi}{\omega} \quad \rightarrow \quad \omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{\frac{\pi}{4}} = 8\ s^{-1}$

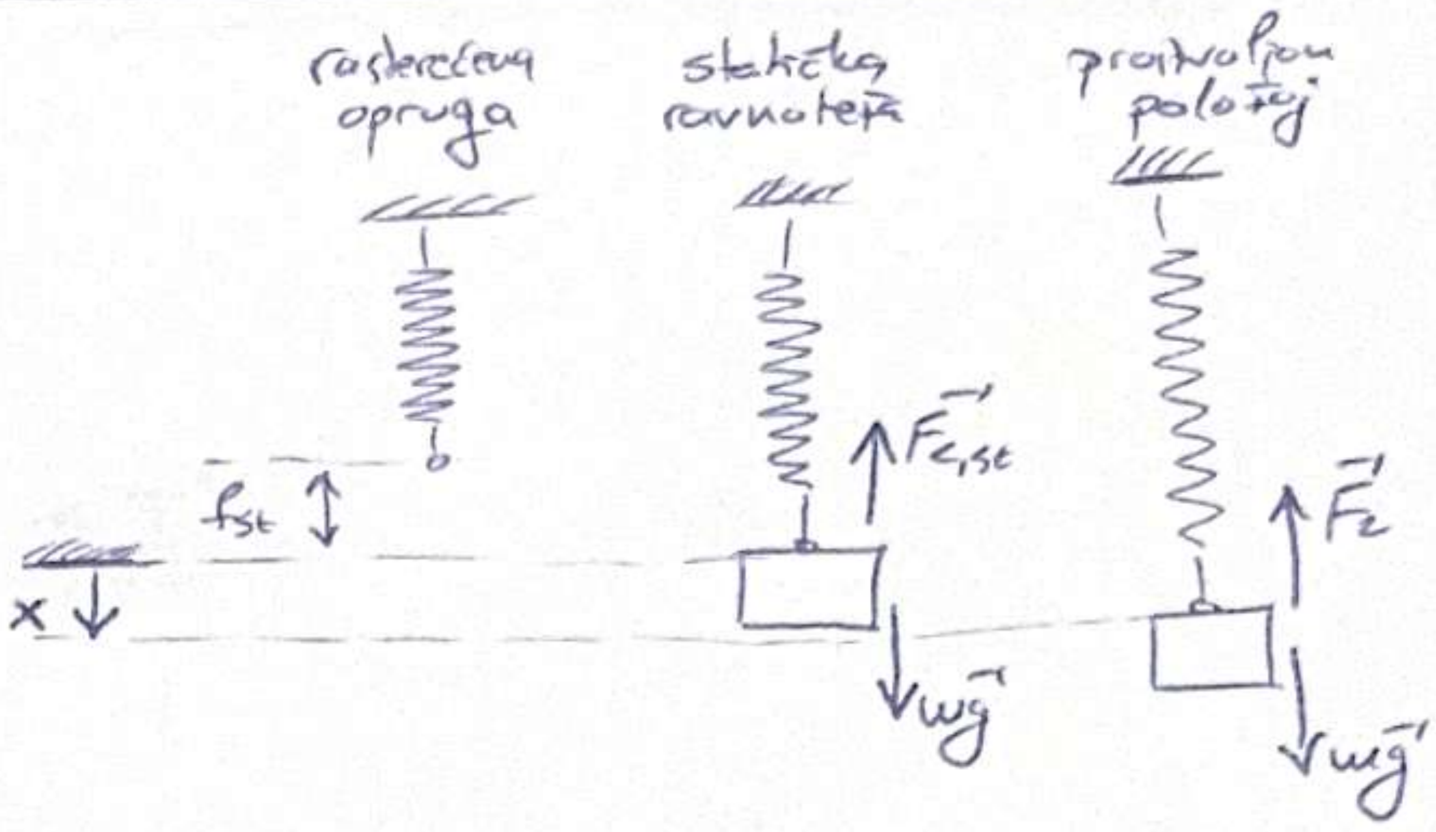
$\omega = \sqrt{\frac{c}{m}} \quad \rightarrow \quad \omega^2 = \frac{c}{m} \quad \rightarrow \quad c = \omega^2 \cdot m = \omega^2 \frac{Q}{g}$

$F = c \cdot \Delta l = \omega^2 \frac{Q}{g} \cdot \Delta l = 64 \cdot \frac{4{,}9}{9{,}81} \cdot 0{,}01 = 0{,}32\ N$

$\boxed{F = 0{,}32\ N}$

(4.3) $x = 10 \sin(7t)$

$\Delta \ell = ?$ - za $x=0$



$(f_{st} = ?)$

$\omega = 7 \quad \Rightarrow \quad \omega^2 = \frac{c}{m} = 49$

statička ravnoteža

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

x: $0 = mg - F_{c,st}$

$0 = mg - c f_{st}$

$$f_{st} = \frac{mg}{c} = g \cdot \frac{1}{\frac{c}{m}} = g \cdot \frac{1}{\omega^2} = \frac{9{,}81}{49} = 0{,}2\ m$$

$$\boxed{f_{st} = 20\ cm}$$

4.4.

opruga: c

$P = ?$ – težina

period $T_\omega = T$

$$T_\omega = \frac{2\pi}{\omega} \quad \rightarrow \quad \omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{T}$$

$$\omega^2 = \frac{c}{m} \quad \rightarrow \quad m = \frac{c}{\omega^2} = \frac{c}{\frac{4\pi^2}{T^2}} = \frac{cT^2}{4\pi^2}$$

$$P = mg \quad \rightarrow \quad \boxed{P = \frac{cgT^2}{4\pi^2}}$$

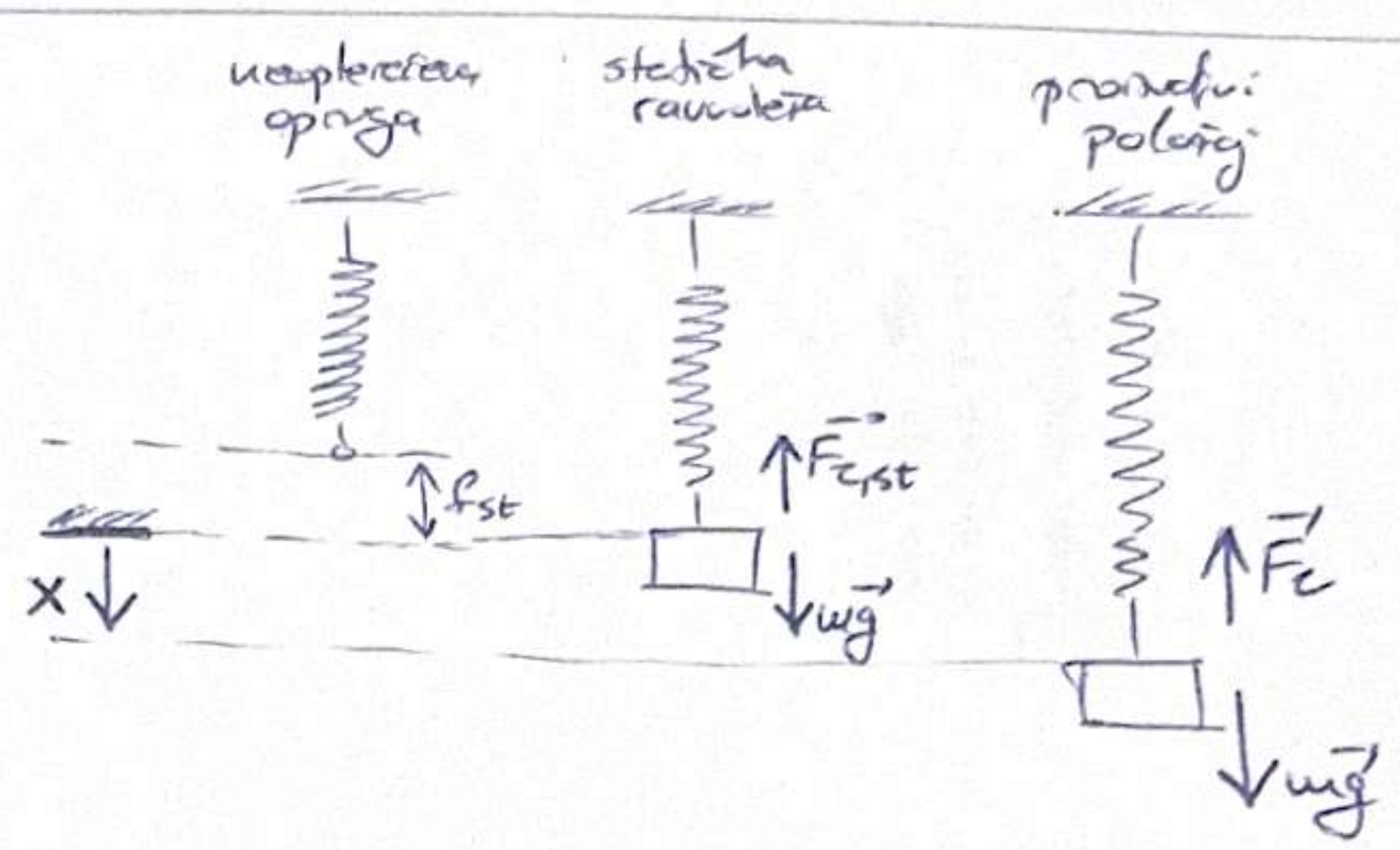
4.5.

$f_{st} = 10\ cm = 0{,}1\ m$

t_0: $x_0 = 15\ cm = 0{,}15\ m$

$v_0 = 99\ \frac{cm}{s} = 99 \cdot \frac{0{,}01\ m}{s} = 0{,}99\ \frac{m}{s}$

$x(t) = ?$



statička ravnoteža:

$m\vec a = m\vec g + \vec F_{c,st}$

x: $0 = mg - F_{c,st}$

$0 = mg - c f_{st}$

$mg = c f_{st}$

$\frac{c}{m} = \frac{g}{f_{st}} = \frac{9{,}81}{0{,}1} = 98{,}1\ s^{-2}$

$\omega^2 = 98{,}1\ s^{-2} \rightarrow \omega = 9{,}9\ s^{-1}$

proizvoljni položaj

$m\vec a = m\vec g + \vec F_c$

x: $m\ddot x = mg - F_c$

$m\ddot x = mg - c(f_{st} + x)$

$m\ddot x = \cancel{mg - c f_{st}} - cx \quad /:m$

$\ddot x + \frac{c}{m} x = 0$

$\ddot x + \omega^2 x = 0 \rightarrow \lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot x = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = 0{,}15 \rightarrow C_1 = 0{,}15$

$\dot x(t_0) = 0 + C_2 \cdot 9{,}9 \cdot 1 = 0{,}99 \rightarrow C_2 = \frac{0{,}99}{9{,}9} = 0{,}1$

$$\boxed{x = 0{,}15 \cos(9{,}9 t) + 0{,}1 \sin(9{,}9 t)}$$

4.6.

$T_{\omega_1} = T$

oprugu izdelimo na 3 dela, jedan odbacimo, a druga dva vežemo paralelno

$T_{\omega_2} = ?$

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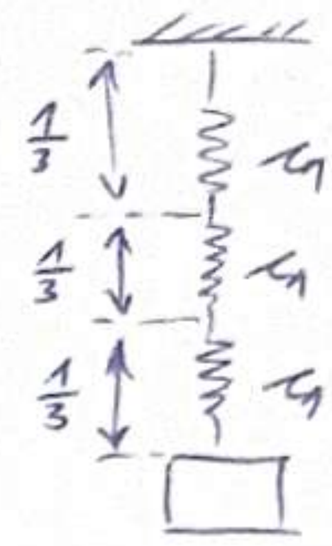
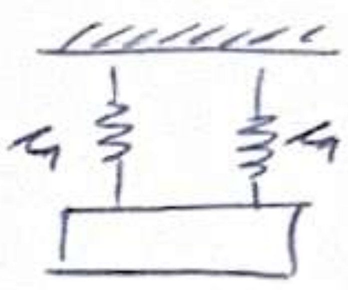
c_1, c_2, c_3 – redna veza



$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_1} + \frac{1}{c_1} = \frac{3}{c_1}$$

$$c_1 = \frac{3c}{1}$$

$$c_2 = c_1 + c_1 = 2c_1 = 6c$$



$$c_2 = 6c$$

$$T_{\omega_1} = T = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\frac{c}{m}}}$$

$$T_{\omega_2} = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{\frac{c_2}{m}}} = \frac{2\pi}{\sqrt{\frac{6c}{m}}} = \frac{2\pi}{\sqrt{6} \cdot \sqrt{\frac{c}{m}}} = \frac{T}{\sqrt{6}}$$

$$\boxed{T_{\omega_2} = \frac{T}{\sqrt{6}}}$$

(4.7) $M: m$

c_1, c_2, c_3

$T_\omega = ?$

$$c_e = c_1 + c_2 + c_3$$

$$T_\omega = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{c_e}{m}}} = 2\pi \cdot \sqrt{\frac{m}{c_e}}$$

$$\boxed{T_\omega = 2\pi \cdot \sqrt{\frac{m}{c_1 + c_2 + c_3}}}$$

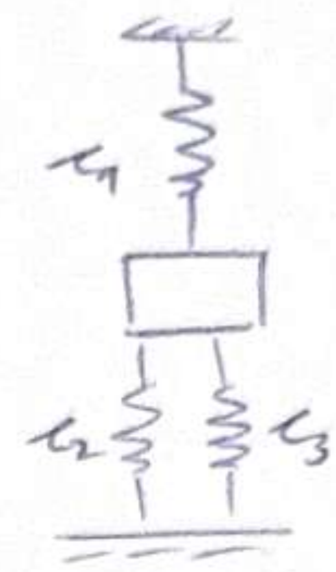
4.8.

$m = 1\ kg$

$c_1 = 2\frac{N}{cm} = 2\frac{N}{0{,}01m} = 200\frac{N}{m}$

$c_2 = c_3 = 7\frac{N}{cm} = 700\frac{N}{m}$

$\omega = ?$



$c_e = c_1 + c_2 + c_3 = 1600\frac{N}{m}$

$\omega = \sqrt{\frac{c}{m}} = \sqrt{\frac{1600}{1}}$

$\boxed{\omega = 40\ s^{-1}}$

4.11.

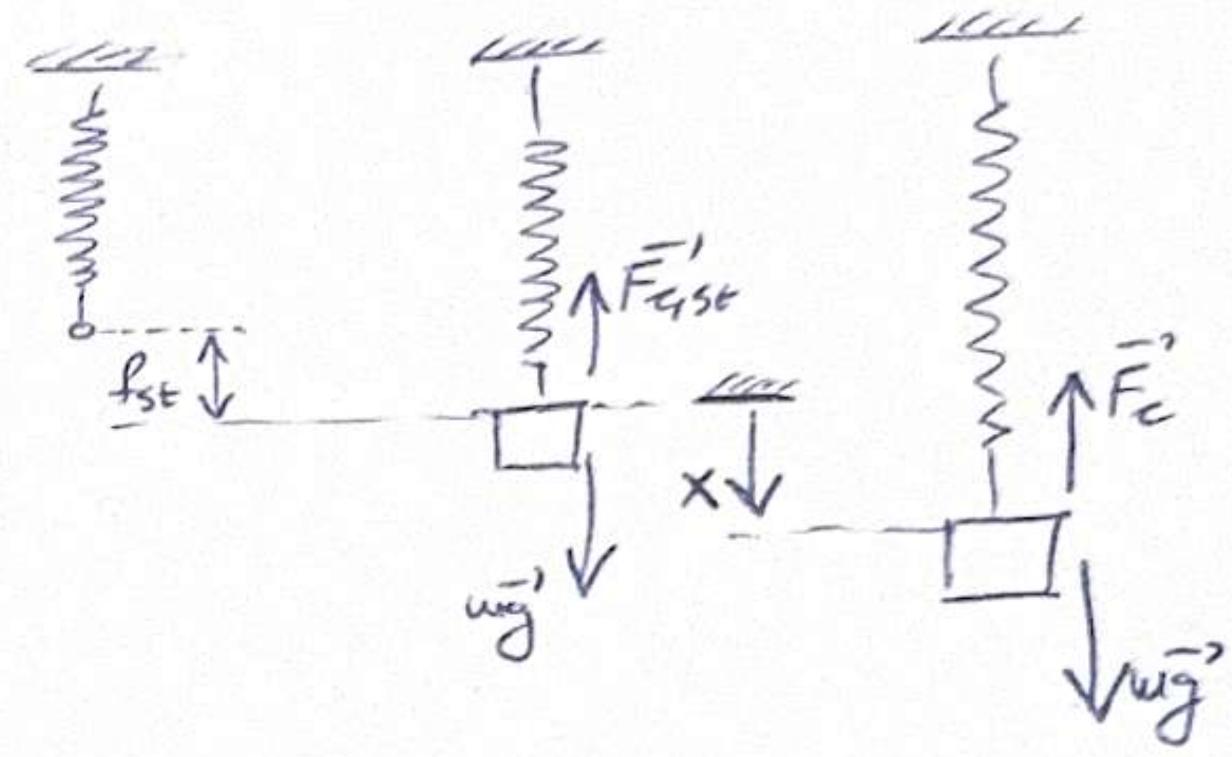
$m = 16 \text{ kg}$

$c = 4 \frac{N}{cm} = 4 \frac{N}{0{,}01 m} = 400 \frac{N}{m}$

$t_0:$ $x_0 = 8 \text{ cm} = 0{,}08 \text{ m}$

$v_0 = ?$

$a = 10 \text{ cm} = 0{,}1 \text{ m}$



statička ravnoteža

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

$x:$ $0 = mg - F_{c,st}$

$$0 = mg - c f_{st}$$

proizvoljan položaj

$$m\vec{a} = m\vec{g} + \vec{F}_c$$

$x:$ $m\ddot{x} = mg - F_c$

$$m\ddot{x} = mg - c(f_{st} + x)$$

$$m\ddot{x} = \cancel{mg - c f_{st}} - cx \qquad /:m$$

$$\ddot{x} + \frac{c}{m} x = 0 \quad , \quad \omega^2 = \frac{c}{m} = \frac{400}{16} = 25 \quad , \quad \omega = 5$$

$$\ddot{x} + \omega^2 x = 0 \quad \to \quad \lambda^2 + \omega^2 = 0 \quad \to \quad \lambda_{1,2} = 0 \pm \omega i$$

$$x = C_1 \cos \omega t + C_2 \sin \omega t$$

$$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$$

$$x(t_0) = C_1 \cdot 1 + 0 = 0{,}08 \text{ m} \quad \to \quad C_1 = 0{,}08$$

$$\dot{x}(t_0) = 0 + C_2 \omega \cdot 1 = v_0 \quad \to \quad C_2 = \frac{v_0}{\omega} = \frac{v_0}{5}$$

$$a^2 = C_1^2 + C_2^2 \quad \to \quad 0{,}01 = 0{,}064 + \frac{v_0^2}{25}$$

$$v_0^2 = (0{,}01 - 0{,}064) \cdot 25 = 0{,}09$$

$$\boxed{v_0 = 0{,}3 \frac{m}{s}}$$

4.12

$m = 1\,kg$

vertikalne oscilacije

$z = 2\sin(kt+1)$ [cm] $= 0{,}02\sin(kt+1)$ [m]

$v_{max} = 4\,\frac{cm}{s} = 4 \cdot \frac{0{,}01\,m}{s} = 0{,}04\,\frac{m}{s}$

$c = ?$

možemo ovo da razložimo:

$z = 0{,}02\sin(kt+1) = 0{,}02\sin(kt)\cos(1) + 0{,}02\cos(kt)\sin(1)$

u svakom slučaju, $\underline{\underline{\omega = k}}$

$\omega^2 = \frac{c}{m} \Rightarrow c = \omega^2 \cdot m = k^2 \cdot m$, $k = ?$

$\dot{z} = 0{,}02\,k\cos(kt+1)$

$\dot{z}_{max} = 0{,}02\,k \cdot 1 = 0{,}04\,\frac{m}{s} \rightarrow k = \frac{0{,}04}{0{,}02} = 2$

$\boxed{c = 4 \cdot 1 = 4\,\frac{N}{m}}$

(4.13) $T_\omega = \frac{\pi}{2}\,s$

$\alpha = ?$ – fazna razlika

$t_0:$ $x_0 = 1\,cm = 0{,}01\,m$

$\dot{x}_0 = 4\,\frac{cm}{s} = 0{,}04\,\frac{m}{s}$

$T_\omega = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4\,s^{-1}$

$x = C_1 \cos(\omega t) + C_2 \sin(\omega t)$

$\dot{x} = -C_1 \omega \sin(\omega t) + C_2 \omega \cos(\omega t)$

$x(t_0) = C_1 \cdot 1 + 0 = 0{,}01 \rightarrow C_1 = 0{,}01$

$\dot{x}(t_0) = 0 + C_2 \cdot \omega \cdot 1 = 0{,}04 \rightarrow C_2 = \frac{0{,}04}{\omega} = \frac{0{,}04}{4} = 0{,}01$

$\alpha = \operatorname{arctg} \frac{C_2}{C_1} = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\boxed{\alpha = \frac{\pi}{4}}$

(4.14) $z = A \sin(kt)$

vertikalne oscilacije

$f_{st} = ?$

---

statička ravnoteža

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

z: $0 = mg - cf_{st}$

$$f_{st} = \frac{mg}{c} = \frac{g}{\frac{c}{m}} = \frac{g}{\omega^2}, \quad \omega = k$$

$$\boxed{f_{st} = \frac{g}{k^2}}$$

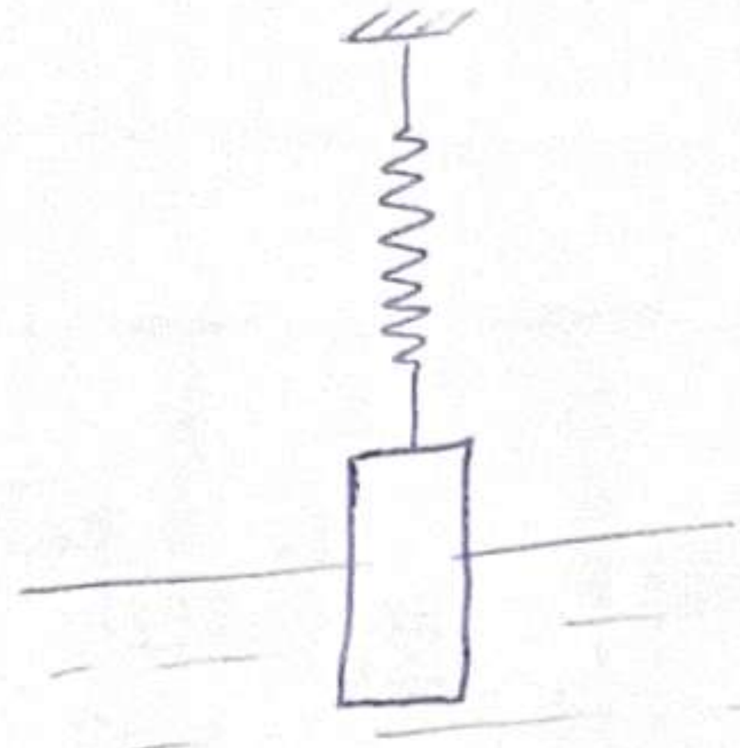
4.15. P, h, S

c

položaj ravnoteže: težište je na nivou površine tečnosti

μ - specifična težina tečnosti

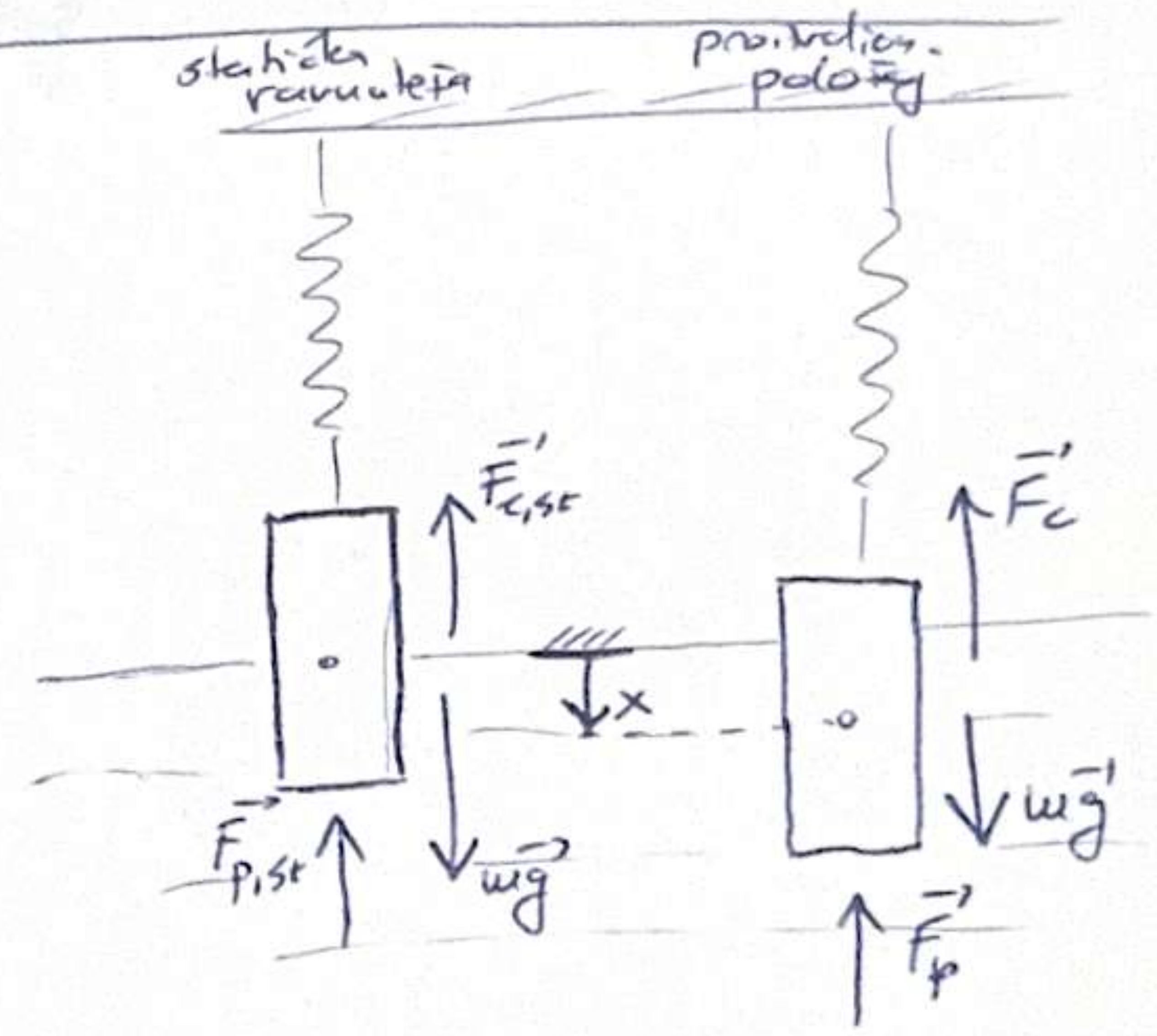
t_0: potopljen do $\frac{3}{4}h$
bez poč. brzine



