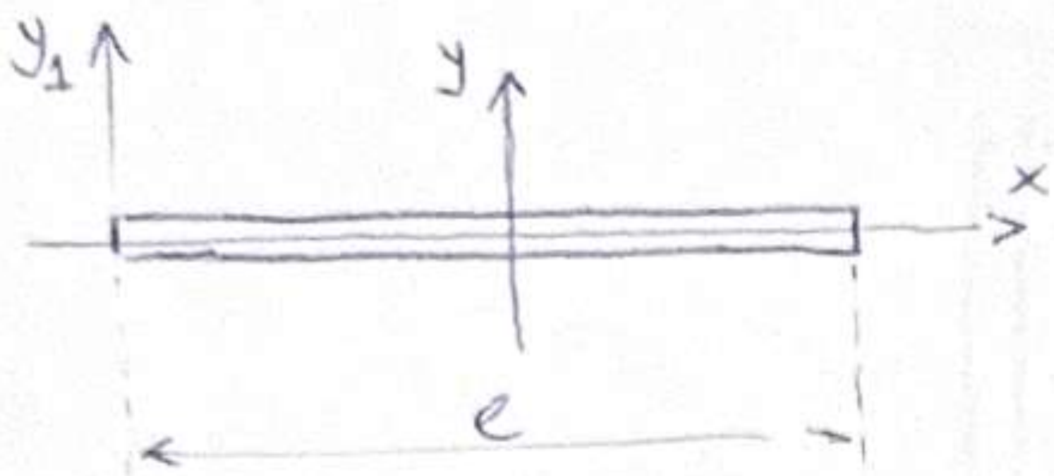
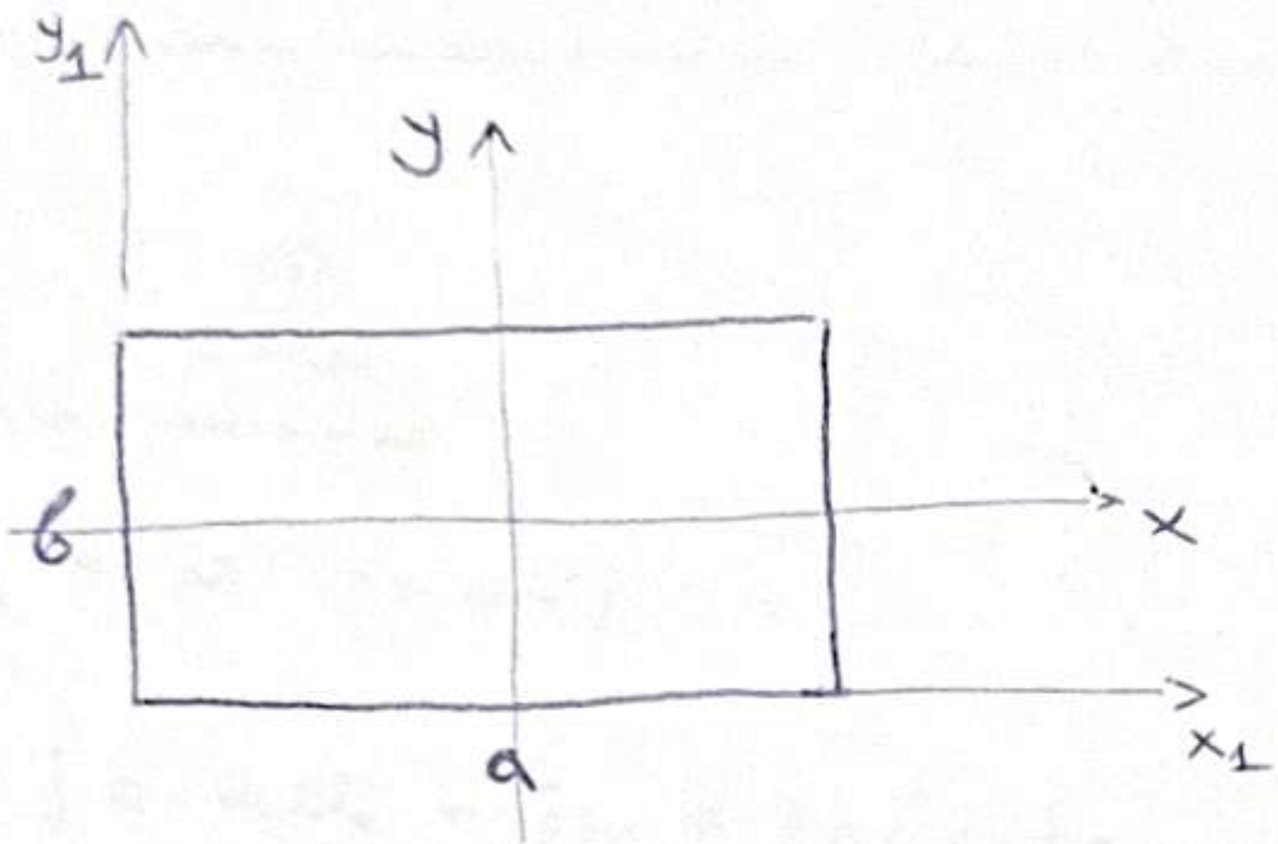


$J_x = 0$

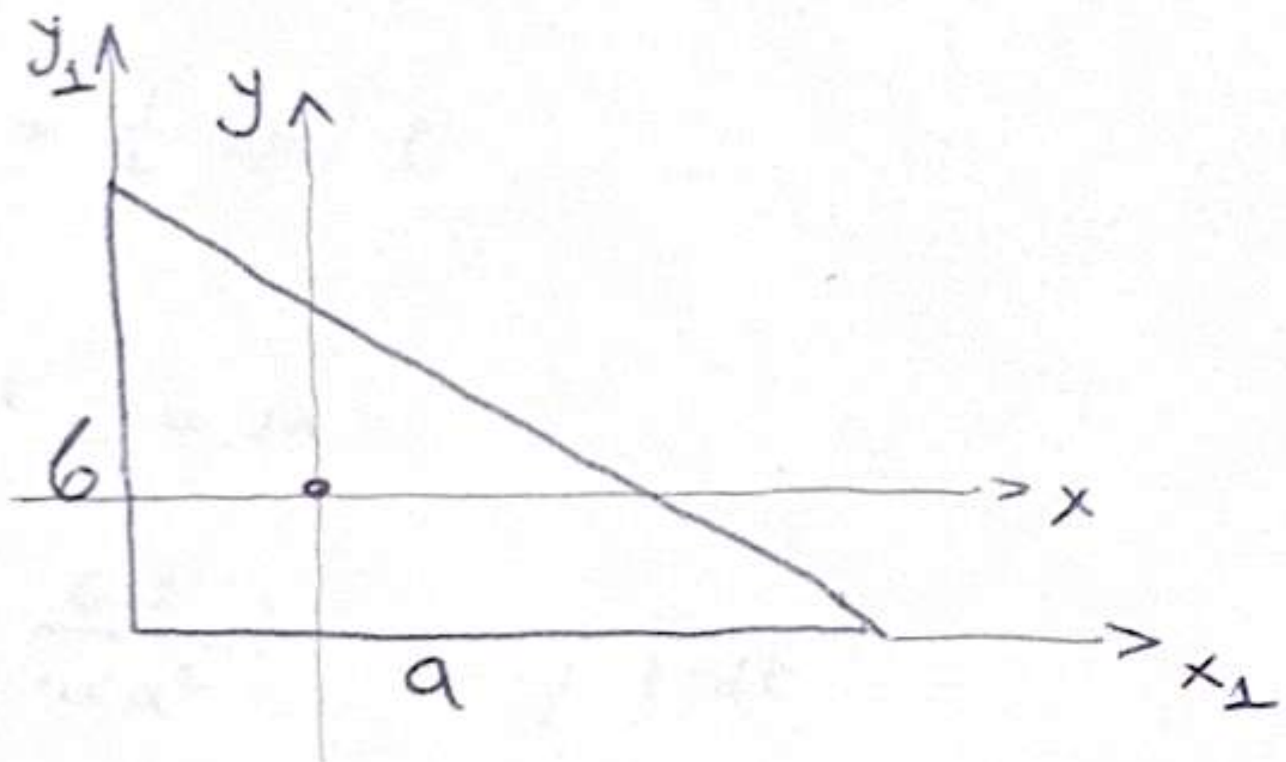
$J_y = \frac{1}{12} m l^2$

$J_{y_1} = \frac{1}{3} m l^2$



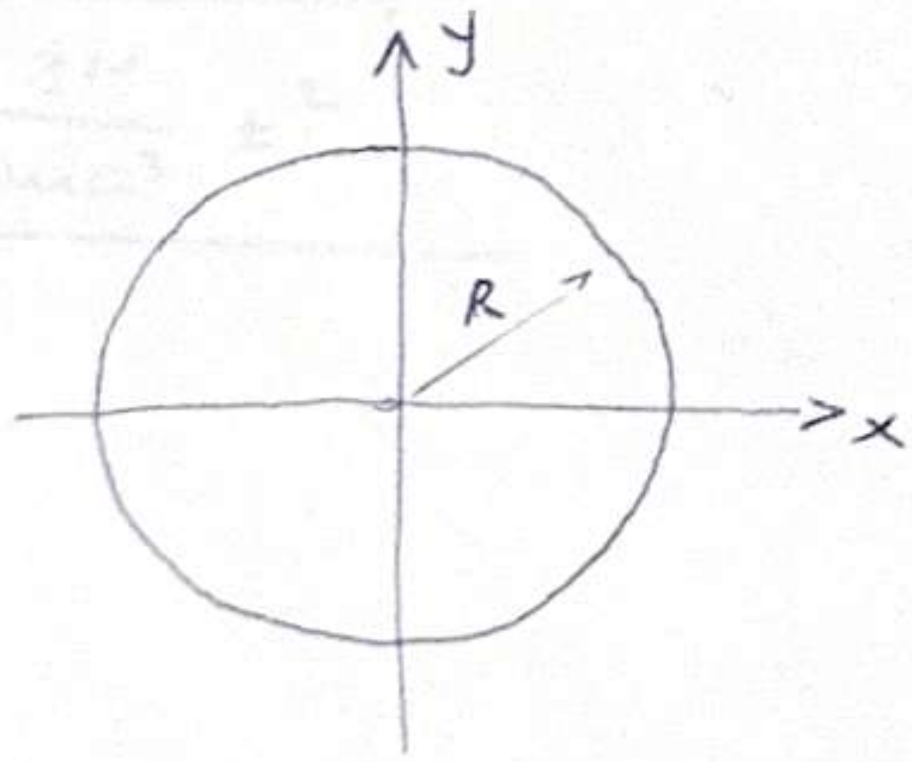
$J_x = \frac{1}{12} m b^2$, $J_y = \frac{1}{12} m a^2$, $J_z = \frac{1}{12} m (a^2 + b^2)$

$J_{x_1} = \frac{1}{3} m b^2$, $J_{y_1} = \frac{1}{3} m a^2$

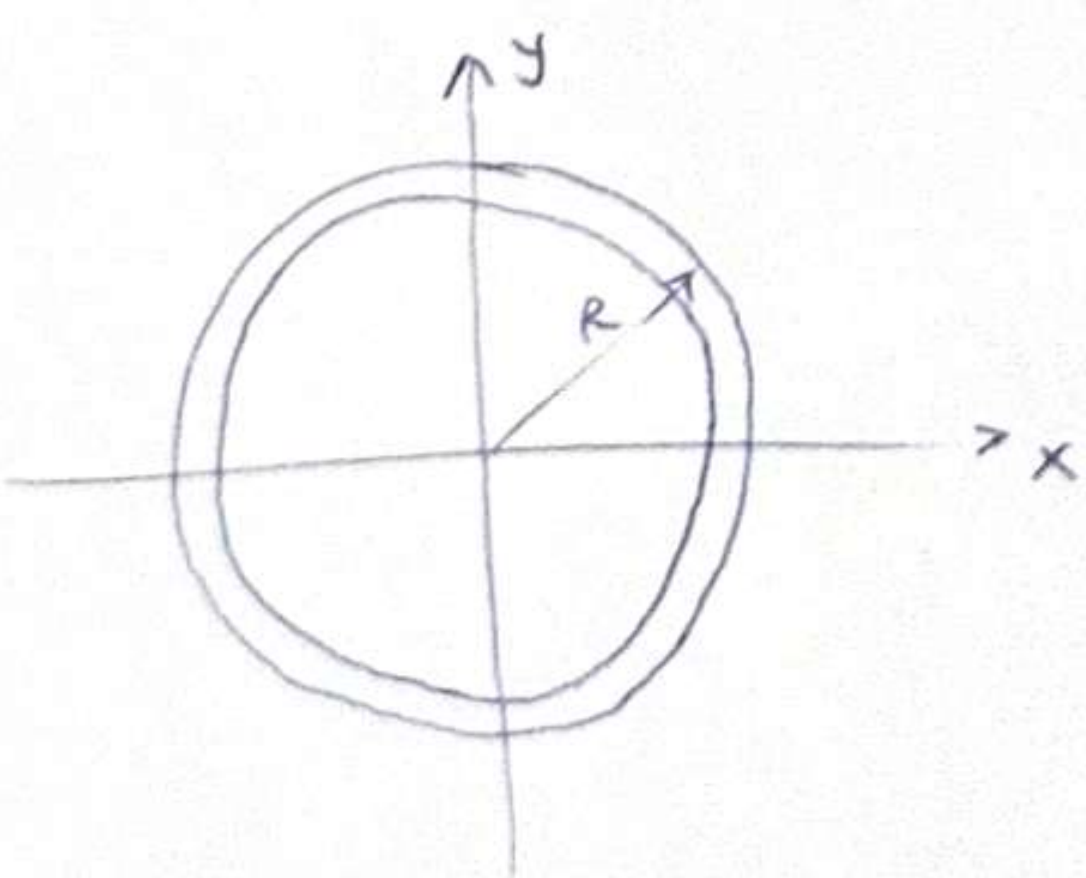


$J_{x\bullet} = \frac{1}{18} m b^2$, $J_{y\bullet} = \frac{1}{18} m a^2$, $J_z = \frac{1}{18} m (a^2 + b^2)$

$J_{x_1} = \frac{1}{6} m b^2$, $J_{y_1} = \frac{1}{6} m a^2$

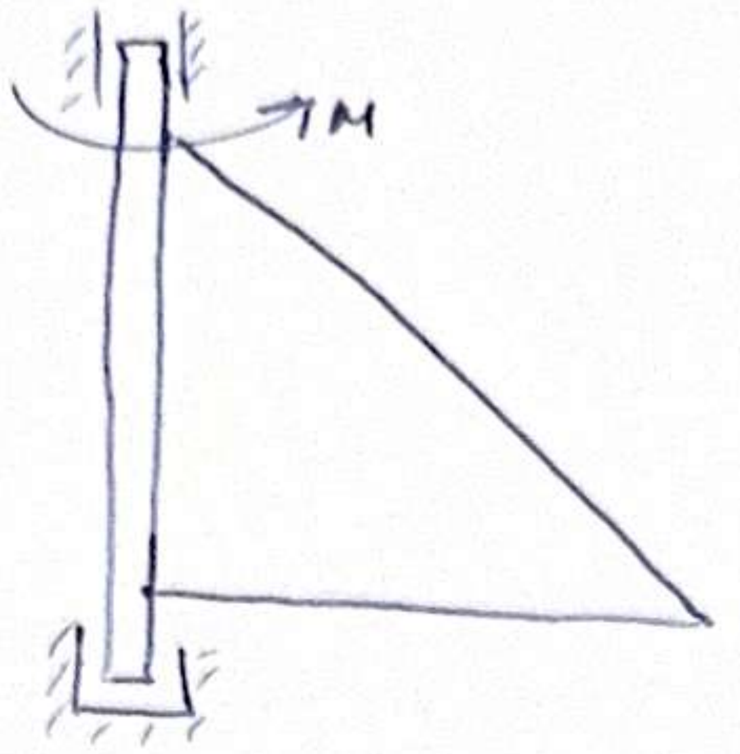


$J_x = \frac{1}{4} m R^2$, $J_y = \frac{1}{4} m R^2$, $J_z = \frac{1}{2} m R^2$



$J_x = \frac{1}{2} m R^2$, $J_y = \frac{1}{2} m R^2$, $J_z = m R^2$

10.1



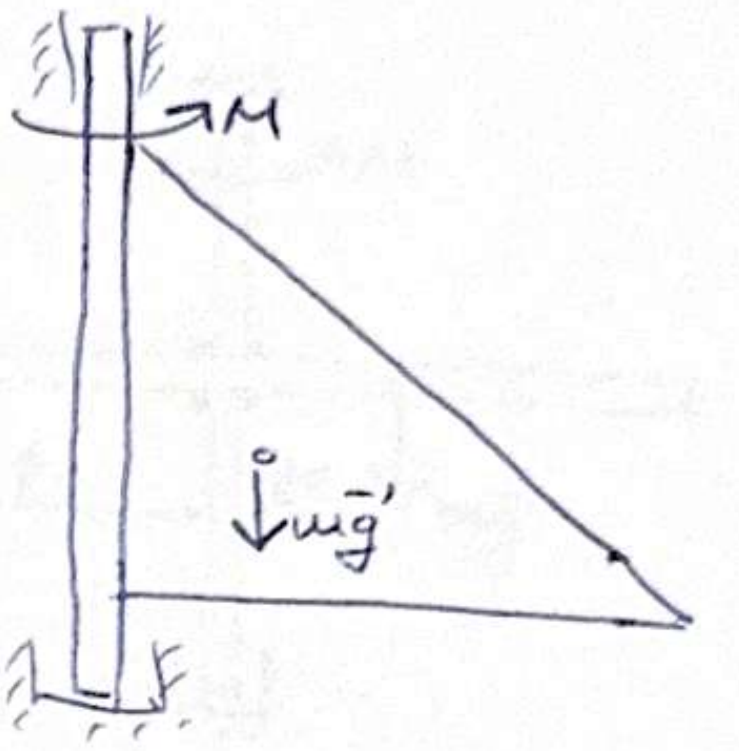
m

$a \times a$

$M = const.$

$\varphi = ?$

t_0: stanje mirovanja



$$\frac{d\vec{L}_{Oz}}{dt} = \Sigma \vec{M}_{Oz}$$

$$\Sigma M_{Oz} = M \quad (= const.)$$

$$L_{Oz} = \frac{1}{3} a \cdot m v_c + J\omega = \frac{1}{3} a \cdot m \cdot \frac{1}{3} a\omega + \frac{1}{18} m a^2 \omega$$

$$L_{Oz} = \frac{1}{6} m a^2 \omega$$

$$\frac{dL_{Oz}}{dt} = \frac{1}{6} m a^2 \varepsilon$$

$$\frac{1}{6} m a^2 \varepsilon = M$$

$$\varepsilon = \frac{6M}{ma^2} \quad / \int dt$$

$$\omega = \frac{6M}{ma^2} t \quad / \int dt$$

$$\boxed{\varphi = \frac{3M}{ma^2} t^2}$$

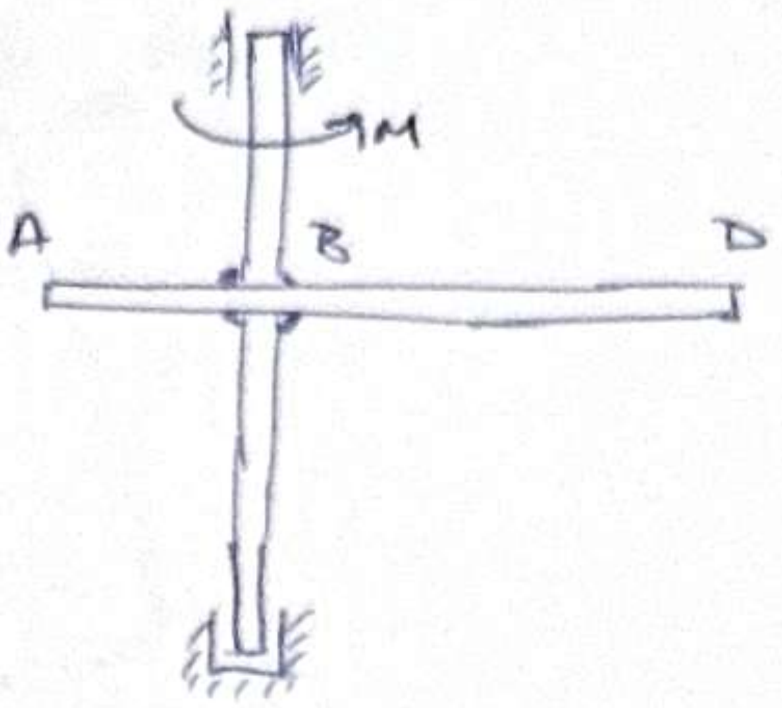
10.2

AD: $m = 1\ kg$, $l = 2\ m$

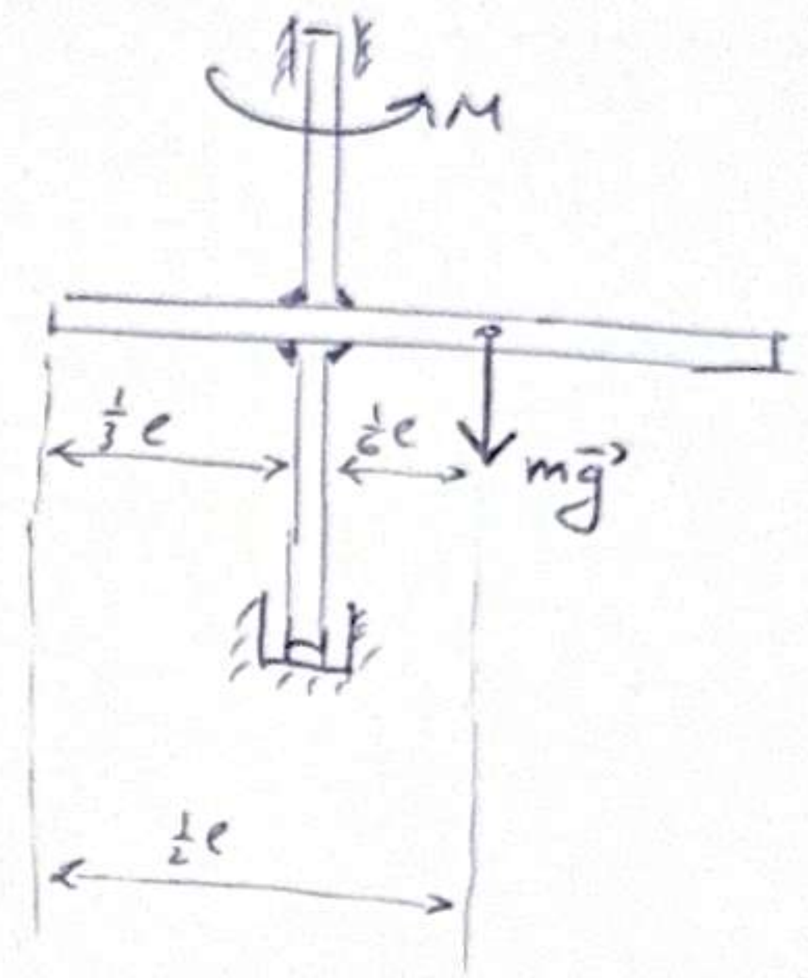
$\overline{AB} = \frac{1}{3} l$

$M = 4\ kNm$

$\varepsilon = ?$



$$\frac{dL_{Oz}}{dt} = \sum M_{Oz}$$



$\sum M_{Oz} = M \quad (= const)$

$L_{Oz} = \frac{1}{6} l \cdot m v_C + J_C \omega = \frac{1}{6} l m \cdot \frac{1}{6} l \omega + \frac{1}{12} m l^2 \omega =$

$L_{Oz} = \frac{1}{9} m l^2 \omega$

$\frac{dL_{Oz}}{dt} = \frac{1}{9} m l^2 \varepsilon$

$\frac{1}{9} m l^2 \varepsilon = M$

$\varepsilon = \frac{9 \cdot M}{m l^2} = \frac{9 \cdot 4}{1 \cdot 2^2} = 9$

$\boxed{\varepsilon = 9\ s^{-2}}$

10. 3

$m = 10 \text{ kg}$

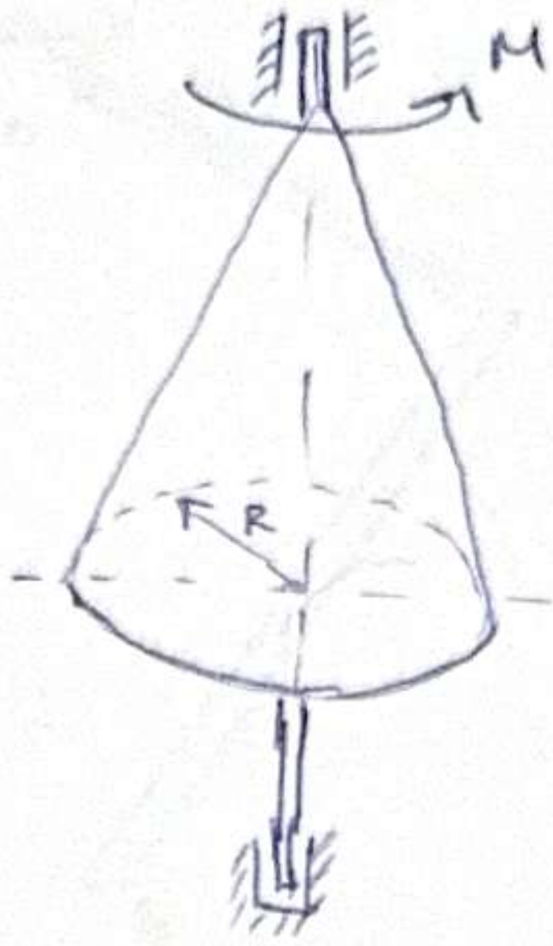
$R = 0{,}2 \text{ m}$

$M = 6 \text{ Nm}$

$t_0 = 0$: mirovanje

$t_1 = ?$

$\omega(t_1) = 20 \text{ s}^{-1}$



$$\frac{d\overset{\curvearrowleft}{L}_{O_7}}{dt} = \Sigma \overset{\curvearrowleft}{M}_{O_7}$$

$$\Sigma \overset{\curvearrowleft}{M}_{O_7} = M \quad (= \text{const.})$$

$$L_{O_7} = 0 + J_{C_7}\,\omega = \frac{3}{10} m R^2 \omega$$

$$\frac{dL_{O_7}}{dt} = \frac{3}{10} m R^2 \varepsilon$$

$$\frac{3}{10} m R^2 \varepsilon = M$$

$$\varepsilon = \frac{10 M}{3 m R^2} = \frac{10 \cdot 6}{3 \cdot \cancel{10} \cdot 0{,}2^2} = 50 \qquad / \int dt$$

$$\omega = 50\, t$$

$$\omega(t_1) = 50 \cdot t_1 = 20 \quad \Rightarrow \quad t_1 = \frac{2}{5} \quad \Rightarrow \quad \boxed{t_1 = 0{,}4 \text{ s}}$$

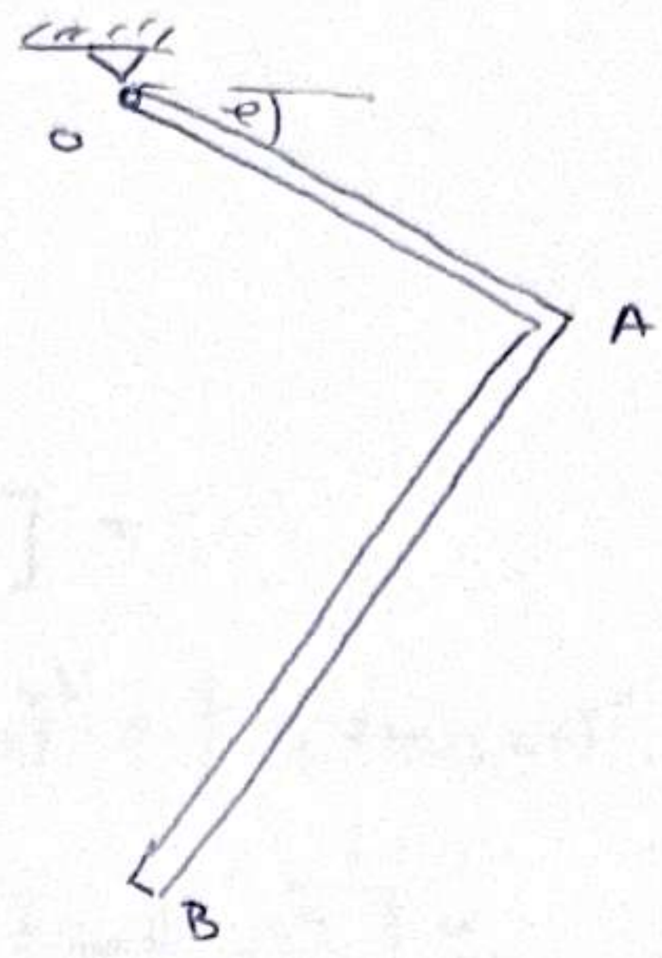
(10. 5.)