$x(t) = ?$

Sila potiska je jednaka težini istisnute tečnosti:

$F_p = Q_t = \mu \cdot V$, V - zapremina dela tela koji je potopljen



stat. ravnoteža:

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st} + \vec{F}_{p,st}$$

x: $0 = mg - c f_{st} - \mu \cdot S \cdot \left(\frac{h}{2}\right)$

proizvoljan položaj

$$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_p$$

x: $m\ddot{x} = \cancel{mg} - c(\cancel{f_{st}} + x) - \mu \cdot S \cdot \left(\cancel{\frac{h}{2}} + x\right) \quad /: m$

$\ddot{x} + \frac{c}{m}x + \frac{\mu S}{m}x = 0$, $\omega^2 = \frac{c + \mu S}{m}$

$\ddot{x} + \omega^2 x = 0 \quad \rightarrow \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm \omega i$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 + 0 = \frac{1}{4}h \quad \rightarrow \quad C_1 = \frac{1}{4}h$

$\dot{x}(t_0) = 0 + C_2 = 0 \quad \rightarrow \quad C_2 = 0$

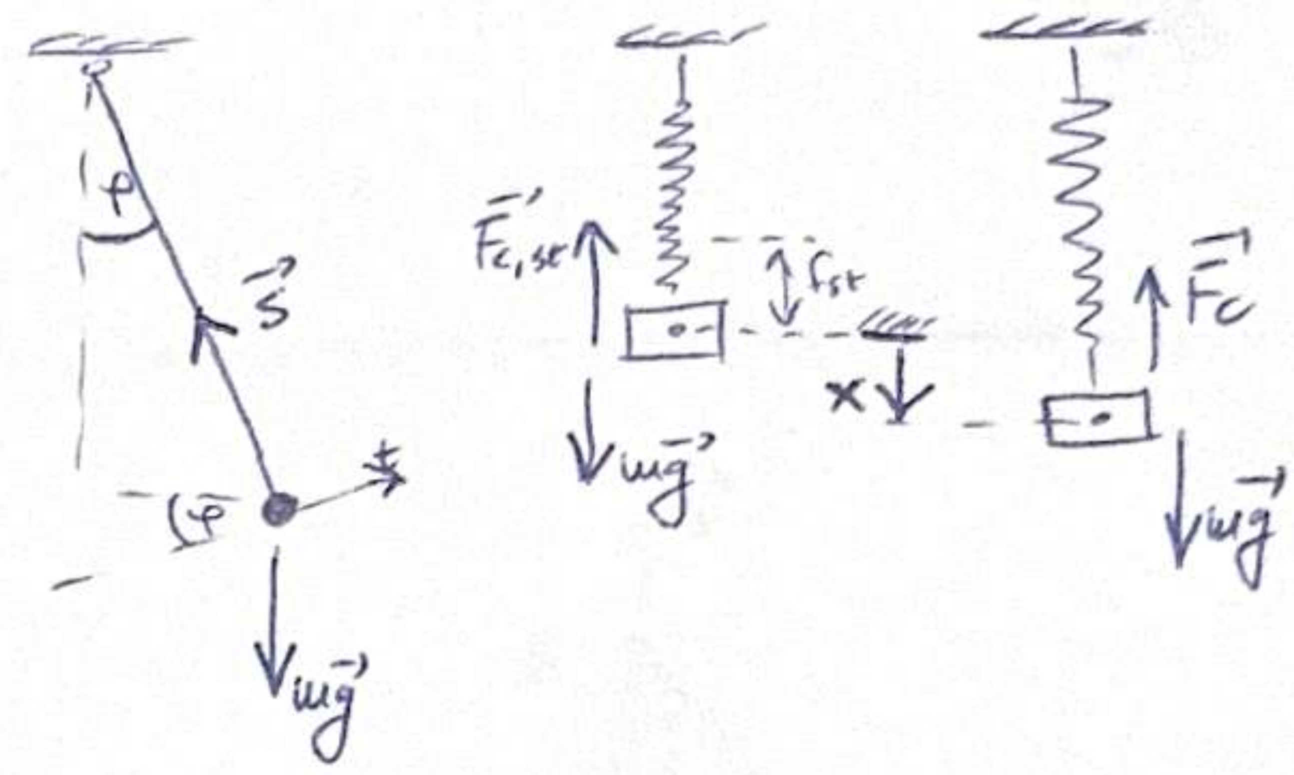
$$x = \frac{1}{4}h \cos\left(\sqrt{\frac{c + \mu S}{m}}\, t\right)$$

$$\boxed{x = \frac{1}{4}h \cos\left(\sqrt{\frac{(c + \mu S)g}{P}}\, t\right)}$$

4.17.

$l = ?$ - da li oscilacije matematičkog klatna imale isti period oscilovanja kao vertikalne oscilacije istog tereta obešenog o oprugu c

m, c, male oscilacije



klatno:

$$m\vec{a} = m\vec{g} + \vec{S}$$

t: $ma_t = -mg\sin\varphi$

$$\cancel{m}\, l\ddot{\varphi} = -\cancel{m}g\varphi$$

$$\ddot{\varphi} + \frac{g}{l}\varphi = 0 \quad , \quad \omega_1^2 = \frac{g}{l}$$

$$T_{\omega_1} = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$$

vertikalne oscilacije

st. ravn:

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

x: $0 = mg - cf_{st}$

proizvoljni položaj:

$$m\vec{a} = m\vec{g} + \vec{F}_c$$

x: $m\ddot{x} = \cancel{mg} - c(\cancel{f_{st}} + x)$

$$\ddot{x} + \frac{c}{m}x = 0 \quad , \quad \omega^2 = \sqrt{\frac{c}{m}}$$

$$T_{\omega_2} = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{\frac{c}{m}}} = 2\pi\sqrt{\frac{m}{c}}$$

$$T_{\omega_1} = T_{\omega_2} \Rightarrow \cancel{2\pi}\sqrt{\frac{l}{g}} = \cancel{2\pi}\sqrt{\frac{m}{c}} \quad /^2 \quad \Rightarrow \frac{l}{g} = \frac{m}{c}$$

$$\boxed{l = \frac{mg}{c}}$$

4.18.

l - dužina klatna

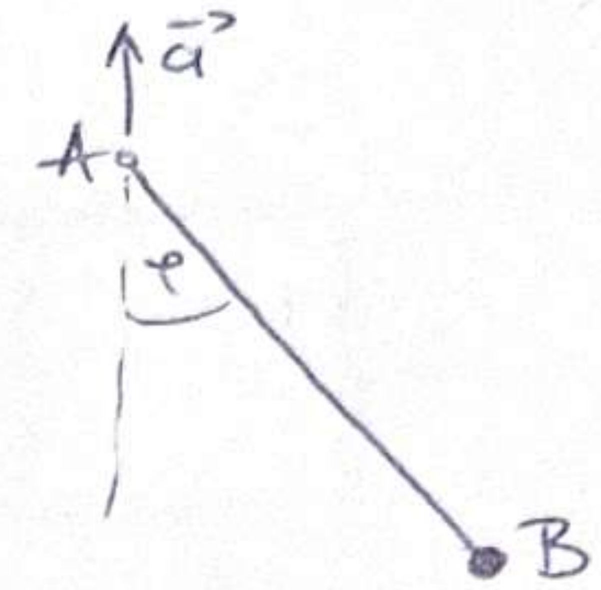
a - ubrzanje tačke o koju je obešeno klatno

$T_\omega = ?$

male oscilacije

a) ubrzanje a je naviše

b) ubrzanje a je naniže $(a<g)$

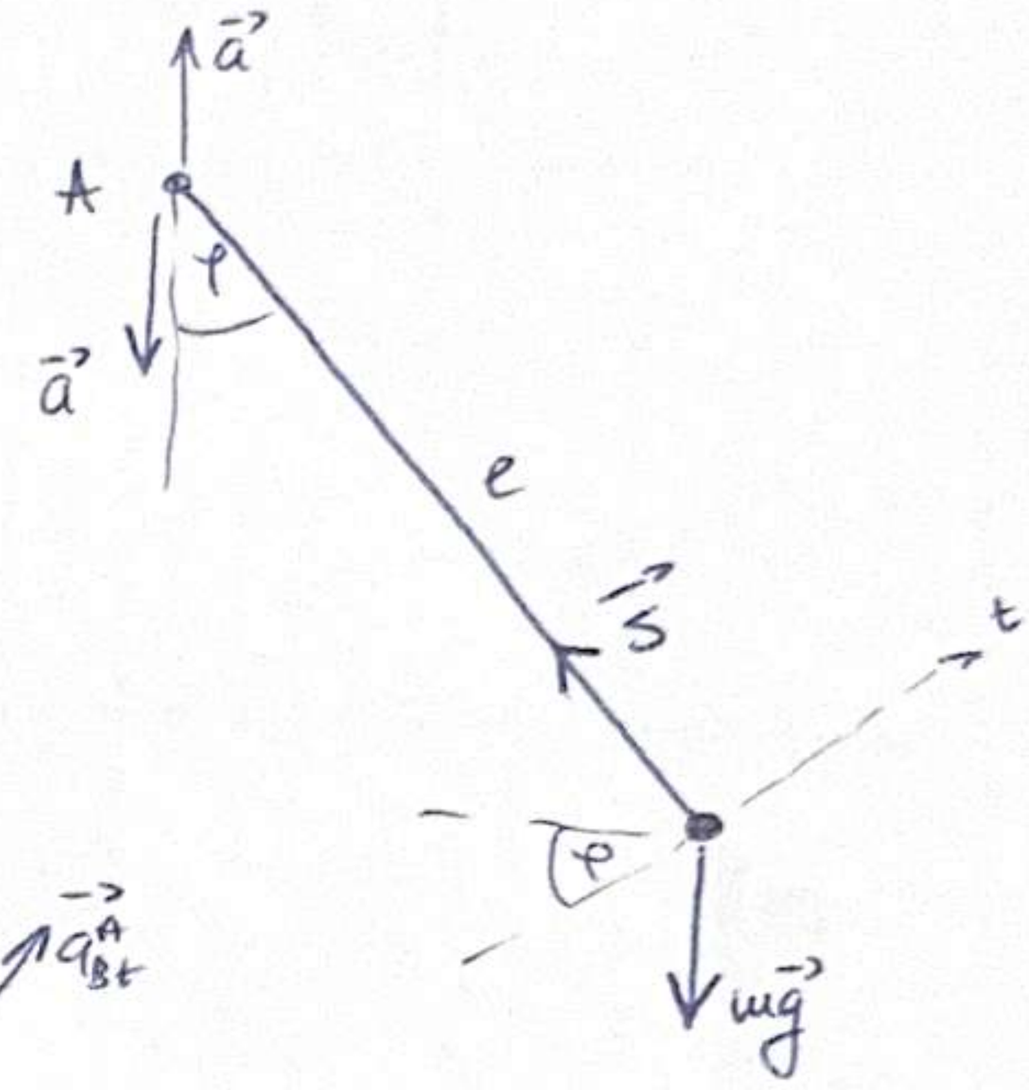


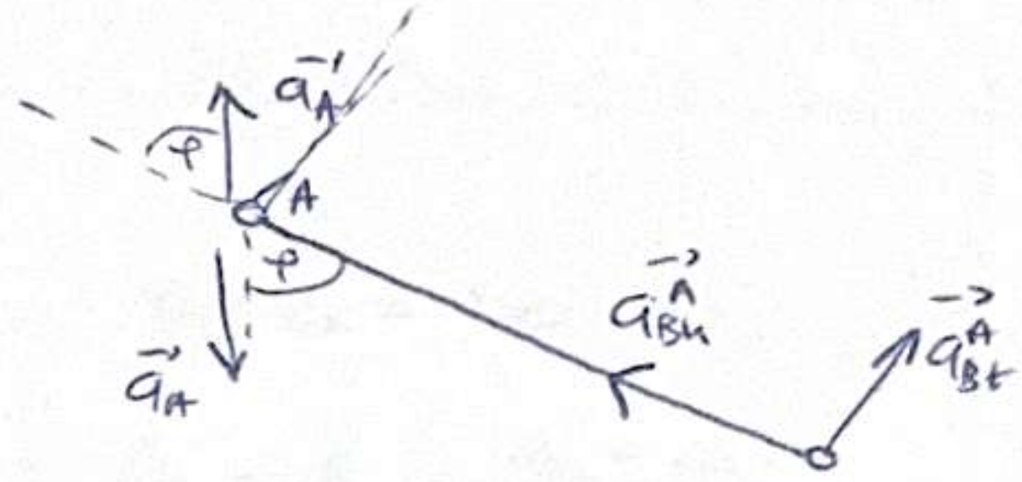
$$m\vec{a} = m\vec{g} + \vec{S}$$

t: $m a_t = -mg \sin\varphi$

oprez: $a_t \neq l \cdot \ddot{\varphi}$!

$$\vec{a}_B = \vec{a}_A + \vec{a}_{Bn}^A + \vec{a}_{Bt}^A$$





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a) t: $a_{Bt} = a_A \sin\varphi + 0 + a_{Bt}^A$

$$a_t = a \cdot \varphi + l\ddot{\varphi}$$

$$\cancel{m} a\varphi + \cancel{m} l\ddot{\varphi} = -\cancel{m} g\varphi \quad /:l$$

$$\ddot{\varphi} + \left(\frac{a}{l} + \frac{g}{l}\right)\varphi = 0 \quad , \quad \omega^2 = \frac{a+g}{l}$$

$$T_{\omega_1} = \frac{2\pi}{\omega} \rightarrow \boxed{T_{\omega_1} = 2\pi\sqrt{\frac{l}{a+g}}}$$

b) t: $a_{Bt} = -a_A \sin\varphi + a_{Bt}^A$

$$a_t = -a\varphi + l\ddot{\varphi}$$

$$-ma\varphi + ml\ddot{\varphi} = -mg\varphi \quad /:l$$

$$\ddot{\varphi} + \left(\frac{g}{l} - \frac{a}{l}\right)\varphi = 0 \quad , \quad \omega^2 = \frac{g-a}{l}$$

$$T_{\omega_2} = \frac{2\pi}{\omega} \rightarrow \boxed{T_{\omega_2} = 2\pi\sqrt{\frac{l}{g-a}}}$$

(4.19.) pravi kružni valjak

G – težina

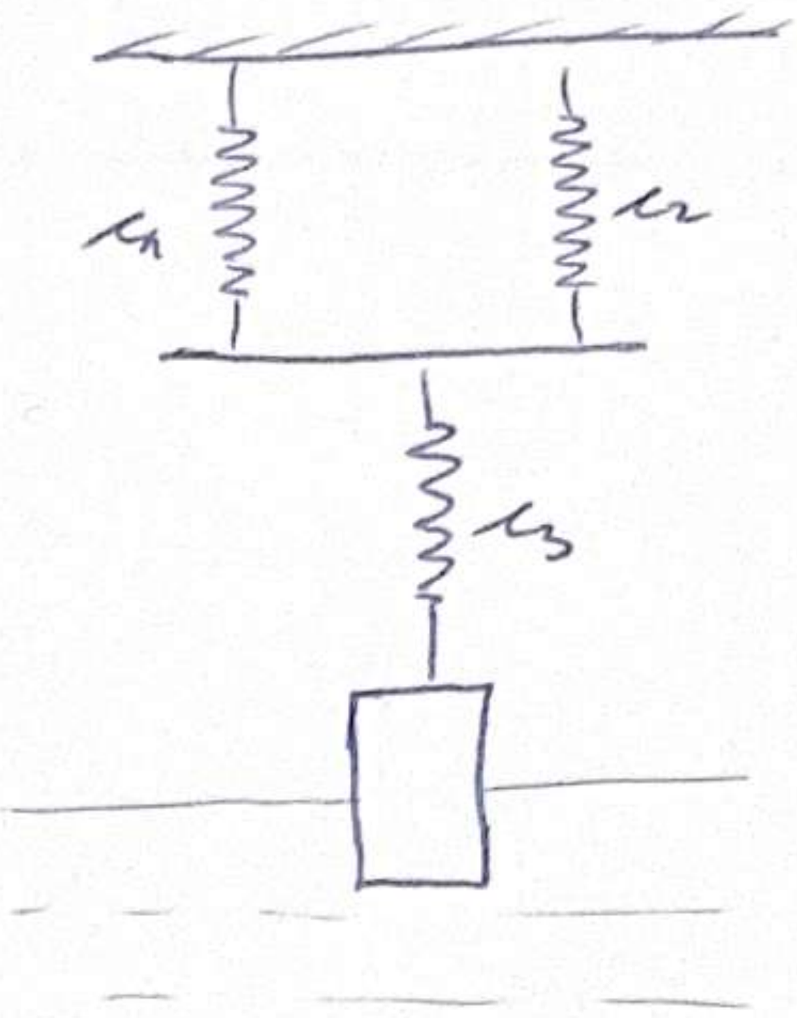
r – poluprečnik osnove

h – visina

$c_1 = c_2 = c \quad , \quad c_3 = 2c$

težina specifične težine μ

ravnotežni položaj: valjak potopljen do polovine svoje težine



t_0: potopljen $\frac{2}{3}h \quad , \quad v_0 = 0$

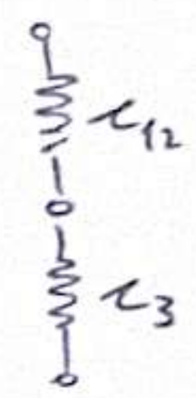
$x = ?$

---

1 : 2: paralelna veza $\rightarrow \quad c_{e12} = c_1 + c_2 = 2c$

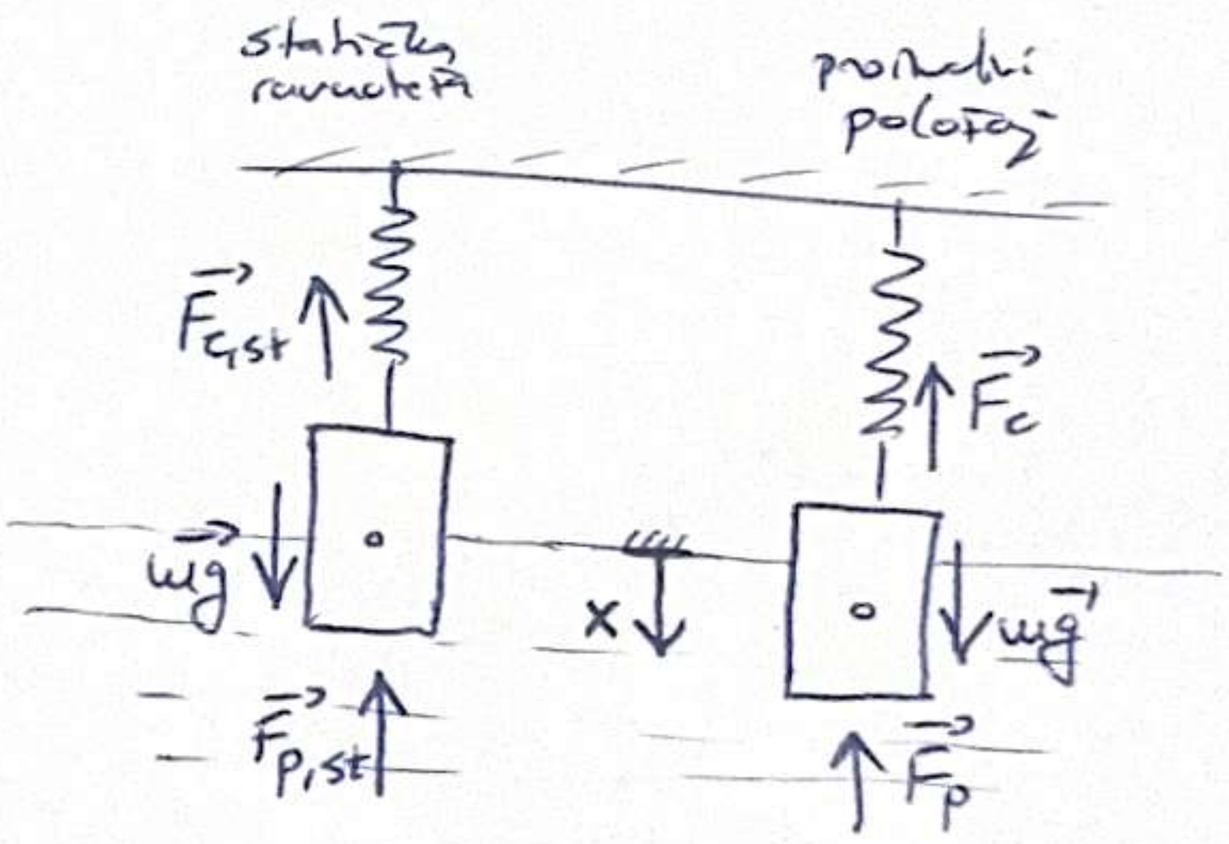
12 : 3: redna veza $\rightarrow \quad \frac{1}{c_e} = \frac{1}{c_{12}} + \frac{1}{c_3} = \frac{c_3 + c_{12}}{c_{12} \cdot c_3}$

$\hookrightarrow \quad c_e = \frac{c_{12} \cdot c_3}{c_{12} + c_3} = \frac{2c \cdot 2c}{2c + 2c} = c$



statička ravnoteža

(napomena: $F_p = Q_t = \mu \cdot V_p$)



$$m\vec{a} = m\vec{g} + \vec{F}_{c,st} + \vec{F}_{p,st}$$

x: $\quad 0 = mg - c f_{st} - \mu (r^2 \pi) \frac{h}{2}$

proizvoljni položaj

$$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_p$$

x: $\quad m\ddot{x} = \cancel{mg} - c(\cancel{f_{st}} + x) - \mu (r^2\pi)(\cancel{\tfrac{h}{2}} + x) \quad /:m$