OA: m, l

AB: $2m, 2l$

$t_0=0$: OA je horiz. mirovanje

$\omega(\varphi)$? $\varepsilon(\varphi)$? $\vec{N}(\varphi)$?



Zadatak je moguće uraditi na 3 načina:

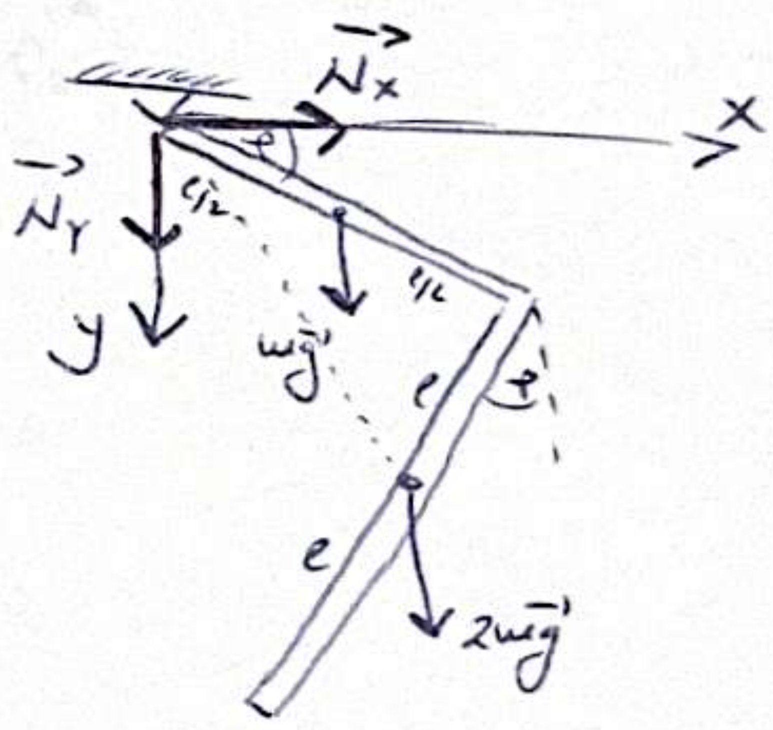
1) $\frac{dL_{Oz}}{dt} = \Sigma M_{Oz}$ → odavde dobijemo ε, zatim <u>integralimo</u> da dobijemo ω

2) $T - T_0 = A$ → odavde dobijamo ω, zatim radimo <u>izvod</u> da dobijemo ε

3) $\frac{dT}{dt} = \frac{d'A}{dt}$ → odavde dobijamo ε, zatim <u>integralimo</u> da dobijemo ω

Uvek je lakše raditi izvod nego integraliti. Ali svejedno ću na oba načina uraditi:

<u>I način</u>



$$\frac{dL_{Oz}}{dt} = \Sigma M_{Oz}$$

$$\Sigma M_{Oz} = mg\cdot\frac{l}{2}\cos\varphi + 2mg(l\cos\varphi - l\sin\varphi) = \frac{5}{2}mgl\cos\varphi - 2mgl\sin\varphi$$

$$L_{Oz} = \left[\frac{l}{2}\cdot m v_1 + J_1\cdot\omega\right] + \left[l\sqrt{2}\cdot 2m v_2 + J_2\omega\right] =$$

$$L_{Oz} = \left[\frac{l}{2} m \frac{l}{2}\omega + \frac{1}{12}ml^2\omega\right] + \left[l\sqrt{2}\cdot 2m\cdot l\sqrt{2}\cdot\omega + \frac{1}{12}2m(2l)^2\omega\right] =$$

$$L_{Oz} = \frac{1}{3}ml^2\omega + \frac{14}{3}ml^2\omega = 5ml^2\omega$$

$$\frac{dL_{Oz}}{dt} = 5ml^2\varepsilon$$

$$5ml^2\varepsilon = \frac{5}{2}mgl\cos\varphi - 2mgl\sin\varphi \Rightarrow \boxed{\varepsilon = \frac{1}{10}\frac{g}{l}(5\cos\varphi - 4\sin\varphi)}$$

$$\varepsilon = \frac{d\omega}{dt}\cdot\frac{d\varphi}{d\varphi} = \frac{d\omega\cdot\omega}{d\varphi} = \frac{1}{10}\frac{g}{l}(5\cos\varphi - 4\sin\varphi)$$

$$\omega\, d\omega = \frac{1}{10}\frac{g}{l}(5\cos\varphi - 4\sin\varphi)\,d\varphi \qquad /\int$$

$$\frac{1}{2}\omega^2 = \frac{1}{10}\frac{g}{l}(5\sin\varphi + 4\cos\varphi) + C_1$$

$$\omega = \sqrt{\frac{1}{5}\frac{g}{l}(5\sin\varphi + 4\cos\varphi) + C_1}$$

$$\omega(t_0=0) = \sqrt{\frac{1}{5}\frac{g}{l}(0+4) + C_1} = 0 \Rightarrow C_1 = -\frac{4}{5}\frac{g}{l} \Rightarrow \boxed{\omega = \sqrt{\frac{1}{5}\frac{g}{l}(5\sin\varphi + 4\cos\varphi - 4)}}$$

II način

$T - T_0 = A$

$T_0 = 0$

$$T = \left[\frac{1}{2} m v_1^2 + \frac{1}{2} J_1 \omega^2\right] + \left[\frac{1}{2} 2m v_2^2 + \frac{1}{2} J_2 \omega^2\right] =$$

$$T = \left[\frac{1}{2} m \left(\frac{l}{2}\omega\right)^2 + \frac{1}{2}\frac{1}{12} m l^2 \omega^2\right] + \left[\frac{1}{2} 2m \left(l\sqrt{2}\,\omega\right)^2 + \frac{1}{2}\frac{1}{12} 2m (2l)^2 \omega^2\right] =$$

$$T = \left[\frac{1}{8} m l^2 \omega^2 + \frac{1}{24} m l^2 \omega^2\right] + \left[2 m l^2 \omega^2 + \frac{1}{3} m l^2 \omega^2\right] =$$

$$T = \frac{1}{6} m l^2 \omega^2 + \frac{7}{3} m l^2 \omega^2 = \frac{5}{2} m l^2 \omega^2$$

$$A = \int m g\, dy_1 + \int 2 m g\, dy_2 =$$

$$\boxed{\begin{aligned} y_1 &= \frac{1}{2} l \sin\varphi \quad \Rightarrow \quad \frac{dy_1}{\cancel{dt}} = \frac{1}{2} l \cos\varphi \cdot \frac{d\varphi}{\cancel{dt}} \\ y_2 &= l \sin\varphi + l\cos\varphi \quad \Rightarrow \quad \frac{dy_2}{\cancel{dt}} = l\cos\varphi \frac{d\varphi}{\cancel{dt}} - l \sin\varphi \frac{d\varphi}{\cancel{dt}} \end{aligned}}$$

$$A = \int_{\varphi_0 = 0}^{\varphi} m g \cdot \frac{1}{2} l \cos\varphi\, d\varphi + \int_{\varphi_0 = 0}^{\varphi} 2 m g \cdot (l\cos\varphi - l \sin\varphi)\, d\varphi =$$

$$A = \frac{1}{2} m g l (\sin\varphi - \cancel{\sin 0}^{0}) + 2 m g l (\sin\varphi - \cancel{\sin 0}^{0}) + 2 m g l (\cos\varphi - \cancel{\cos 0}^{1})$$

$$A = \frac{5}{2} m g l \sin\varphi + 2 m g l \cos\varphi - 2 m g l$$

$$\frac{5}{2} \cancel{m} l^{\cancel{2}} \omega^2 = \frac{5}{2} \cancel{m} g \cancel{l} \sin\varphi + 2 \cancel{m} g \cancel{l} \cos\varphi - 2 \cancel{m} g \cancel{l}$$

$$\boxed{\omega = \sqrt{\frac{1}{5}\frac{g}{l}(5\sin\varphi + 4\cos\varphi - 4)}}$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{1}{2 \cdot \sqrt{\cdots}} \cdot \frac{1}{5}\frac{g}{l}(5\cos\varphi - 4\sin\varphi) \cdot \cancel{\omega}$$

$$\boxed{\varepsilon = \frac{1}{10}\frac{g}{l}(5\cos\varphi - 4\sin\varphi)}$$

Nastavak (na prvi način ili na drugi način):

$$M\vec{a}_C = m\vec{g} + 2m\vec{g} + \vec{N}_X + \vec{N}_Y$$

$$m\vec{a}_1 + 2m\vec{a}_2 = m\vec{g} + 2m\vec{g} + \vec{N}_X + \vec{N}_Y$$

x: $m\ddot{x}_1 + 2m\ddot{x}_2 = N_X$

$x_1 = \frac{1}{2}l\cos\varphi \Rightarrow \dot{x}_1 = -\frac{1}{2}l\sin\varphi\cdot\omega \Rightarrow \ddot{x}_1 = -\frac{1}{2}\omega^2 l\cos\varphi$

$x_2 = l\cos\varphi - l\sin\varphi \Rightarrow \dot{x}_2 = -l\omega\sin\varphi - l\omega\cos\varphi \Rightarrow \ddot{x}_2 = \omega^2 l\sin\varphi - \omega^2 l\cos\varphi$

$$N_X = -\frac{1}{2}ml\omega^2\cos\varphi + 2ml\omega^2\sin\varphi - 2ml\omega^2\cos\varphi$$

$$\boxed{N_X = 2ml\omega^2\sin\varphi - \frac{5}{2}ml\omega^2\cos\varphi}$$

y: $m\ddot{y}_1 + 2m\ddot{y}_2 = mg + 2mg + N_Y$

$y_1 = \frac{1}{2}l\sin\varphi \Rightarrow \dot{y}_1 = \frac{1}{2}l\omega\cos\varphi \Rightarrow \ddot{y}_1 = -\frac{1}{2}l\omega^2\sin\varphi$

$y_2 = l\sin\varphi + l\cos\varphi \Rightarrow \dot{y}_2 = l\omega\cos\varphi - l\omega\sin\varphi \Rightarrow \ddot{y}_2 = -l\omega^2\sin\varphi - l\omega^2\cos\varphi$

$$-\frac{1}{2}ml\omega^2\sin\varphi - 2ml\omega^2\sin\varphi - 2ml\omega^2\cos\varphi = 3mg + N_Y$$

$$\boxed{N_Y = -3mg - 2ml\omega^2\cos\varphi - \frac{5}{2}ml\omega^2\sin\varphi}$$

(10.6)

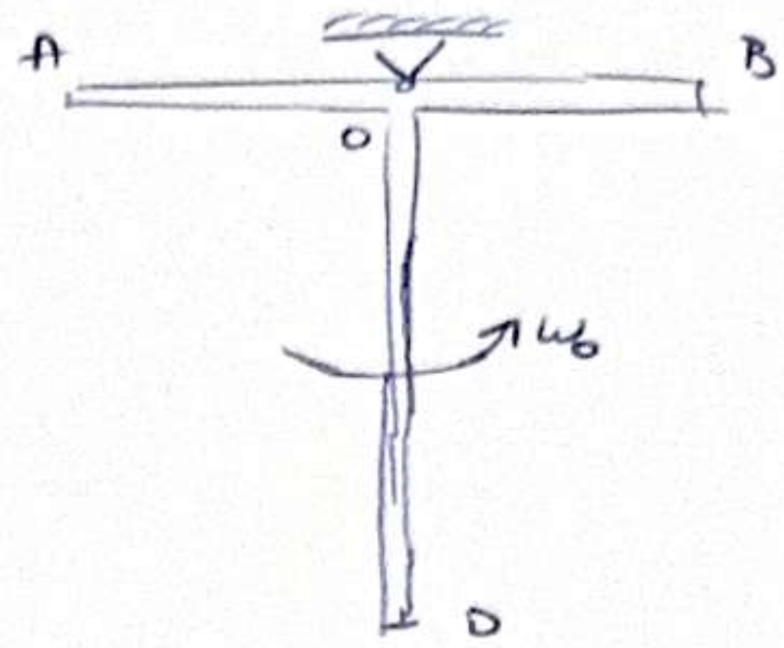
OD: m, ℓ

AB: m, ℓ

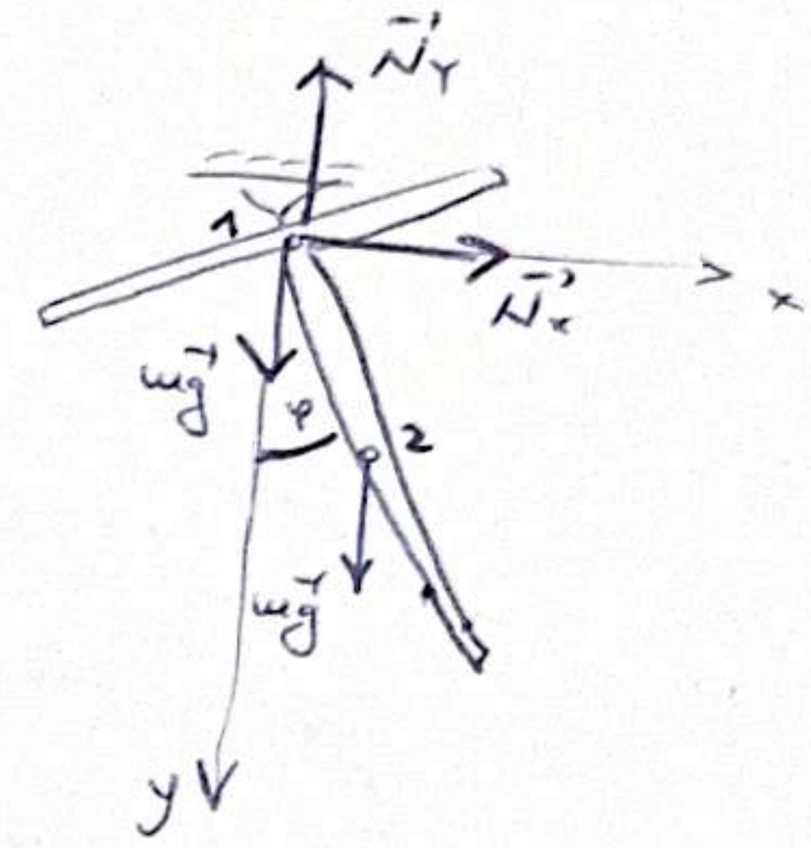
$\varepsilon(\varphi)$? $\omega(\varphi)$? $N(\varphi)$?

t_0: ravnotežni položaj

$\omega_0 = \sqrt{\frac{12}{5}\frac{g}{\ell}}$



Napomena: opet može na više načina, pošto u prethodnom zadatku nismo koristili treći način, sada ćemo.



$$\frac{dT}{dt} = \frac{d'A}{dt}$$

$$T = \left[0 + \frac{1}{2}J_1\omega^2\right] + \left[\frac{1}{2}m v_1^2 + \frac{1}{2}J_2\omega^2\right] =$$

$$T = \left[\frac{1}{2}\frac{1}{12}m\ell^2\omega^2\right] + \left[\frac{1}{2}m\left(\frac{1}{2}\omega\ell\right)^2 + \frac{1}{2}\frac{1}{12}m\ell^2\omega^2\right] =$$

$$T = \frac{1}{24}m\ell^2\omega^2 + \frac{1}{8}m\ell^2\omega^2 + \frac{1}{24}m\ell^2\omega^2 =$$

$$T = \frac{5}{24}m\ell^2\omega^2$$

$$\frac{dT}{dt} = \frac{5}{12}m\ell^2\omega\varepsilon$$

$$d'A = 0 + mg\,dy_2 = -\frac{1}{2}mg\ell\sin\varphi\,d\varphi$$

$$\boxed{y_2 = \frac{1}{2}\ell\cos\varphi \quad \Rightarrow \quad \frac{dy_2}{\cancel{dt}} = -\frac{1}{2}\ell\sin\varphi\frac{d\varphi}{\cancel{dt}}}$$

$$\frac{d'A}{dt} = -\frac{1}{2}mg\ell\sin\varphi\cdot\omega$$

$$\frac{5}{12}m\ell^2\omega\,\varepsilon = -\frac{1}{2}mg\ell\sin\varphi$$

$$\boxed{\varepsilon = -\frac{6}{5}\frac{g}{\ell}\sin\varphi}$$

$$\varepsilon = \frac{d\omega}{dt}\frac{d\varphi}{d\varphi} = \frac{d\omega\cdot\omega}{d\varphi} = -\frac{6}{5}\frac{g}{\ell}\sin\varphi$$

$$\omega\,d\omega = -\frac{6}{5}\frac{g}{\ell}\sin\varphi\,d\varphi \qquad /\int dt$$

$$\frac{1}{2}\omega^2 = \frac{6}{5}\frac{g}{\ell}\cos\varphi + C_1$$

$$\omega = \sqrt{\frac{12}{5}\frac{g}{\ell}\cos\varphi + C_1}$$

$$\omega(t_0=0) = \sqrt{\frac{12}{5}\frac{g}{\ell}\cdot 1 + C_1} = \sqrt{\frac{12}{5}\frac{g}{\ell}} \quad \Rightarrow \quad \underline{C_1 = 0}$$

$$\boxed{\omega = \sqrt{\frac{12}{5}\frac{g}{\ell}}}$$

$$M\vec{a}_C = m\vec{g} + m\vec{g} + \vec{N}_X + \vec{N}_Y$$

$$m\vec{a}_1 + m\vec{a}_2 = m\vec{g} + m\vec{g} + \vec{N}_X + \vec{N}_Y$$

$x:$ $\quad m\ddot{x}_1 + m\ddot{x}_2 = N_X$

$x_1 = 0 \Rightarrow \dot{x}_1 = 0 \Rightarrow \ddot{x}_1 = 0$

$x_2 = \frac{1}{2}\ell\sin\varphi \Rightarrow \dot{x}_2 = +\frac{1}{2}\ell\omega\cos\varphi \Rightarrow \ddot{x}_2 = -\frac{1}{2}\ell\omega^2\sin\varphi$

$$\boxed{N_X = -\frac{1}{2}m\ell\omega^2\sin\varphi}$$

$y:$ $\quad m\ddot{y}_1 + m\ddot{y}_2 = mg + mg - N_Y$

$y_1 = 0 \Rightarrow \dot{y}_1 = 0 \Rightarrow \ddot{y}_1 = 0$

$y_2 = \frac{1}{2}\ell\cos\varphi \Rightarrow \dot{y}_2 = -\frac{1}{2}\ell\omega\sin\varphi \Rightarrow \ddot{y}_2 = -\frac{1}{2}\ell\omega^2\cos\varphi$

$$0 - \frac{1}{2}m\ell\omega^2\cos\varphi = 2mg - N_Y$$

$$\boxed{N_Y = 2mg + \frac{1}{2}m\ell\omega^2\cos\varphi}$$

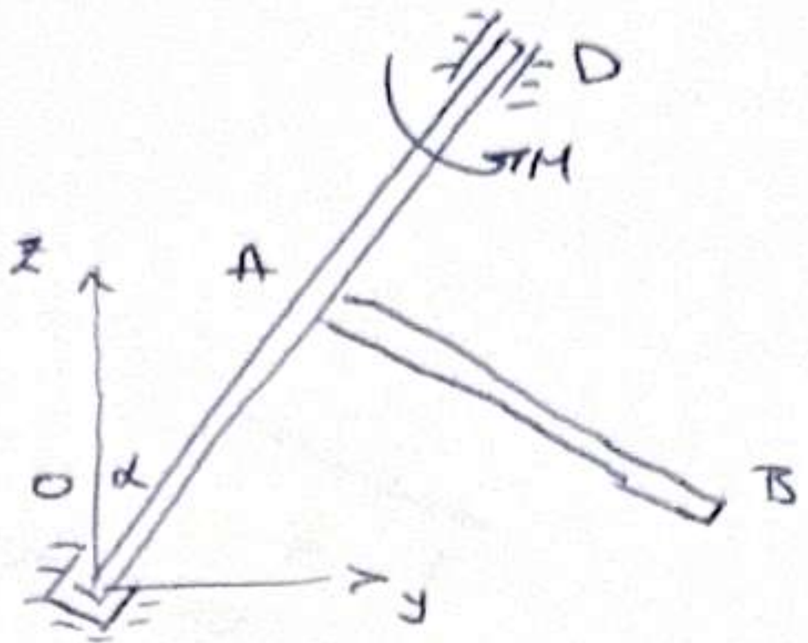
(10.7.)