$\ddot{x} + \frac{c}{m} x + \frac{\mu r^2 \pi}{m} x = 0 \quad , \quad \omega^2 = \frac{c + \mu r^2 \pi}{m} = \frac{(c + \mu r^2 \pi) g}{G}$

$\ddot{x} + \omega^2 x = 0 \quad \rightarrow \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm \omega i$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = - C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = \left(\frac{2}{3}h - \frac{1}{2}h\right) = \frac{1}{6}h \quad \rightarrow \quad C_1 = \frac{1}{6}h$

$\dot{x}(t_0) = 0 + C_2 \cdot 1 = 0 \quad \rightarrow \quad C_2 = 0$

$$\boxed{x = \frac{1}{6} h \cos\left(\sqrt{\frac{(c + \mu r^2 \pi) g}{G}}\, t\right)}$$

4.20. nastavak na prethodni zadatak

$v_0 = ?$ - da bi potpuno izronio iz vode

$x(t_0) = 0$

$\dot{x}(t_0) = v_0 = ?$

$a = \frac{h}{2}$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = 0 \quad \rightarrow \quad C_1 = 0$

$\dot{x}(t_0) = 0 + C_2 \omega \cdot 1 = v_0 \quad \rightarrow \quad C_2 = \frac{v_0}{\omega}$

$a^2 = C_1^2 + C_2^2$

$\frac{h^2}{4} = \frac{v_0^2}{\omega^2}$

$v_0 = \frac{h \cdot \omega}{2}$

$$\boxed{v_0 \geq \frac{h}{2} \sqrt{\frac{(c + \mu r^2 \pi) g}{G}}}$$

(4.21) telo mase m

opruga krutosti c

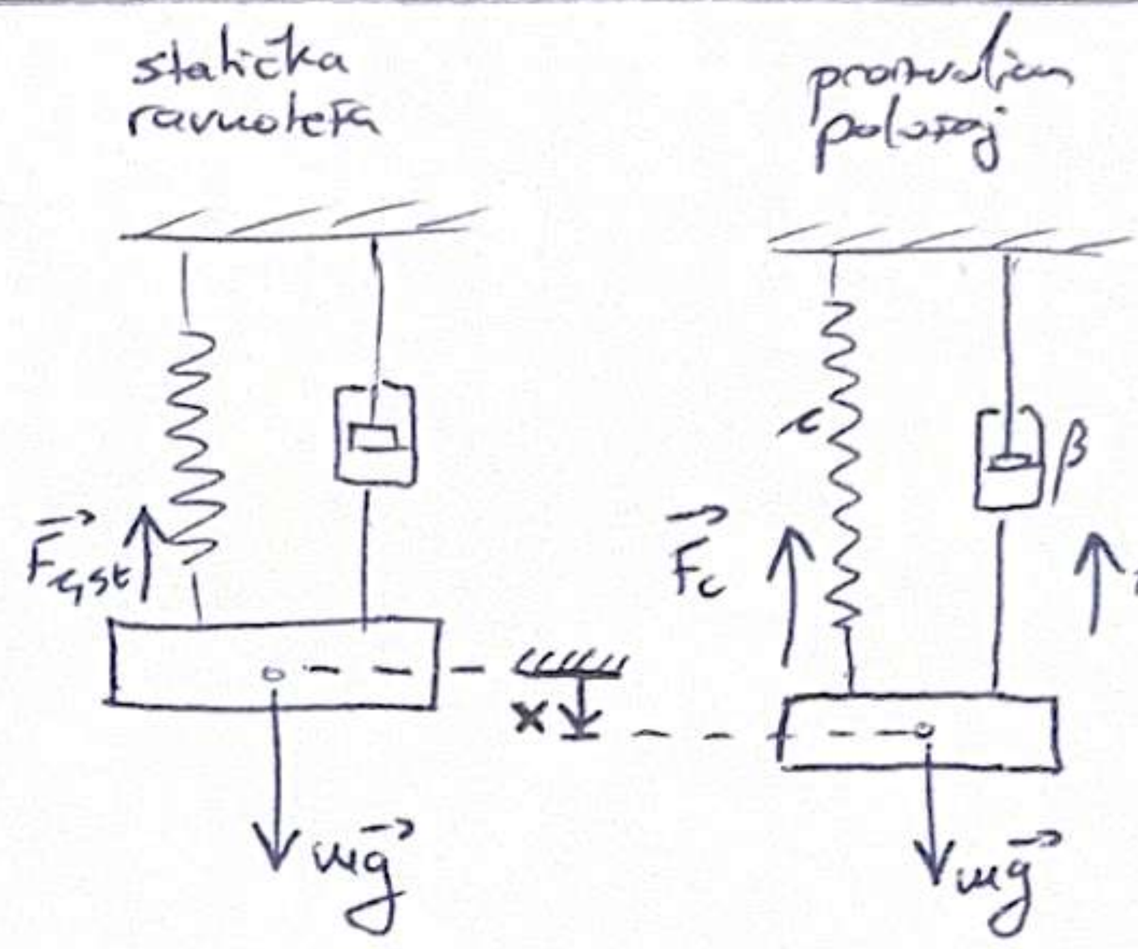
vertikalne oscilacije

otpor β

t_0: x_0 - vertikalno naniže

v_0 - (verovatno naniže?)

$x(t) = ?$



statička ravnoteža

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

x: $0 = mg - c f_{st}$

proizvoljni položaj

$$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_\beta$$

x: $m\ddot{x} = \cancel{mg} - c(\cancel{f_{st}} + x) - \beta\dot{x}$ $\quad /:m$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 0$, $\omega^2 = \frac{c}{m}$, $2\delta = \frac{\beta}{m}$

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = 0$

$\lambda^2 + 2\delta\lambda + \omega^2 = 0 \quad \to \quad \lambda_{1/2} = \frac{-2\delta \pm \sqrt{4\delta^2 - 4\omega^2}}{2} = -\delta \pm \sqrt{\delta^2 - \omega^2}$

1) $\delta < \omega \quad \to \quad -p^2 = \delta^2 - \omega^2$

$\lambda_{1/2} = -\delta \pm pi$

$x = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$

$\dot{x} = -\delta e^{-\delta t}(C_1 \cos pt + C_2 \sin pt) +$
$+ e^{-\delta t} \cdot p \cdot (-C_1 \sin pt + C_2 \cos pt)$

$x(t_0) = 1 \cdot (C_1 + 0) = x_0 \quad \to \quad C_1 = x_0$

$\dot{x}(t_0) = -\delta(x_0 + 0) + p(0 + C_2) \to C_2 = \frac{v_0 + \delta x_0}{p}$

$$\boxed{x = e^{-\delta t}\left(x_0 \cos pt + \frac{v_0 + \delta x_0}{p}\sin pt\right)}$$

2) $\delta = \omega \quad \to \quad \lambda_{1/2} = -\delta$ (dvostruko rešenje)

$x = e^{-\delta t}(C_1 t + C_2)$

$\dot{x} = -\delta e^{-\delta t}(C_1 t + C_2) + C_1 e^{-\delta t}$

$x(t_0) = 1(0 + C_2) = x_0 \quad \to \quad C_2 = x_0$

$\dot{x}(t_0) = -\delta(0 + x_0) + C_1 = v_0 \quad \to \quad C_1 = v_0 + x_0\delta$

$$\boxed{x = e^{-\delta t}\left((v_0 + x_0\delta)t + x_0\right)}$$

3) $\delta > \omega \quad \to \quad \nu^2 = \delta^2 - \omega^2$

$x = e^{-\delta t}(C_1 \cosh \nu t + C_2 \sinh \nu t)$

$\dot{x} = -\delta e^{-\delta t}(C_1 \cosh \nu t + C_2 \sinh \nu t) + e^{-\delta t} \cdot \nu (C_1 \sinh \nu t + C_2 \cosh \nu t)$

$x(t_0) = 1(C_1 + 0) = x_0 \quad \to \quad C_1 = x_0$

$\dot{x}(t_0) = -\delta(x_0 + 0) + \nu(0 + C_2) = v_0 \quad \to \quad C_2 = \frac{v_0 + x_0\delta}{\nu}$

$$\boxed{x = e^{-\delta t}\left(x_0 \cosh \nu t + \frac{v_0 + x_0\delta}{\nu}\sinh \nu t\right)}$$

(4.22) prigušene oscilacije imaju period oscilovanja 25 % veći od odgovarajućih slobodnih

nači: $\frac{\delta}{\omega}$?

$$T_\omega = \frac{2\pi}{\omega}$$

$$T_p = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\omega^2-\delta^2}}$$

$$\frac{T_p}{T_\omega} = 1{,}25 = \frac{5}{4}$$

$$\frac{\frac{2\pi}{\sqrt{\omega^2-\delta^2}}}{\frac{2\pi}{\omega}} = \frac{5}{4} \quad /^2$$

$$\frac{\omega^2}{\omega^2-\delta^2} = \frac{25}{16}$$

$$25\omega^2 - 25\delta^2 = 16\omega^2$$

$$9\omega^2 = 25\delta^2$$

$$\frac{\delta^2}{\omega^2} = \frac{9}{25} \quad /\sqrt{\ }$$

$$\boxed{\frac{\delta}{\omega} = \frac{3}{5}}$$

(4.23.) prigušene oscilacije

$D = e^{0,75\pi}$ - dekrement

naći $\frac{T_p}{T_\omega}$ odgovarajućih slobodnih oscilacija bez prigušenja

---

$$D = e^{\delta \frac{T_p}{2}} = e^{0,75\pi} \quad \Rightarrow \quad \delta \cdot \frac{T_p}{2} = 0,75\,\pi \quad \Rightarrow \quad \delta \cdot \frac{2\pi}{2p} = 0,75\,\pi$$

$$\delta \cdot \frac{1}{\sqrt{\omega^2 - \delta^2}} = \frac{3}{4} \quad \Rightarrow \quad \frac{\delta^2}{\omega^2 - \delta^2} = \frac{9}{16} \quad \Rightarrow \quad 16\,\delta^2 = 9\,\omega^2 - 9\,\delta^2$$

$$25\,\delta^2 = 9\,\omega^2 \quad \Rightarrow \quad \frac{\omega^2}{\delta^2} = \frac{25}{9} \quad \Rightarrow \quad \frac{\omega}{\delta} = \frac{5}{3}$$

$$\frac{T_p}{T_\omega} = \frac{\frac{2\pi}{\sqrt{\omega^2 - \delta^2}}}{\frac{2\pi}{\omega}} \quad /^2$$

$$\frac{T_p^2}{T_\omega^2} = \frac{\omega^2}{\omega^2 - \delta^2} = \frac{1}{\frac{\omega^2}{\omega^2} - \frac{\delta^2}{\omega^2}} = \frac{1}{1 - \left(\frac{3}{5}\right)^2} = \frac{1}{1 - \frac{9}{25}} = \frac{1}{\frac{16}{25}} = \frac{25}{16}$$

$$\boxed{\frac{T_p}{T_\omega} = \frac{5}{4}}$$

4.24. $Q = 1N$

prigušene oscilacije: $T_p = 0{,}102\ s$

$$\beta = \frac{0{,}02\ N}{1\ \frac{cm}{s}} = \frac{0{,}02\ N}{\frac{0{,}01\ m}{s}} = 2\ \frac{Ns}{m}$$

$$\frac{R_1}{R_5} = ?$$

$$D = \frac{R_n}{R_{n+1}} = e^{\delta \frac{T_p}{2}}$$

$$2\delta = \frac{\beta}{m} \Rightarrow \delta = \frac{\beta}{2m} = \frac{\beta g}{2Q}$$

$$D = e^{\frac{\beta g}{2Q} \cdot \frac{T_p}{2}} = e^{0{,}5}$$

$$R_2 = \frac{R_1}{D}$$

$$R_3 = \frac{R_2}{D} = \frac{R_1}{D^2}$$

$$R_4 = \frac{R_3}{D} = \frac{R_1}{D^3}$$

$$R_5 = \frac{R_4}{D} = \frac{R_1}{D^4}$$

$$\frac{R_1}{R_5} = \frac{R_1}{\frac{R_1}{D^4}} = D^4 = \left(e^{0{,}5}\right)^4 = e^2$$

$$\boxed{\frac{R_1}{R_5} = e^2}$$

4-25. $\frac{R_1}{R_7} = 10$

$\mathbb{D} = ?$

$$R_7 = \frac{R_1}{D^6}$$

$$\frac{R_1}{R_7} = \frac{R_1}{\frac{R_1}{D^6}} = 10 \quad \rightarrow \quad D^6 = 10 \quad \rightarrow \quad D = (10)^{\frac{1}{6}}$$

$$\mathbb{D} = \ln D = \ln (10)^{1/6} = \frac{1}{6} \ln 10$$

$$\boxed{\mathbb{D} = \frac{1}{6} \ln 10}$$

4.26. $m = 1\ kg$

$T_p = 0{,}25\ s$

$\mathbb{D} = 0{,}5$

$F_p = ?$ - za $v = 1\frac{cm}{s} = 0{,}01\frac{m}{s}$

$F_p = \beta \cdot \dot{x} = \beta \cdot 0{,}01$, $\beta = ?$

$2\delta = \frac{\beta}{m} \rightarrow \beta = 2\delta \cdot m$, $\delta = ?$

$\mathbb{D} = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \cdot \frac{T_p}{2} = 0{,}5 \rightarrow \delta = \frac{2 \cdot 0{,}5}{T_p} = \frac{1}{0{,}25} = 4$

$\beta = 2 \cdot 4 \cdot 1 = 8$

$F_p = 8 \cdot 0{,}01 = 0{,}08\ N$

$\boxed{F_p = 0{,}08\ N}$

(4.27.) $T_p = \frac{\pi}{2}$ s

$\mathbb{D} = \frac{\pi}{10}$

$x(t) = ?$

t_0: $x = 3$ cm $= 0{,}03$ m

$v_0 = 0$

$$x = e^{-\delta t}\left(C_1 \cos(pt) + C_2 \sin(pt)\right) \quad , \quad \delta = ? \ , \ p = ?$$

$$\dot{x} = -\delta e^{-\delta t}\left(C_1 \cos pt + C_2 \sin pt\right) + p e^{-\delta t}\left(-C_1 \sin pt + C_2 \cos pt\right)$$

$$\mathbb{D} = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \frac{T_p}{2} \quad \to \quad \delta = \frac{2 \cdot \mathbb{D}}{T_p} = \frac{2 \cdot \frac{\pi}{10}}{\frac{\pi}{2}} = \frac{4}{10} = \frac{2}{5}$$

$$T_p = \frac{2\pi}{p} \quad \to \quad p = \frac{2\pi}{T_p} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$x(t_0) = 1 \cdot (C_1 \cdot 1 + 0) = 0{,}03 \quad \to \quad C_1 = 0{,}03$$

$$\dot{x}(t_0) = -\frac{2}{5} \cdot 1\left(0{,}03 \cdot 1 + 0\right) + 4 \cdot 1\left(0 + C_2 \cdot 1\right) = 0$$

$$\to \quad -\frac{2}{5} \cdot \frac{3}{100} + 4C_2 = 0 \quad \to \quad 4C_2 = \frac{3}{250} \quad \to \quad C_2 = \frac{3}{1000} = 0{,}003$$

$$\boxed{x(t) = e^{-\frac{2}{5}t}\left(0{,}03 \cos 4t + 0{,}003 \sin 4t\right)}$$

4.28. $\frac{T_p}{T_\omega} = ?$

$\mathbb{D} = \frac{5\pi}{12}$

$$\frac{T_p}{T_\omega} = \frac{\frac{2\pi}{p}}{\frac{2\pi}{\omega}} = \frac{\omega}{p} = \frac{\omega}{\sqrt{\omega^2 - \delta^2}} \quad /^2$$

$$\frac{T_p^2}{T_\omega^2} = \frac{\omega^2}{\omega^2 - \delta^2} = \frac{1}{1 - \frac{\delta^2}{\omega^2}} \quad , \quad \frac{\delta}{\omega} = ?$$

$$\mathbb{D} = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \frac{T_p}{2}$$

$$T_p = \frac{\mathbb{D} \cdot 2}{\delta} = \frac{5}{6} \frac{\pi}{\delta}$$

$$T_p = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\omega^2 - \delta^2}}$$

$$\frac{2\pi}{\sqrt{\omega^2 - \delta^2}} = \frac{5}{6} \frac{\pi}{\delta} \quad /^2$$

$$\frac{4}{\omega^2 - \delta^2} = \frac{25}{36} \frac{1}{\delta^2}$$

$$144\delta^2 = 25\omega^2 - 25\delta^2$$

$$169\delta^2 = 25\omega^2$$

$$\frac{\delta^2}{\omega^2} = \frac{25}{169}$$

$$\frac{T_p^2}{T_\omega^2} = \frac{1}{1 - \frac{\delta^2}{\omega^2}} = \frac{1}{1 - \frac{25}{169}} = \frac{1}{\frac{144}{169}} = \frac{169}{144}$$

$$\boxed{\frac{T_p}{T_\omega} = \frac{13}{12}}$$

4.29. $m = 0{,}3$ kg

$x = 24\cos 13t$ - slobodne neprigušene

$x(t) = ?$ - slobodne prigušene pri istim početnim uslovima

$R = 3v \; [N \frac{m}{s}]$ - sila otpora

$\omega = 13$

$R = \beta \cdot v \quad \to \quad \underline{\underline{\beta = 3}}$

početni uslovi:

$x(t_0) = 24 \cdot 1 = 24$

$\dot{x} = -24 \cdot 13 \sin(13t)$

$\dot{x}(t_0) = 0$

$$x(t) = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$$

$$2\delta = \frac{\beta}{m} = \frac{3}{0{,}3} = 10 \quad \to \quad \underline{\underline{\delta = 5}}$$

$$p^2 = \omega^2 - \delta^2 = 169 - 25 = 144 \quad \to \quad \underline{\underline{p = 12}}$$

$$x(t) = e^{-5t}(C_1 \cos 12t + C_2 \sin 12t)$$

$$\dot{x}(t) = -5e^{-5t}(C_1 \cos 12t + C_2 \sin 12t) + 12e^{-5t}(-C_1 \sin 12t + C_2 \cos 12t)$$

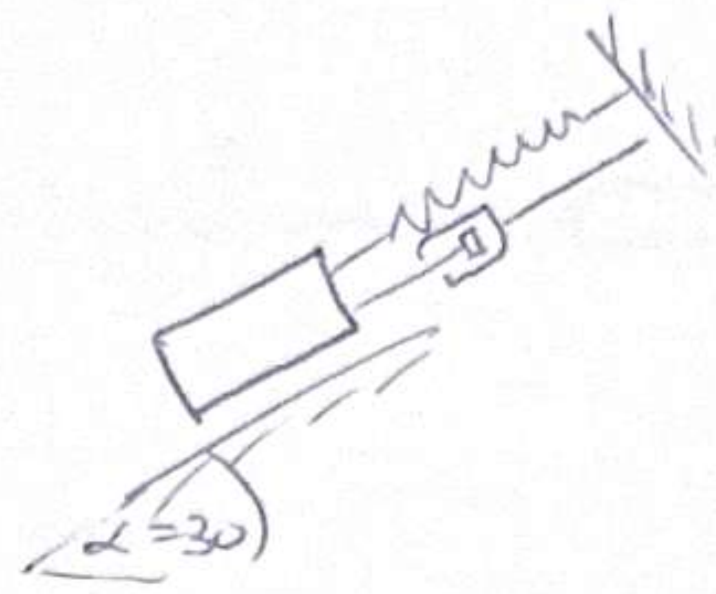
$$x(t_0) = 1 \cdot (C_1 + 0) = 24 \quad \to \quad C_1 = 24$$

$$\dot{x}(t_0) = -5 \cdot 1(24 \cdot 1 + 0) + 12 \cdot 1(0 + C_2 \cdot 1) = 0 \quad \to \quad 12C_2 = 120 \quad \to \quad C_2 = 10$$

$$\boxed{x(t) = e^{-5t}(24\cos(12t) + 10\sin(12t))}$$

(4.30.) $m, c, \alpha = 30°, \beta$

$\beta = ?$ - da li: $T_p = 2T_\omega$



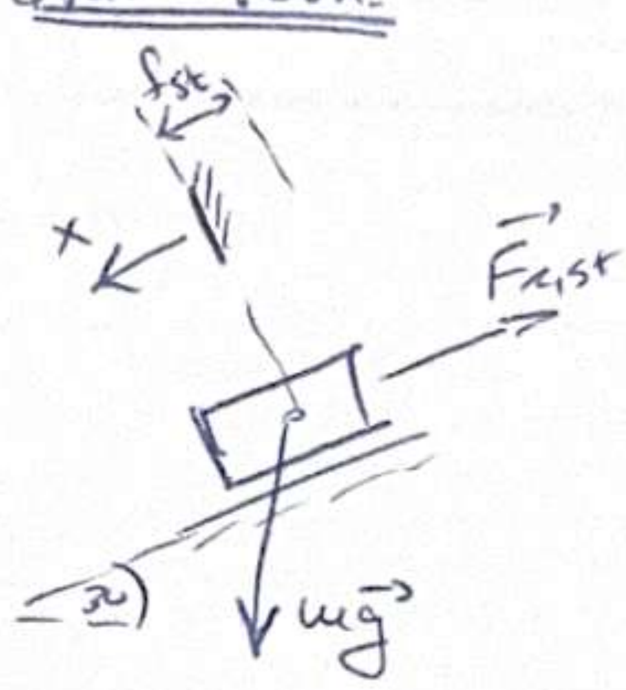
$T_\omega = \frac{2\pi}{\omega}$, $T_p = \frac{2\pi}{p}$, $p = \sqrt{\omega^2 - \delta^2}$, ali ...