AB : m, l

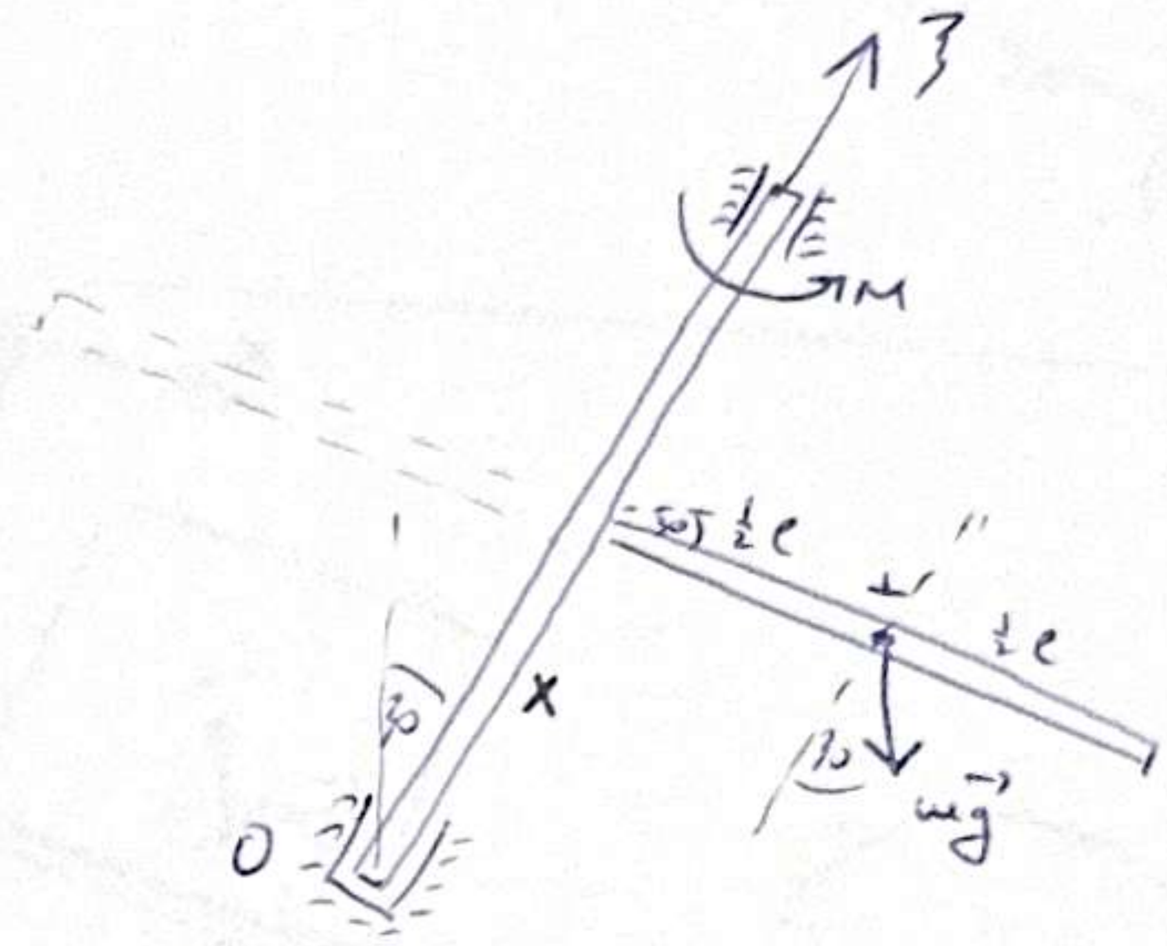
$\alpha = 30^\circ$

$M = \text{const.}$

$\varepsilon(\varphi) = ?$



$$\frac{dT}{dt} = \frac{d'A}{dt}$$



$$T = \frac{1}{2} m v_1^2 + \frac{1}{2} J_1 \omega^2 =$$

$$T = \frac{1}{2} m \left(\frac{1}{2} l \omega\right)^2 + \frac{1}{2}\frac{1}{12} m l^2 \omega^2 =$$

$$T = \frac{1}{8} m l^2 \omega^2 + \frac{1}{24} m l^2 \omega^2 =$$

$$T = \frac{1}{6} m l^2 \omega^2$$

$$\frac{dT}{dt} = \frac{1}{3} m l^2 \omega \varepsilon$$

$$d'A = -mg\, dz_1 + M\, d\varphi = + \frac{1}{2} mgl \cos\varphi\, d\varphi + M\, d\varphi$$

$$z_1 = x - \frac{1}{2} l \sin 30 \sin\varphi \Rightarrow \frac{dz_1}{dt} = -\frac{1}{4} l \cos\varphi \frac{d\varphi}{dt}$$

$$\frac{d'A}{dt} = \frac{1}{4} mgl \cos\varphi \cdot \omega + M\omega$$

$$\frac{1}{3} m l^2 \omega \varepsilon = \frac{1}{4} mgl \cos\varphi\, \omega + M\omega$$

$$\boxed{\varepsilon = \frac{3}{4}\frac{g}{l}\cos\varphi + \frac{3M}{ml^2}}$$

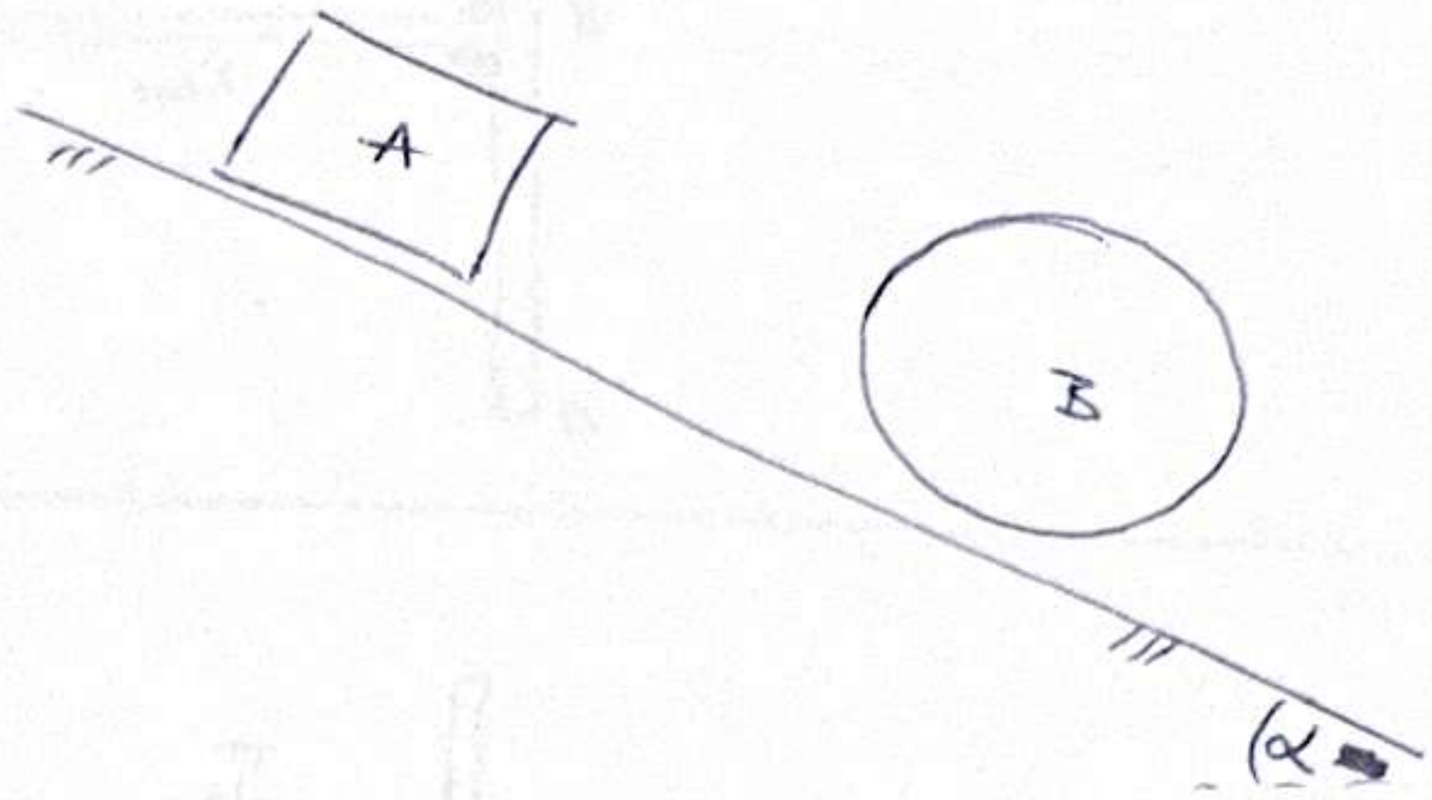
(10. B.)

A: klizi bez trenja

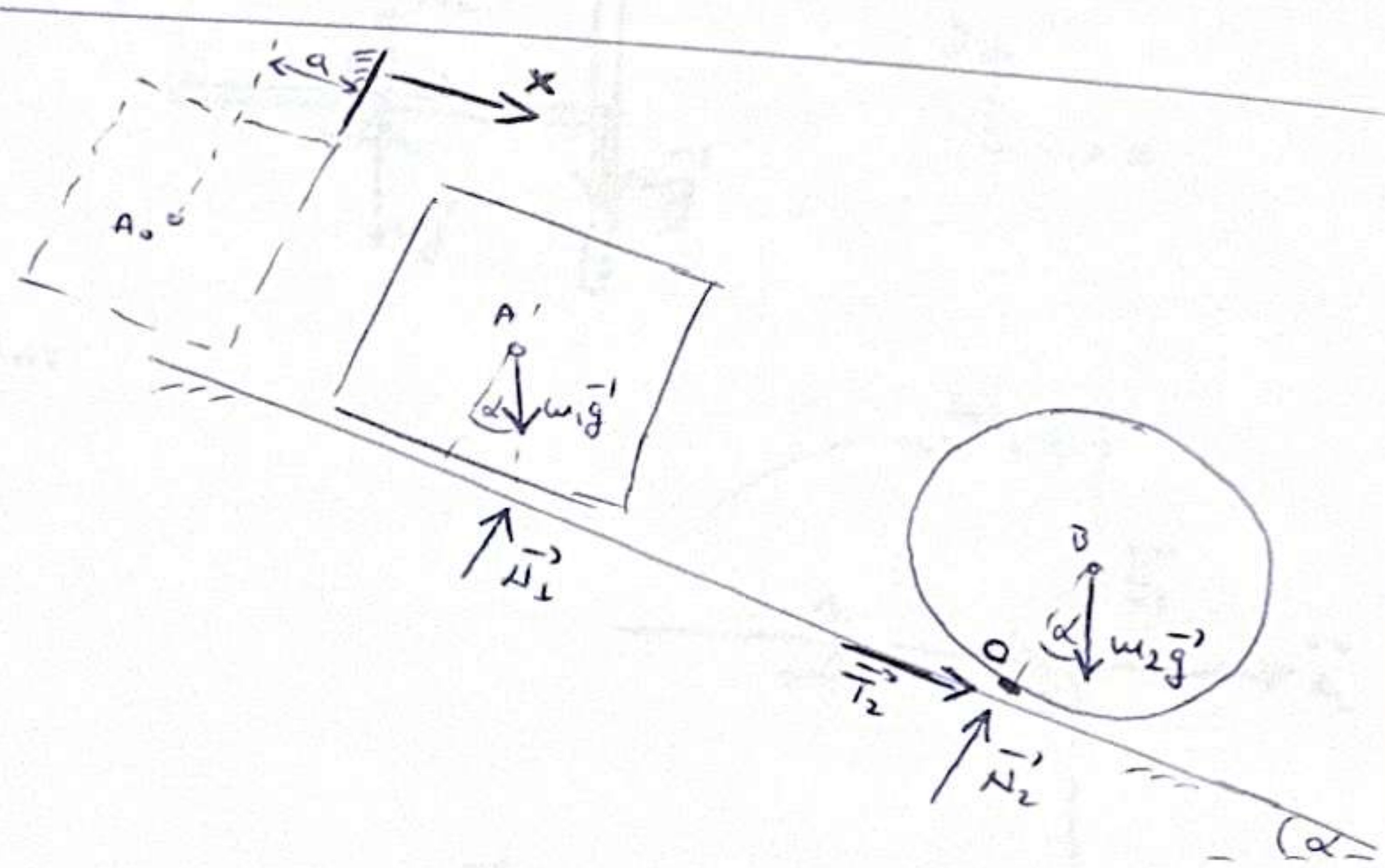
B: kotrlja se bez klizanja (cilindar)

t_0: najkraće rastojanje između A i B je e mirovanje

$t_1 = ?$ – trenutak kada A sustigne B



telo 1



$$m_1 \vec{a}_1 = m_1 \vec{g} + \vec{N}_1$$

$x: \; m_1 \ddot{x}_1 = m_1 g \sin\alpha \quad / \int dt$

$\dot{x}_1 = g \sin\alpha \cdot t + \cancel{C_1}^{0} \quad / \int dt$

$x_1 = \frac{1}{2} g \sin\alpha \cdot t^2 + C_2 , \quad C_2 = -a$

$$x_1 = \frac{1}{2} g \sin\alpha \cdot t^2 - a$$

telo 2

$$\frac{d\overset{\curvearrowright}{L}_{0z}}{dt} = \Sigma \overset{\curvearrowright}{M}_{0z}$$

– na ovaj način ćemo izbeći pojavljivanje nepoznate sile T, da smo radili ovo za centar diska onda bi se pojavila T, pa bi morali još j-na da uvedemo (drugi Njutnov zakon)

$\Sigma M_{0z} = m_2 g \sin\alpha \cdot R$

$L_{0z} = R \cdot m_2 v_2 + J_2 \omega = R \cdot m_2 R\omega + m_2 R^2 \omega = 2 m_2 R^2 \omega$

$\frac{dL_{0z}}{dt} = 2 m_2 R^2 \varepsilon$

$2 \cancel{m_2} R^{\cancel{2}} \varepsilon = \cancel{m_2} g \sin\alpha \cdot \cancel{R}$

$2 \cdot \ddot{x}_2 = g \sin\alpha$

$\ddot{x}_2 = \frac{1}{2} g \sin\alpha \quad / \int dt$

$\dot{x}_2 = \frac{1}{2} g \sin\alpha \cdot t + \cancel{C_1}^{0} \quad / \int dt$

$x_2 = \frac{1}{4} g \sin\alpha \cdot t^2 + C_2 , \quad C_2 = e + R$

$$x_2 = \frac{1}{4} g \sin\alpha \cdot t^2 + e + R$$

~~za trenutak t_1 važi:~~

za trenutak t_1 važi: $x_1(t_1) + a = x_2(t_1) - R$

$\frac{1}{2} g \sin\alpha \cdot t^2 \cancel{- a} \cancel{+ a} = \frac{1}{4} g \sin\alpha \cdot t^2 + e \cancel{+R} \cancel{-R}$

$\frac{1}{4} g \sin\alpha \cdot t^2 = e$

$$\boxed{t = \sqrt{\frac{4e}{g \sin\alpha}}}$$

(10.9.)

AB: m, ℓ

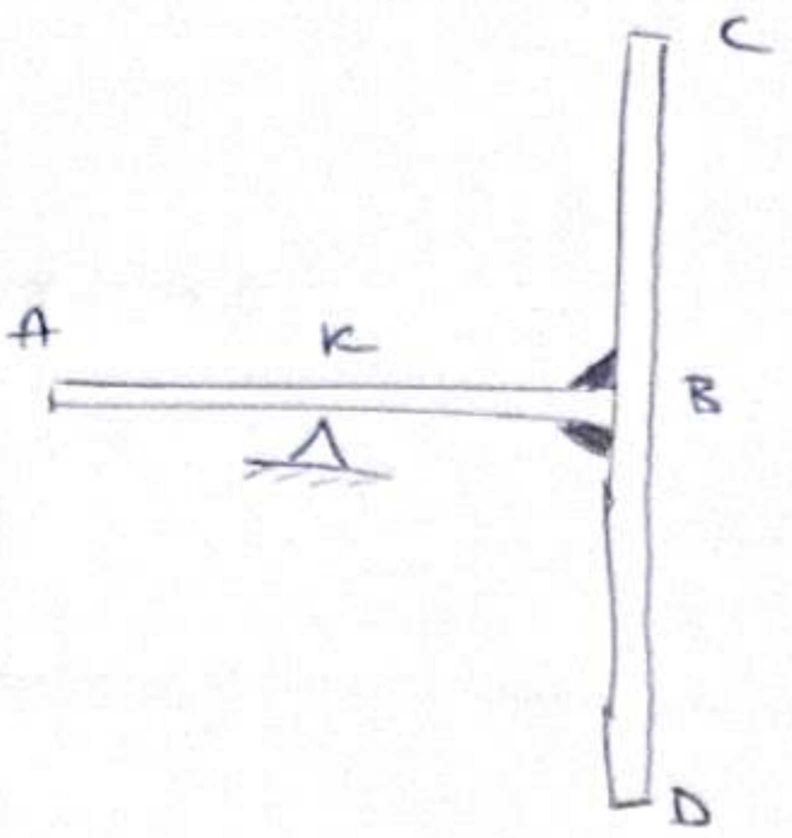
CD: m, ℓ

t_0: AB je horizontalan
sistem miruje

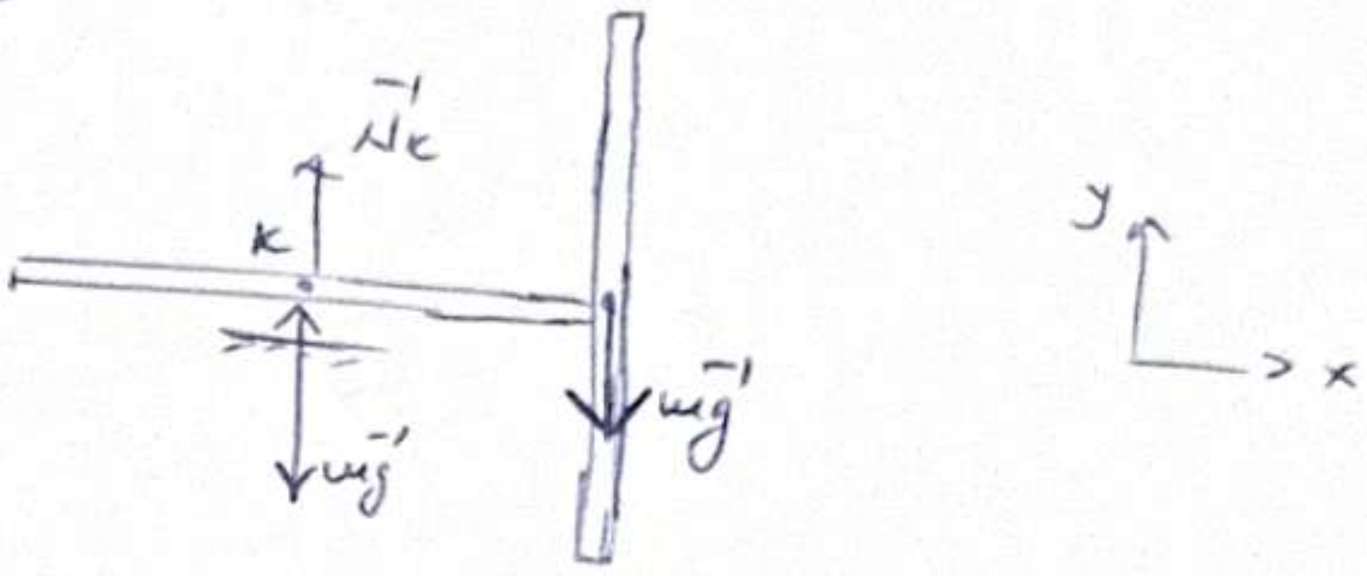
štap AB je samo naslonjen na oslonac

$N_K(t_0) = N_K = ?$

trenje zanemariti



trenutak t_0



Posmatramo samo početni trenutak t_0:

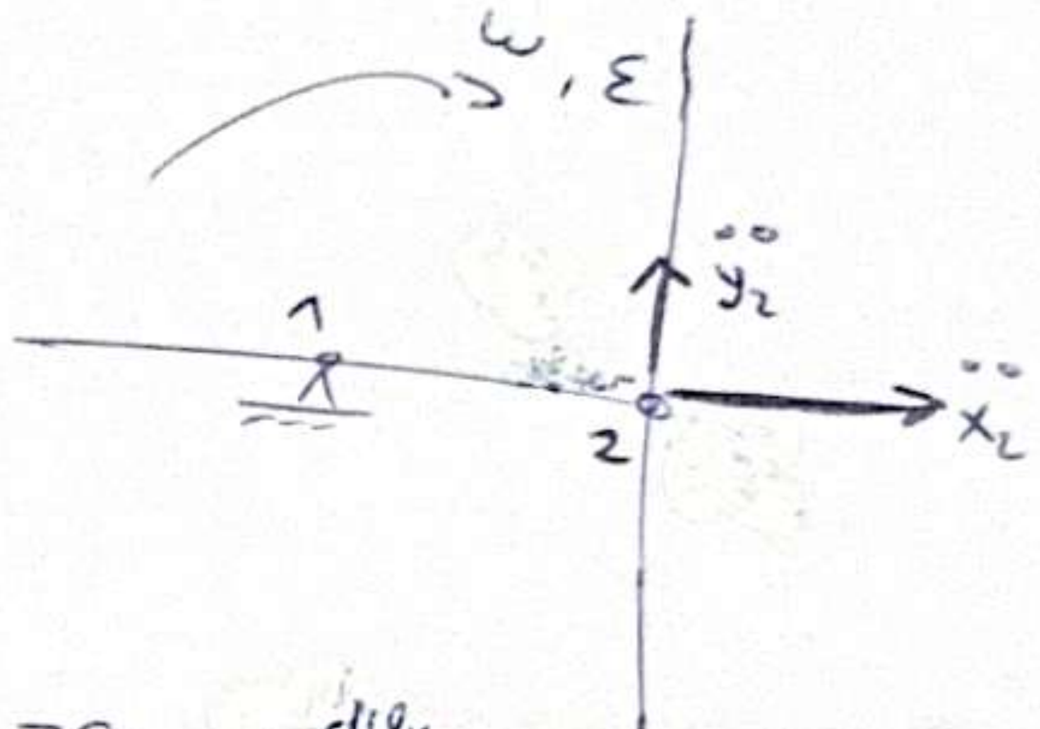
$$M\vec{a}_c = m\vec{g} + m\vec{g} + \vec{N}_K$$

$$y: \quad M\ddot{y}_c = -2mg + N_K$$

$$m_1 \cancel{\ddot{y}_1} + m_2 \ddot{y}_2 = -2mg + N_K$$

$$N_K = m\ddot{y}_2 + 2mg$$

$$N_K = 2mg - \frac{1}{2} m\ell\varepsilon \quad (1) \quad \begin{pmatrix} \varepsilon = ? \end{pmatrix}$$



$$\ddot{x}_2 = -a_{2n} + \frac{dv_K}{dt} = \ldots \text{ (ne treba nam)}$$

$$\ddot{y}_2 = -a_{2t} = -\frac{1}{2}\ell\varepsilon$$

$$\frac{dL_{Kz}}{dt} = \sum M_{Kz}$$

$$\sum M_{Kz} = \frac{1}{2} mg\ell$$

$$L_{Kz} = [0 + J_1 \cdot \omega] + [\tfrac{1}{2}\ell \cdot m v_2 + J_2\omega] = [\tfrac{1}{12} m\ell^2\omega] + [\tfrac{1}{2} m\ell \cdot \tfrac{\omega\ell}{2} + \tfrac{1}{12} m\ell^2\omega] = \tfrac{5}{12} m\ell^2\omega$$

$$\frac{dL_{Kz}}{dt} = \frac{5}{12} m\ell^2\varepsilon$$

$$\rightarrow \quad \frac{5}{12} m\ell^2\varepsilon = \frac{1}{2} mg\ell$$

$$\varepsilon = \frac{6}{5}\frac{g}{\ell}$$

$$(1) \Rightarrow \quad N_K = 2mg - \frac{1}{2} m\ell \cdot \frac{6}{5}\frac{g}{\ell}$$

$$\Rightarrow \quad \boxed{N_K = \frac{7}{5} mg}$$

(10.10)

prsten: $m = 2\ kg$, $R = 0,5\ m$

$x_c = 5 \cos 2t$, $y_c = 5 \sin 2t$, $\varphi = 4t^2$

$X_R^S = ?$, $Y_R^S = ?$, $M_{Ct}^S = ?$

$m \vec{a}_c = \vec{X}_R^S + \vec{Y}_R^S$

x: $m \ddot{x}_c = X_R^S$

$m \cdot (-20 \cdot \cos 2t) = X_R^S$

$$\boxed{X_R^S = -40 \cos 2t \quad [N]}$$

y: $m \ddot{y}_c = Y_R^S$

$m(-20 \sin 2t) = Y_R^S$

$$\boxed{Y_R^S = -40 \sin 2t \quad [N]}$$

$\frac{d\overset{\frown}{L}_{Ct}}{dt} = \Sigma \overset{\frown}{M}_{Ct}$

$\Sigma M_{Ct} = M_{Ct}^S$

$L_{Ct} = mR^2 \cdot \omega$

$\frac{dL_{Ct}}{dt} = mR^2 \varepsilon$

$mR^2 \varepsilon = M_{Ct}$

$$\boxed{M_{Ct} = 4 \quad [Nm]}$$

(10.11.)

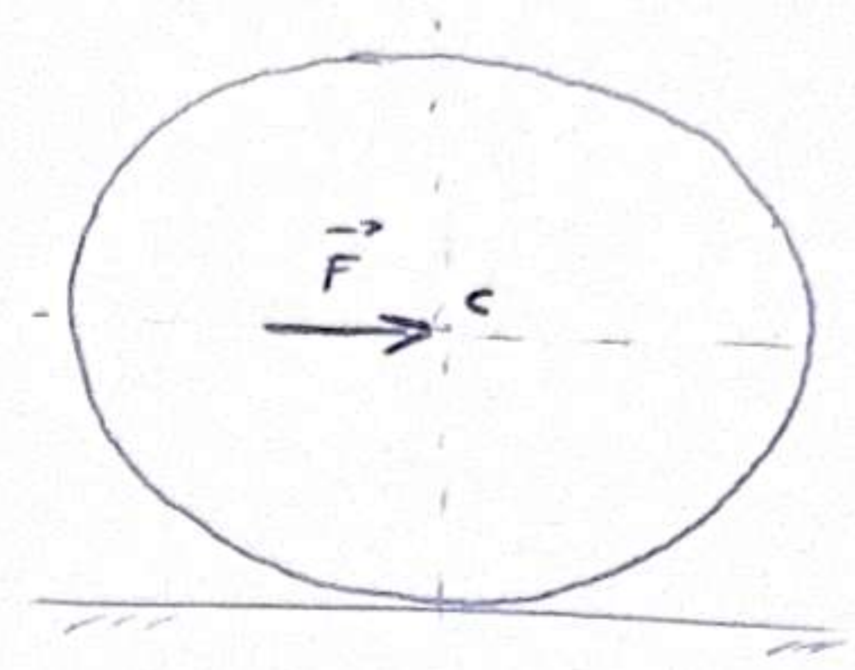
prsten: m, R

kotrljanje bez klizanja

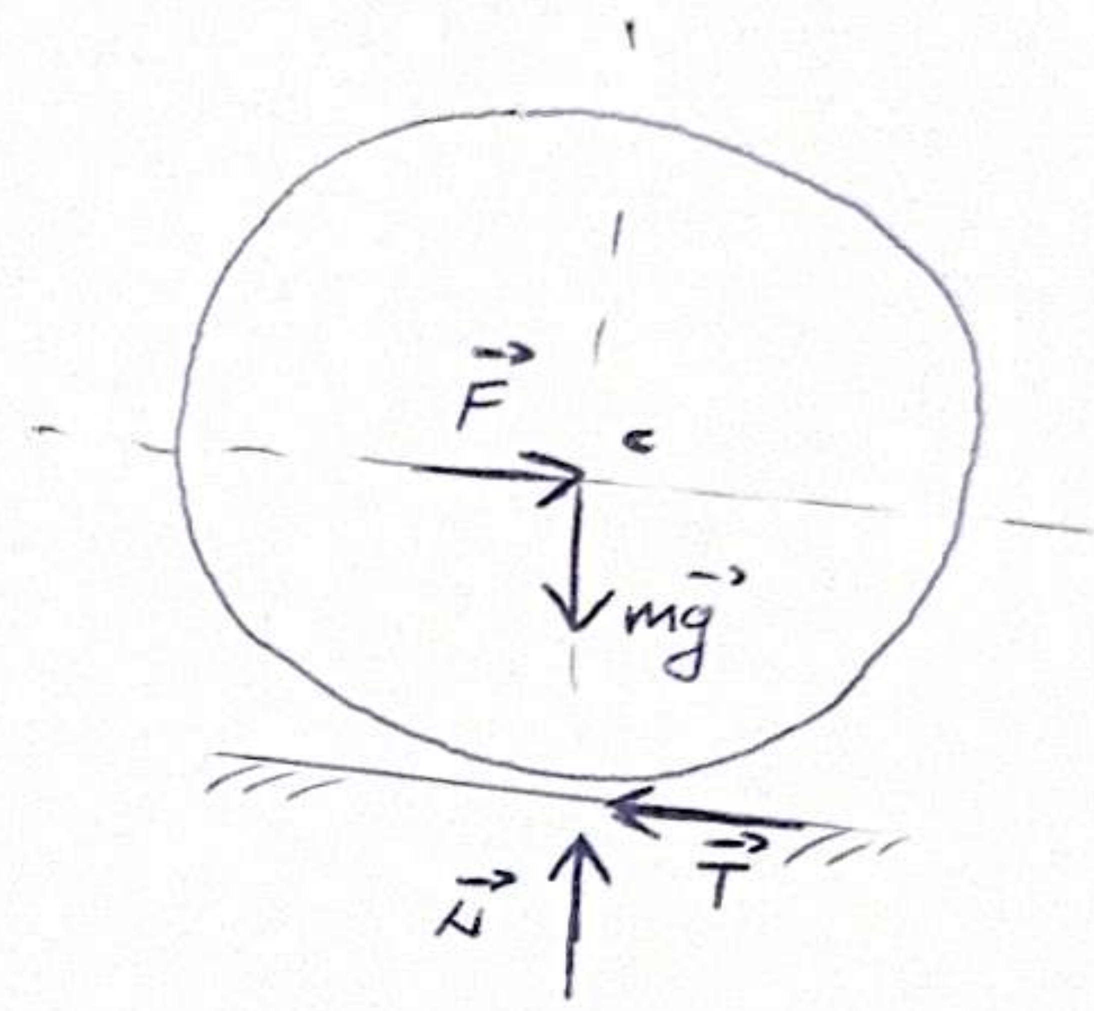
$F = mg$

$t_0 = 0$: mirovanje

$x_c = ?$



$$m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N} + \vec{T}$$



x: $m\ddot{x}_c = F - T$

$m\ddot{x}_c = mg - T$ (1) $\left(\begin{matrix}\ddot{x}_c = ?\\ T = ?\end{matrix}\right)$

y: ... (ne treba nam)

$$\frac{d\overset{\curvearrowright}{L}_{cz}}{dt} = \Sigma \overset{\curvearrowright}{M}_{cz}$$

$\Sigma M_{cz} = T \cdot R$

$L_{cz} = J\omega = mR^2\omega$

$\frac{dL_{cz}}{dt} = mR^2\varepsilon$

$mR^{\cancel{2}}\varepsilon = T \cdot \cancel{R}$

$m\ddot{x}_c = T$

(1) $\Rightarrow$ $m\ddot{x}_c = mg - m\ddot{x}_c$

$\Rightarrow$ $\ddot{x}_c = \frac{1}{2}g$ $\quad /\int$

$\Rightarrow$ $\dot{x}_c = \frac{1}{2}gt$ $\quad /\int$

$\Rightarrow$ $\boxed{x_c = \frac{1}{4}gt^2}$

10. 12

(nastavak na 11. zad)

$T = m\ddot{x}_c = \frac{1}{2} mg \quad (1)$

$T = \mu \cdot N \; , \; N = ? \quad (2)$

$m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N} + \vec{T}$

$y: \quad 0 = mg - N$

$N = mg$

$(1) = (2)$

$\frac{1}{2} mg = \mu \cdot N$

$\frac{1}{2} mg = \mu \cdot mg$

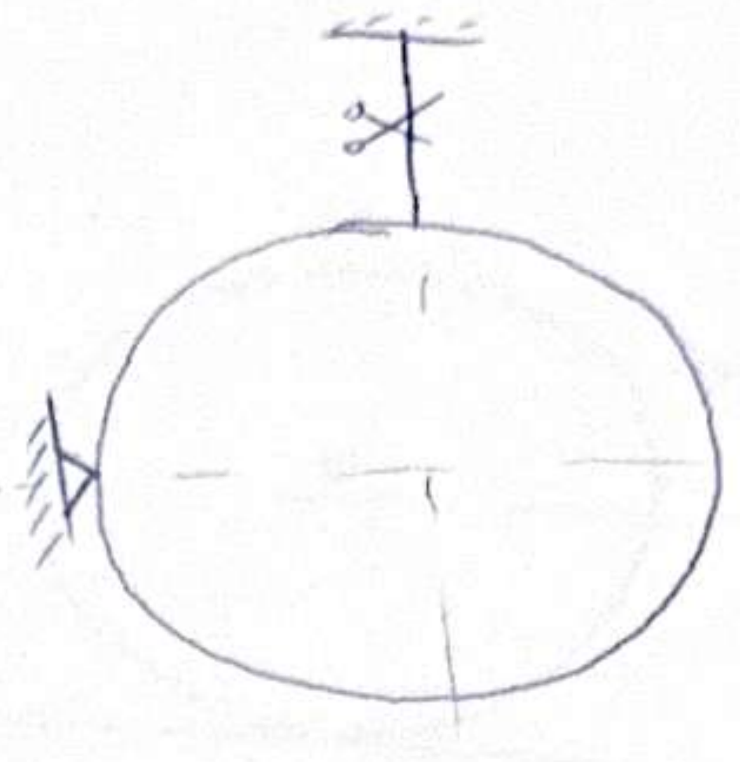
$\mu = \frac{1}{2}$ – granični slučaj da ne dođe do proklizavanja

$\boxed{\mu_{min} = \frac{1}{2}}$

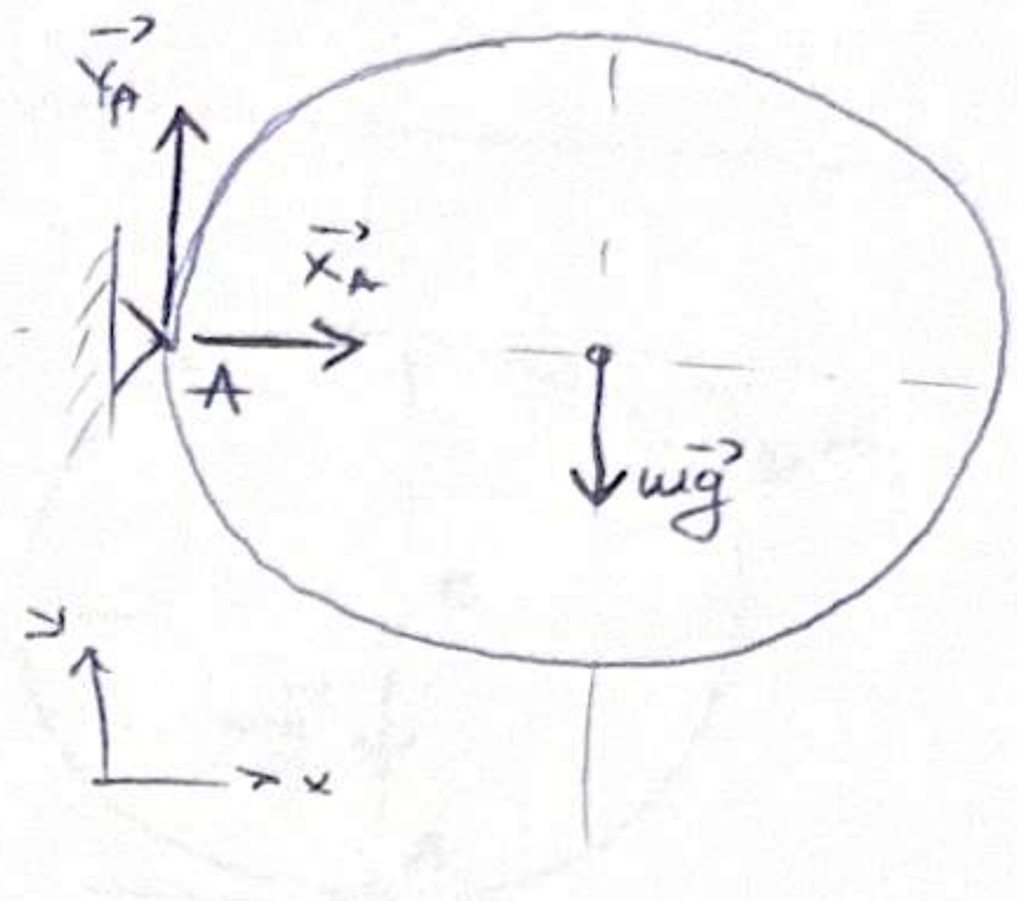
10.13.

disk: m, r

X_A? , Y_A? – neposredno nakon presecanja kanapa



Posmatramo sve za taj početni trenutak, kanap je presečen, a telo je mirovalo.



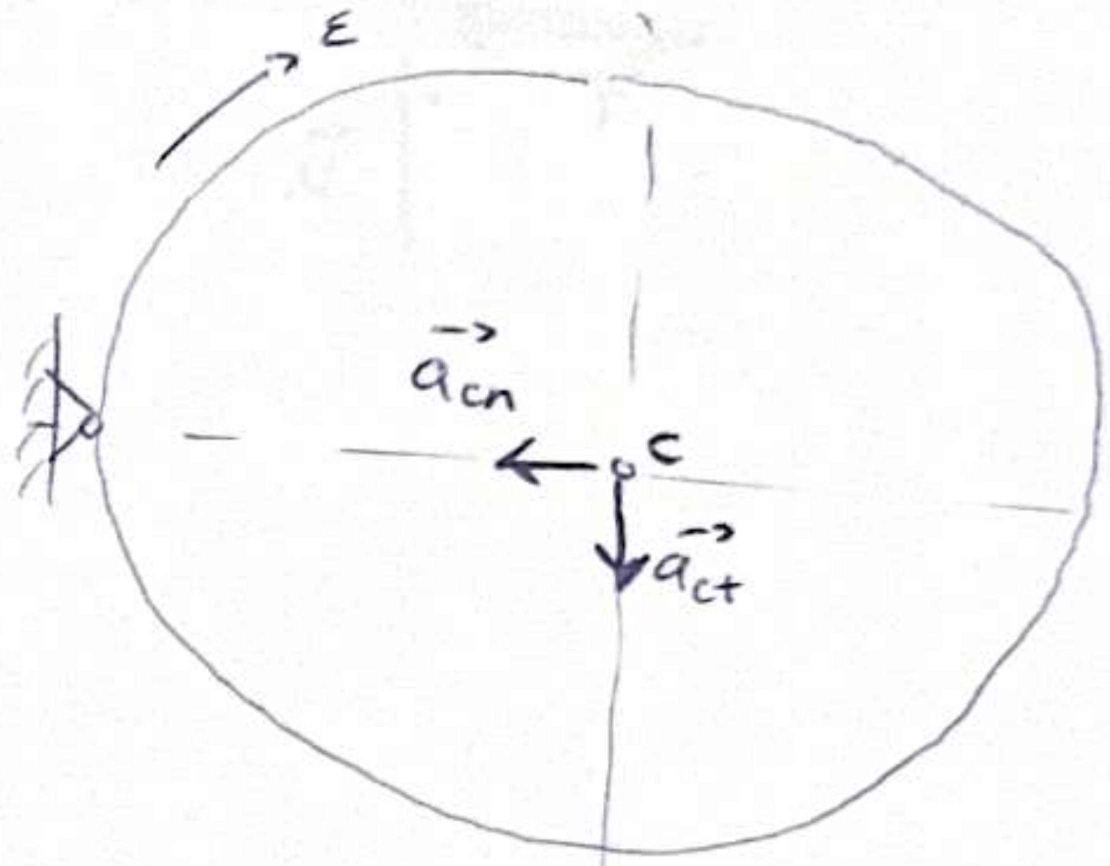
$$m\vec{a}_C = m\vec{g} + \vec{X}_A + \vec{Y}_A$$

x: $m\cancel{\ddot{x}_C}^{\,0} = X_A$

$$\boxed{X_A = 0}$$

y: $m\ddot{y}_C = mg - Y_A$

$Y_A = mg - mR\varepsilon$ (1) ($\varepsilon = ?$)



$\ddot{x}_C = -a_{Cn} = -\frac{v^2}{R} = 0$

$\ddot{y}_C = -a_{Ct} = -\varepsilon \cdot R$

$$\frac{d\overset{\frown}{L_{Az}}}{dt} = \Sigma \overset{\frown}{M_{Az}}$$

– nećemo imati nepoznatih sila, a pojaviće se ε, pa ćemo ga izračunati. (mogli smo da raspišemo i za središte diska, dobili bi j-nu sa dve nepoznate, i sa prethodnom j-nom bi to rešili)

$\Sigma M_{Az} = mg \cdot r$

$L_{Az} = r \cdot m v_C + J_C \omega = rmr\omega + \frac{1}{2}mr^2\omega = \frac{3}{2}mr^2\omega$

$\frac{dL_{Az}}{dt} = \frac{3}{2}mr^2\varepsilon$

$\frac{3}{2}\cancel{m}r^{\cancel{2}}\varepsilon = \cancel{m}g\cancel{r}$

$$\varepsilon = \frac{2}{3}\frac{g}{r}$$

(1) $\Rightarrow$ $Y_A = mg - m \cdot \varepsilon \cdot r$

$\Rightarrow$ $Y_A = mg - \frac{2}{3}m \cdot \frac{g}{\cancel{r}}\cancel{r}$

$\Rightarrow$ $\boxed{Y_A = \frac{1}{3}mg}$

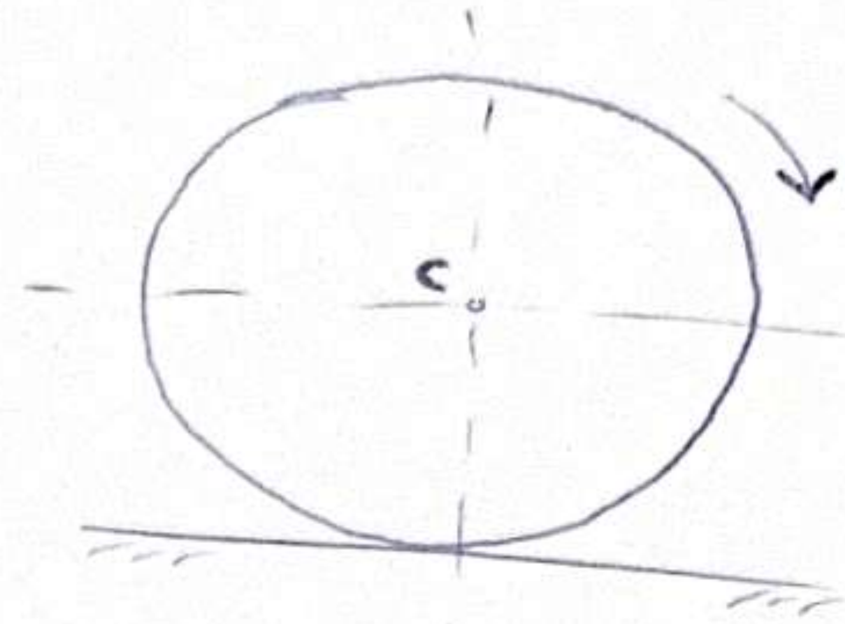
10. 14.

disk: m, r, kotrlja se bez klizanja

$M = const.$

$t_0 = 0: \quad v_{C0} = 0$

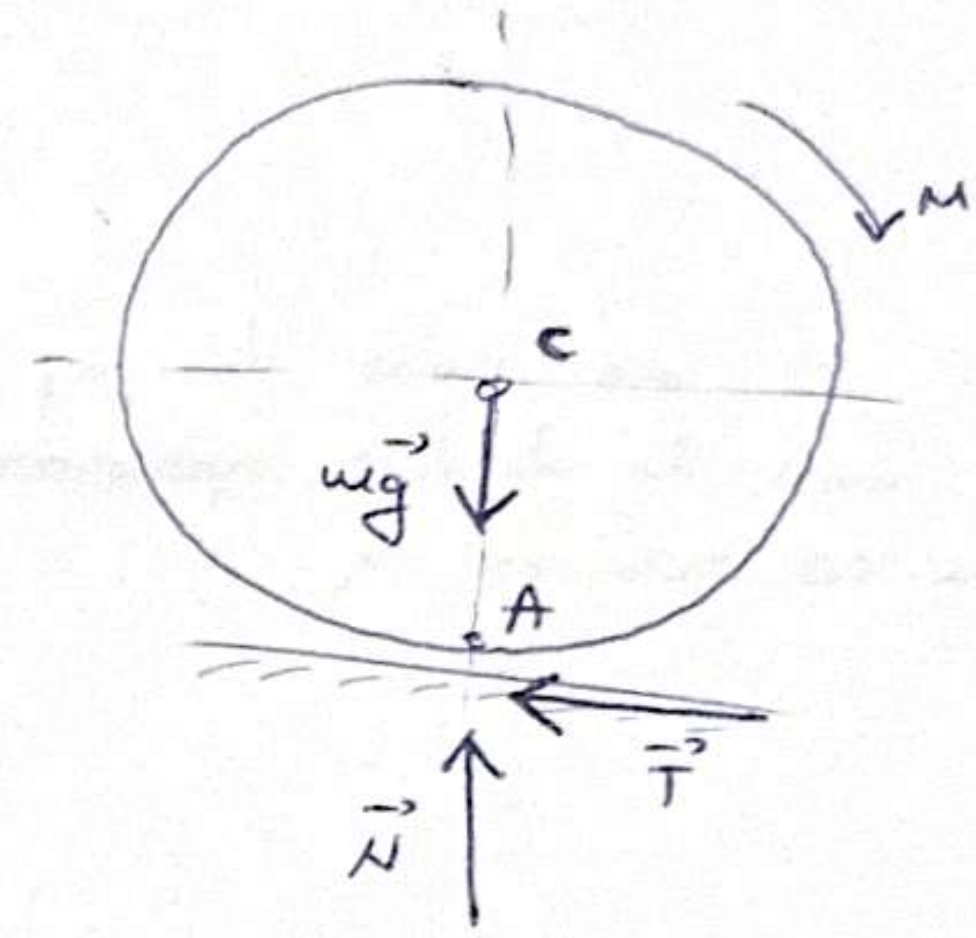
$x_C = ?$



Imamo dva načina:

1) $\dfrac{dL_{Cz}}{dt} = \Sigma M_{Cz}$, $+$ II N.z. – pojaviće se dve nepoznate (ε, T), pa ćemo morati i drugi njutnov z. da raspišemo + još jednu j-nu

2) $\dfrac{dL_{Az}}{dt} = \Sigma M_{Az}$ – pojaviće se samo jedna nepoznata (ε)



Primenjujemo da je drugi način brži:

$$\frac{dL_{Az}}{dt} = \Sigma M_{Az}$$

$$\Sigma M_{Az} = M$$

$$L_{Az} = r \cdot m v_C + J_C \cdot \omega = r \cdot m \cdot r\omega + \frac{1}{2} m r^2 \omega = \frac{3}{2} m r^2 \omega$$

$$\frac{dL_{Az}}{dt} = \frac{3}{2} m r^2 \varepsilon$$

$$\frac{3}{2} m r^2 \varepsilon = M$$

$$\varepsilon = \frac{2}{3} \frac{M}{m r^2}$$

$$\ddot{x}_C = r \cdot \varepsilon \Rightarrow \ddot{x}_C = \frac{2}{3} \frac{M}{m r} \qquad /\int$$

$$\dot{x}_C = \frac{2}{3} \frac{M}{m r} t + \cancel{C_1}^{\,0} \qquad /\int$$

$$\boxed{x_C = \frac{1}{3} \frac{M}{m r} t^2}$$

10.15

(nastavak na 14.)

$$m\vec{a}_c = m\vec{g} + \vec{N} + \vec{T}$$

$y: \quad 0 = mg - N$

$$N = mg$$

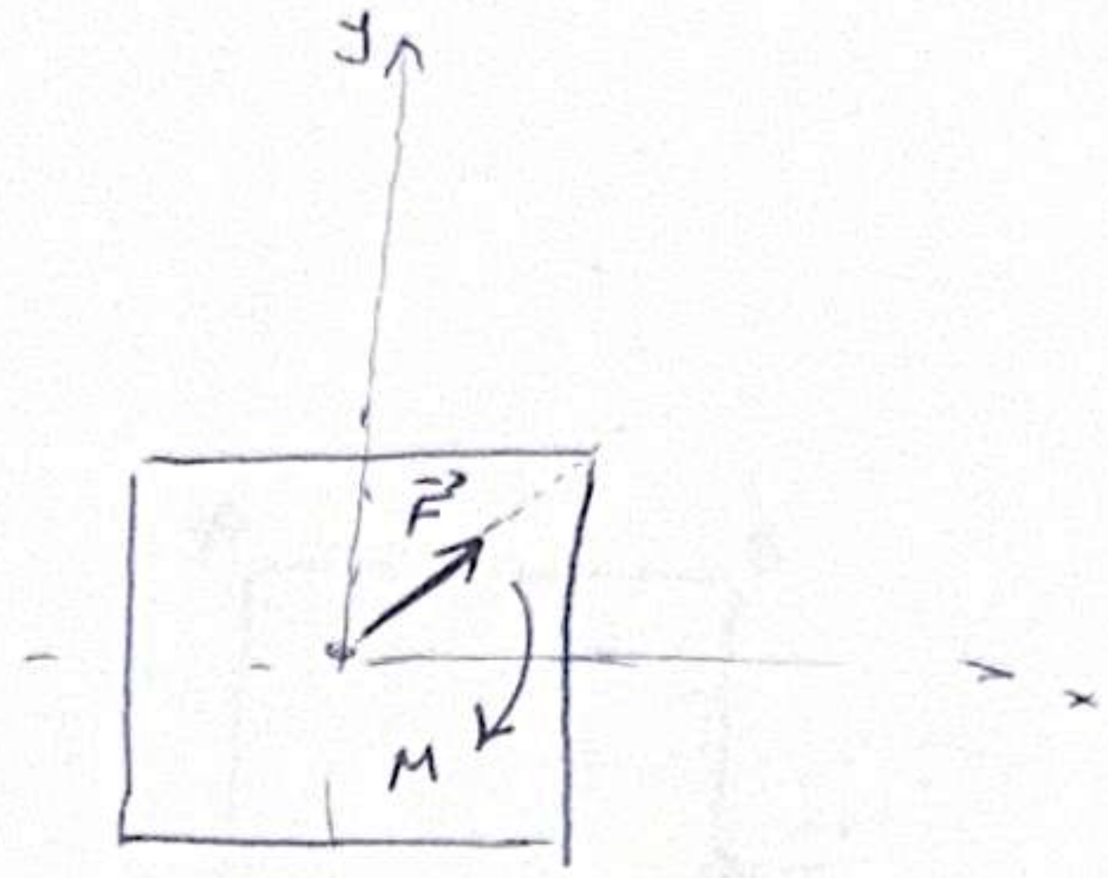
$x: \quad m\ddot{x}_c = -T$

$\frac{2}{3}\frac{M}{r} = +\mu_0 N$ - očigledno je da smo pogrešili smer sile T, pa ćemo nastaviti ~~...~~ zadatak ali ćemo okrenuti smer (jer μ ne može biti manje od 0)

$$\frac{2}{3}\frac{M}{r} = \mu_0 mg$$

$$\boxed{\mu_{min} = \frac{2}{3}\frac{M}{mgr}}$$

(10.16.)



kvadratna ploča

$m = 1\ kg$, $\ell = 1\ m$ - dužina stranice

horizontalna ravan

t_0: mirovanje, položaj na slici

$M = 2\ Nm$, $F = 5\sqrt{2}\ N$

$x_c(t_1)$? $y_c(t_1)$? $\varphi(t_1)$?

$t_1 = 2\ s$

$m\vec{a}_c = \vec{F}$

x: $m\ddot{x}_c = F\frac{\sqrt{2}}{2}$

$1 \cdot \ddot{x}_c = 5\cancel{\sqrt{2}}\frac{\cancel{\sqrt{2}}}{\cancel{2}}$ //∫

$\dot{x}_c = 5t + \cancel{C_1}^{0}$ //∫

$x_c = 2{,}5t^2 + \cancel{C_2}^{0}$

$\boxed{x_c(t_1 = 2s) = 10\ [m]}$

y: $m\ddot{y}_c = F\frac{\sqrt{2}}{2}$

...