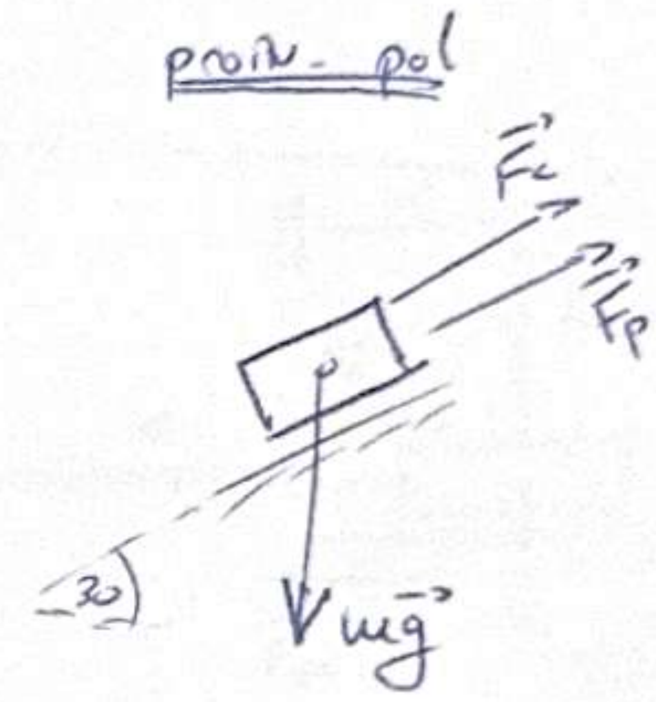
da li su su $\omega^2 = \frac{c}{m}$? i da li je $2\delta = \frac{\beta}{m}$? jer je pod uglom ...

stat. ravn.:

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

$x:$ $0 = mg\sin 30 - c f_{st}$





proiz. položaj:

$$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_{p_1}$$

$x:$ $m\ddot{x} = mg\sin 30 - c(x + f_{st}) - \beta\dot{x}$ $/:m$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 0$ => ok, liče $2\delta = \frac{\beta}{m}$ i $\omega^2 = \frac{c}{m}$

$\frac{T_p}{T_\omega} = 2$ -> $\frac{\frac{2\pi}{p}}{\frac{2\pi}{\omega}} = 2$ -> $\frac{\omega}{p} = 2$

$\frac{\omega}{\sqrt{\omega^2 - \delta^2}} = 2$ -> $\frac{\omega^2}{\omega^2 - \delta^2} = 4$ -> $4\omega^2 - 4\delta^2 = \omega^2$

$4\delta^2 = 3\omega^2$ -> $4 \cdot \left(\frac{\beta}{2m}\right)^2 = 3\frac{c}{m}$ -> $\frac{\beta^2}{m^2} = 3\frac{c}{m}$

$$\boxed{\beta = \sqrt{3cm}}$$

(4.31) $Q = 20\,N$

$c_1 = 1{,}5\frac{N}{cm} = 1{,}5\frac{N}{0{,}01\,m} = 150\frac{N}{m}$

$c_2 = 2{,}5\frac{N}{cm} = 250\frac{N}{m}$

položaj 0: neuapregnute opruge

t_0: $x_0 = 15\,cm = 0{,}15\,m$, $v_0 = 0$

t_1: telo se zaustavilo

$t_1 = ?$, $x(t_1) = ?$

$\mu_1 = 0{,}2$ – koef. trenja mirovanja

$\mu_2 = 0{,}1$ – koef. trenja pri kretanju

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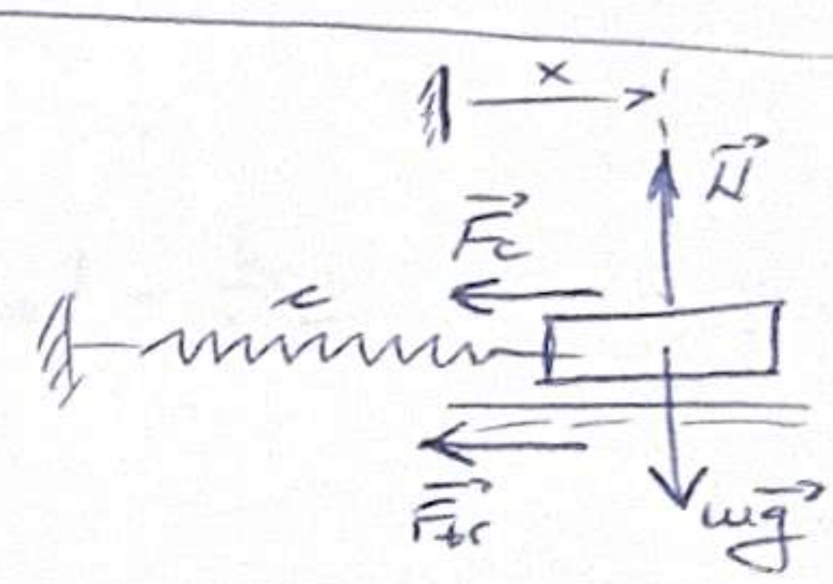
paralelne opruge: $c_e = c_1 + c_2 = 400 = c$

$f_{st} = 0$ (horizontalne oscilacije)



uslov da se telo zaustavi:

↳ da u amplitudnom položaju bude $\Sigma F_{x_i} = 0$

$\Sigma F_{x_i} = 0 \Rightarrow F_c - F_{tr} = c \cdot x_k - \mu_1 \cdot N = 0$, $N = mg$ (jer je $m\ddot{y} = mg - N = 0$)

$\Rightarrow x_k = \frac{\mu_1 \cdot mg}{c} = 0{,}01\,m$

zadatak: naći $x_1, x_2, \dots, x_k$! (to su amplitude, tj. koordinate pri svakom zaustavljanju)

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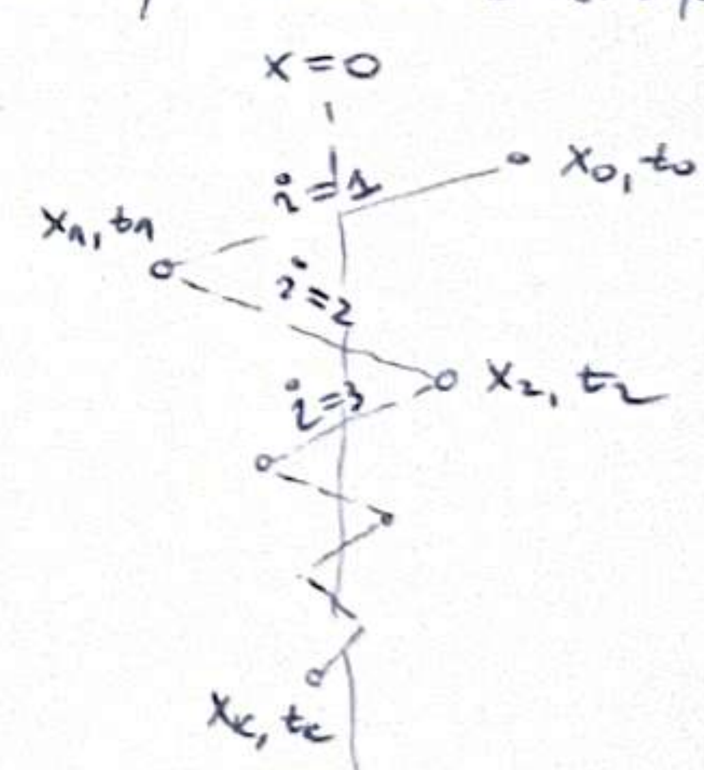
svaki prolaz od jednog do drugog amplitudnog položaja je druga dif. j-na, jer sila trenja deluje u suprotnom smeru



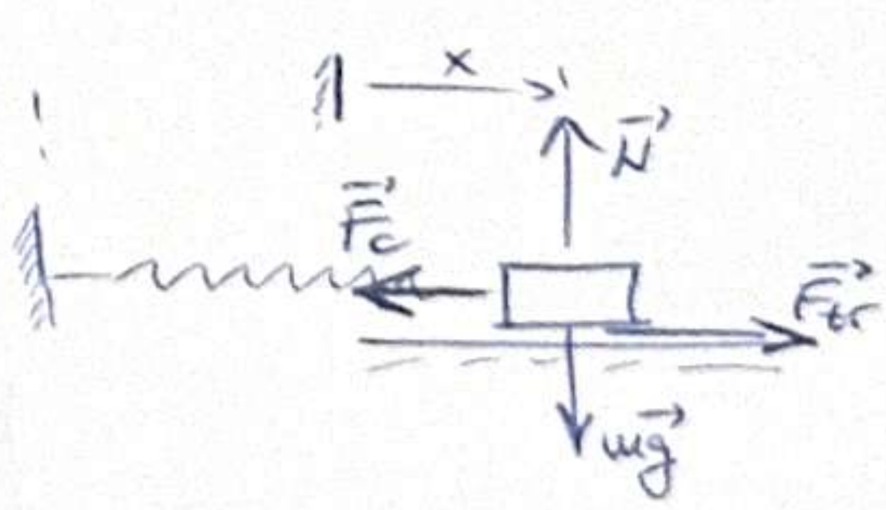
posmatrajmo i-ti prolaz, uzećemo da je $x > 0$, $\dot{x} < 0$:

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F_c} + \vec{F_{tr}}$ ← j-na koja važi za i-ti prolaz, od t_{i-1} do t_i

x: $m\ddot{x} = -cx + \mu_2 mg$ $\quad /:m$

$\ddot{x} + \frac{c}{m}x = \mu_2 g$

$\ddot{x} + \omega^2 x = h$



H: $\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i$

$x_h = A_i \cos(\omega t) + B \sin(\omega t)$

za t_{i-1} → desni amplitudni položaj
$\cos(\omega t_{i-1}) = +1$
$\sin(\omega t_{i-1}) = 0$

za t_i → levi amplitudni položaj
$\cos(\omega t_i) = -1$
$\sin(\omega t_i) = 0$

$P: \quad \mathcal{b}(t) = \mu_2 g = e^{0t}(\mu_2 g \cos 0t + 0 \sin 0t)$

$x_p = t^0 e^{0t}(C \cos 0t + D \sin 0t) = C \quad , \quad \ddot{x}_p = 0$

$0 + w^2 C = h \quad \rightarrow \quad C = \frac{h}{w^2}$

$x = A_i \cos(wt) + B_i \sin(wt) + \frac{h}{w^2}$

$\dot{x} = -wA_i \sin(wt) + wB_i \cos(wt)$

na osnovu početnih uslova i-tog prolaza:

$\dot{x}(t_{i-1}) = -0 + w_i B_i \cdot 1 = 0 \quad \rightarrow \quad \underline{B_i = 0}$

$x(t_{i-1}) = A_i \cdot 1 + 0 + \frac{h}{w^2} = |x_{i-1}| \quad \rightarrow \quad A_i = |x_{i-1}| - \frac{h}{w^2}$

na osnovu krajnjih uslova i-tog prolaza:

$x(t_i) = A_i \cdot (-1) + 0 + \frac{h}{w^2} = -|x_i| \quad \rightarrow \quad A_i = |x_i| + \frac{h}{w^2}$

$|x_i| + \frac{h}{w^2} = |x_{i-1}| - \frac{h}{w^2}$

$|x_{i-1}| = |x_i| + \frac{2h}{w^2}$

$|x_{i-2}| = |x_i| + \frac{4h}{w^2}$

$\dots$

$|x_0| = |x_i| + 2i \cdot \frac{h}{w^2}$

$|x_i| = x_0 - 2i\frac{h}{w^2} = 0{,}15 - (2i) \cdot \frac{\mu_2 g}{w^2} = 0{,}15 - 2i \cdot 0{,}005$

$|x_k| = 0{,}15 - 2k \cdot 0{,}005$

$0{,}01 \leq 0{,}15 - (2k) \cdot 0{,}005$

$k = \frac{0{,}15 - 0{,}01}{0{,}005 \cdot 2} = 14 \quad \Rightarrow \quad \underline{k = 14}$

$x_k = 0{,}15 - 2 \cdot 14 \cdot 0{,}005 = 0{,}01 \text{ m} \quad \rightarrow \quad \underline{x_k = 1 \text{ cm}}$

$t_k = k \cdot \frac{T_w}{2} = 14 \cdot \frac{2\pi}{2w} = 14 \cdot \frac{\pi}{14} \quad \Rightarrow \quad \boxed{t_k = \pi}$

4.32.

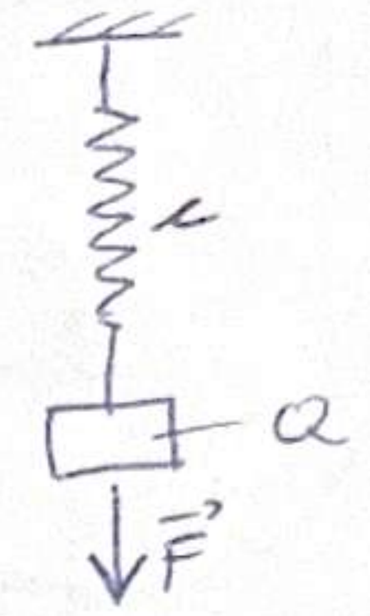
$Q = 2{,}45\ N$

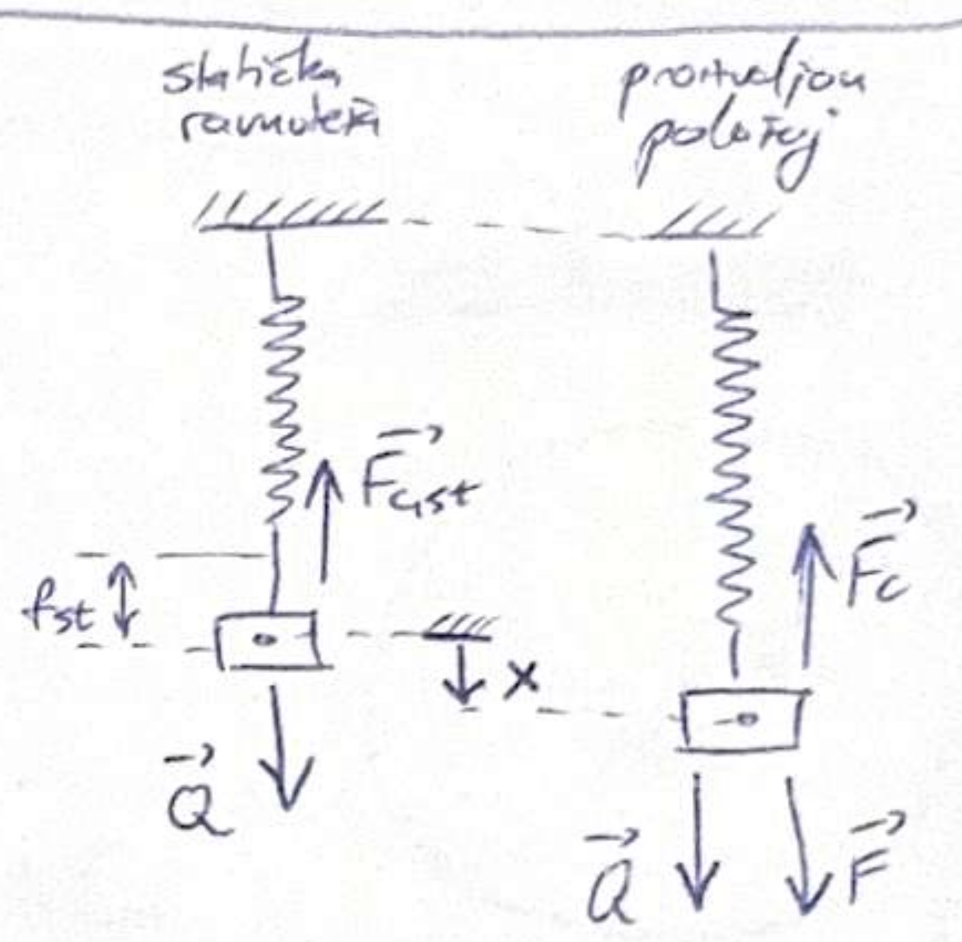
$c = 1\frac{N}{cm} = 100\frac{N}{m}$

$F = 1{,}8\sin(16t)$

j-na prinudnih osc?

vertikalne oscilacije





statička ravnoteža:

$$\underline{m\vec{a} = \vec{Q} + \vec{F}_{c,st}}$$

$$x:\quad \underline{\underline{0 = Q - c f_{st}}}$$

proizvoljan položaj:

$$\underline{m\vec{a} = \vec{Q} + \vec{F}_c + \vec{F}}$$

$$x:\quad m\ddot{x} = Q - c(f_{st} + x) + 1{,}8\sin(16t) \qquad /:m$$

$$\ddot{x} + \frac{c}{m}x = \frac{1{,}8}{m}\sin(16t), \qquad \omega^2 = \frac{c}{m} = \frac{cg}{Q} = 400, \qquad \omega = 20, \qquad h = \frac{1{,}8}{m} = \frac{1{,}8g}{Q} = 7{,}207$$

$$\ddot{x} + \omega^2 x = h\sin 16t$$

sopstvene oscilacije $\rightarrow$ homogeno rešenje (nema potrebe da ga tražimo, jer se traže prinudne oscilacije. Bitno je samo da zaključimo da neće doći do rezonance (neće jer $\lambda^2 + \omega^2 = 0 \rightarrow \lambda = 0 \pm \omega i$) $(\omega = 20)$

prinudne oscilacije $\rightarrow$ partikularno rešenje

$$P:\quad b(t) = h\sin(16t) = e^{0t}(0\cos(16t) + h\sin(16t))$$

$$x_p = t^0 e^{0t}(A\cos(16t) + B\sin(16t)), \qquad \ddot{x}_p = -256(A\cos\Omega t + B\sin\Omega t)$$

$$\cos(16t)(-256A + 400A) + \sin(16t)(-256B + 400B) = 0\cos 16t + h\sin 16t$$

$$A = 0, \qquad 144B = h \rightarrow B = 0{,}05$$

$$\boxed{x_p = 0{,}05\cdot\sin(16t)}$$

4.33 disk: r

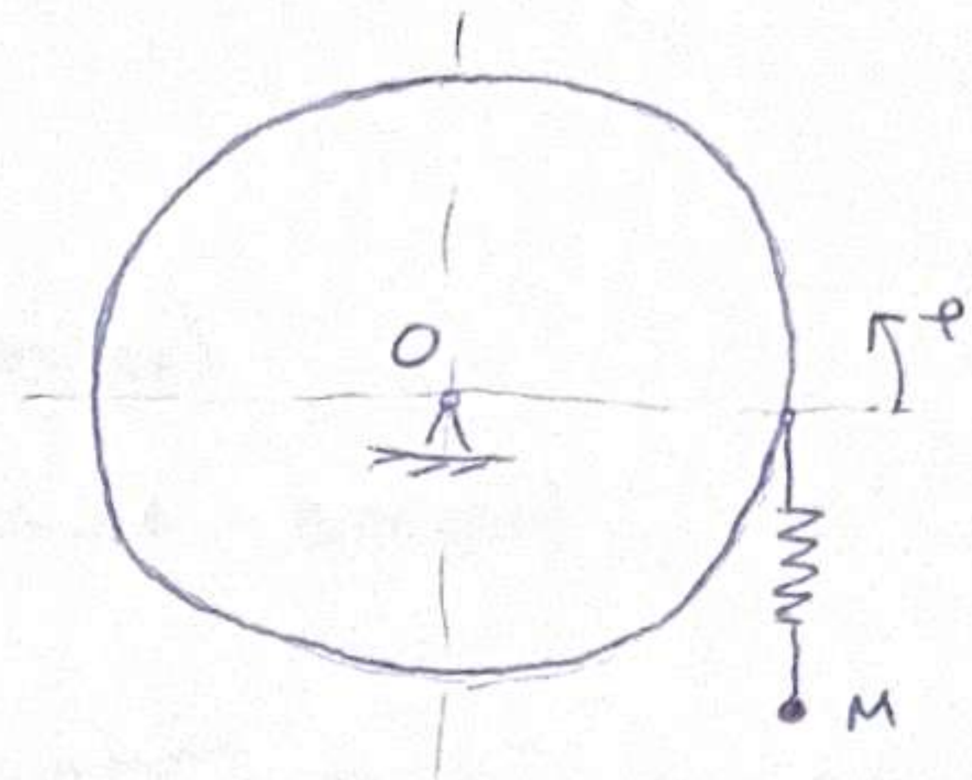
M: m

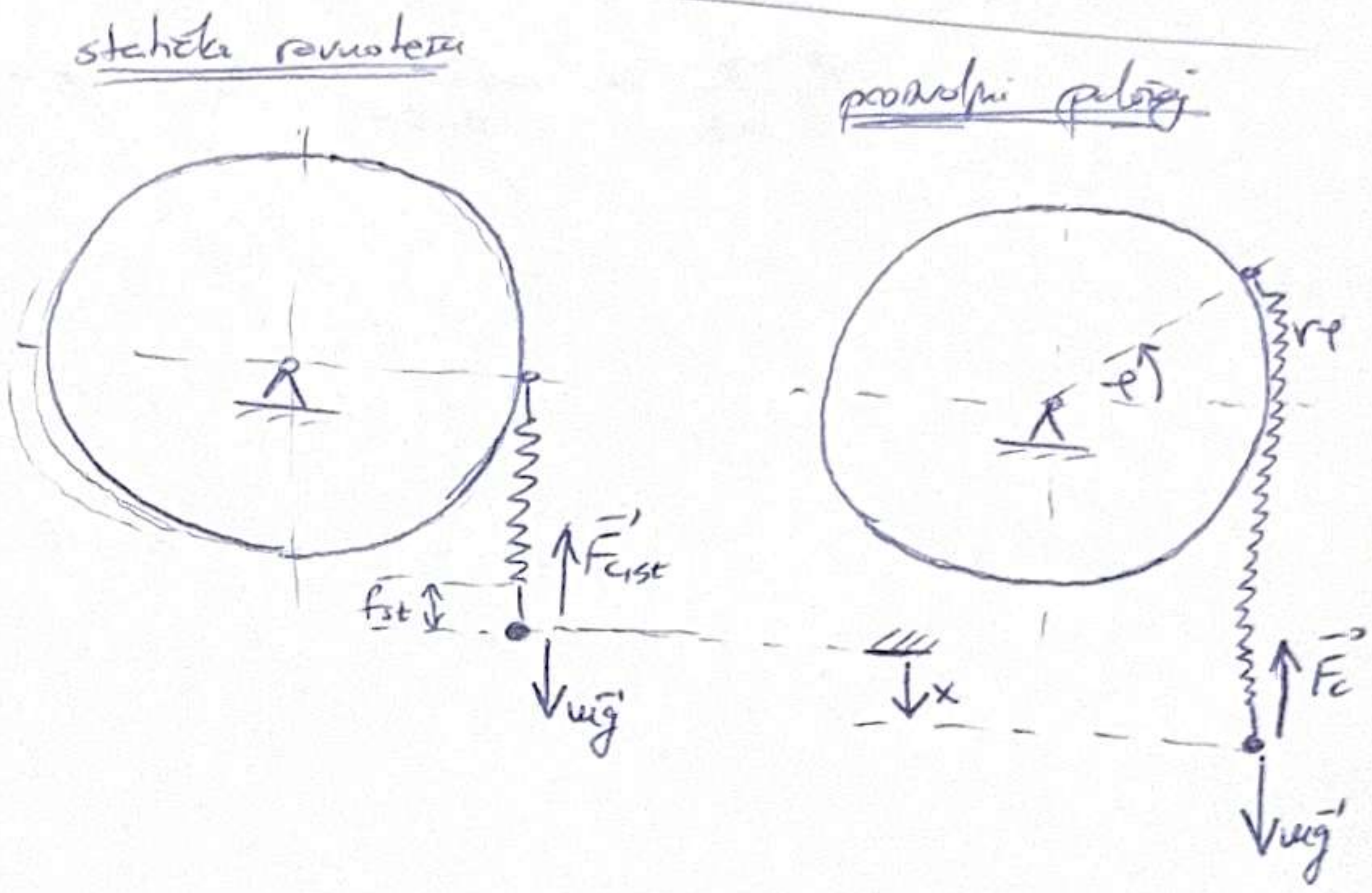
$\varphi = \varphi_0 \sin \Omega t$ – oscilacije diska

f-na prinudnih osc?

1) $\Omega \neq \sqrt{\frac{c}{m}}$

2) $\Omega = \sqrt{\frac{c}{m}}$





statička ravnoteža:

$$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$$

x: $\quad 0 = mg - cf_{st}$

proizvoljni položaj

$$m\vec{a} = m\vec{g} + \vec{F}_c$$

x: $\quad m\ddot{x} = mg - c(x + f_{st} + r\varphi)$

$m\ddot{x} = -cx - cr\varphi_0 \sin(\Omega t) \qquad /:m$

$\ddot{x} + \frac{c}{m} = -\frac{c}{m} r\varphi_0 \sin(\Omega t) \qquad , \quad \omega^2 = \frac{c}{m} \quad , \quad h = -\frac{c}{m} r\varphi_0$

$\ddot{x} + \omega^2 x = h \sin(\Omega t)$

H: $\quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = 0 \pm \omega i \qquad \begin{pmatrix} \alpha = 0 \\ \beta = \omega \end{pmatrix}$

P: $\quad b(t) = h \sin(\Omega t) = e^{0t}(0 \cos \Omega t + h \sin \Omega t) \qquad \begin{pmatrix} a = 0 \\ b = \Omega \end{pmatrix}$

1) $\omega \neq \Omega$

$x_p = t^0 e^{0t}(A \cos \Omega t + B \sin \Omega t) \quad , \quad \ddot{x}_p = -\Omega^2(A \cos \Omega t + B \sin \Omega t)$

$\cos \Omega t(-A\Omega^2 + \omega^2 A) + \sin \Omega t(-B\Omega^2 + B\omega^2) = 0 \cos \Omega t + h \sin \Omega t$

$A = 0 \quad , \qquad B(\omega^2 - \Omega^2) = h \quad \rightarrow \quad B = \frac{h}{\omega^2 - \Omega^2}$

$x_p = B \sin \Omega t = \frac{h}{\omega^2 - \Omega^2} \sin \Omega t = -\frac{cr\varphi_0}{m(\frac{c}{m} - \Omega^2)} \sin \Omega t$

$$\boxed{x_p = \frac{cr\varphi_0}{m\Omega^2 - c} \sin(\Omega t)}$$

2) $\omega = \Omega$

$$x_p = t^1 e^{0t}(A\cos\Omega t + B\sin\Omega t)$$

$$\dot{x}_p = (A\cos\Omega t + B\sin\Omega t) + t\Omega(-A\sin\Omega t + B\cos\Omega t)$$

$$\ddot{x}_p = \Omega(-A\sin\Omega t + B\cos\Omega t)\cdot 2 + t\Omega^2(-A\sin\Omega t - B\cos\Omega t)$$

$$\cos\Omega t(2\Omega B \cancel{- t\Omega^2 A + tA\omega^2}) + \sin\Omega t(-2\Omega A \cancel{- t\Omega^2 B + tB\omega^2}) = 0\cos\Omega t + h\sin\Omega t$$

$B = 0$, $-2\Omega A = h$ $\rightarrow$ $A = \dfrac{h}{-2\Omega} = \dfrac{\varepsilon r \varphi_0}{2m\Omega}$

$$\boxed{x_p = \frac{\varepsilon r \varphi_0}{2m\Omega} t\cos\Omega t}$$

4.34 $v_x = 30\,\frac{m}{s} = const.$

$A = ?$ – amplituda prinudnih oscilacija

$G = 2\ kN$

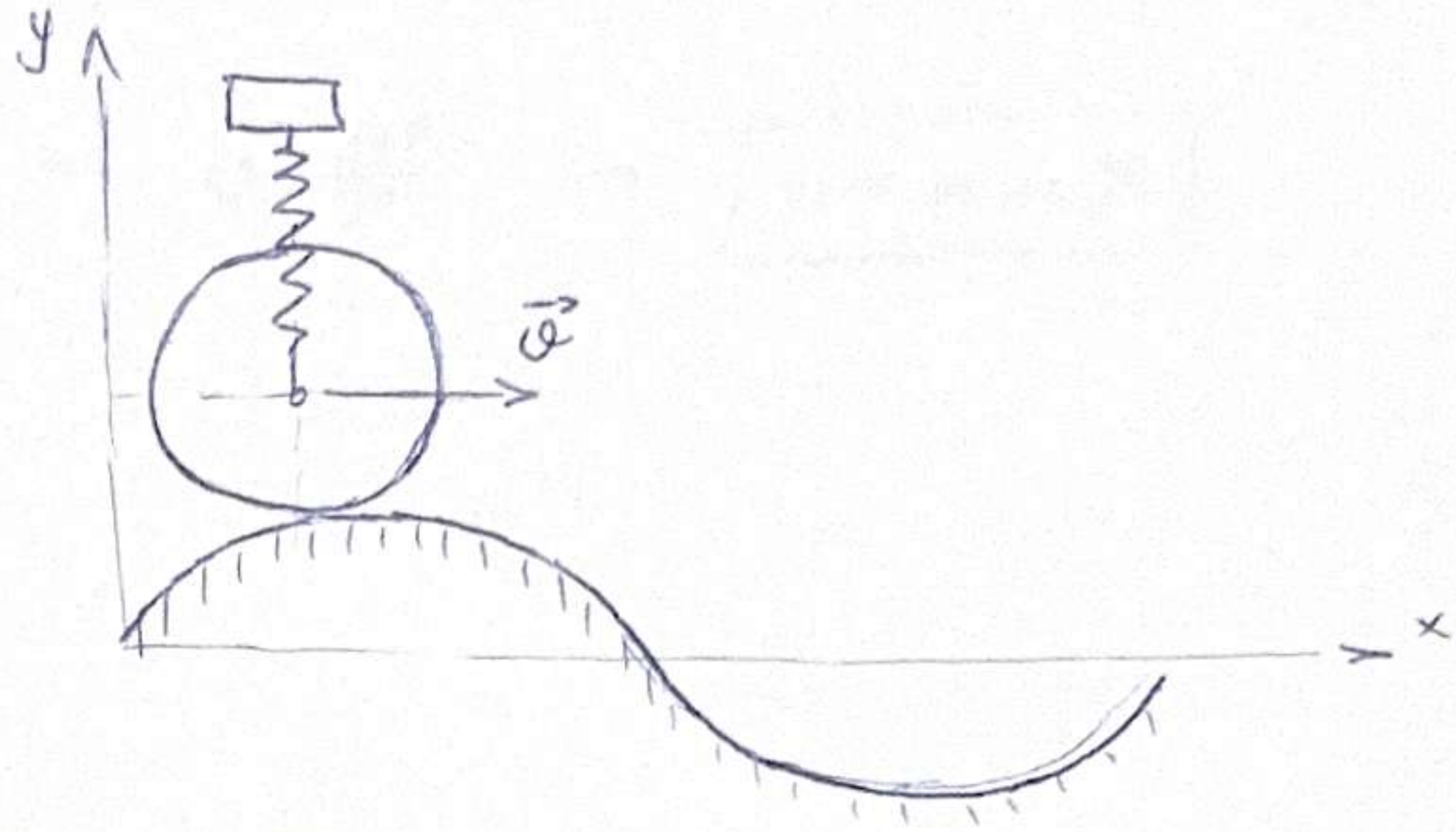
$c = 40\,\frac{kN}{cm}$

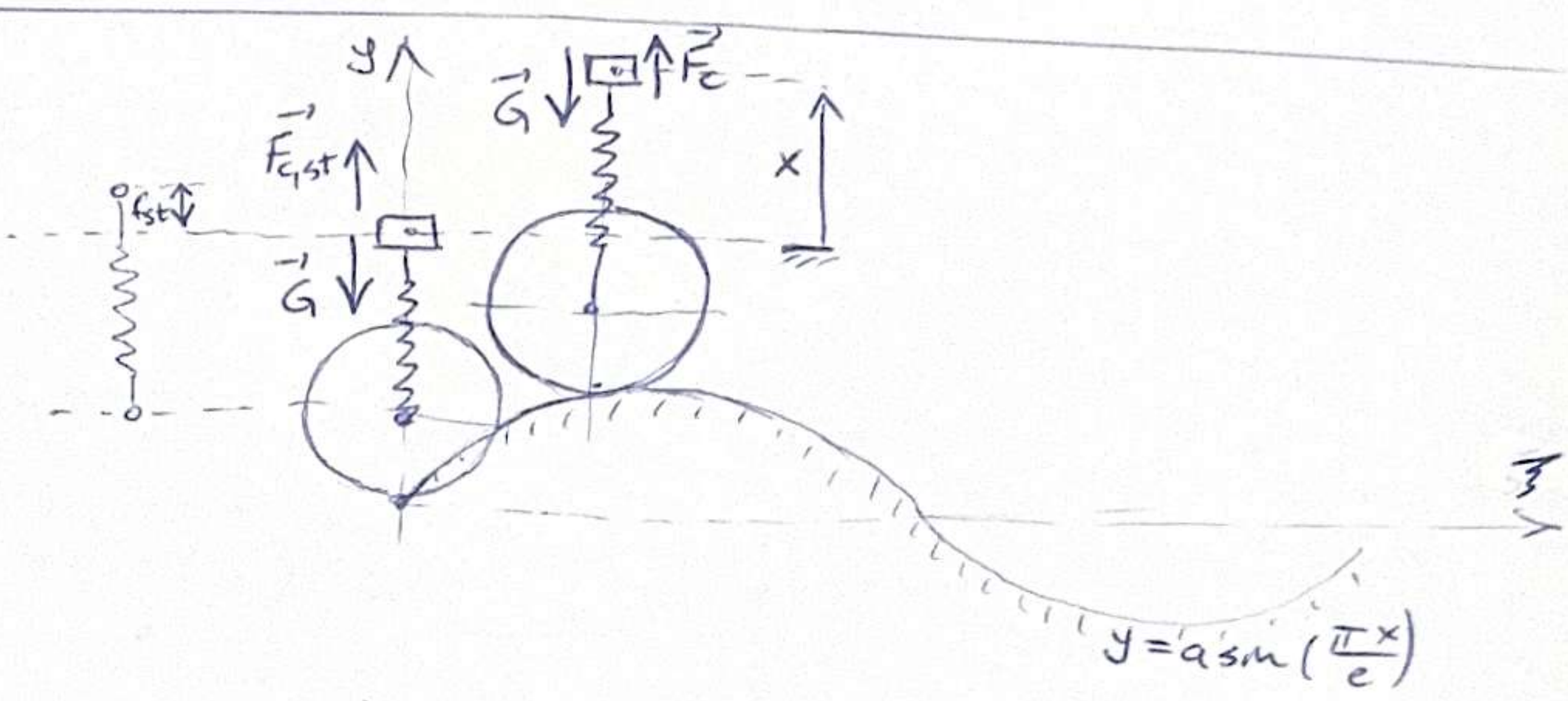
$y = a \sin\left(\frac{\pi x}{l}\right)$

$a = 1{,}5\ cm = 0{,}015\ m$

$l = 2{,}5\ m$

(zanemariti dimenzije točkova)





st. ravn.

$$m\vec{a} = \vec{G} + \vec{F}_{c,st}$$

$$x: \quad 0 = -G + c f_{st}$$

proizvoljan položaj:

$$m\vec{a} = \vec{G} + \vec{F}_c$$

$$x: \quad m\ddot{x} = \cancel{-G} + c(y + \cancel{f_{st}} - x)$$

$$m\ddot{x} + cx = cy \quad /:m$$

$$\ddot{x} + \frac{c}{m}x = \frac{c}{m} a \sin\left(\frac{\pi}{l}\right) \quad , \quad \omega^2 = \frac{c}{m} = \frac{cg}{G} = 196{,}2 \quad , \quad \omega = 14 \quad , \quad h = \frac{c}{m}\cdot a = 2{,}94$$

$$\ddot{x} + \omega^2 x = h \sin(\Omega t) \qquad \xi = \int v_x dt = 30\,t \quad , \quad \Omega = \frac{\pi \cdot 30}{l} = 12\pi$$

$$H: \quad \lambda^2 + \omega^2 \cdot 1 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm i\omega \quad \left(\begin{matrix}\alpha = 0\\ \beta = \omega = 14\end{matrix}\right)$$

$$P: \quad f(t) = e^{0t}\left(0\cos(\Omega t) + h\sin(\Omega t)\right) \quad \left(\begin{matrix}a = 0\\ b = \Omega = 12\pi\end{matrix}\right)$$

$$x_p = t^0 e^{0t}(A\cos\Omega t + B\sin\Omega t) \quad , \quad \ddot{x}_p = -\Omega^2(A\cos\Omega t + B\sin\Omega t)$$

$$\cos\Omega t\,(-A\Omega^2 + A\omega^2) + \sin\Omega t\,(-B\Omega^2 + B\omega^2) = 0\cos\Omega t + h\sin\Omega t$$

$$A = 0 \quad , \quad B = \frac{h}{\omega^2 - \Omega^2} \quad \rightarrow \quad x_p = \frac{h}{\omega^2 - \Omega^2}\sin(\Omega t)$$

$$X_{max} = \frac{h}{|\omega^2 - \Omega^2|} = \frac{2{,}94}{|196{,}2 - 144\pi^2|} = 0{,}0024\ m \quad \Rightarrow \quad \boxed{X_{max} = 0{,}24\ cm}$$

(4.35.) nastavak na prethodni zadatak

$v = ?$ - da je: $\Omega = \omega$

$$\Omega = \frac{\pi x}{l\, t} = \frac{\pi v}{l} \qquad \omega = 14 \quad \Bigg\} \Rightarrow \frac{\pi v}{l} = 14 \Rightarrow v = \frac{14 l}{\pi} \Rightarrow \boxed{v = 11{,}14\, \frac{m}{s}}$$

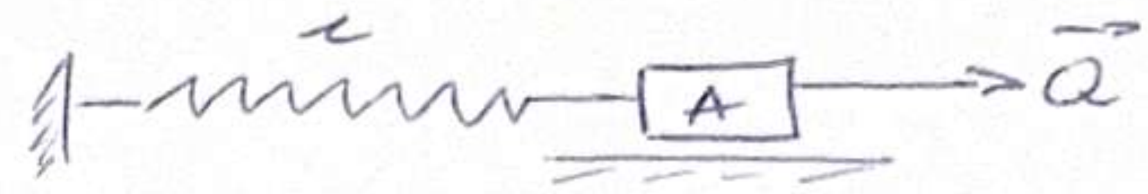
437.

$m = 1\,kg$

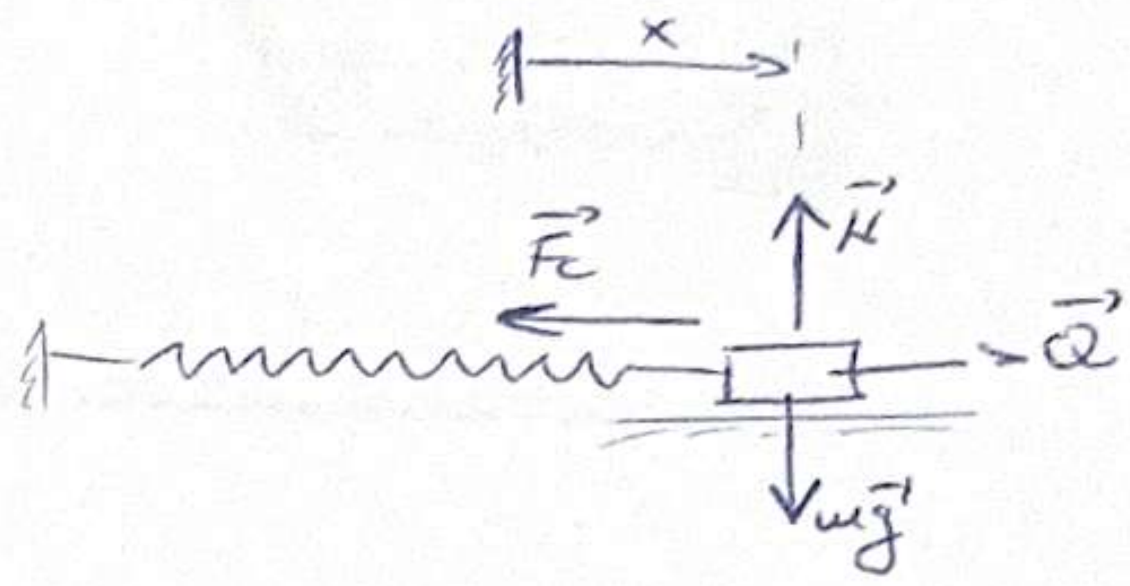
$c = 0{,}1\,\frac{kN}{m} = 100\,\frac{N}{m}$

$Q = 10\sin(10t + \frac{1}{3})$ $[N, s]$

$x_p = ?$



$f_{st} = 0$ (hor. oscilacije)



$$\underline{m\vec{a} = m\vec{g} + \vec{N} + \vec{F_c} + \vec{Q}}$$

$x:$ $m\ddot{x} = -cx + Q \quad /:m$

$$\ddot{x} + \frac{c}{m}x = 10\sin(10t + \tfrac{1}{3}), \quad \omega^2 = \frac{c}{m} = 100, \quad \omega = 10, \quad h = 10, \quad \Omega = 10, \quad \varphi = \frac{1}{3}$$

$$\ddot{x} + \omega^2 x = h\sin(\Omega t + \varphi)$$

$H:$ $\lambda^2 + \omega^2 \cdot 1 = 0 \;\to\; \lambda_{1/2} = 0 \pm \omega i \quad \begin{pmatrix}\alpha = 0 \\ \beta = \omega = 10\end{pmatrix}$

$P:$ $G(t) = e^{0t}\left(P_1\cos(\Omega t + \varphi) + P_2\sin(\Omega t + \varphi)\right) \quad \begin{pmatrix}a = 0 \\ \beta = \Omega = 10\end{pmatrix} \Rightarrow$ rezonansa!

$$x_p = t^1 e^{0t}\left(A\cos(\Omega t + \varphi) + B\sin(\Omega t + \varphi)\right)$$

$$\dot{x}_p = \left(A\cos(\Omega t + \varphi) + B\sin(\Omega t + \varphi)\right) + \Omega t\left(-A\sin(\Omega t + \varphi) + B\cos(\Omega t + \varphi)\right)$$

$$\ddot{x}_p = \Omega\left(-A\sin(\Omega t + \varphi) + B\cos(\Omega t + \varphi)\right)\cdot 2 + \Omega^2 t\left(-A\cos(\Omega t + \varphi) - B\sin(\Omega t + \varphi)\right)$$

$$\cos(\Omega t + \varphi)\left(2B\Omega \cancel{- \Omega^2 Bt + B\omega^2 t}\right) + \sin(\Omega t + \varphi)\left(-2A\Omega \;\; B\Omega^2 t + B\omega^2 t\right) = 0\cos(\Omega t + \varphi) + h\sin(\Omega t + \varphi)$$

$$B = 0, \quad A = \frac{h}{-2\Omega} = \frac{10}{-2\cdot 10} = -\frac{1}{2}$$

$$\boxed{x_p = -0{,}5\,t\sin(10t + \tfrac{1}{3})}$$

4.38.

OA: $\Omega = \sqrt{\frac{c}{m}} = konst.$

opruga: c

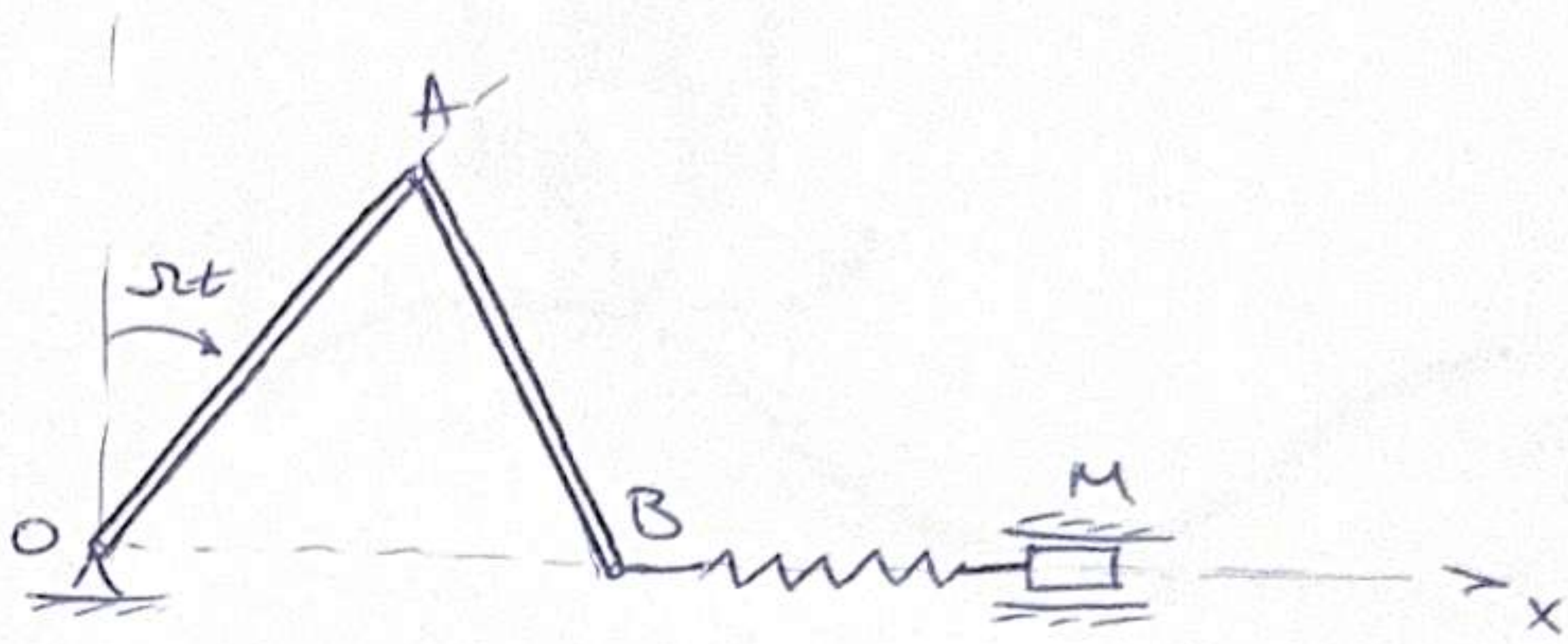
M: m

$t_0 = 0$: opruga neuopregnuta

M mirovalo

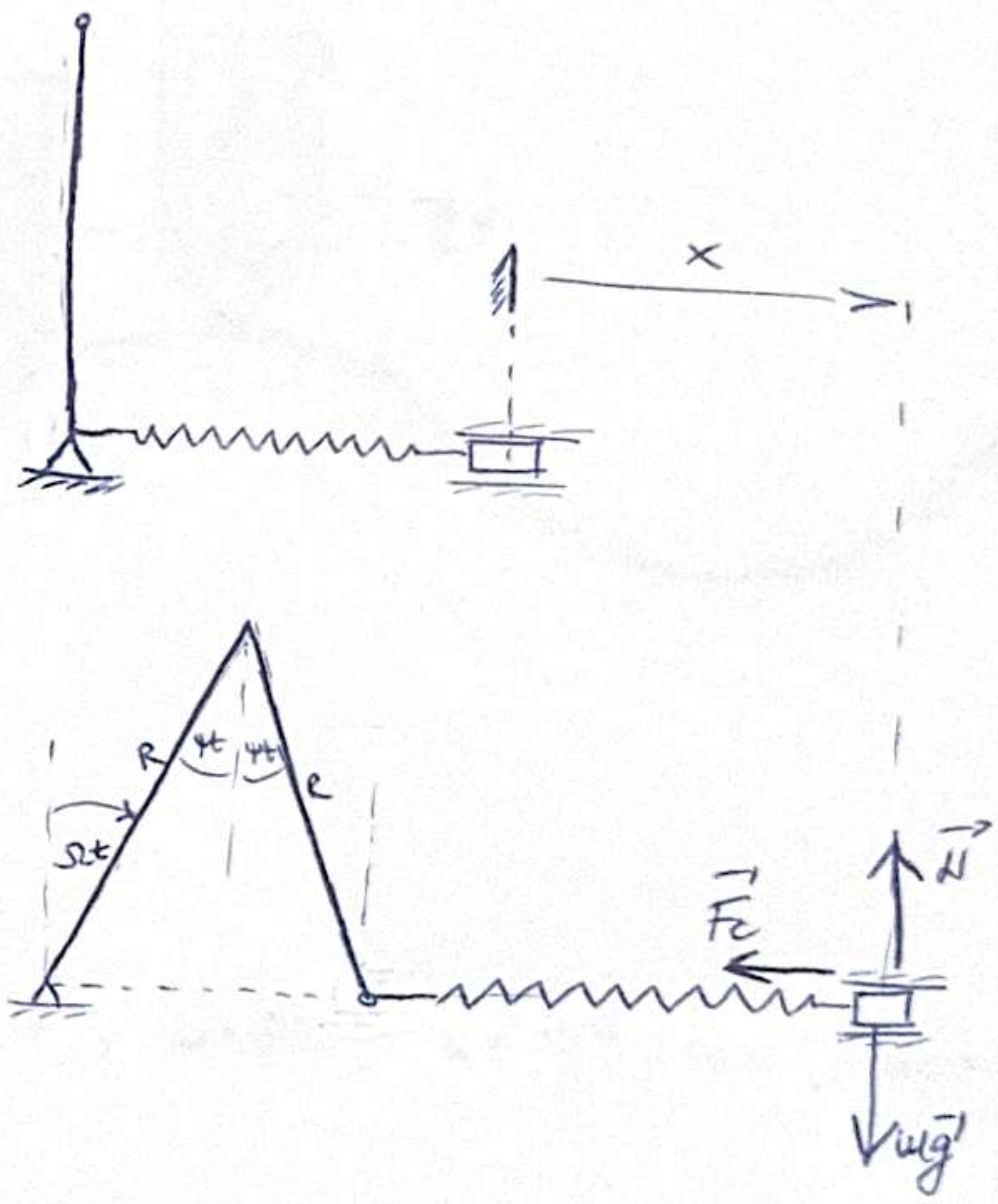
$x(t) - ?$, koord. početak je u početnoj položaj točke M

$\overline{OA} = \overline{AB} = R$



$f_{st} = 0$ (hor. osc.)

$$\underline{m\vec{a} = m\vec{g} + \vec{N} + \vec{F_c}}$$



x: $m\ddot{x} = -c(-2R\sin\Omega t + x) \quad /:m$

$$\ddot{x} + \frac{c}{m}x = \frac{2Rc}{m}\sin\Omega t \quad , \quad \omega^2 = \frac{c}{m} \; , \quad h = 2R\omega^2$$

$$\ddot{x} + \omega^2 x = h\sin\Omega t$$

H: $\lambda^2 + \omega^2 \cdot 1 = 0 \quad \to \quad \lambda_{1,2} = 0 \pm \omega i \quad \left(\begin{matrix}\alpha = 0 \\ \beta = \omega = \sqrt{\frac{c}{m}}\end{matrix}\right)$

$$x_h = C_1\cos\omega t + C_2\sin\omega t$$

P: $b(t) = h\sin\Omega t = e^{0t}(0\cos\Omega t + h\sin\Omega t) \quad , \quad \begin{matrix}a = 0 \\ b = \Omega = \sqrt{\frac{c}{m}}\end{matrix}$

$$x_p = t^1 e^{0t}(A\cos\Omega t + B\sin\Omega t)$$

$$\dot{x}_p = A\cos\Omega t + B\sin\Omega t + t\Omega(-A\sin\Omega t + B\cos\Omega t)$$

$$\ddot{x}_p = -2\Omega A\sin\Omega t + 2B\Omega\cos\Omega t + t\Omega^2(-A\cos\Omega t - B\sin\Omega t)$$

$$\cos\Omega t\left(2B\Omega - \cancel{At\Omega^2} + \cancel{At\omega^2}\right) + \sin\Omega t\left(-2A\Omega - \cancel{Bt\Omega^2} + \cancel{Bt\omega^2}\right) = 0\cos\Omega t + h\sin\Omega t$$

$$B = 0 \; , \quad A = -\frac{h}{2\Omega} = -\frac{2R\omega^2}{2\Omega} = -R\omega$$

$$x = C_1\cos\omega t + C_2\sin\omega t - t\cdot R\omega\cos\Omega t$$

$$\dot{x} = -C_1\omega\sin\omega t + C_2\omega\cos\omega t - R\omega\cos\Omega t + t\cdot R\omega^2\sin\Omega t$$