$\boxed{y_c(t_1 = 2s) = 10\ [m]}$

$\frac{d\overset{\frown}{L}_{cz}}{dt} = \Sigma \overset{\frown}{M}_{cz}$

$\Sigma M_{cz} = M$

$L_{cz} = 0 + J_c\omega = \frac{1}{6}m\ell^2\omega$

$\frac{dL_{cz}}{dt} = \frac{1}{6}m\ell^2\varepsilon$

$\frac{1}{6}m\ell^2\varepsilon = M$

$\varepsilon = \frac{6M}{m\ell^2} = \frac{6 \cdot 2}{1 \cdot 1^2} = 12$ //∫

$\omega = 12t + \cancel{C_1}^{0}$ //∫

$\varphi = 6t^2 + \cancel{C_2}^{0}$ => $\boxed{\varphi(t_1 = 2s) = 24\ rad}$

(10. 17)

kvadratna ploča

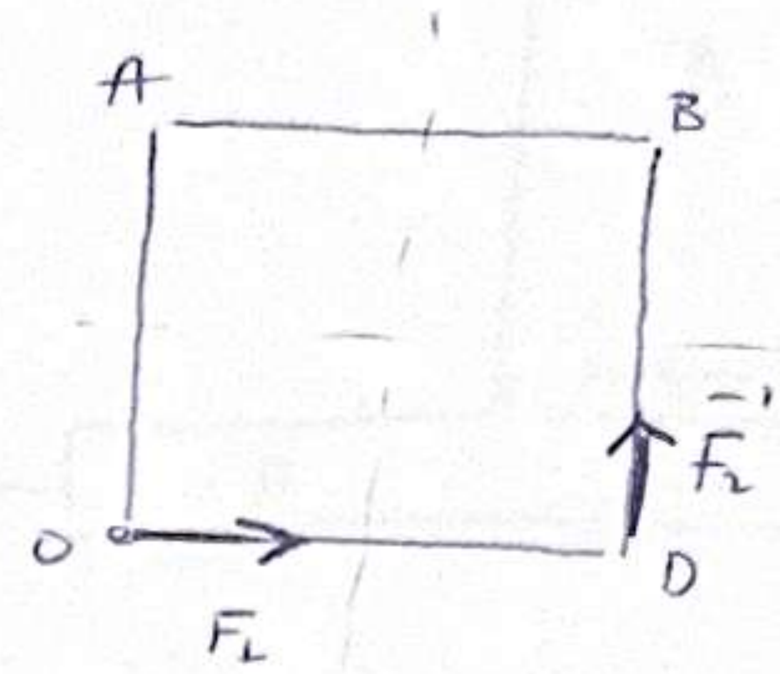
m, a - stranica

F_1, F_2 - pravci se poklapaju sa stranicama OD i DB, respektivno (const. sile)

t_0: ploča miruje, položaj na slici

$\ddot{x}_c$? $\ddot{y}_c$?

horiz. ravan



$$m\vec{a}_c = \vec{F_1} + \vec{F_2}$$

x: $m\ddot{x}_c = F_1 \cos\varphi - F_2 \sin\varphi$ (1)

y: $m\ddot{y}_c = F_1 \sin\varphi + F_2 \cos\varphi$ (2)

$\Big\}$ $\varphi = ?$

$$\frac{d\overleftarrow{L}_{C\tau}}{dt} = \sum \overleftarrow{M}_{C\tau}$$

$$\sum M_{C\tau} = F_1 \cdot \tfrac{1}{2}a + F_2 \cdot \tfrac{1}{2}a = \tfrac{1}{2}a(F_1 + F_2)$$

$$L_{C\tau} = 0 + J_c\omega = \tfrac{1}{6}ma^2\omega$$

$$\frac{dL_{C\tau}}{dt} = \tfrac{1}{6}ma^2\varepsilon$$

$$\tfrac{1}{6}ma^2\varepsilon = \tfrac{1}{2}a(F_1 + F_2)$$

$$\varepsilon = \frac{3(F_1+F_2)}{ma} \quad //$$

$$\omega = \frac{3(F_1+F_2)}{ma}t + \cancel{C_1}^{0} \quad //$$

$$\varphi = \frac{3}{2}\frac{(F_1+F_2)}{ma}t^2 + \cancel{C_2}^{0}$$

(1) $\Rightarrow$ $\boxed{\ddot{x}_c = \frac{1}{m}\left(F_1 \cos\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right) - F_2 \sin\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right)\right)}$

(2) $\Rightarrow$ $\boxed{\ddot{y}_c = \frac{1}{m}\left(F_1 \sin\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right) + F_2 \cos\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right)\right)}$

10.20

A: $2m$

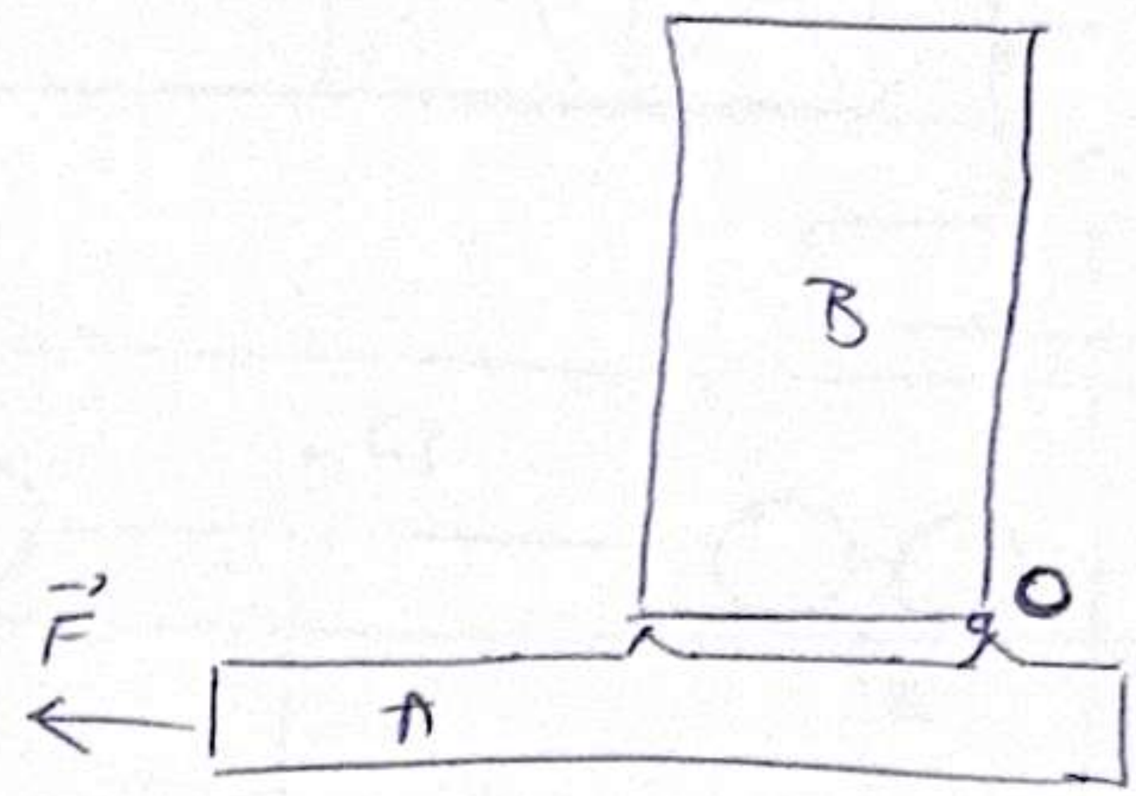
B: m, $a \times 2a$

$F = kt$

t_0: sistem miruje, stranica a je horiz.

$t_1 = ?$ – trenutak kada B počinje da se obrće

j-na kretanja tela A do t_1?



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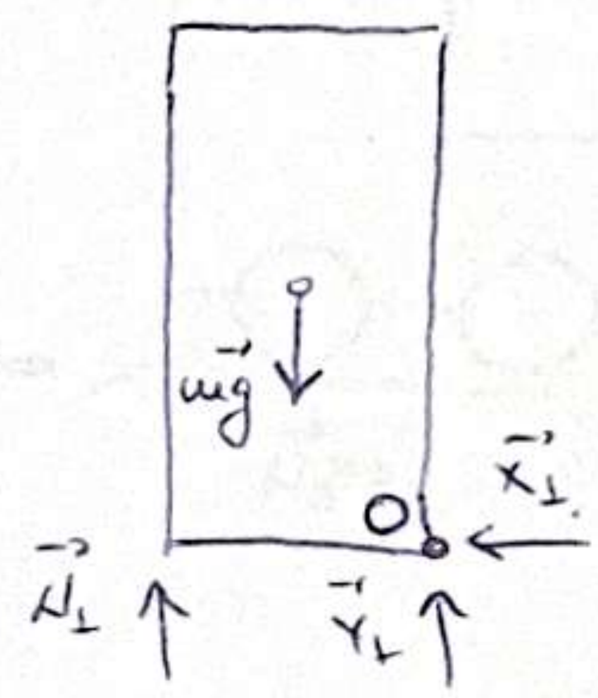
uslov za t_1: $\underline{\underline{N_1(t_1) = 0}}$

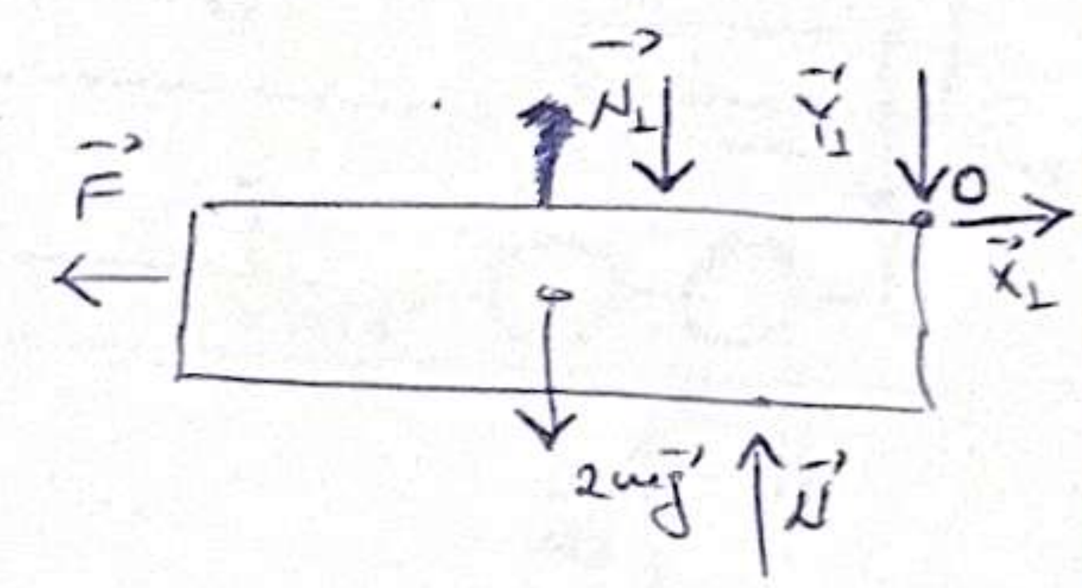
=> posmatramo opšti trenutak t, nađemo $N_1(t)$, izjednačimo sa nulom : dobijemo t_1

t

telo B



telo A

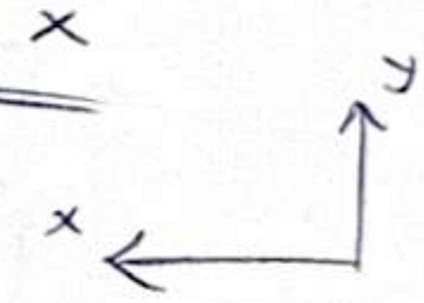


$$m\vec{a}_B = m\vec{g} + \vec{N}_1 + \vec{X}_1 + \vec{Y}_1$$

x: $\underline{\underline{m\ddot{x} = X_1}}$ (1)

y: $\underline{\underline{0 = -mg + N_1 + Y_1}}$ (2)

kinematska veza: $\underline{\underline{\ddot{x}_A = \ddot{x}_B = \ddot{x}}}$



$$\frac{d\vec{L}_{Oz}}{dt} = \sum \vec{M}_{Oz}$$

$\sum M_{Oz} = N_1 \cdot a - mg \cdot \frac{1}{2}a$

$L_{Oz} = -a \cdot m\dot{x}$

$\frac{dL_{Oz}}{dt} = -ma\ddot{x}$

$-ma\ddot{x} = N_1 \cdot a - \frac{1}{2} mga$

$\underline{\underline{N_1 = \frac{1}{2}mg - m\ddot{x}}}$, $\ddot{x}(t) = ?$

$$2m\vec{a}_A = 2m\vec{g} + \vec{F} + \vec{N} + \vec{N}_1 + \vec{X}_1 + \vec{Y}_1$$

y: – – (nepotrebno)

x: $2m\ddot{x} = F - X_1$

$2m\ddot{x} = kt - m\ddot{x}$

$\underline{\ddot{x} = \frac{1}{3}\frac{k}{m}t}$ //

$\dot{x} = \frac{1}{6}\frac{k}{m}t^2 + \cancel{C_1}$ //

$$\boxed{x = \frac{1}{18}\frac{k}{m}t^3}$$

(3) => $N_1 = \frac{1}{2}mg - \frac{1}{3}kt$

$N_1(t_1) = \frac{1}{2}mg - \frac{1}{3}kt_1 = 0$

=> $\boxed{t_1 = \frac{3}{2}\frac{mg}{k}}$

10.21

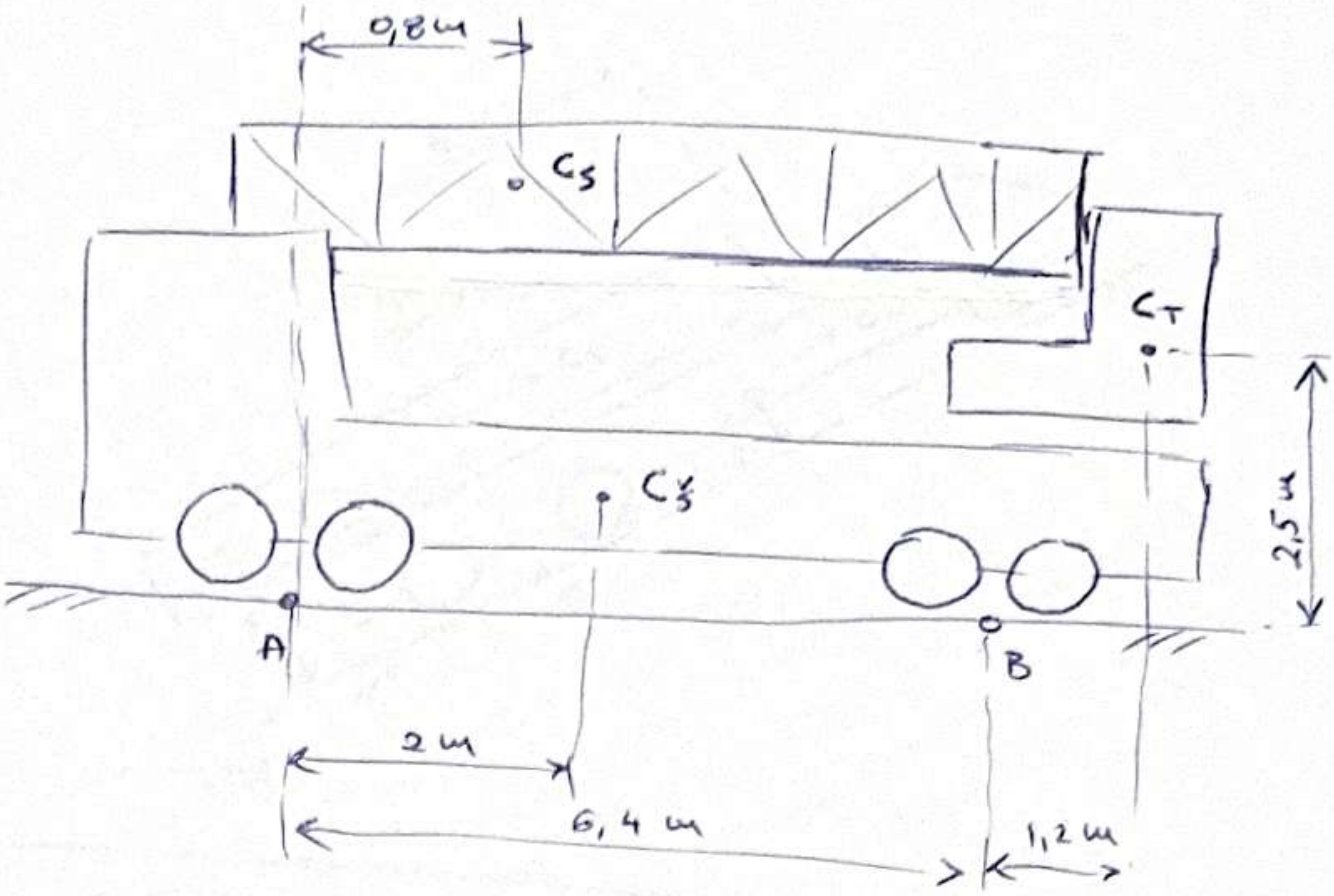
$m_t = ?$ - da ne dođe do prevrtanja

$m_s = 4t$

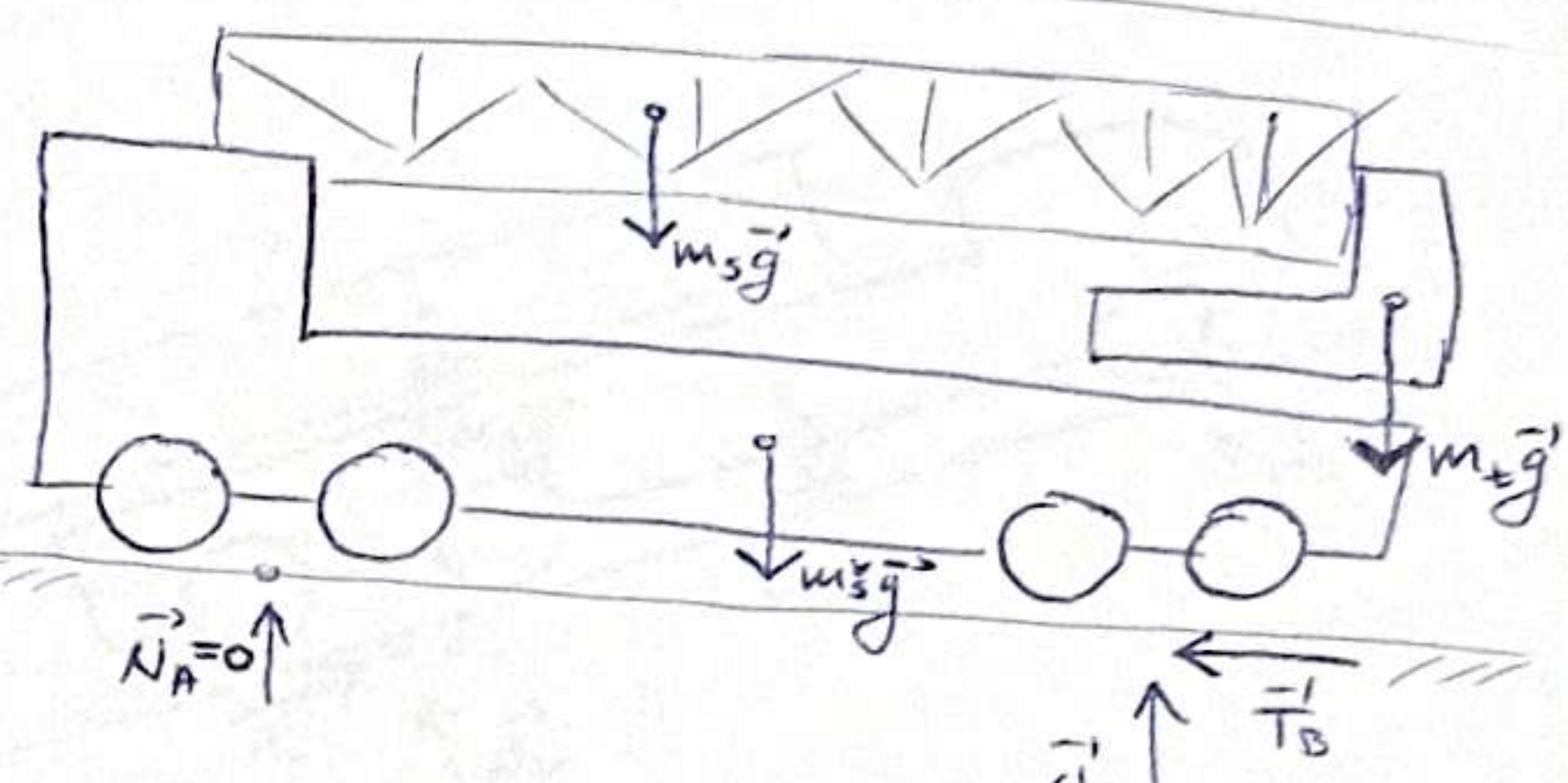
$m_ž = 8t$

$a_{max} = 2\,m/s$

pogon na zadnjim točkovima



uslov da dođe do odvajanja: $\underline{N_A = 0}$



$$\frac{d\vec{L}_B}{dt} = \sum \vec{M}_B$$

$$\sum M_B = \cancel{N_A}^{0}\cdot 6{,}4 - m_s \cdot g \cdot 5{,}6 - m_ž \cdot g \cdot 4{,}4 + m_t \cdot g \cdot 1{,}2 =$$

$$\sum M_B = +(-4 \cdot 5{,}6 - 8 \cdot 4{,}4 + m_t \cdot 1{,}2)\, g$$

$$\sum M_B = -(57{,}6 - m_t \cdot 1{,}2)\, g$$

$$L_B = [-3{,}8 \cdot m_s \cdot v + J_s\, \omega] + [-1{,}1 \cdot m_ž \cdot v + J_ž \cdot \omega] + [-2{,}5 \cdot m_t \cdot v + J_t \cdot \omega]$$

$$\frac{dL_B}{dt} = [-3{,}8 \cdot 4 \cdot a + J_s \cancel{\varepsilon}^{0}] + [-1{,}1 \cdot 8 \cdot a + J_ž \cancel{\varepsilon}^{0}] + [-2{,}5\, m_t \cdot a + J_t \cancel{\varepsilon}^{0}]$$

$$\frac{dL_B}{dt} = -(24 + 2{,}5 \cdot m_t) \cdot a = -(24 + 2{,}5\, m_t) \cdot 2 = -(48 + 5\, m_t)$$

$$-(48 + 5\, m_t) = -(57{,}6 - m_t \cdot 1{,}2)\, g$$

$$48 + 5\, m_t = 57{,}6 \cdot g - m_t \cdot g \cdot 1{,}2$$

$$m_t\,(5 + 1{,}2 \cdot g) = 57{,}6 \cdot g - 48$$

$$m_t = \frac{57{,}6 \cdot g - 48}{5 + 1{,}2 \cdot g}$$

$$\boxed{m_t = 30\,829\ kg}$$

(10.22.)