$x(t_0) = 0 \quad \to \quad C_1 \cdot 1 + 0 + 0 = 0 \quad \to \quad C_1 = 0$

$\dot{x}(t_0) = 0 \quad \to \quad 0 + C_2\omega - R\omega = 0 \quad \to \quad C_2 = R$

$$\boxed{x = R\sin\left(\sqrt{\frac{c}{m}}\,t\right) - t\cdot R\sqrt{\frac{c}{m}}\cos\left(\sqrt{\frac{c}{m}}\,t\right)}$$

4.39

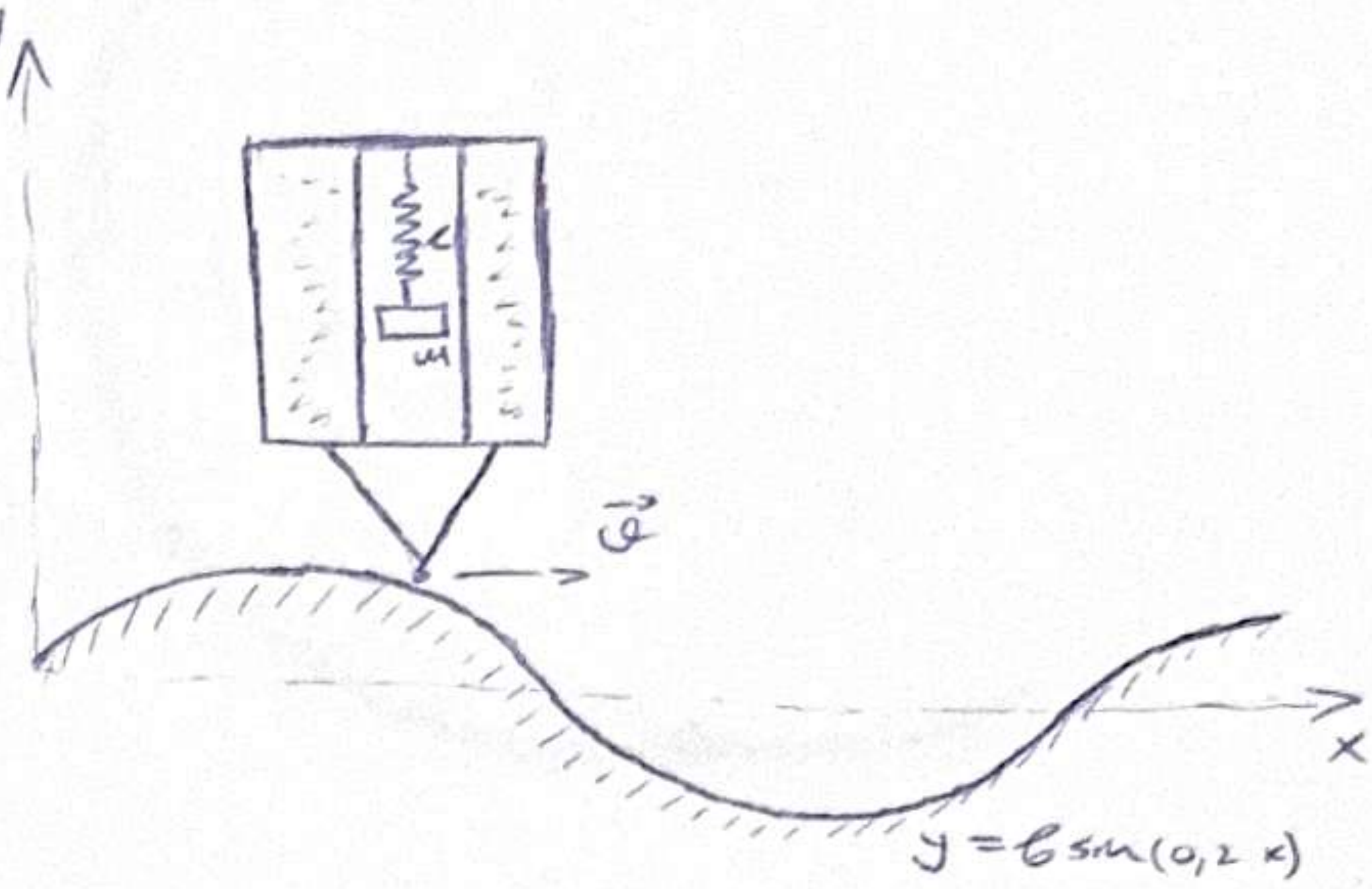
$v_x = 10\ \frac{m}{s}$

$y = b \sin(0{,}2x)$

$c = 0{,}1\ \frac{kN}{m} = 100\ \frac{N}{m}$

$m = 10\ kg$

odrediti b, ako je amplituda prinudnih vertikalnih osc. $A = 2cm = 0{,}02\ m$

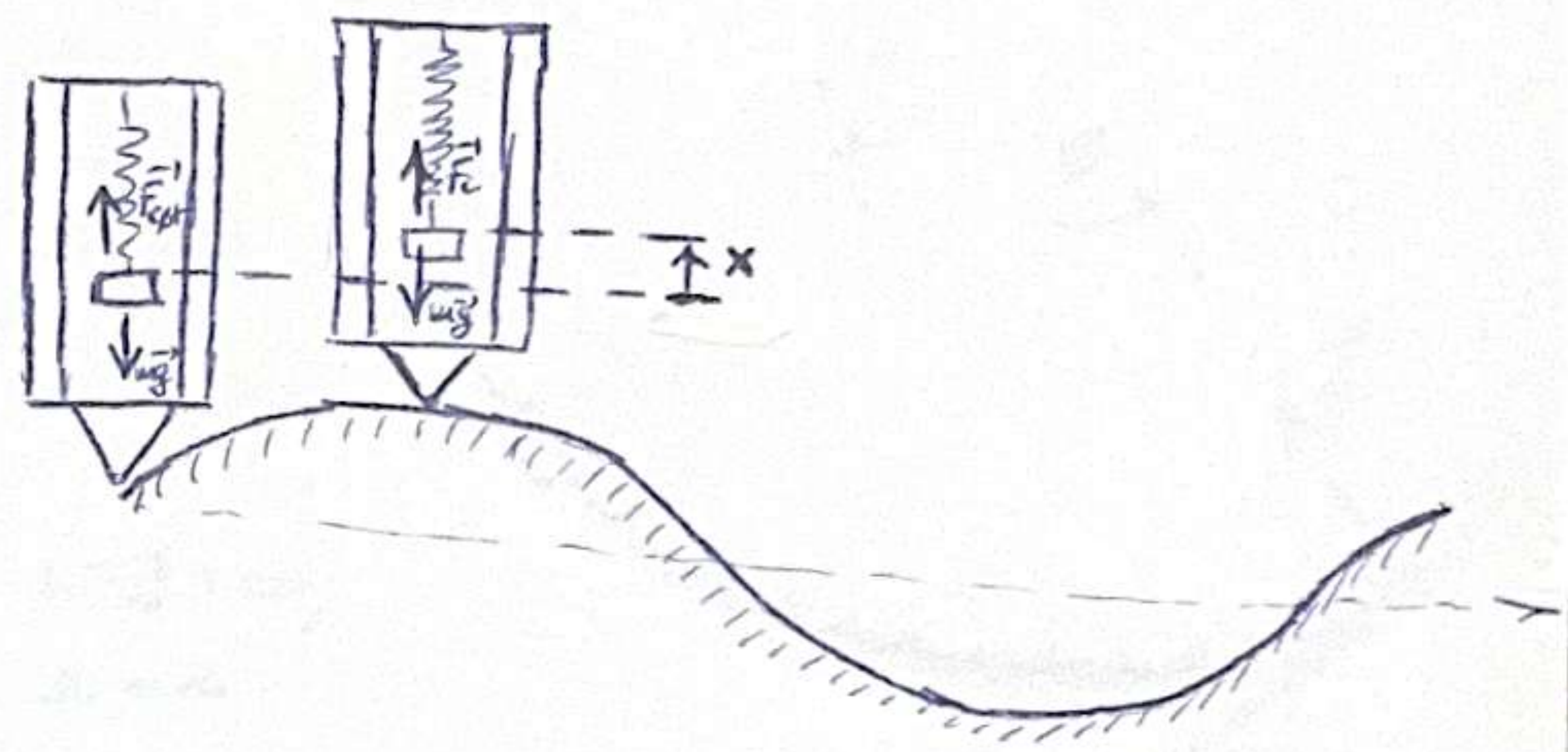


$y = b\sin(0{,}2x) = b\sin(0{,}2vt) = b\sin(2t)$

stat. ravn.

$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$

x: $\quad 0 = -mg + c f_{st}$



proizv. položaj

$m\vec{a} = m\vec{g} + \vec{F}_c$

x: $\quad m\ddot{x} = \cancel{-mg} + c(y - x + \cancel{f_{st}}) \quad /:m$

$\ddot{x} + \frac{c}{m}x = \frac{cb}{m}\sin(2t) \quad , \quad \omega^2 = \frac{c}{m} = \frac{100}{10} = 10 \ , \ \omega = \sqrt{10} \ , \ h = \frac{cb}{m} = 10b \ , \ \Omega = 2$

$\ddot{x} + \omega^2 x = h\sin(\Omega t)$

H: $\quad \lambda^2 + \omega^2 \cdot 1 = 0 \ \rightarrow \ \lambda_{1,2} = 0 \pm i\omega = 0 \pm i\sqrt{10}$

P: $\quad x_p = t^0 e^{0t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{x}_p = -A\Omega\sin\Omega t + B\Omega\cos\Omega t$

$\ddot{x}_p = -A\Omega^2\cos\Omega t - B\Omega^2\sin\Omega t$

$\cos\Omega t(-A\Omega^2 + A\omega^2) + \sin\Omega t(-B\Omega^2 + B\omega^2) = 0\cos\Omega t + h\sin\Omega t$

$B(\omega^2 - \Omega^2) = h \ \rightarrow \ B = \frac{h}{\omega^2 - \Omega^2} \quad , \quad A = 0$

$x_p = \frac{h}{\omega^2 - \Omega^2}\sin\Omega t \quad , \quad A = \frac{h}{\omega^2 - \Omega^2} = 0{,}02$

$\frac{10b}{10 - 4} = 0{,}02 \ \Rightarrow \ b = \frac{0{,}02 \cdot 6}{10} = 0{,}012\ m \ \Rightarrow \ \boxed{b = 1{,}2\ cm}$

4.40. horizontalna ravan

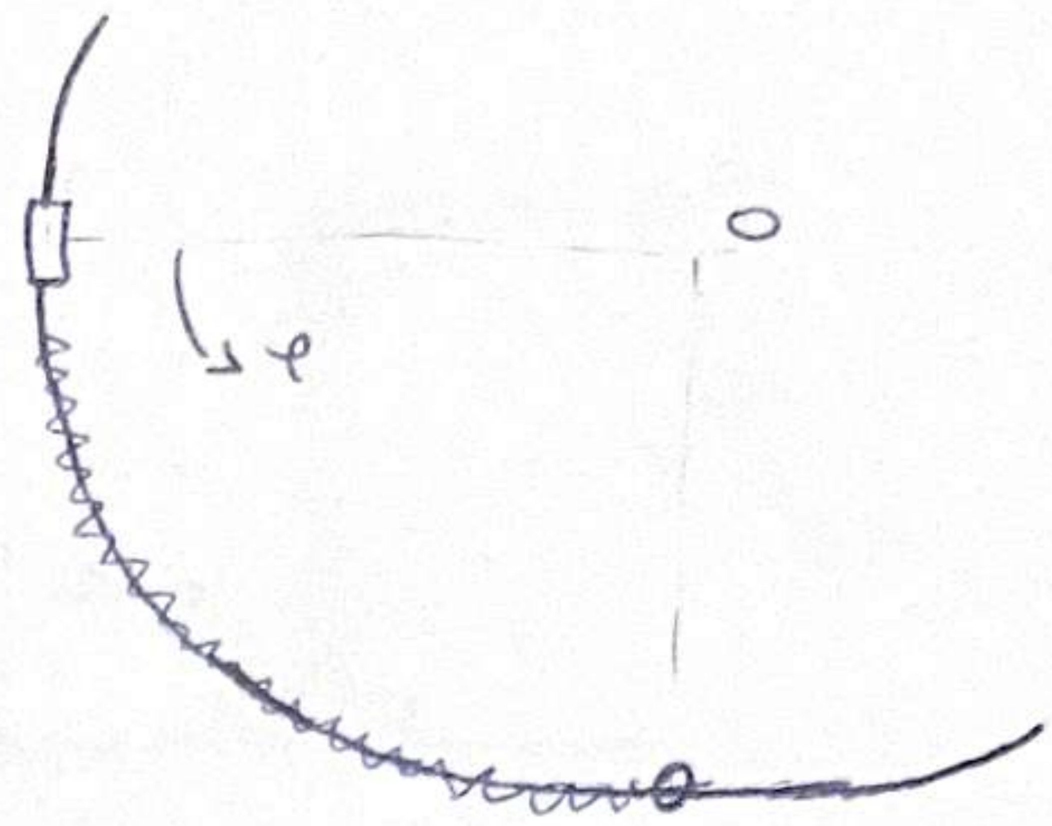
$m = 1\,kg$

$c = 1 \frac{N}{cm} = 100 \frac{N}{m}$

$\varphi = \sin(10t)$

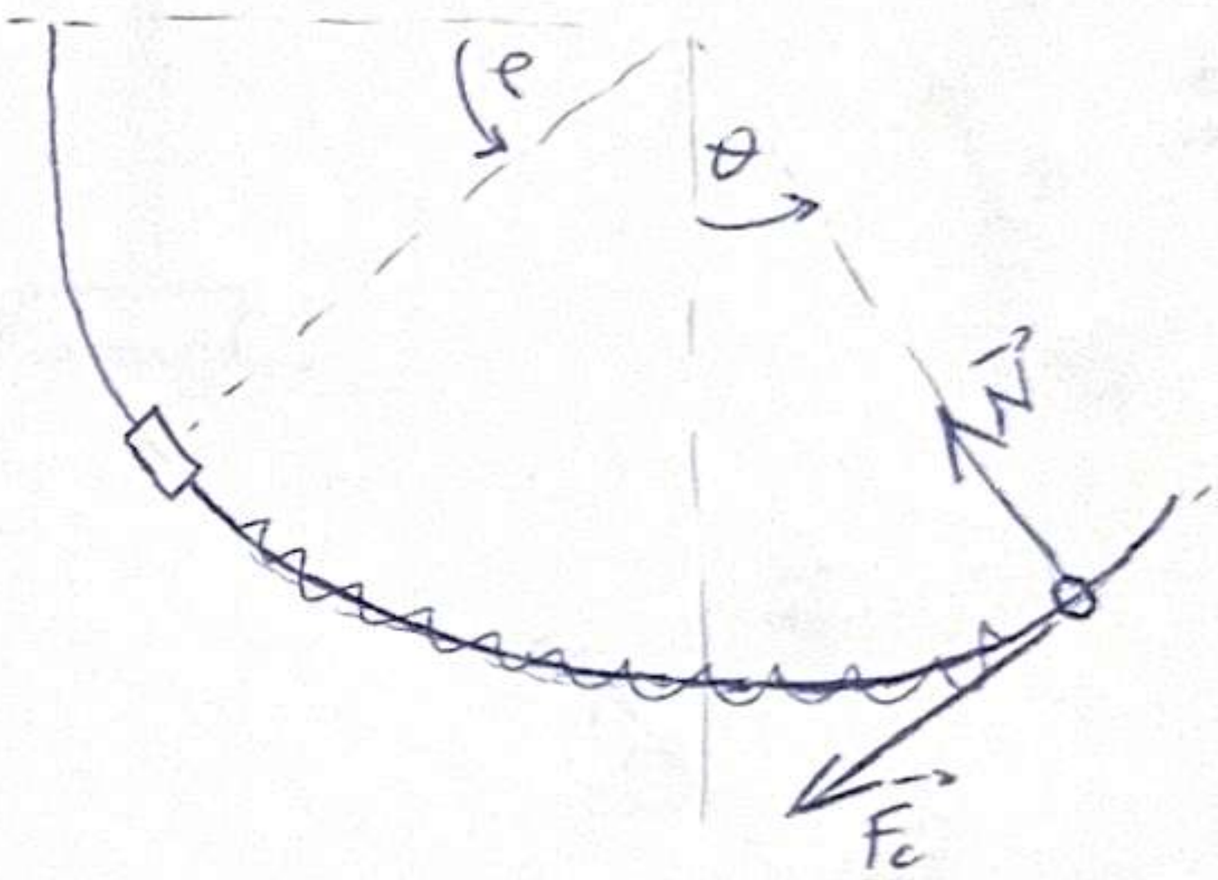
t_0: opruga nenapregnuta
tačka mirovala

j-na kretanja tačke?



$\theta_{st} = 0$ (horizontalna ravan)

$m\vec{a} = \vec{F_c} + \vec{N}$



$t: \quad m a_t = -c(\theta \cdot R - \varphi \cdot R)$

$m R \ddot{\theta} = -c\theta R + cR\varphi \quad /: mR$

$\ddot{\theta} + \frac{c}{m}\theta = \frac{c}{m}\sin(10t)$, $\quad \omega^2 = \frac{c}{m} = 100$, $\quad h = \frac{c}{m} = 100$,

$\ddot{\theta} + \omega^2\theta = h\sin(\Omega t)$ $\quad \omega = 10$, $\quad \Omega = 10$

$H: \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = 0 \pm i\omega$, $\quad \alpha = 0, \ \beta = \omega = 10 \quad \Rightarrow \quad \theta_h = C_1\cos(\omega t) + C_2\sin(\omega t)$

$P: \quad b(t) = e^{0t}(0\cos(\Omega t) + h\sin(\Omega t))$, $\quad a = 0, \ b = \Omega = 10 \quad \Rightarrow$ rezonansa!

$\theta_p = t^1 e^{0t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{\theta}_p = A\cos\Omega t + B\sin\Omega t + t\Omega(-A\sin\Omega t + B\cos\Omega t)$

$\ddot{\theta}_p = -2A\Omega\sin\Omega t + 2B\Omega\cos\Omega t + t\Omega^2(-A\cos\Omega t - B\sin\Omega t)$

$\cos\Omega t\,(2B\Omega - \cancel{At\Omega^2} + \cancel{At\omega^2}) + \sin\Omega t\,(-2A\Omega - \cancel{Bt\Omega^2} + \cancel{Bt\omega^2}) = 0\cos\Omega t + h\sin\Omega t$

$B = 0$, $\quad -2A\Omega = h \quad \Rightarrow \quad A = -\frac{h}{2\Omega} = -\frac{100}{2\cdot 10} = -5$

$\theta = C_1\cos 10t + C_2\sin 10t - 5t\cos 10t$

$\dot{\theta} = -10C_1\sin 10t + 10C_2\cos 10t - 5\cos 10t + 50t\sin 10t$

$\theta(t_0) = C_1\cdot 1 + 0 - 0 = 0 \quad \rightarrow \quad \underline{C_1 = 0}$

$\dot{\theta}(t_0) = 0 + 10C_2\cdot 1 - 5 + 0 = 0 \quad \rightarrow \quad \underline{C_2 = \frac{1}{2}}$

$\boxed{\theta = 0{,}5\sin(10t) - 5t\cos(10t)}$

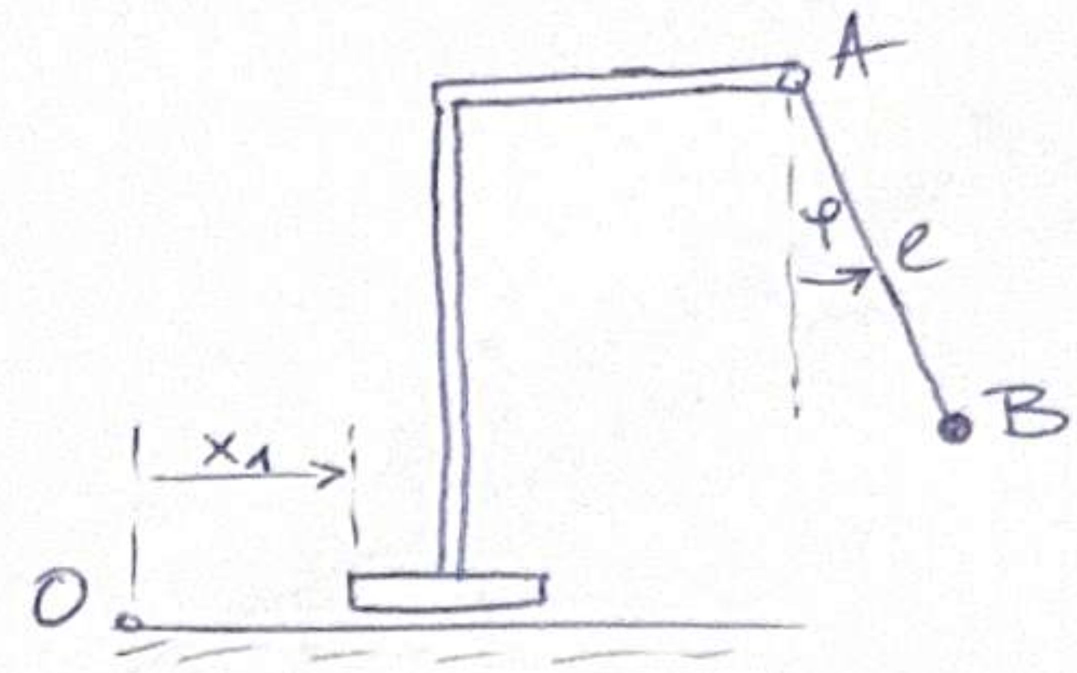
(4.41) $x_1 = a\sin(\Omega t)$

ℓ – dužina klatna

$\omega = 0{,}2\,\Omega$ – frekvencija oscilovanja matematičkog klatna

$\varphi_p = ?$ – j-na prinudnih osc.