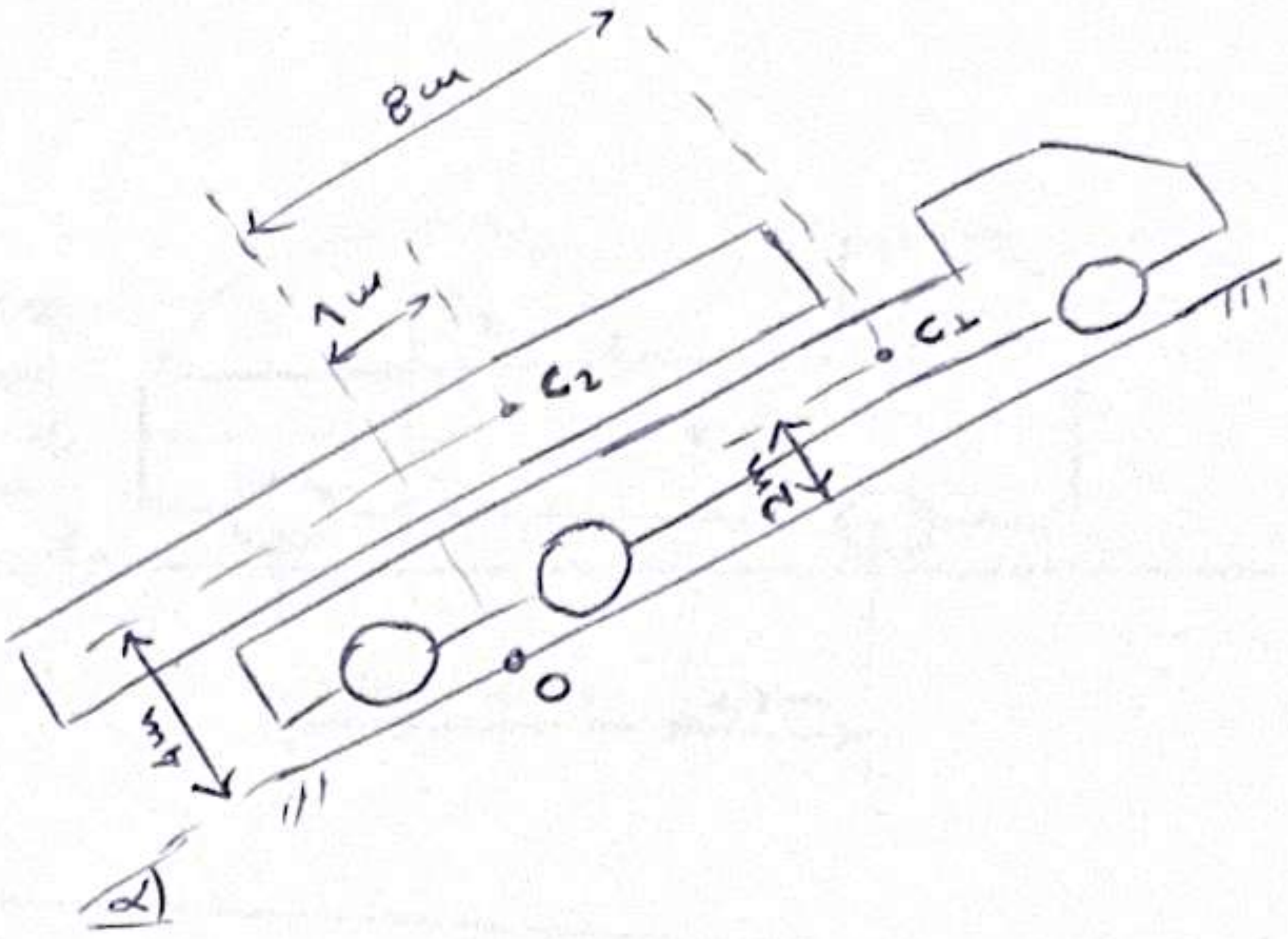
$m_1 = 25\,t$

$m_2 = 18\,t$

$\alpha = 15°$

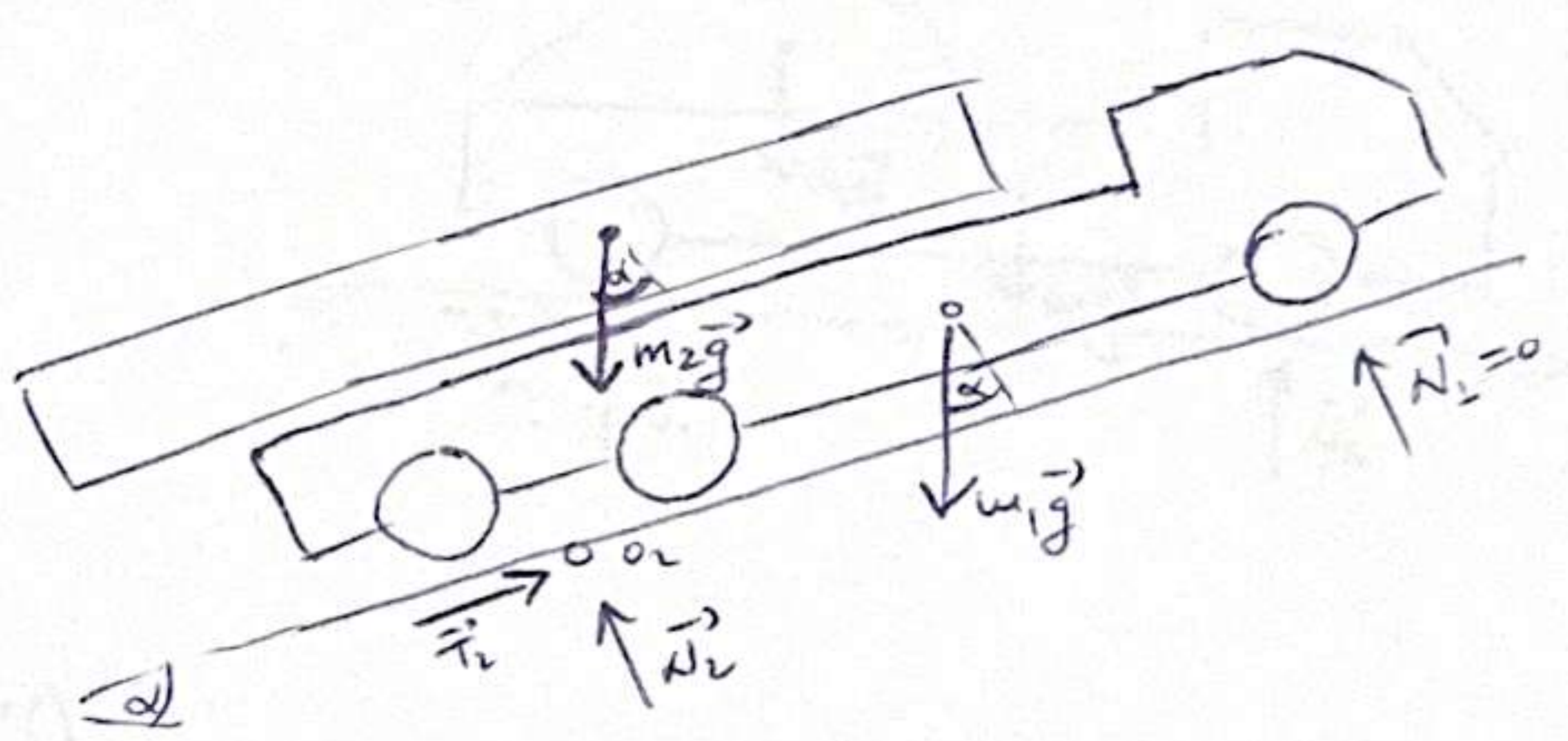
pogon na zadnjim točkovima

$a_{max} = ?$ – da ne dođe do prevrtanja



uslov: $\underline{\underline{N_1 = 0}}$

$$\frac{dL_{O_2 z}}{dt} = \sum M_{O_2 z}$$



$$\sum M_{O_2 z} = -m_2 g \cos\alpha \cdot 1 + m_2 g \sin\alpha \cdot 4 - m_1 g \cos\alpha \cdot 8 + m_1 g \sin\alpha \cdot 2 =$$

$$\sum M_{O_2 z} = -1\,740,716$$

$$L_{O_2 z} = [-4 \cdot m_2 v + J_2 \omega] + [-2 m_1 v + J_1 \omega]$$

$$\frac{dL_{O_2 z}}{dt} = [-4 m_2 a_{max} + \cancel{J_2 \varepsilon}^{0}] + [-2 m_1 a_{max} + \cancel{J_1 \varepsilon}^{0}] =$$

$$\frac{dL_{O_2 z}}{dt} = -122 \cdot a_{max}$$

$$-122 \cdot a_{max} = -1\,740,716$$

$$\boxed{a_{max} = 14,27\ [m/s^2]}$$

— približno tačno rešenju u zbirci

10.23.

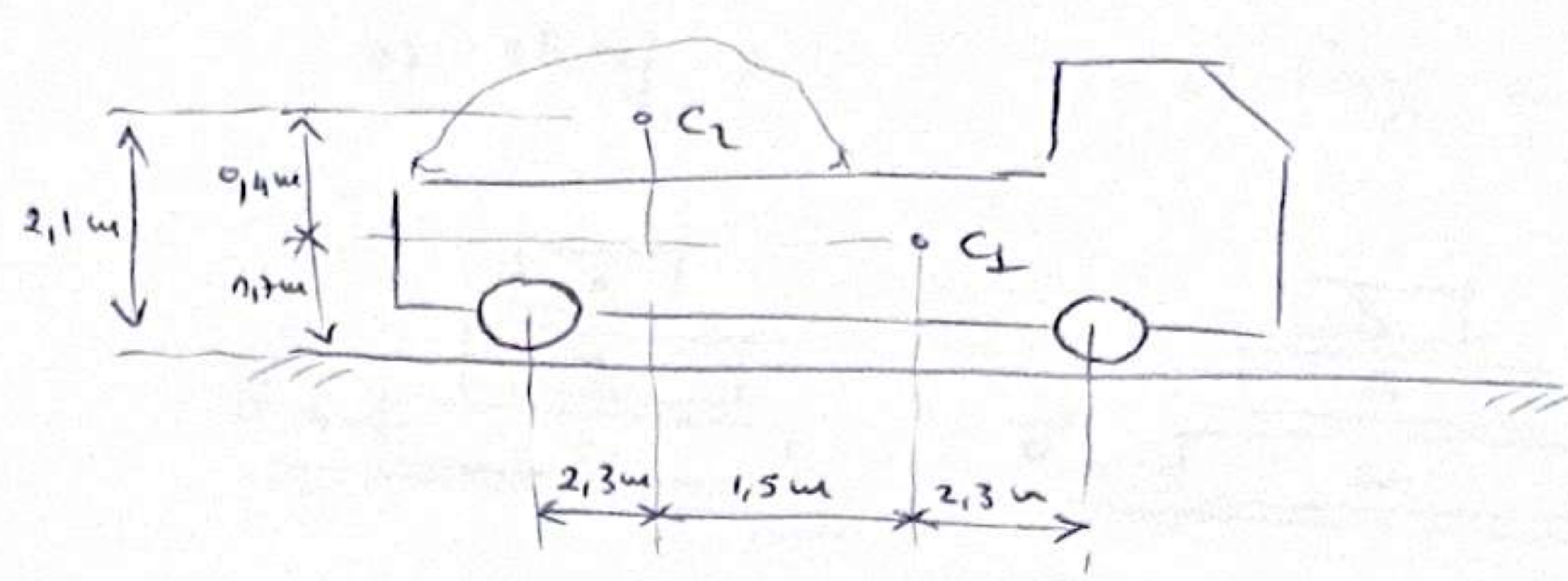
$a_{max} = ?$

m_1

$m_2 = m_1$

$\mu = 0{,}6$

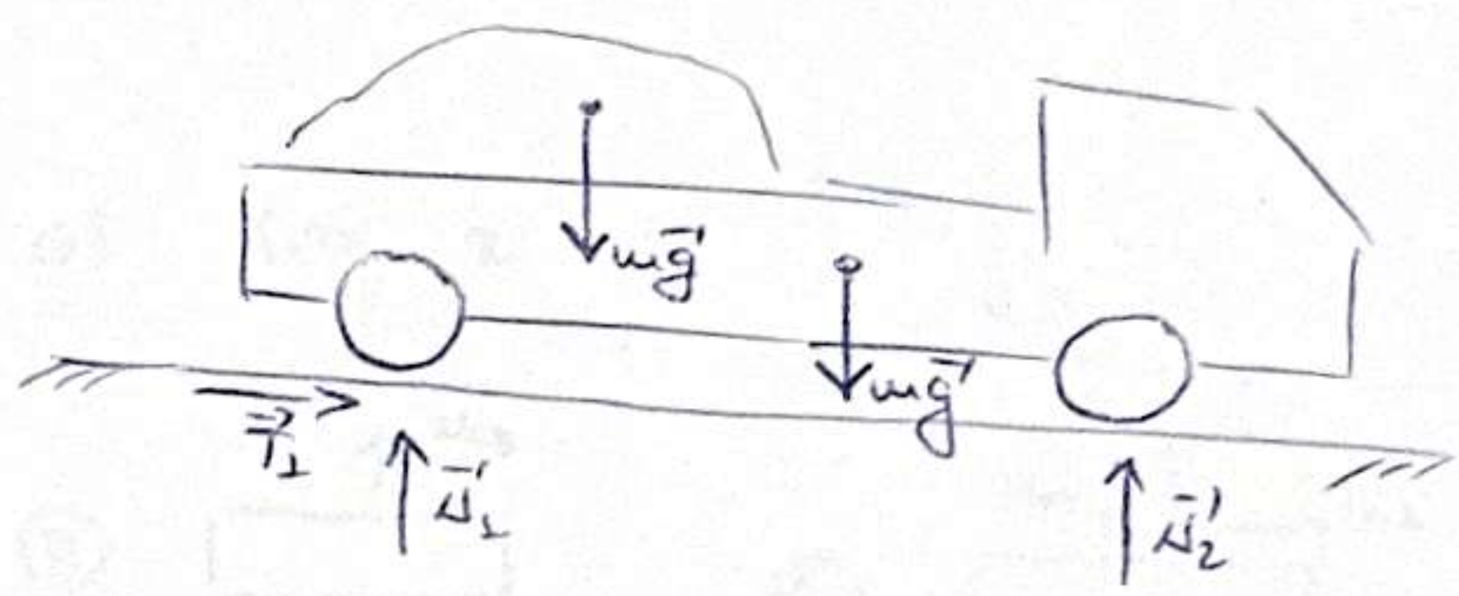
pogon na zadnjim točkovima



granični uslov: $T_1 = N_1 \cdot \mu$ – tada je a_{max}

(kada je $a < a_{max}$, onda je $T_1 < N_1 \cdot \mu$)

$$M\vec{a}_c = \vec{N}_1 + \vec{T}_1 + \vec{N}_2 + m\vec{g} + m\vec{g}$$



x: $\quad 2m\ddot{x}_c = T_1$

$$2m\,a_{max} = \mu \cdot N_1 \quad (1) \quad \begin{pmatrix} N_1 = ? \\ a_{max} = ? \end{pmatrix}$$

y: $\quad 0 = N_1 + N_2 - mg - mg \quad (2) \quad \begin{pmatrix} N_1 = ? \\ N_2 = ? \end{pmatrix}$

$$\frac{d\overrightarrow{L_{1z}}}{dt} = \Sigma \overrightarrow{M_{1z}}$$

$$\Sigma M_{1z} = mg \cdot 2{,}3 + mg \cdot 3{,}8 - N_2 \cdot 6{,}1 = 6{,}1(mg - N_2) = 6{,}1(mg - 2mg + N_1) = 6{,}1(N_1 - mg)$$

$$L_{1z} = [1{,}7\, m\,v + J_1\omega] + [2{,}1\, m\,v + J_2\omega]$$

$$\frac{dL_{1z}}{dt} = 3{,}8 \cdot m \cdot a_{max} + \cancel{J_1\varepsilon}^{\,0} + \cancel{J_2\varepsilon}^{\,0}$$

$$3{,}8 \cdot m \cdot a_{max} = 6{,}1\,(N_1 - mg)$$

$$3{,}8\, m\, a_{max} = 6{,}1\left(\frac{2m\,a_{max}}{\mu} - mg\right) \quad /:m$$

$$3{,}8\, m\, a_{max} = \frac{6{,}1 \cdot 2}{0{,}6} \cdot a_{max} - 6{,}1 \cdot g$$

$$a_{max}\left(\frac{12{,}2}{0{,}6} - 3{,}8\right) = 6{,}1 \cdot g$$

$$a_{max} = \frac{6{,}1 \cdot g}{\frac{12{,}2}{0{,}6} - 3{,}8}$$

$$\boxed{a_{max} = 3{,}62\,[m/s^2]}$$ – približno rešenju u zbirci

10.25

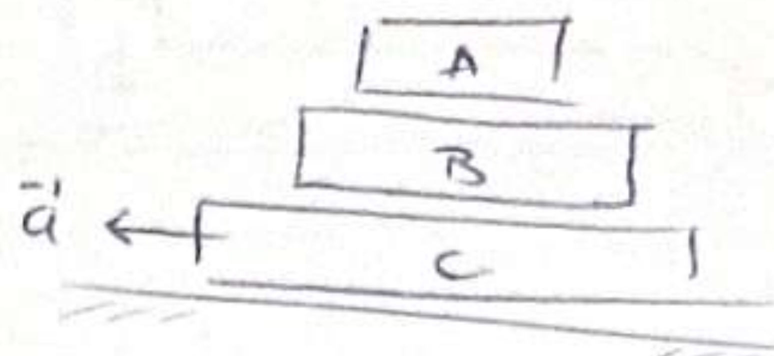
$a_{max} = ?$ - iz uslova da A i B miruju u odnosu na C

$\mu_{AB} = 0,4$, $\mu_{BC} = 0,3$, $\mu_{AC} = 0,5$

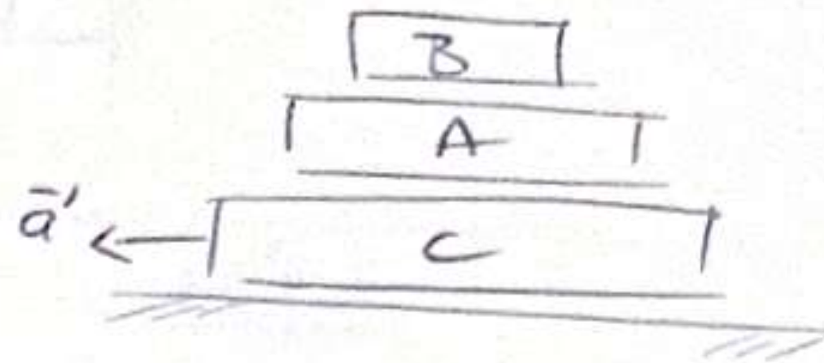
a) slučaj 1

b) slučaj 2

a) slučaj 1

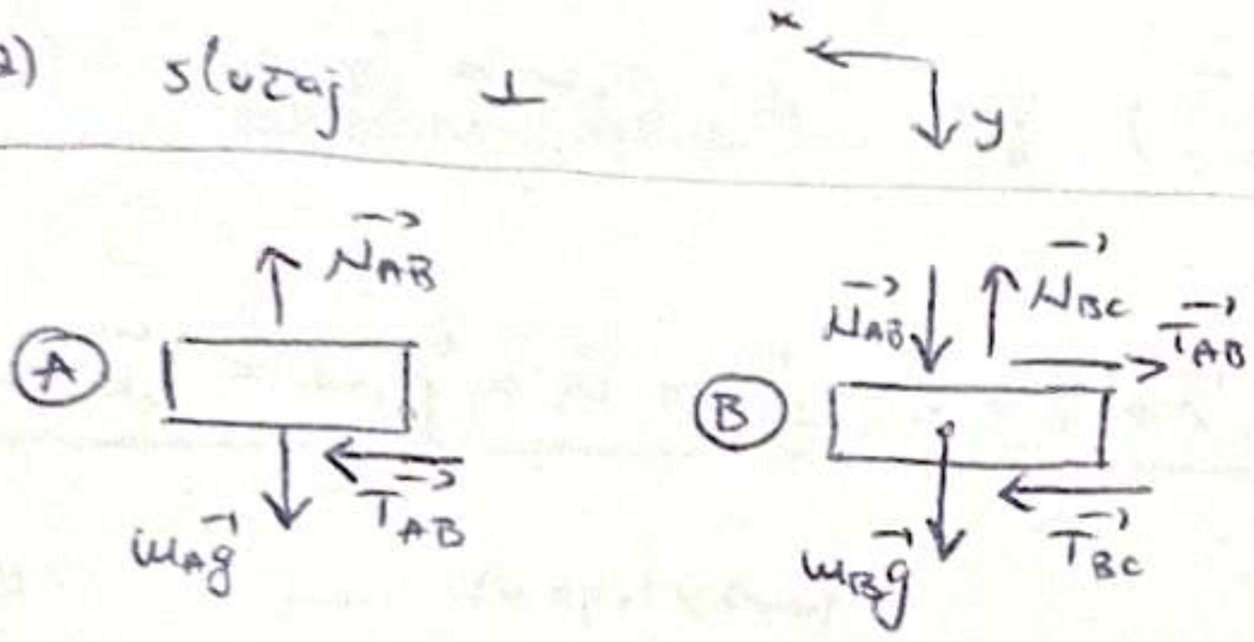


b) slučaj 2



Kinematska veza: $a_A = a_B = a_C = a$

a) slučaj 1



$$m_A \vec{a}_A = m_A \vec{g} + \vec{N}_{AB} + \vec{T}_{AB}$$

y: $0 = m_A g - N_{AB} \Rightarrow N_{AB} = m_A g$

$\Rightarrow T_{AB} = \mu_{AB} \cdot m_A g$

x: $m_A a = T_{AB}$

$\cancel{m_A} a = \mu_{AB} \cancel{m_A} g$

$a_{max1} = \mu_{AB} \cdot g$

$a_{max1} = 0,4\, g$

$$m_B \vec{a}_B = m_B \vec{g} + \vec{N}_{AB} + \vec{N}_{BC} + \vec{T}_{AB} + \vec{T}_{BC}$$

y: $0 = m_B g + N_{AB} - N_{BC}$

$0 = m_B g + m_A g - N_{BC} \Rightarrow N_{BC} = (m_A + m_B) g$

$\Rightarrow T_{BC} = \mu_{BC} (m_A + m_B) g$

x: $m_B a = -T_{AB} + T_{BC}$

$m_B a = -m_A \cdot a + \mu_{BC} (m_A + m_B) g$

$\cancel{(m_A + m_B)} a = \mu_{BC} \cancel{(m_A + m_B)} g$

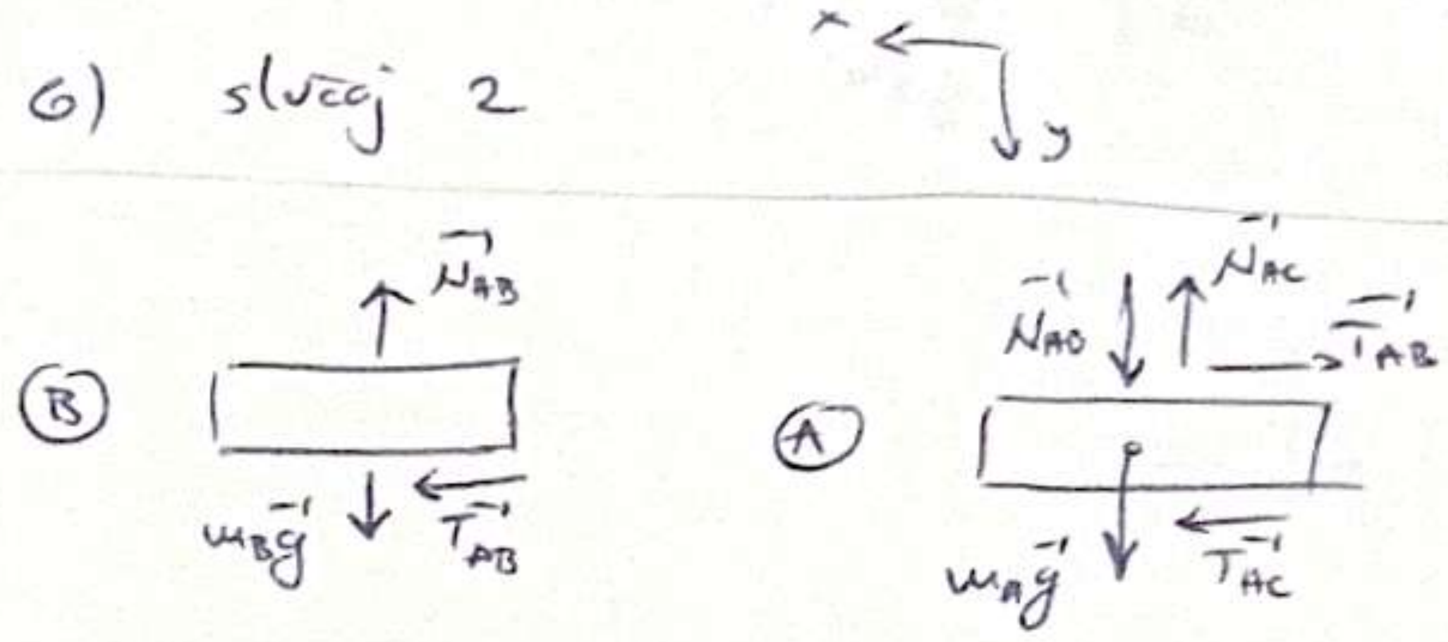
$a_{max2} = \mu_{BC} \cdot g$

$a_{max2} = 0,3\, g$

$a_{max} = \min(a_{max1}, a_{max2})$

$\boxed{a_{max} = 0,3\, g}$

b) slučaj 2



$$m_B \vec{a}_B = m_B \vec{g} + \vec{N}_{AB} + \vec{T}_{AB}$$

y: $0 = m_B g - N_{AB} \Rightarrow N_{AB} = m_B g$

$\Rightarrow T_{AB} = \mu_{AB} \cdot m_B g$

x: $m_B a = T_{AB}$

$\cancel{m_B} a = \mu_{AB} \cdot \cancel{m_B} g$

$a_{max1} = \mu_{AB} \cdot g$

$a_{max1} = 0,4 \cdot g$

$$m_A \vec{a}_A = m_A \vec{g} + \vec{N}_{AB} + \vec{N}_{AC} + \vec{T}_{AB} + \vec{T}_{AC}$$

y: $0 = m_A g + N_{AB} - N_{AC}$

$0 = m_A g + m_B g - N_{AC} \Rightarrow N_{AC} = (m_A + m_B) g$

$\Rightarrow T_{AC} = \mu_{AC} (m_A + m_B) g$

x: $m_A a = -T_{AB} + T_{AC}$

$m_A a = -m_B a + \mu_{AC} (m_A + m_B) g$

$\cancel{(m_A + m_B)} a = \mu_{AC} \cancel{(m_A + m_B)} g$

$a_{max2} = \mu_{AC} \cdot g$

$a_{max2} = 0,5 \cdot g$

$a_{max} = \min(a_{max1}, a_{max2})$

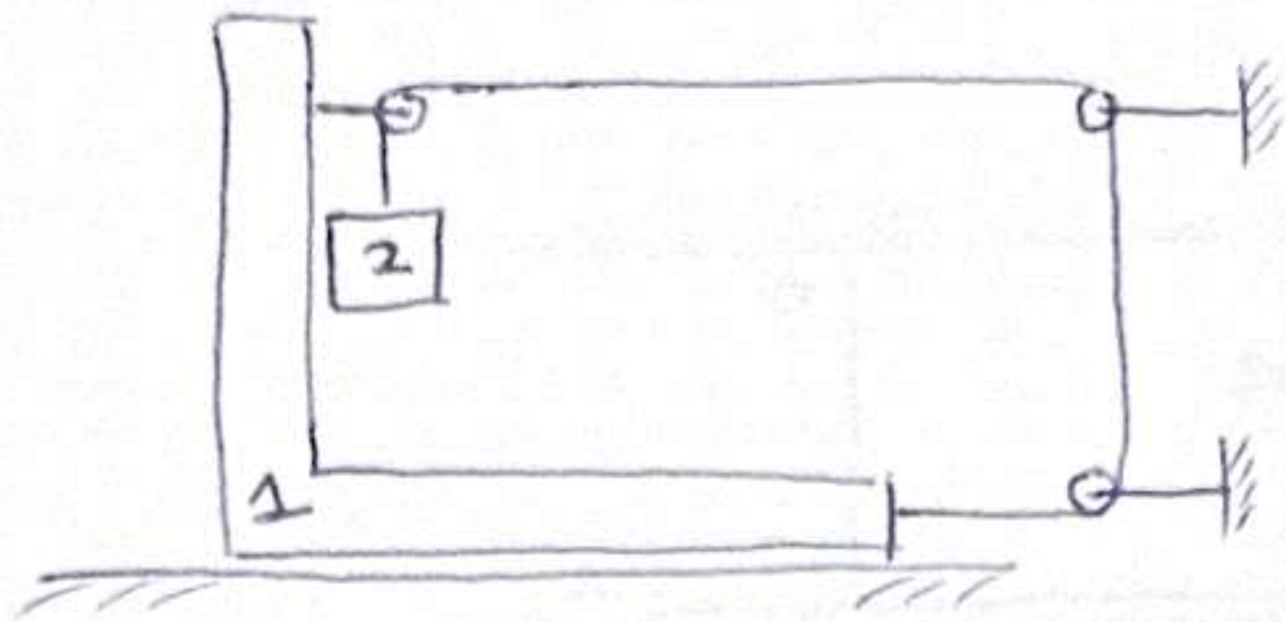
$\boxed{a_{max} = 0,5\, g}$

(10.29)

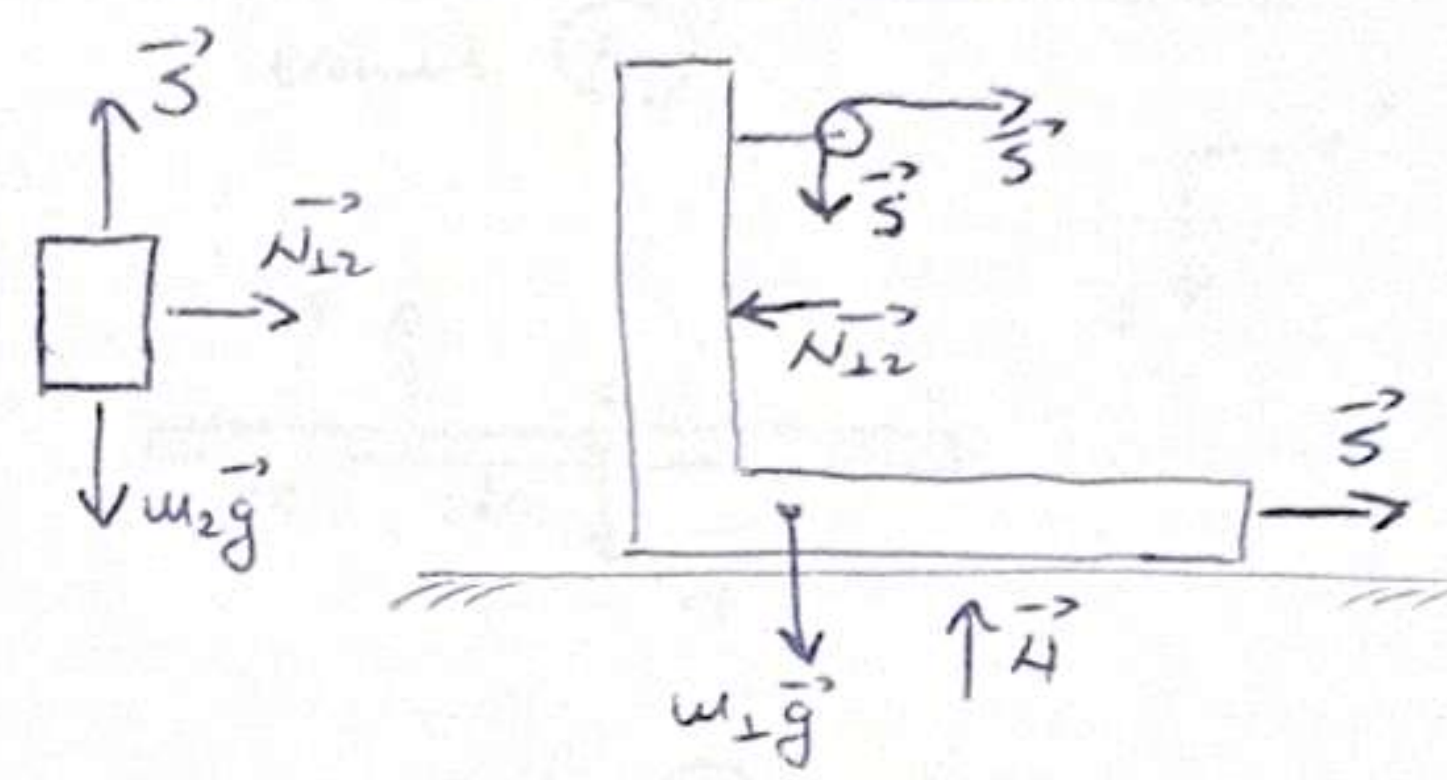
1: m_1

2: m_2

a_1 ? S ?



telo 1

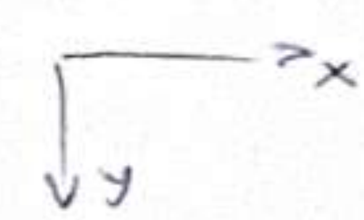


$$\underline{m_2 \vec{a}_2 = m_2 \vec{g} + \vec{S} + \vec{N}_{12}}$$

x: $m_2 \ddot{x}_2 = N_{12}$, $\underline{\ddot{x}_1 = \ddot{x}_2 = \ddot{x}}$

$\underline{m_2 \ddot{x} = N_{12}}$ (1) ($\ddot{x} = ?$ $N_{12} = ?$)

y: $\underline{m_2 \ddot{y}_2 = m_2 g - S}$ (2) ($\ddot{y}_2 = ?$ $S = ?$)



$$\underline{m_1 \vec{a}_1 = m_1 \vec{g} + \vec{N} + \vec{N}_{12} + \vec{S} + \vec{S} + \vec{S}'}$$

y: — (nepotrebno)

x: $m_1 \ddot{x} = -N_{12} + S + S$

$m_1 \ddot{x} = -m_2 \ddot{x} + 2S$

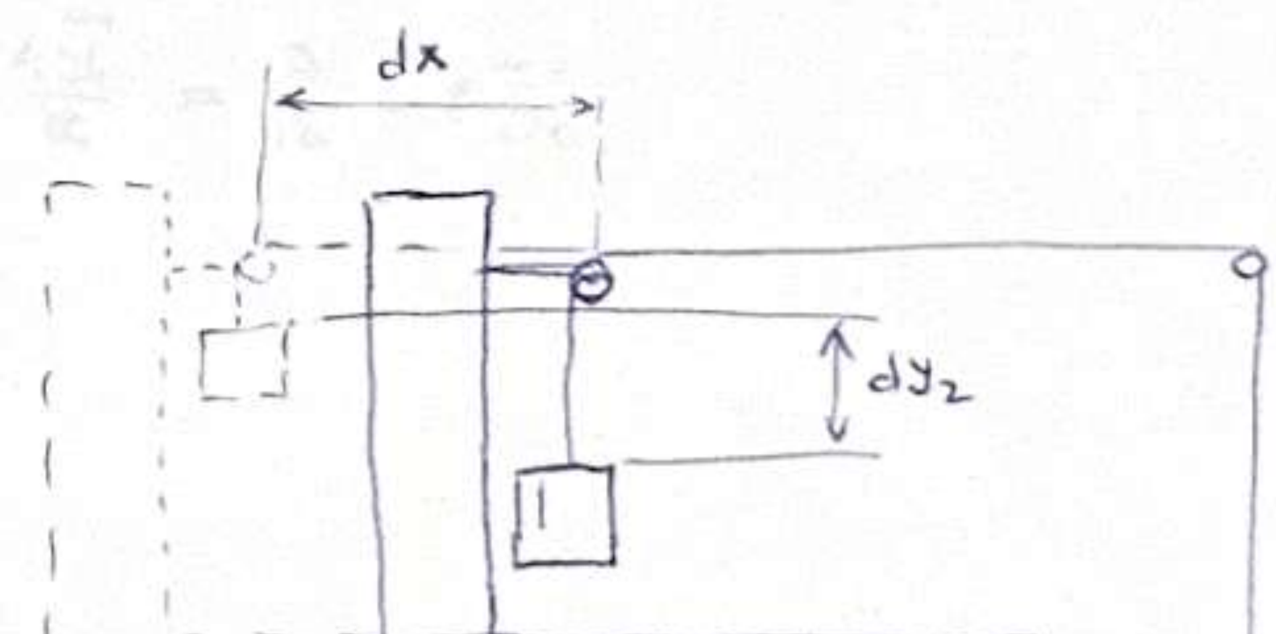
$\underline{(m_1 + m_2)\ddot{x} = 2S}$ (3)

kinematska veza:

$\ell = const.$

$\ell - dx - dx + dy_2 = const.$ $\quad / \cdot \frac{1}{dt} \quad / \cdot \frac{d}{dt}$

$\underline{2\ddot{x} = \ddot{y}_2}$ (3)



(2) $\Rightarrow$ $m_2 \cdot 2\ddot{x} = m_2 g - \frac{m_1 + m_2}{2}\ddot{x}$ $\quad / \cdot 2$

$4 m_2 \ddot{x} = 2 m_2 g - m_1 \ddot{x} - m_2 \ddot{x}$

$\ddot{x}(5 m_2 + m_1) = 2 m_2 g$

$$\boxed{\ddot{x} = \frac{2 m_2}{m_1 + 5 m_2} g}$$

(1) $\Rightarrow$ $S = \frac{m_1 + m_2}{2}\ddot{x} = \frac{m_1 + m_2}{2} \cdot \frac{2 m_2}{m_1 + 5 m_2} g$

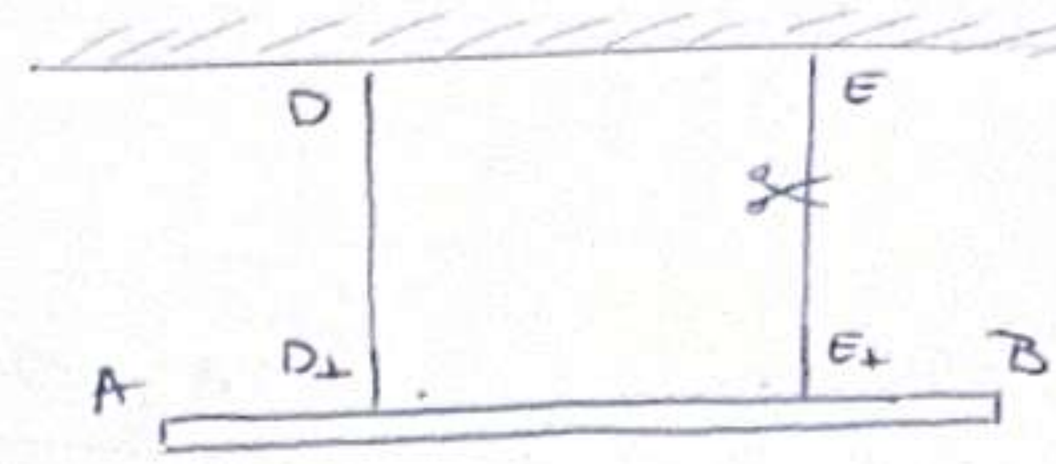
$$\boxed{S = \frac{m_1 + m_2}{m_1 + 5 m_2} m_2 g}$$

(10. 80.)

AB: $\quad Q, \ell$

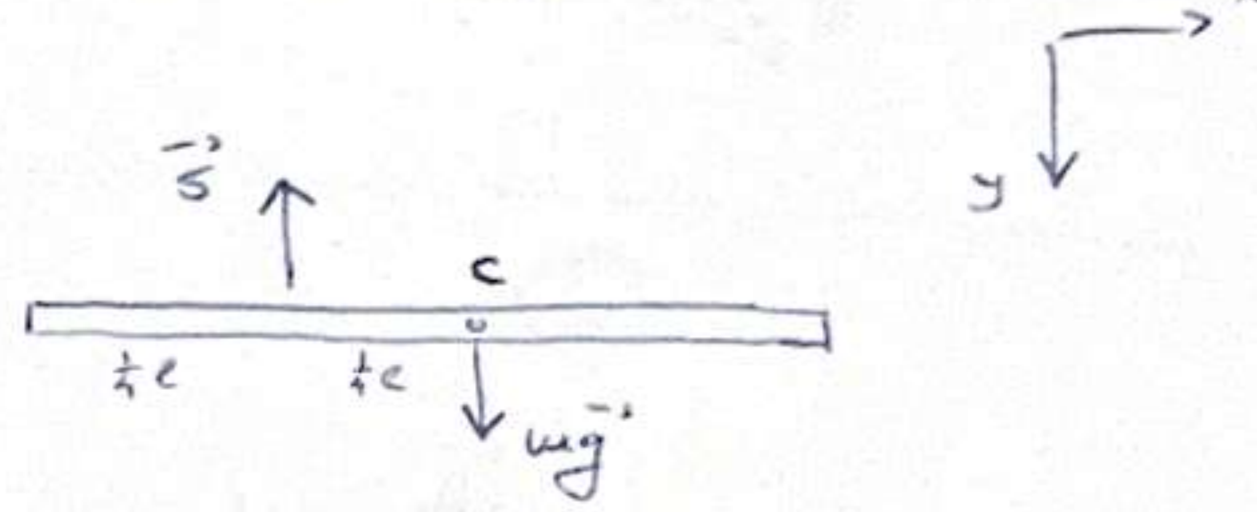
$\overline{AD_1} = \overline{BE_1} = \frac{1}{4}\ell$

$S = S_{DD_1} = ?$ – neposredno nakon presecanja kanapa EE_1



Posmatramo (i raspisujemo j-ne za) trenutak t_0:

trenutak (t_0)



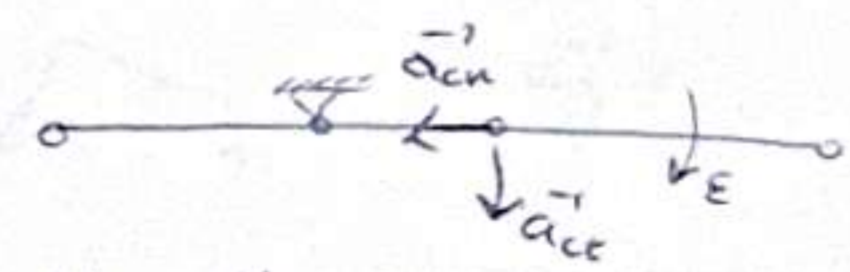
$$m\vec{a}_C = m\vec{g} + \vec{S}$$

x: – – – – (nista)

y: $m\ddot{y}_C = mg - S$

$$S = m(g - \ddot{y}_C) \quad (1)$$

trenutak (t_0) – ubrzanja



$\vec{a}_C = \vec{a}_{Cn} + \vec{a}_{Ct}$

x: $\ddot{x}_C = -a_{Cn} = -\omega^2 \cdot \frac{1}{4}\ell = 0$

y: $\ddot{y}_C = +a_{Ct} = +\varepsilon \cdot \frac{1}{4}\ell$

$$\frac{d\overset{\curvearrowright}{L_{Az}}}{dt} = \sum \overset{\curvearrowright}{M_{Az}}$$

$\sum M_{Az} = \frac{1}{4} mg\ell$

$L_{Az} = \frac{1}{4}\ell \cdot m v_C + J_C \omega$

$$\frac{dL_{Az}}{dt} = \frac{1}{4}\ell \cdot m a_{Ct} + \frac{1}{12} m\ell^2 \varepsilon = \frac{1}{4}\ell \cdot m \ddot{y}_C + \frac{1}{12} m\ell^2 \cdot \frac{4\ddot{y}_C}{\ell} = \frac{7}{12} m\ell \ddot{y}_C$$

$\frac{7}{12} \cancel{m}\cancel{\ell}\ddot{y}_C = \frac{1}{4}\cancel{m}g\cancel{\ell}$

$$\ddot{y}_C = \frac{3}{7} g$$

(1) $\Rightarrow \quad S = m\left(g - \frac{3}{7}g\right) = \frac{Q}{g} \cdot \frac{4}{7} g$

$$\boxed{S = \frac{4}{7} Q}$$

(10.31)

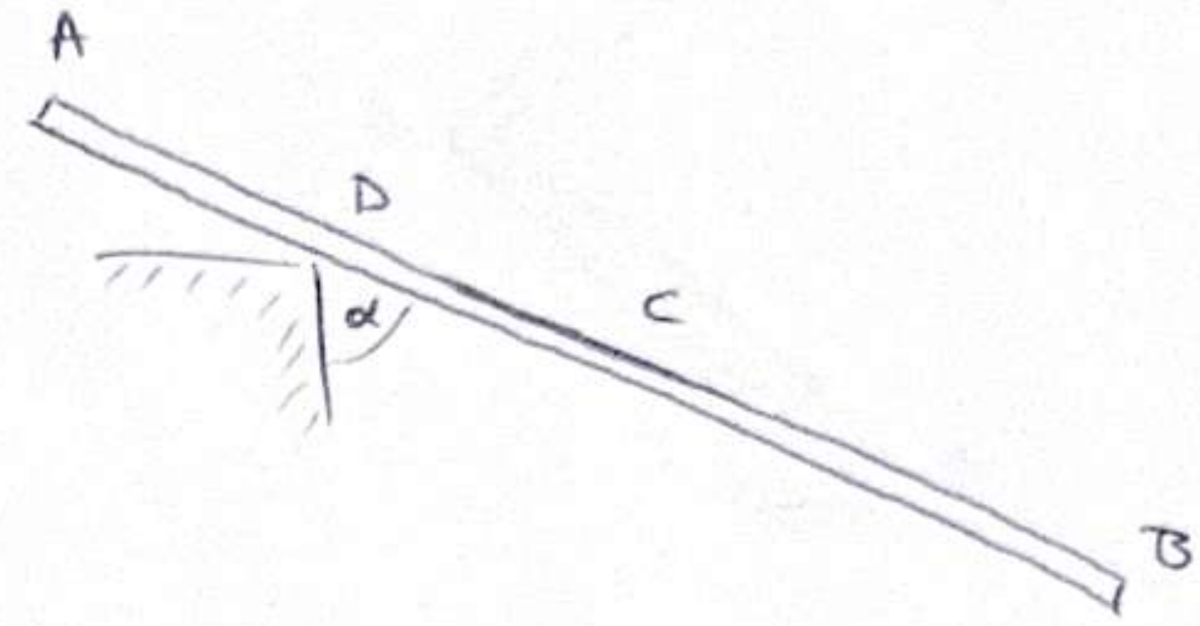
AB: Q, ℓ

$\overline{CD} = a$

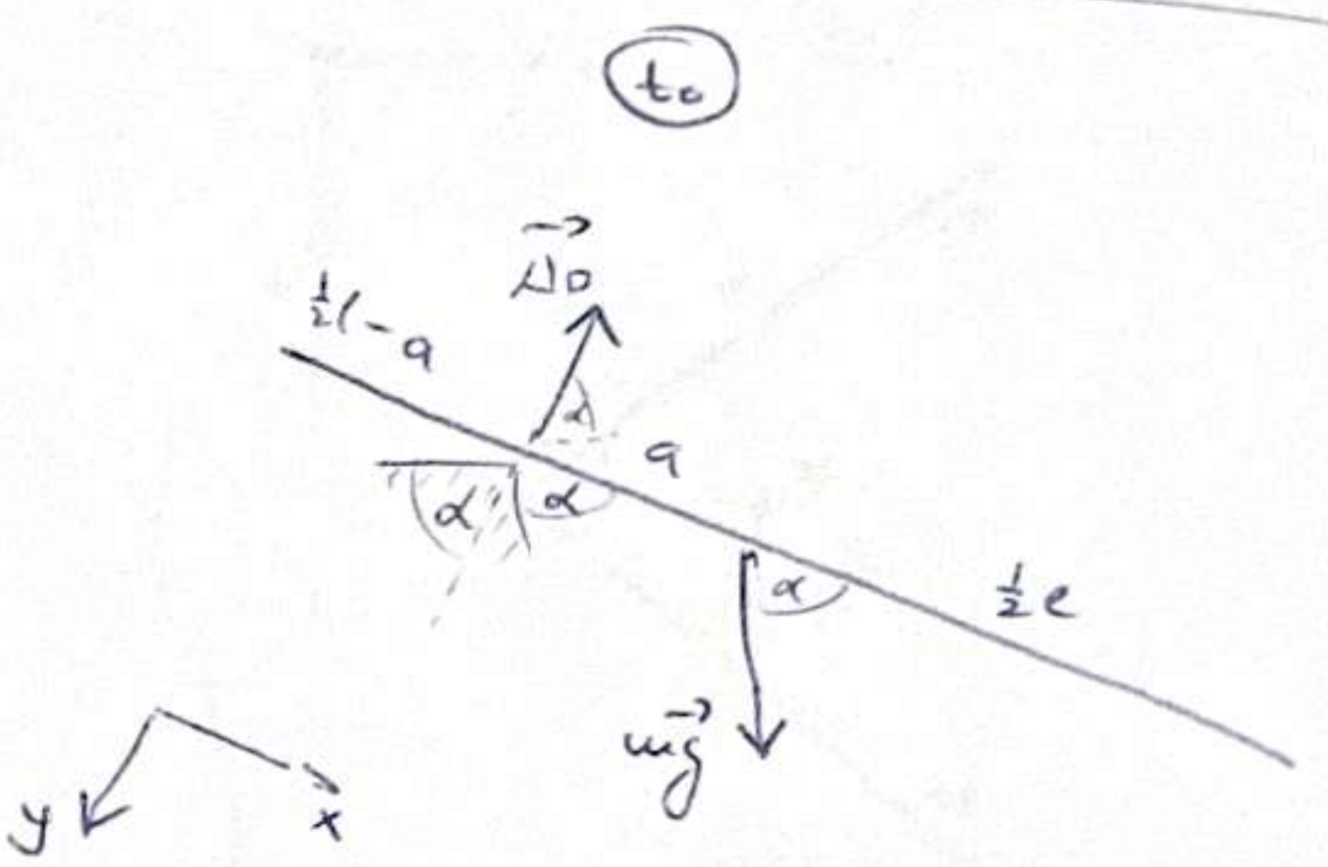
$t_0 = 0$: štap je mirovao, ugao α

$N_D = N_D(t_0) = ?$

$a_c = a_c(t_0) = ?$



Posmatramo (i raspisujemo j-ne za) trenutak (t_0):

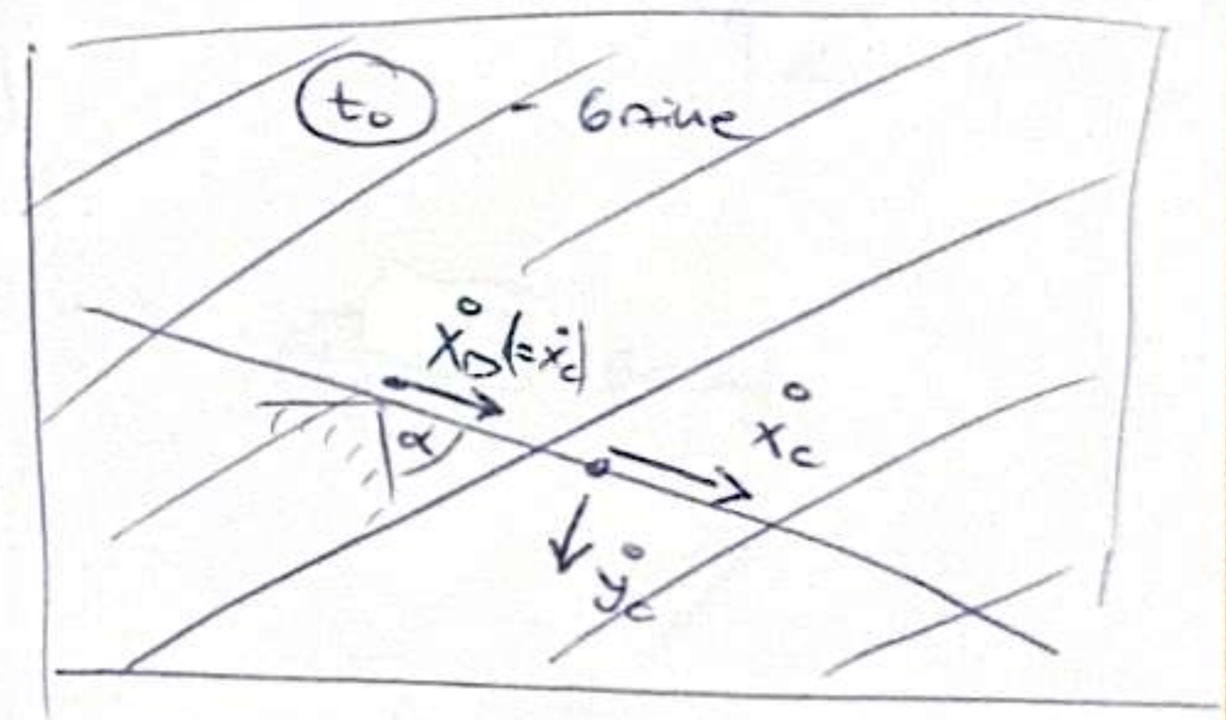


$$\underline{m\vec{a}_c = m\vec{g} + \vec{N}_D}$$

y: $m\ddot{y}_c = mg\sin\alpha - N_D$

$$N_D = m(g\sin\alpha - \ddot{y}_c) \quad (1)$$

ne treba nam ovo



$$\underline{\frac{d\vec{L}_{C\tau}}{dt} = \Sigma \vec{M}_{C\tau}}$$

$\Sigma M_{C\tau} = N_D \cdot a$

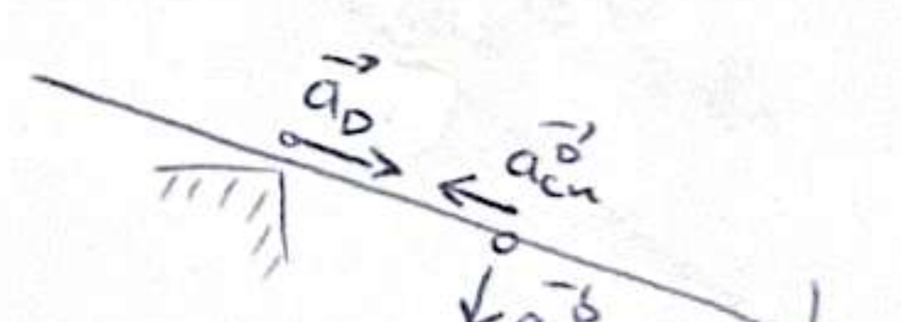
$L_{C\tau} = J_c\omega = \frac{1}{12}m\ell^2\omega$