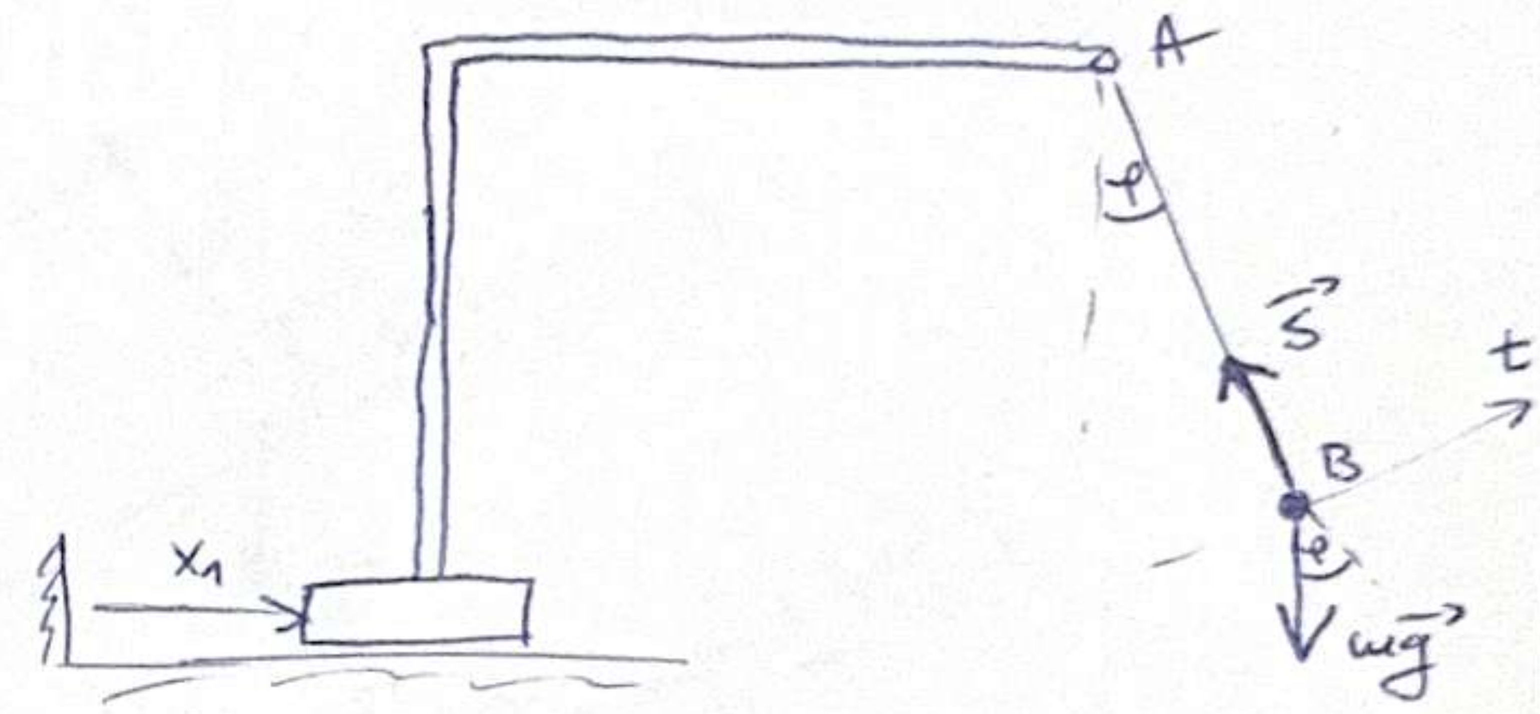
male oscilacije



$$m\vec{a} = m\vec{g} + \vec{S}$$

t: $\quad m a_t = -mg\sin\varphi \quad /:m$

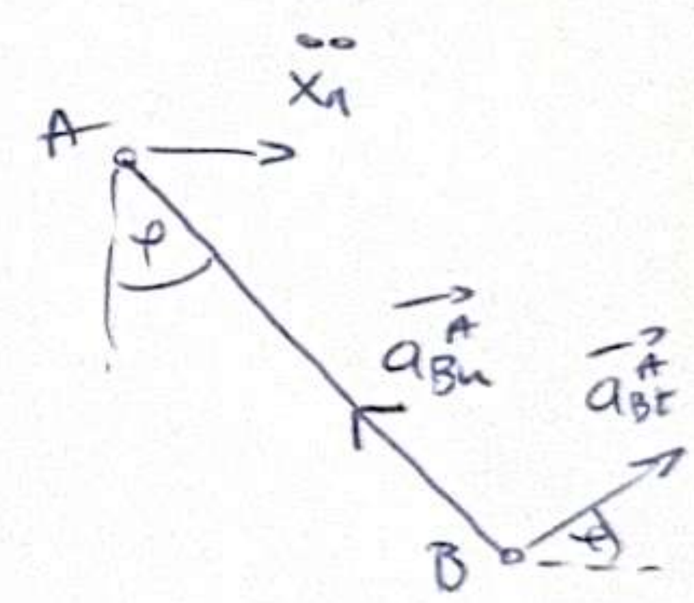
$$a_t = -g\varphi$$



$$\vec{a}_B = \vec{a}_A + \vec{a}^A_{Bn} + \vec{a}^A_{Bt}$$

t: $\quad a_{Bt} = \ddot{x}_1 + a^A_{Bt}$

$$a_t = \ddot{x}_1 + \ell\ddot{\varphi}$$



$\ddot{x}_1 + \ell\ddot{\varphi} = -g\varphi \quad /:\ell$

$\ddot{\varphi} + \frac{g}{\ell}\varphi = \frac{a}{\ell}\Omega^2\sin(\Omega t)$, $\quad \omega^2 = \frac{g}{\ell}$, $\quad \omega = \sqrt{\frac{g}{\ell}} = 0{,}2\,\Omega$, $\quad h = \frac{a}{\ell}\Omega^2$

$\ddot{\varphi} + \omega^2\varphi = h\sin(\Omega t)$

H: nema potrebe da ga računamo (ne traži se, a i vidimo da neće doći do rezonanse)

P: $G(t) = h\sin(\Omega t) = e^{0t}(0\cos(\Omega t) + h\sin(\Omega t))$

$\varphi_p = A\cos(\Omega t) + B\sin(\Omega t)$, $\quad \ddot{\varphi}_p = -A\Omega^2\cos(\Omega t) - B\Omega^2\cos(\Omega t)$

$\cos(\Omega t)(-A\Omega^2 + A\omega^2) + \sin(\Omega t)(-B\Omega^2 + B\omega^2) = 0\cos(\Omega t) + h\sin(\Omega t)$

$A = 0$, $\quad B(\omega^2 - \Omega^2) = h \quad \rightarrow \quad B = \frac{h}{\omega^2 - \Omega^2} = \frac{\frac{a}{\ell}\Omega^2}{(\frac{1}{5}\Omega)^2 - \Omega^2} = \frac{\frac{a}{\ell}\cancel{\Omega^2}}{-\frac{24}{25}\cancel{\Omega^2}} = -\frac{25}{24}\frac{a}{\ell}$

$$\boxed{\varphi_p = -\frac{25}{24}\frac{a}{\ell}\sin(\Omega t)}$$

4.42 $\alpha = 30°$

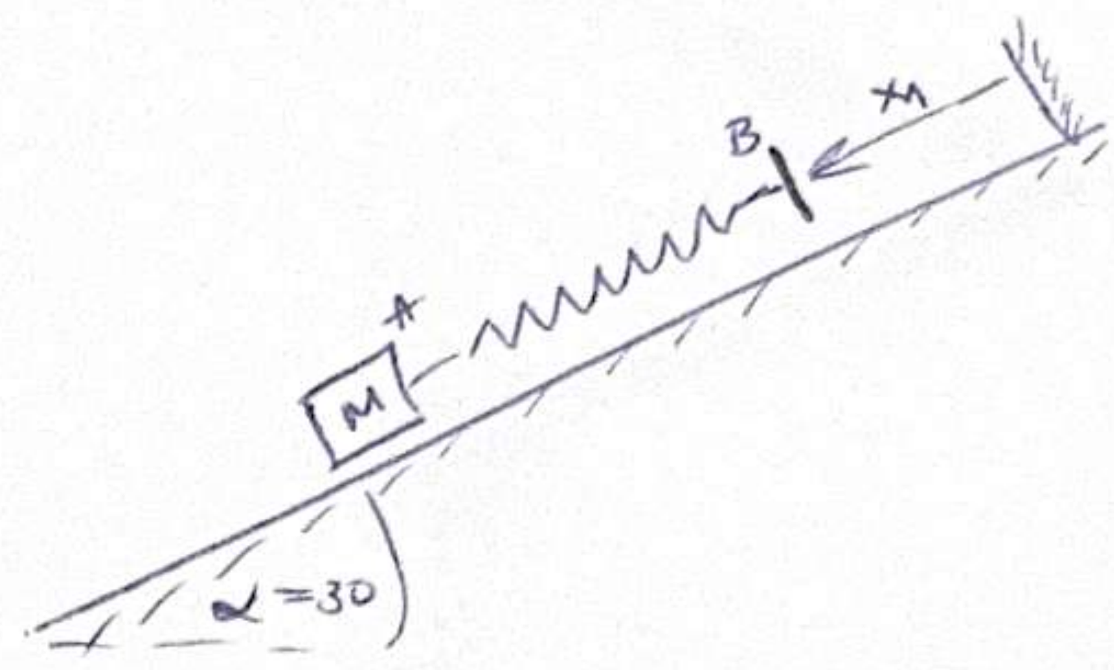
M: $Q = 100\ N$

$c = 50 \frac{N}{cm} = 5000 \frac{N}{m}$

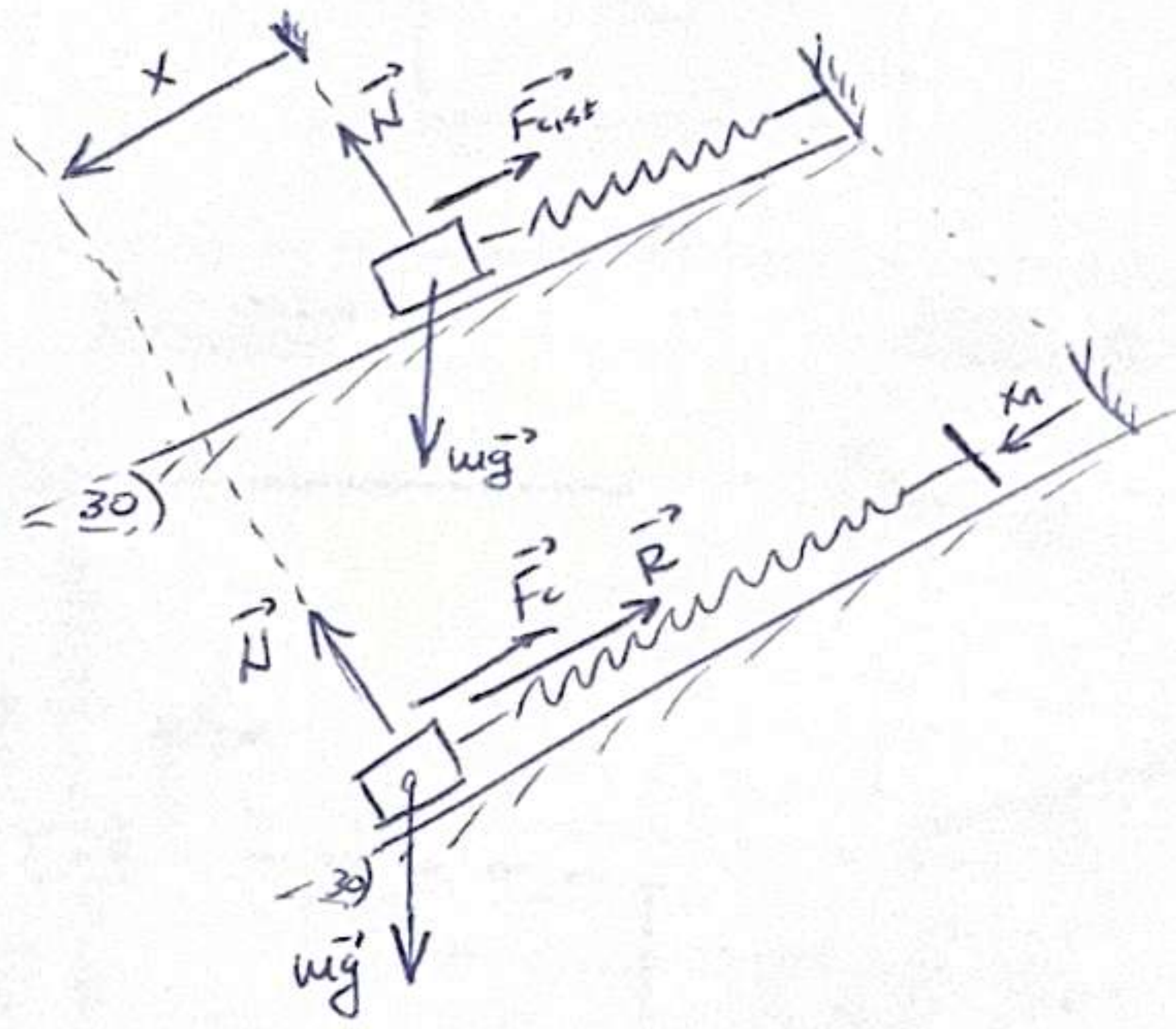
$x_1 = 2 \sin(15t)$

$x_p = ?$

sila otpora: $\vec{R} = -2\vec{v}\ \left[\frac{cm}{s}\right]$



$R = \beta \cdot v \Rightarrow \beta = \frac{R}{v} = 2 \frac{Ns}{cm} = 200 \frac{Ns}{m}$



stat. ravnoteža

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{cst}$

x: $\quad 0 = mg\sin 30 - c f_{st}$

proizv. položaj

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_c + \vec{R}$

x: $\quad m\ddot{x} = mg\sin 30 - c(f_{st} + x - x_1) - \beta\dot{x} \quad /:m$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 2\frac{c}{m}\sin(15t)$, $\omega^2 = \frac{c}{m} = \frac{cg}{Q} = 490{,}5$, $\omega = 22{,}15$,

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h\sin(\Omega t)$ $\quad$ $2\delta = \frac{\beta}{m} = \frac{\beta g}{Q} = 19{,}62$, $h = \frac{2c}{m} = 981$, $\Omega = 15$

H: ne treba nam (ne traži se) (a i očigledno je da nema rezonanse)

P: $\quad f(t) = h\sin(\Omega t) = e^{0t}(0\cos(\Omega t) + h\sin(\Omega t))$

$x_p = t^0 e^{0t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{x}_p = -A\Omega\sin\Omega t + B\Omega\cos\Omega t$

$\ddot{x}_p = -A\Omega^2\cos\Omega t - B\Omega^2\sin\Omega t$

$\cos\Omega t(-A\Omega^2 + B\Omega \cdot 2\delta + A\omega^2) + \sin\Omega t(-B\Omega^2 - A\Omega \cdot 2\delta + B\omega^2) = 0\cos\Omega t + h\sin\Omega t$

$A(\omega^2 - \Omega^2) + 2\delta\Omega \cdot B = 0 \quad \rightarrow \quad B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega}$

$B(\omega^2 - \Omega^2) + 2\delta\Omega \cdot A = h$

$-\frac{A(\omega^2 - \Omega^2)^2}{2\delta\Omega} - 2\delta\Omega \cdot A = h \quad \rightarrow \quad A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -1{,}84 \quad \rightarrow B = 1{,}66$

$$\boxed{x_p = -1{,}84\cos(15t) + 1{,}66\sin(15t)}$$

(4.43) $c = 0{,}5\ \frac{N}{cm} = 50\ \frac{N}{m}$

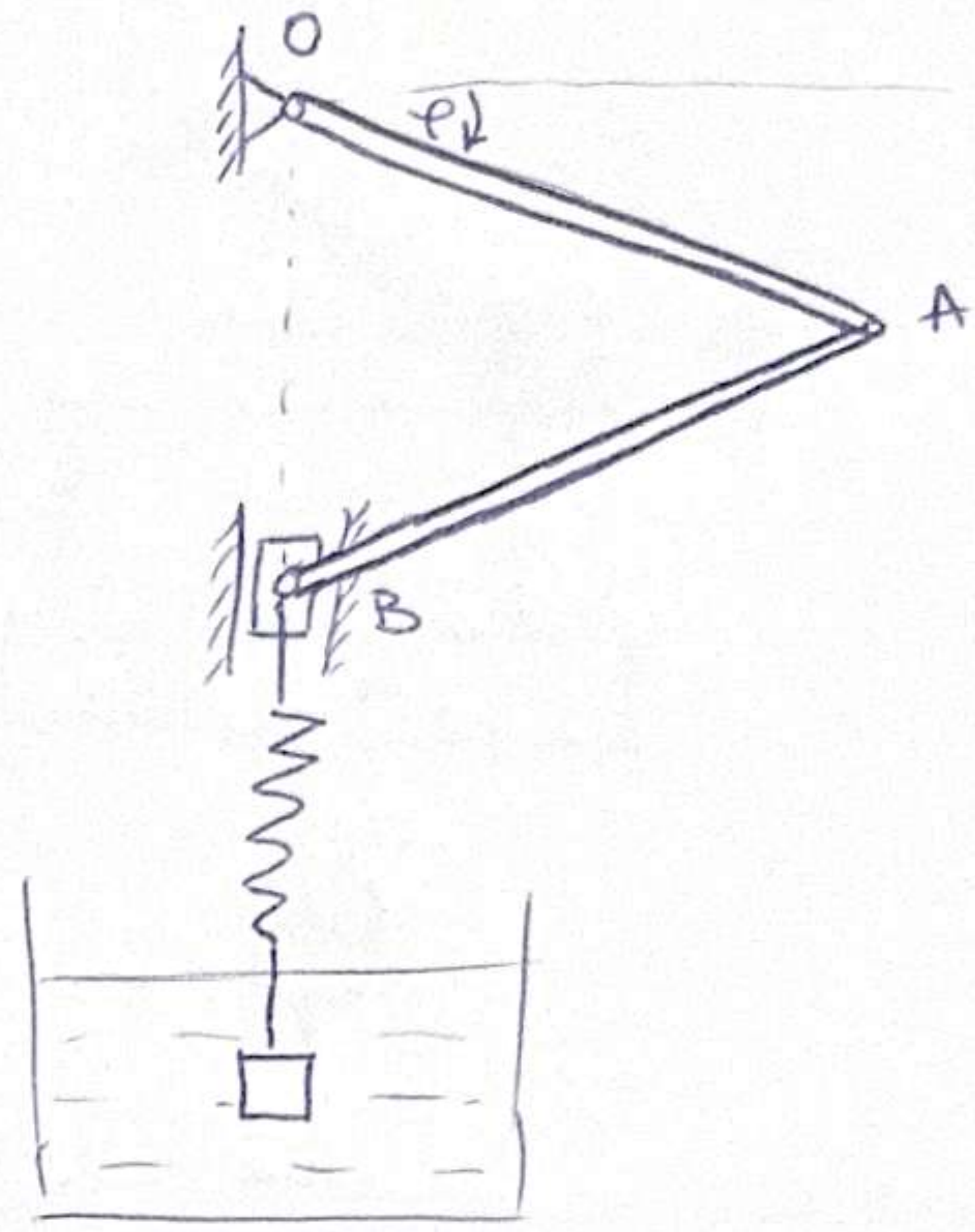
$G = 1\ N$

$\omega = 4\pi$

$\vec{v} = 1\ \frac{cm}{s} = 0{,}01\ \frac{m}{s} \Rightarrow R = 0{,}02\ N$

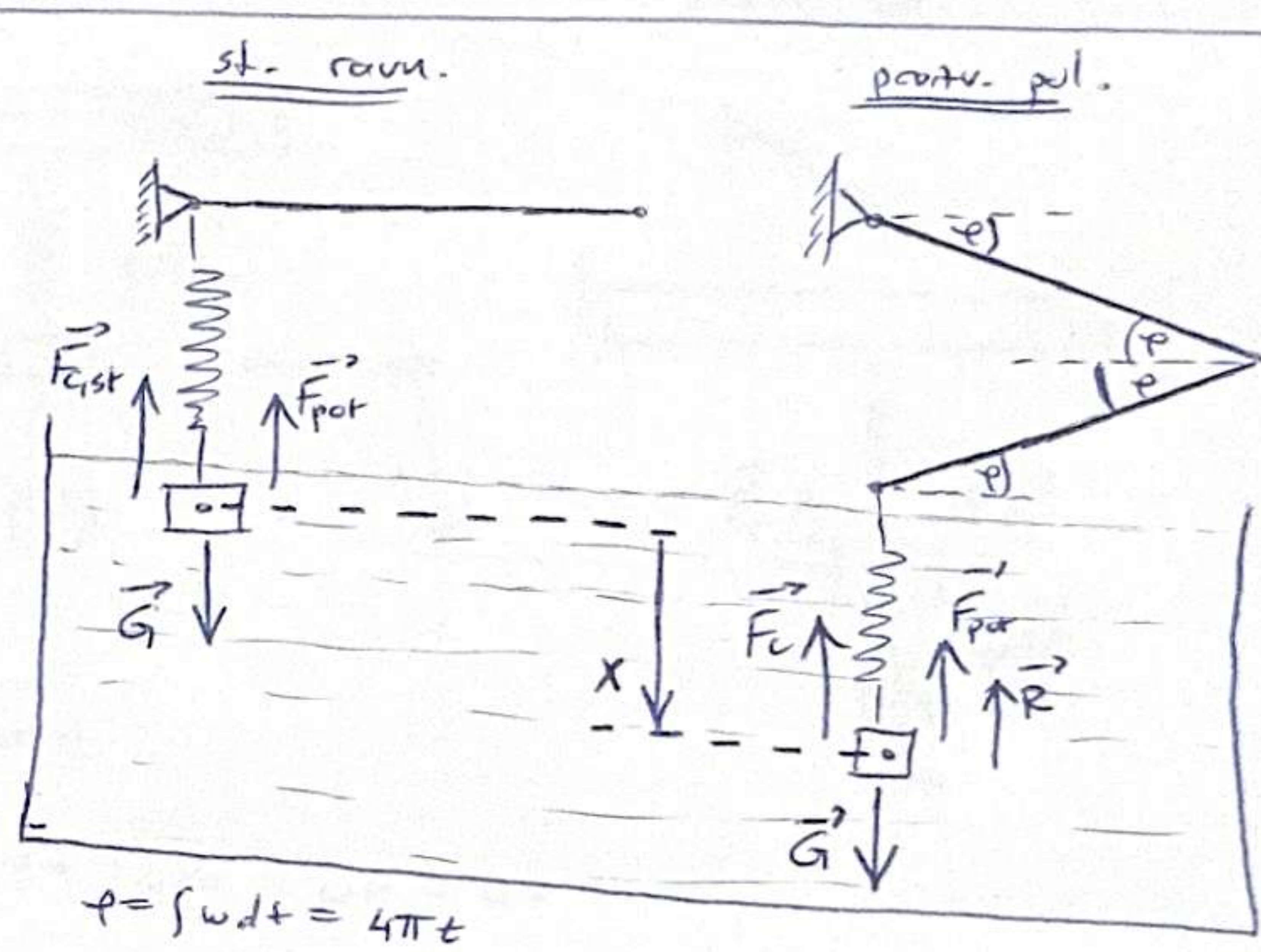
$x_p = ?$ – oko položaja statičke ravnoteže koji odgovara uglu $\varphi = 0$

$\overline{OA} = \overline{AB} = 3\ cm = 0{,}03\ m$



$F_{pot} = \rho_t \cdot V \cdot g = ?$ $(V = ?)$

$F_{pot} = \text{const.}$ (jer je uvek ista zapremina potopljena)



st. ravn.

$$\underline{m\vec{a} = \vec{G} + \vec{F}_{c,st} + \vec{F}_{pot}}$$

$x:\quad 0 = G - c f_{st} - F_{pot}$

$\varphi = \int \omega\, dt = 4\pi t$

prinudni položaj

$$\underline{m\vec{a} = \vec{G} + \vec{F}_c + \vec{F}_{pot} + \vec{R}}$$

$v = 0{,}01\ \frac{m}{s} \Rightarrow R = 0{,}02\ N$

$v = 1\ \frac{m}{s} \Rightarrow R = 2N$

$R = 2 \cdot v = \beta \cdot v \Rightarrow \beta = 2$

$x:\quad m\ddot{x} = G - c(f_{st} + x - 2 \cdot 0{,}03 \sin 4\pi t) - \beta \dot{x}$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = \frac{0{,}06 \cdot c}{m}\sin(4\pi t)$,

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h \sin(\Omega t)$

$\omega^2 = \frac{c}{m} = \frac{c \cdot g}{G} = 490{,}5$, $\omega = 22{,}15$,

$2\delta = \frac{\beta}{m} = \frac{\beta \cdot g}{G} = 19{,}62$, $\Omega = 4\pi$,

$h = \frac{0{,}06\, c}{m} = \frac{0{,}06 \cdot c \cdot g}{G} = 29{,}43$

H: neću da ga računam, ne traži se (očigledno je i da nema rezonanse)

P: (identično rešavanje kao u 4.42.)

$$A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -0{,}0423\ ,\qquad B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega} = 0{,}0571$$

$$\boxed{x_p = -0{,}0423 \cos(4\pi t) + 0{,}0571 \sin(4\pi t)}$$

(4.44.) $Q = 98$

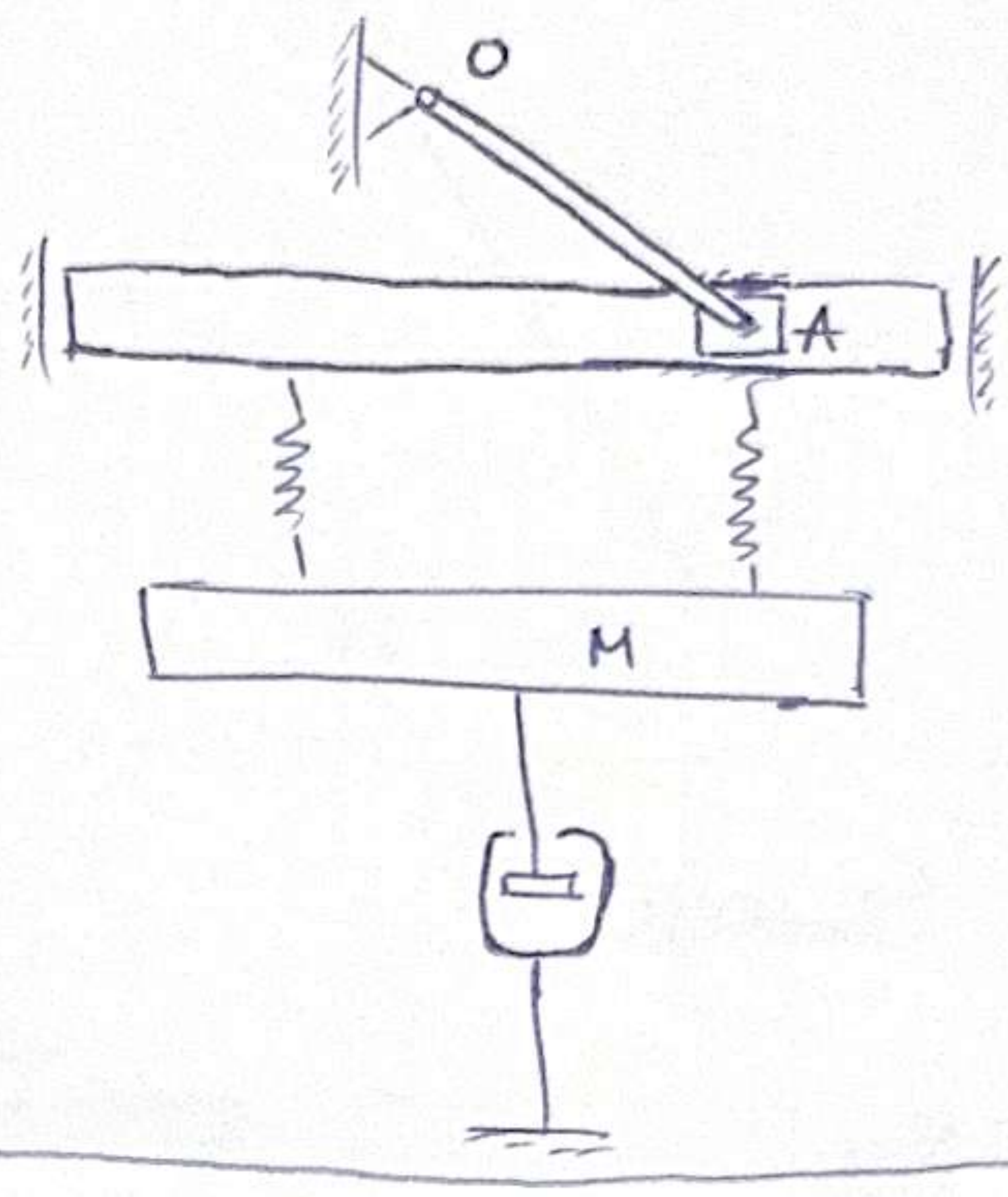
$c = 6{,}8\ \frac{N}{cm}$

$\delta = \omega_s$ - prigušenje

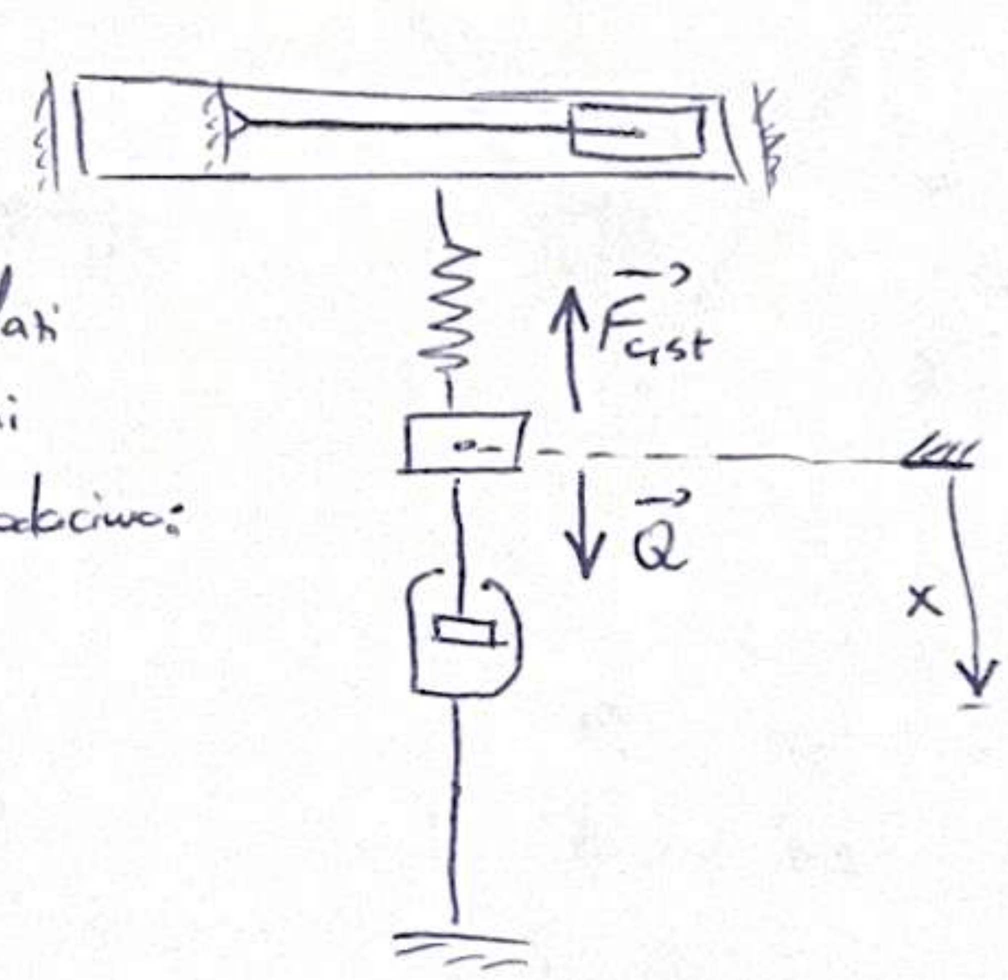
$x_p = ?$

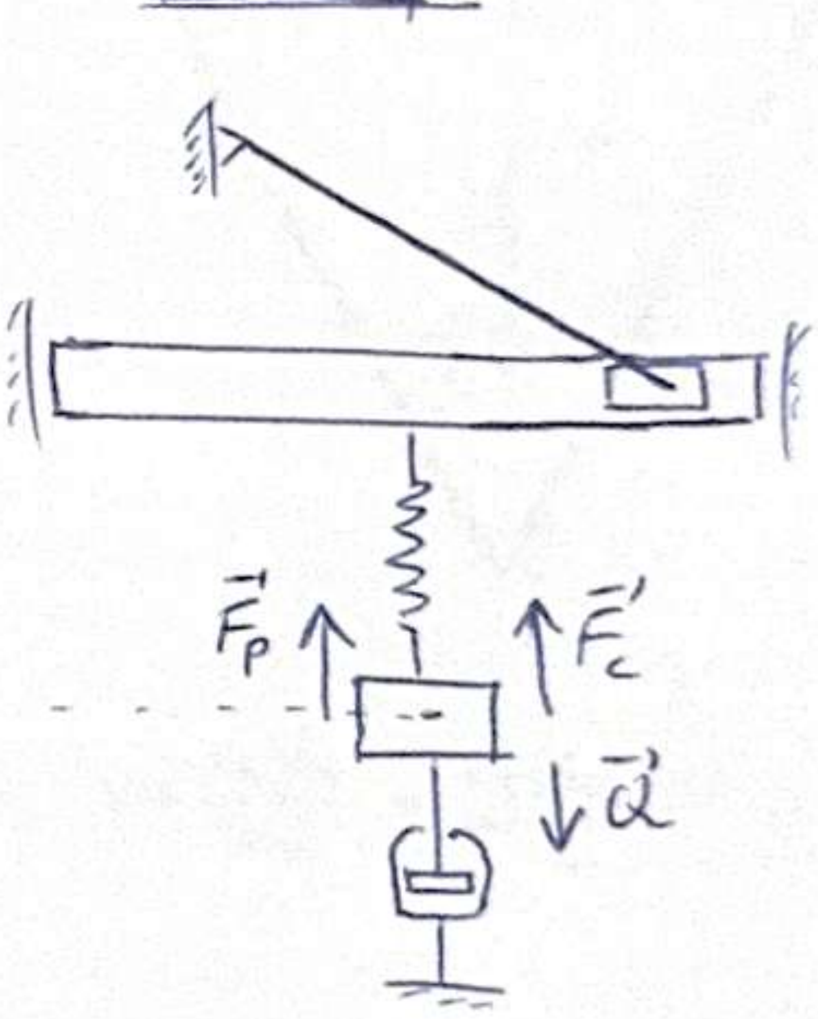
$\overline{OA} = 7{,}5$ cm

$\omega = 6\ s^{-1}$



Napomena:





Analizirajući rezultat koji se nalazi u zbirci sam shvatio da su oni zadatak radili sa sledećim podacima:

$c = 6{,}8\ \frac{N}{m}$, $\overline{OA} = 7{,}5$ m

$c_e = c + c = 2c = 13{,}6\ \frac{N}{m}$

$\varphi = \int \omega\, dt = \omega t = 6t$

st. ravnoteža:

$m\vec a = \vec Q + \vec F_{c,st}$

x: $0 = Q - c f_{st}$

proizvoljni položaj:

$m\vec a = \vec Q + \vec F_c' + \vec F_p$

x: $m\ddot x = Q - c(f_{st} + x - 7{,}5 \sin\varphi) - \beta \cdot \dot x$

$\ddot x + \frac{\beta}{m}\dot x + \frac{c}{m} x = 7{,}5 \cdot \frac{c}{m} \sin(6t)$,

$\ddot x + 2\delta \dot x + \omega^2 x = h \sin(\Omega t)$

$\omega^2 = \frac{c}{m} = \frac{cQ}{g} = 135{,}86$, $\omega = 11{,}656$,

$2\delta = \frac{\beta}{m} = 2\omega = 23{,}312$, $\Omega = 6$

$h = 7{,}5\ \frac{c}{m} = 1018{,}96$

H: nema potrebe da radimo, ne traži se (a očigledno je i da nema rezonanse)

P: (već smo rešavali ovo u zadatku 4.42)

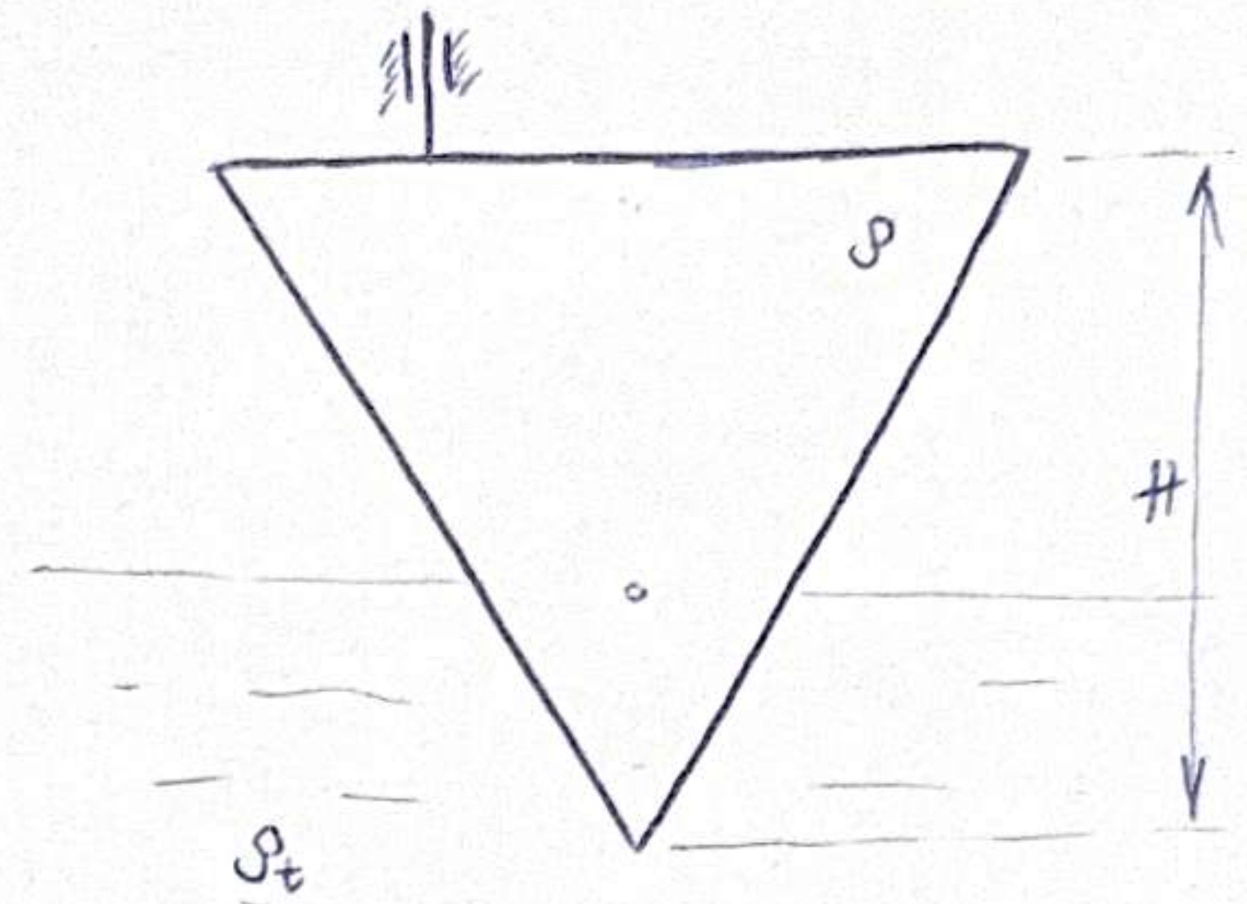
$A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -4{,}83$, $B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega} = 3{,}45$

$\boxed{x_p = -4{,}83 \cos(6t) + 3{,}45 \sin(6t)}$

4.45. plovak: H, ρ, konus

tekućost: ρ_t, $(\rho_t > \rho)$

T_w ? – period malih vertikalnih oscilacija

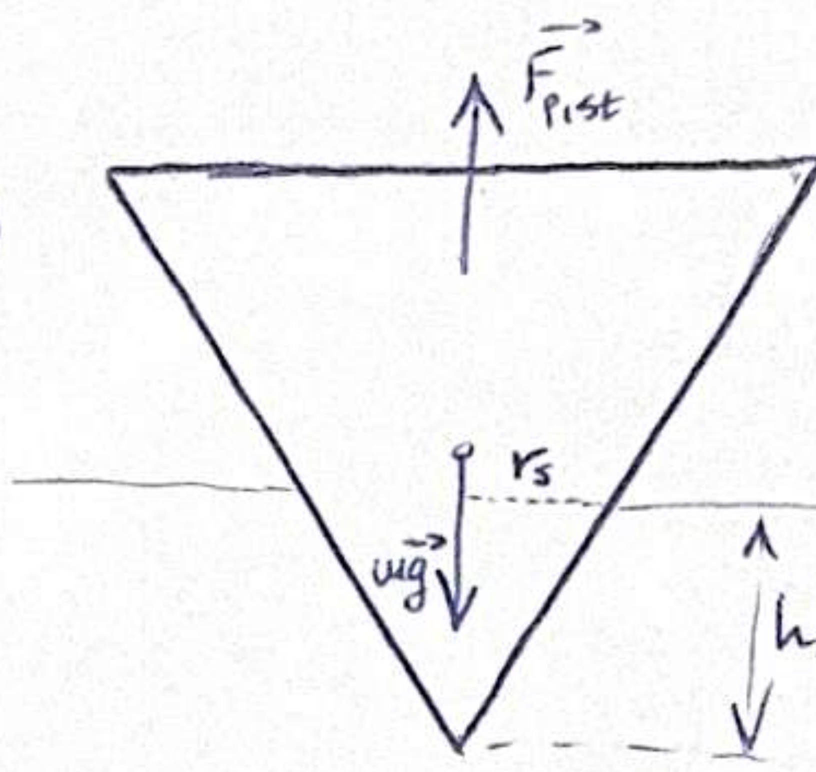


$$T_w = \frac{2\pi}{\omega}, \quad \omega = ?$$

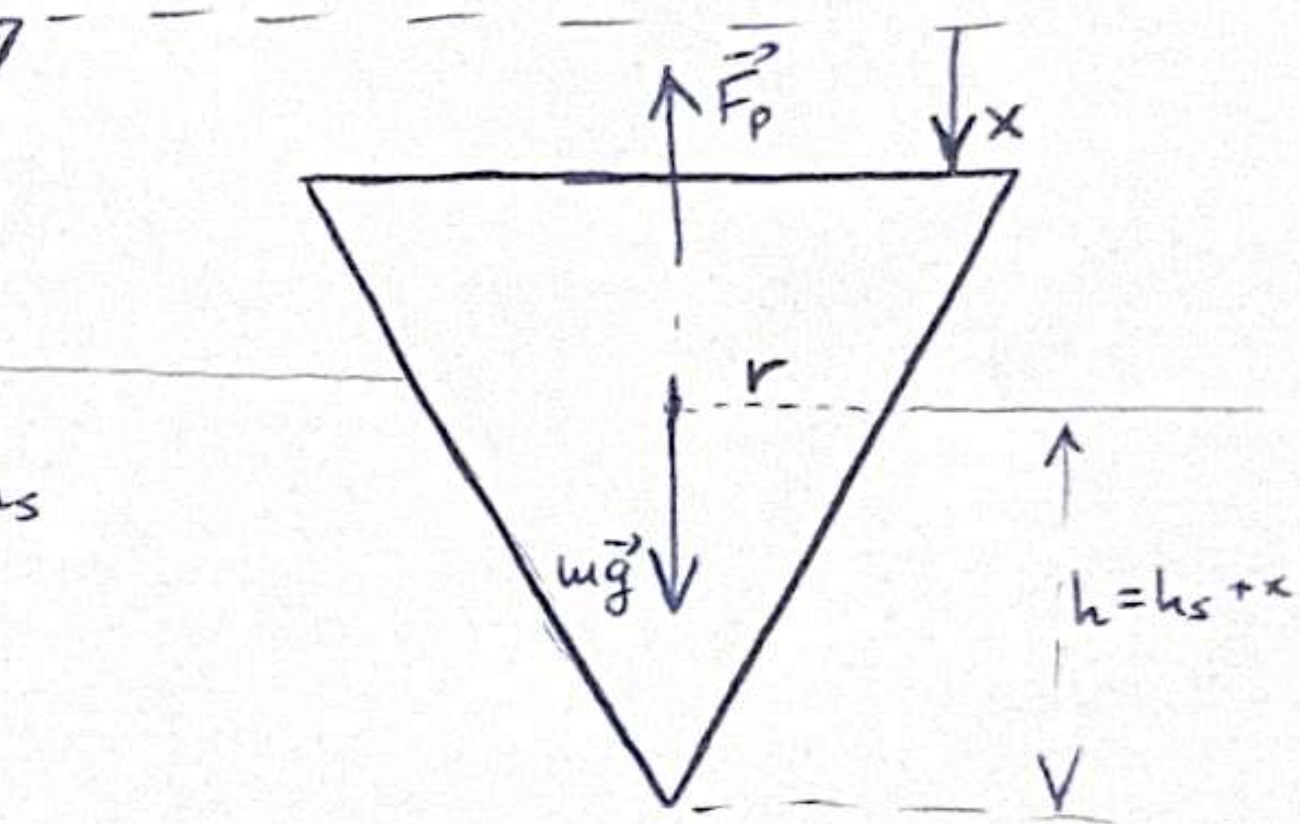
male oscilacije => koord. x je mala =>
=> članovi višeg reda se odbacuju $(x^2, x^3 ...)$
+ $\sin x = x$, $\cos x = 1$, ...

$$F_p = Q_{pot} = \rho_t \cdot V_{pot} \cdot g = \rho_t \cdot \frac{1}{3} r^2 \pi h g$$

stat. ravn.



protiv. pol.



$$r \ldots r_s, \quad h \ldots h_s \quad \Rightarrow \quad r = \frac{r_s \cdot h}{h_s} = \frac{r_s(h_s + x)}{h_s} = r_s\left(1 + \frac{x}{h_s}\right)$$

st. ravn.

$$m\vec{a} = m\vec{g} + \vec{F}_{pist}$$

$$x: \quad 0 = mg - \rho_t \cdot \frac{1}{3} r_s^2 \pi h_s \cdot g \qquad (1)$$

protiv. pol.

$$m\vec{a} = m\vec{g} + \vec{F}_p$$

$$x: \quad m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} r^2 \pi h g$$

$$m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} r_s^2 \left(1 + \frac{x}{h_s}\right)^2 \pi (h_s + x) g$$

$$m\ddot{x} = mg - \rho_t \cdot \frac{1}{3}\left(r_s^2 + 2r_s^2 \frac{x}{h_s} + r_s^2 \frac{x^2}{h_s^2}\right)\pi (h_s + x) g$$

$$m\ddot{x} = \cancel{mg} - \rho_t \cdot \frac{1}{3}\Big(\cancel{r_s^2 \pi h_s} + r_s^2 \pi x + 2r_s^2 \frac{x}{\cancel{h_s}} \pi \cancel{h_s} + 2r_s^2 \cancel{\frac{x}{h_s}\pi x}_{\to x^2 \approx 0} + \cancel{r_s^2 \frac{x^2}{h_s^2} \pi h_s}_{\to x^2 \approx 0} + \cancel{r_s^2 \frac{x^2}{h_s^2} \pi x}_{\to x^3 \approx 0}\Big) g$$

$$m\ddot{x} = -\frac{1}{3}\rho_t (3r_s^2 \pi x) g \qquad /: m$$

$$\ddot{x} + \frac{\rho_t}{m} r_s^2 \pi g x = 0, \quad \omega^2 = \frac{\rho_t}{m} r_s^2 \pi g \quad m = ?, \ r_s = ?$$

$$m = \frac{1}{3}\rho \cdot V = \frac{1}{3}\rho \cdot R^2 \pi \cdot H$$

$$\omega^2 = \frac{3\rho_t}{\rho R^2 \pi H} r_s^2 \pi g = \frac{3\rho_t r_s^2 g}{\rho R^2 H}, \quad r_s = ?, \ R = ?$$

$$r_s \ldots R, \quad h_s \ldots H \quad \Rightarrow \quad r_s = \frac{R h_s}{H}$$

$$\omega^2 = \frac{3\rho_t \cancel{R^2} h_s^2 g}{\rho \cancel{R^2} \cdot H^2 \cdot H} = \frac{3\rho_t h_s^2 g}{\rho H^3}, \quad h_s = ?$$

$$(1) \Rightarrow mg - \rho_t \cdot \frac{1}{3} r_s^2 \pi \cdot h_s \cdot g = 0$$

$$\Rightarrow h_s = \frac{3m}{\rho_t \cdot r_s^2 \cdot \pi} = \frac{\cancel{3} \cdot \cancel{\frac{1}{3}} \rho R^2 \cancel{\pi} H}{\rho_t r_s^2 \cancel{\pi}}$$

$$\Rightarrow h_s = \frac{\rho R^2 H}{\rho_t \cdot \frac{R^2 h_s^2}{H^2}} = \frac{\rho R^2 H^3}{\rho_t R^2 h_s^2}$$

$$\Rightarrow h_s^3 = \frac{\rho H^3}{\rho_t} \Rightarrow h_s = \sqrt[3]{\frac{\rho}{\rho_t}} \cdot H$$

$$\omega^2 = \frac{3\rho_t \sqrt[3]{\frac{\rho}{\rho_t}}\, H^2 g}{\rho \cdot H^3} = \frac{3g}{H} \frac{\rho^{-1}}{\rho_t^{-1}} \cdot \frac{\rho^{\frac{2}{3}}}{\rho_t^{\frac{2}{3}}}$$

$$\omega^2 = \frac{3g}{H} \frac{\rho^{-\frac{1}{3}}}{\rho_t^{-\frac{1}{3}}} = \frac{3g}{H} \sqrt[3]{\frac{\rho_t}{\rho}}$$

$$T_w = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{H}} \cdot \sqrt[6]{\frac{\rho_t}{\rho}}}$$

$$\boxed{T_w = 2\pi \cdot \sqrt{\frac{H}{3g}} \cdot \sqrt[6]{\frac{\rho}{\rho_t}}}$$