$\frac{dL_{C\tau}}{dt} = \frac{1}{12}m\ell^2\varepsilon = \frac{1}{12}m\ell^2\frac{\ddot{y}_c}{a}$

$\frac{1}{12}m\ell^2\frac{\ddot{y}_c}{a} = N_D \cdot a$

$$N_D = \frac{1}{12}m\frac{\ell^2}{a^2}\ddot{y}_c \quad (2)$$

t_0 - ubrzanja



$$\underline{\vec{a}_c = \vec{a}_D + \vec{a}^D_{cn} + \vec{a}^D_{ct}}$$

x: $\ddot{x}_c = \ddot{x}_D - a^D_{cn} = \ddot{x}_D - \cancel{\omega^2}a = \ddot{x}_D$

y: $\ddot{y}_c = a_{ct} = \varepsilon \cdot a$

(1) = (2)

$\frac{1}{12}\cancel{m}\frac{\ell^2}{a^2}\ddot{y}_c = \cancel{m}(g\sin\alpha - \ddot{y}_c) \quad / \cdot 12a^2$

$\ddot{y}_c \cdot \ell^2 + \ddot{y}_c \cdot 12a^2 = g \cdot 12a^2\sin\alpha$

$$\ddot{y}_c = \frac{12a^2\sin\alpha}{12a^2+\ell^2}g$$

(1) => $N_D = m\left(g\sin\alpha - \frac{12a^2\sin\alpha}{12a^2+\ell^2}g\right)$

$N_D = mg\left(\frac{\cancel{12a^2\sin\alpha} + \ell^2\sin\alpha - \cancel{12a^2\sin\alpha}}{12a^2+\ell^2}\right)$

$$\boxed{N_D = Q \cdot \frac{\ell^2\sin\alpha}{12a^2+\ell^2}}$$

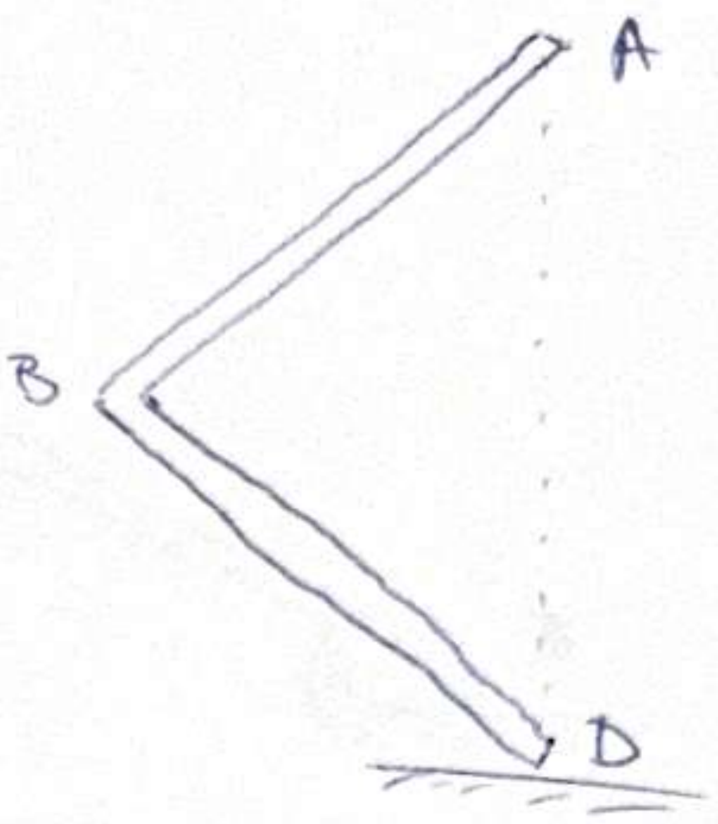
$$\underline{m\vec{a}_c = m\vec{g} + \vec{N}_D}$$

x: $\cancel{m}\ddot{x}_c = \cancel{m}g\cos\alpha$

$$\boxed{a_c = \sqrt{\ddot{x}_c^2 + \ddot{y}_c^2} = \ldots}$$

(10. 32.)

ABD: P, sastavljen od dva jednaka štapa

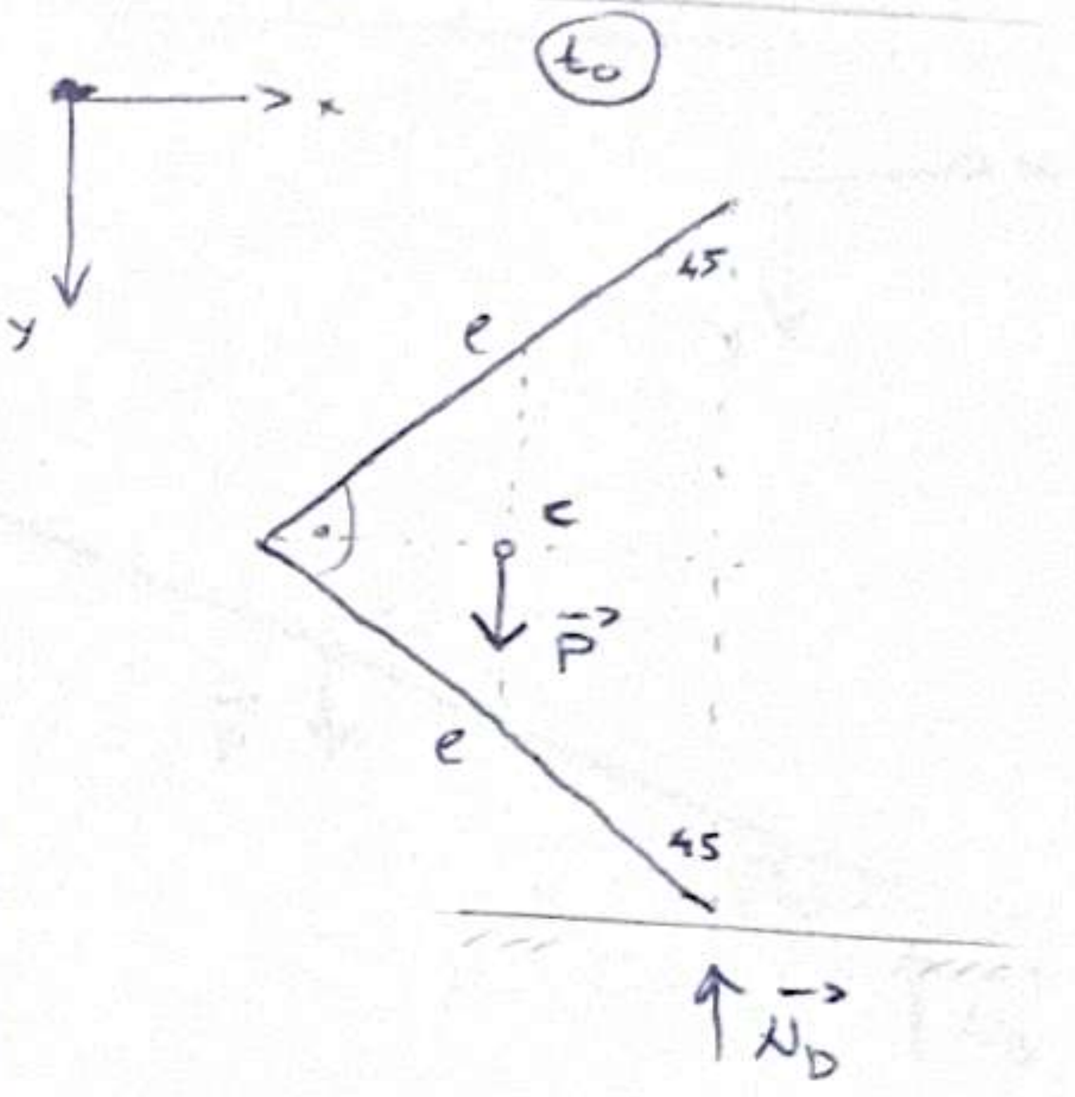


t_0: A : D su u vertikali

bez poč. brzine

$N_D = N_D(t_0) = ?$

Posmatramo (i raspisujemo j-ne za) trenutak t_0:



$m\vec{a_C} = \vec{P} + \vec{N_D}$

x: -- (nebitno)

y: $m\ddot{y}_C = P - N_D$ (1)

$\frac{dL_{Cz}}{dt} = \sum M_{Cz}$

$\sum M_{Cz} = N_D \cdot \frac{1}{2} l \cos 45 = \frac{\sqrt{2}}{4} N_D \cdot l$

$L_{Cz} = J_{Cz} \cdot \omega = 2 \cdot \left[\frac{1}{12} \frac{m}{2} l^2 + \frac{m}{2}\left(\frac{1}{2} l \cos 45\right)^2\right]\omega$

$L_{Cz} = m\left[\frac{1}{12} l^2 + \frac{2}{16} l^2\right]\omega = \frac{5}{24} m l^2 \omega$

$\frac{dL_{Cz}}{dt} = \frac{5}{24} m l^2 \varepsilon = \frac{5}{24} m l^2 \cdot 2\sqrt{2} \frac{\ddot{y}_C}{l} = \frac{5\sqrt{2}}{12} m l \ddot{y}_C$

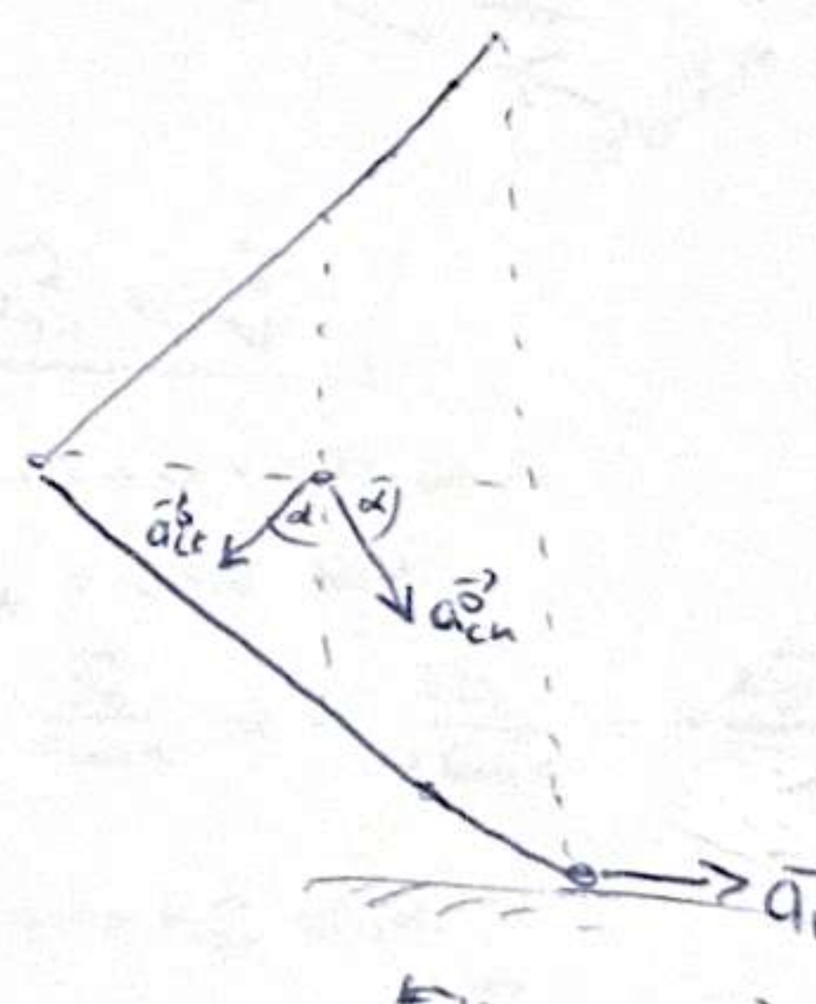
$\frac{5\sqrt{2}}{12} m \cancel{l} \ddot{y}_C = \frac{\sqrt{2}}{4} N_D \cdot \cancel{l}$

$N_D = \frac{5}{3} m\ddot{y}_C = \frac{5}{3}(P - N_D)$

$\frac{8}{\cancel{3}} N_D = \frac{5}{\cancel{3}} P$

$\boxed{N_D = \frac{5}{8} P}$

t_0 - ubrzanja



$\vec{a}_C = \vec{a}_D + \cancel{\vec{a}^D_{Cn}} + \vec{a}^D_{Ct}$ (jer je $\omega = 0$)

x: $\ddot{x}_C = --$ (nebitno)

y: $\ddot{y}_C = 0 + 0 + a^D_{Ct} \cdot \cos\alpha$

$\ddot{y}_C = \varepsilon \cdot \overline{DC} \cdot \frac{1}{2} l \cos 45$

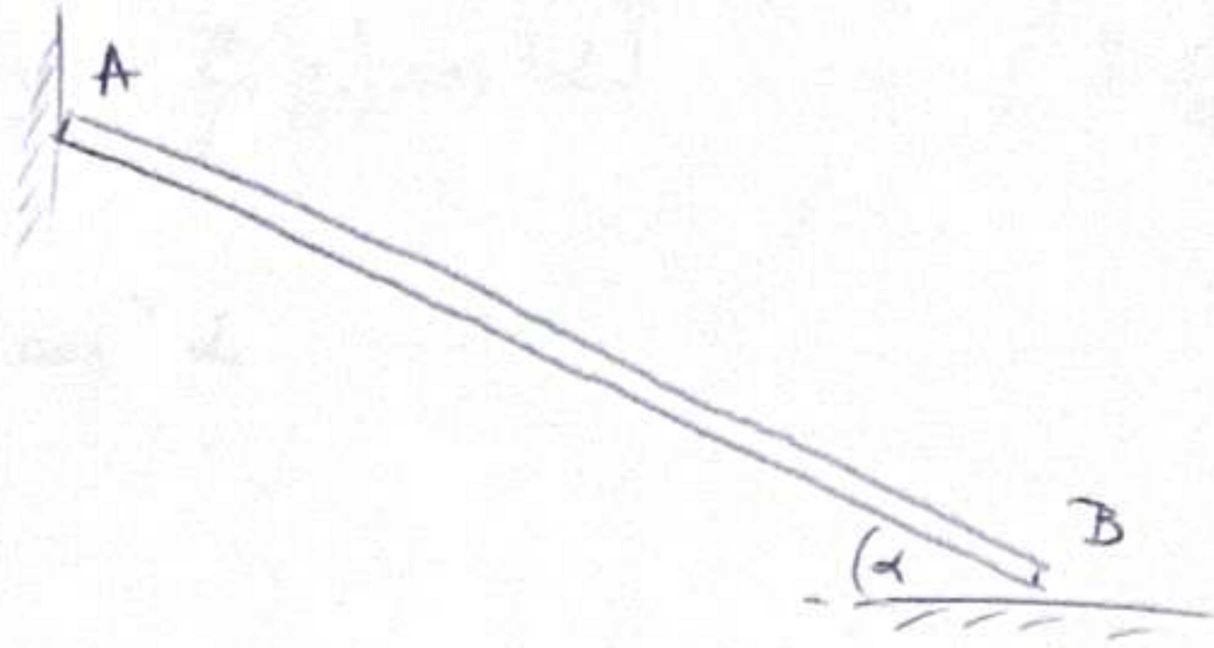
$\ddot{y}_C = \frac{\sqrt{2}}{4} l \varepsilon$

(10. 33)

AB : $\vec{P}$

t: mirovanje

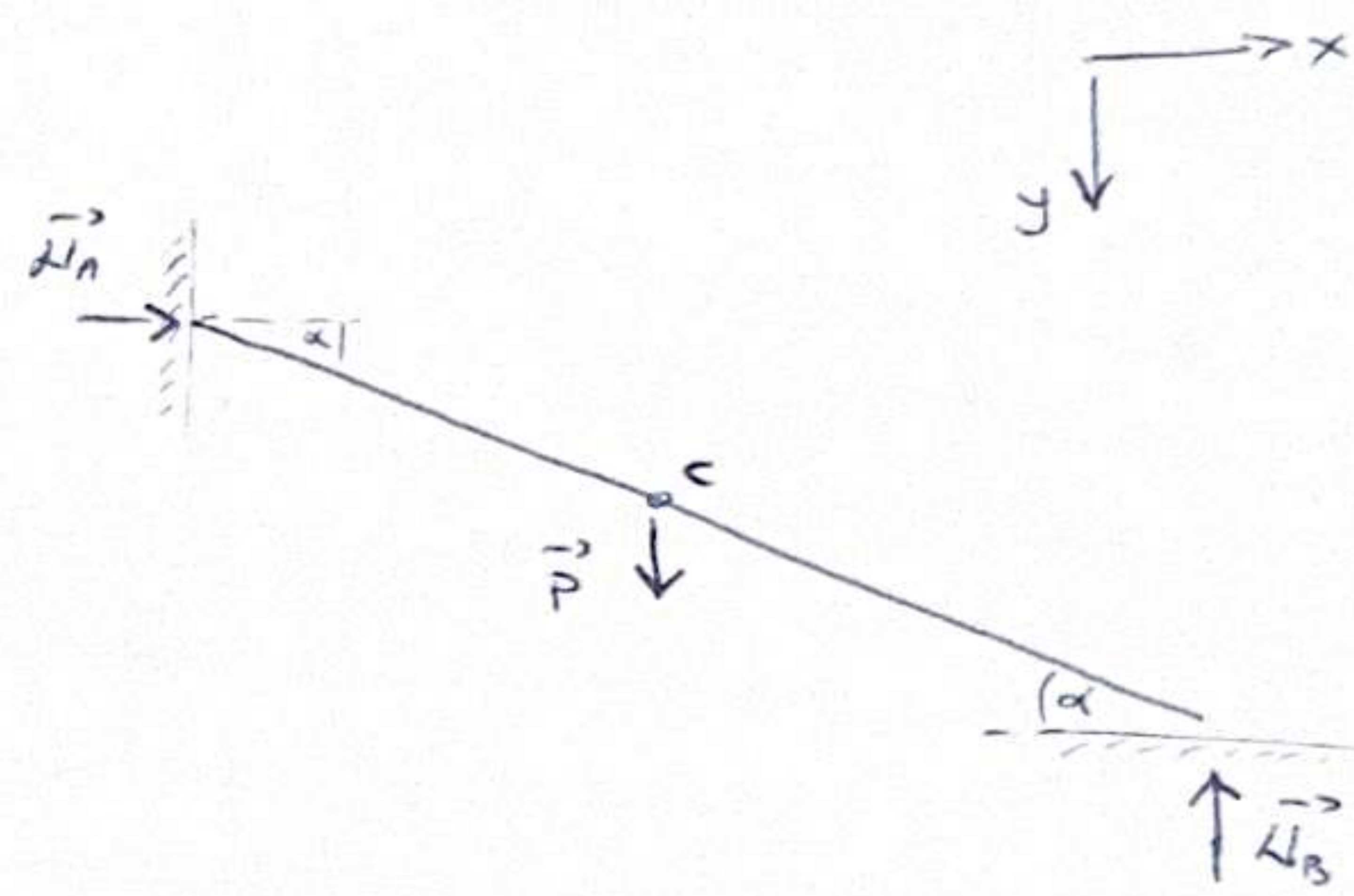
ugao α



$N_A = N_A(t) = ?$

$N_B = N_B(t) = ?$

Posmatramo trenutak t_0:



$$\frac{P}{g}\vec{a}_C = \vec{P} + \vec{N_A} + \vec{N_B}$$

$$x: \quad \frac{P}{g}\ddot{x}_C = N_A$$

$$N_A = \frac{P}{g}\ddot{y}\,tg\alpha \qquad (1)$$

$$y: \quad \frac{P}{g}\ddot{y}_C = -P + N_B$$

$$\frac{P}{g}\ddot{y} = -P + N_B \qquad (2)$$

$$\frac{d\vec{L}_{Ct}}{dt} = \Sigma\vec{M}_{Ct}$$

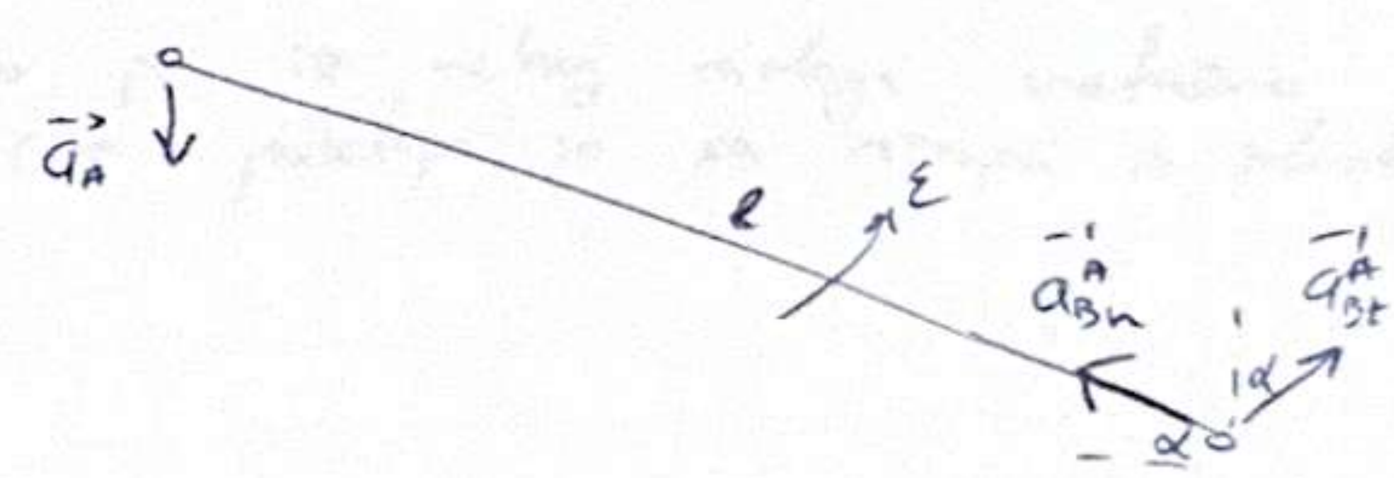
$$\Sigma M_{Ct} = N_A \cdot \frac{1}{2}l\sin\alpha - N_B \cdot \frac{1}{2}l\cos\alpha$$

$$L_{Ct} = J_C \cdot \omega = \frac{1}{12}\frac{P}{g}l^2\omega$$

$$\frac{dL_{Ct}}{dt} = \frac{1}{12}\frac{P}{g}l^2\varepsilon = -\frac{1}{12}\frac{P}{g}l^2\frac{2\ddot{y}}{l\cos\alpha} = -\frac{1}{6}\frac{P}{g}l\frac{\ddot{y}}{\cos\alpha}$$

$$\frac{1}{6}\frac{P}{g}l\ddot{y}\cdot\frac{1}{\cos\alpha} = \frac{1}{2}l\,(N_A\sin\alpha - N_B\cos\alpha) \quad /\cdot\left(-\frac{6\cos\alpha}{l}\right)$$

$$\frac{P}{g}\ddot{y} = 3\,(N_A\sin\alpha\cos\alpha - N_B\cos^2\alpha) \qquad (3)$$



$$\vec{a}_B = \vec{a}_A + \vec{a}^A_{Bn} + \vec{a}^A_{Bt}$$

$$y: \quad 0 = -a_A + 0 + a^A_{Bt}\cos\alpha$$

$$0 = \ddot{y}_A + \varepsilon\cdot l\cos\alpha$$

$$\varepsilon = -\frac{\ddot{y}_A}{l\cos\alpha} = -\frac{2\ddot{y}_C}{l\cos\alpha} = -\frac{2\ddot{y}}{l\cos\alpha}$$

$$x: \quad \ddot{x}_B = 0 + 0 + a^A_{Bt}\sin\alpha$$

$$\ddot{x}_B = \varepsilon\cdot l\cdot\sin\alpha = -\frac{2\ddot{y}}{\cancel{l}\cos\alpha}\cdot\cancel{l}\sin\alpha$$

$$\ddot{x}_B = -2\ddot{y}\,tg\alpha$$

$$\ddot{x}_C = \frac{1}{2}\ddot{x}_B = -\ddot{y}\,tg\alpha$$

(1) i (2) ubacimo u (3)

$$\frac{P}{g}\ddot{y} = 3\left(-\frac{P}{g}\ddot{y}\,tg\alpha\cdot\sin\alpha\cos\alpha - \left(\frac{P}{g}\ddot{y}+\frac{P}{g}g\right)\cos^2\alpha\right) \qquad /:\frac{P}{g}$$

$$\ddot{y}\left(1+3\sin^2\alpha+3\cos^2\alpha\right) = -3g\cos^2\alpha$$

$$4\ddot{y} = -3g\cos^2\alpha$$

$$\ddot{y} = -\frac{3}{4}g\cos^2\alpha$$

$$(1) \Rightarrow \quad N_A = +\frac{P}{g}\cdot\frac{3}{4}g\cos^2\alpha\cdot tg\alpha$$

$$N_A = \frac{P}{\cancel{g}}\cdot\frac{3}{4}\cancel{g}\sin\alpha\cos\alpha\cdot 2\cdot\frac{1}{2}$$

$$\boxed{N_A = \frac{3}{8}P\sin 2\alpha}$$

$$(2) \Rightarrow \quad N_B = -\frac{P}{\cancel{g}}\cdot\frac{3}{4}\cancel{g}\cos^2\alpha + P$$

$$\boxed{N_B = \frac{1}{4}P\left(4-3\cos^2\alpha\right)}$$

— ovo je iz nekog razloga netačno (ne poklapa se sa rešenjem iz zbirke)

10. 36.

horizontalna ravan

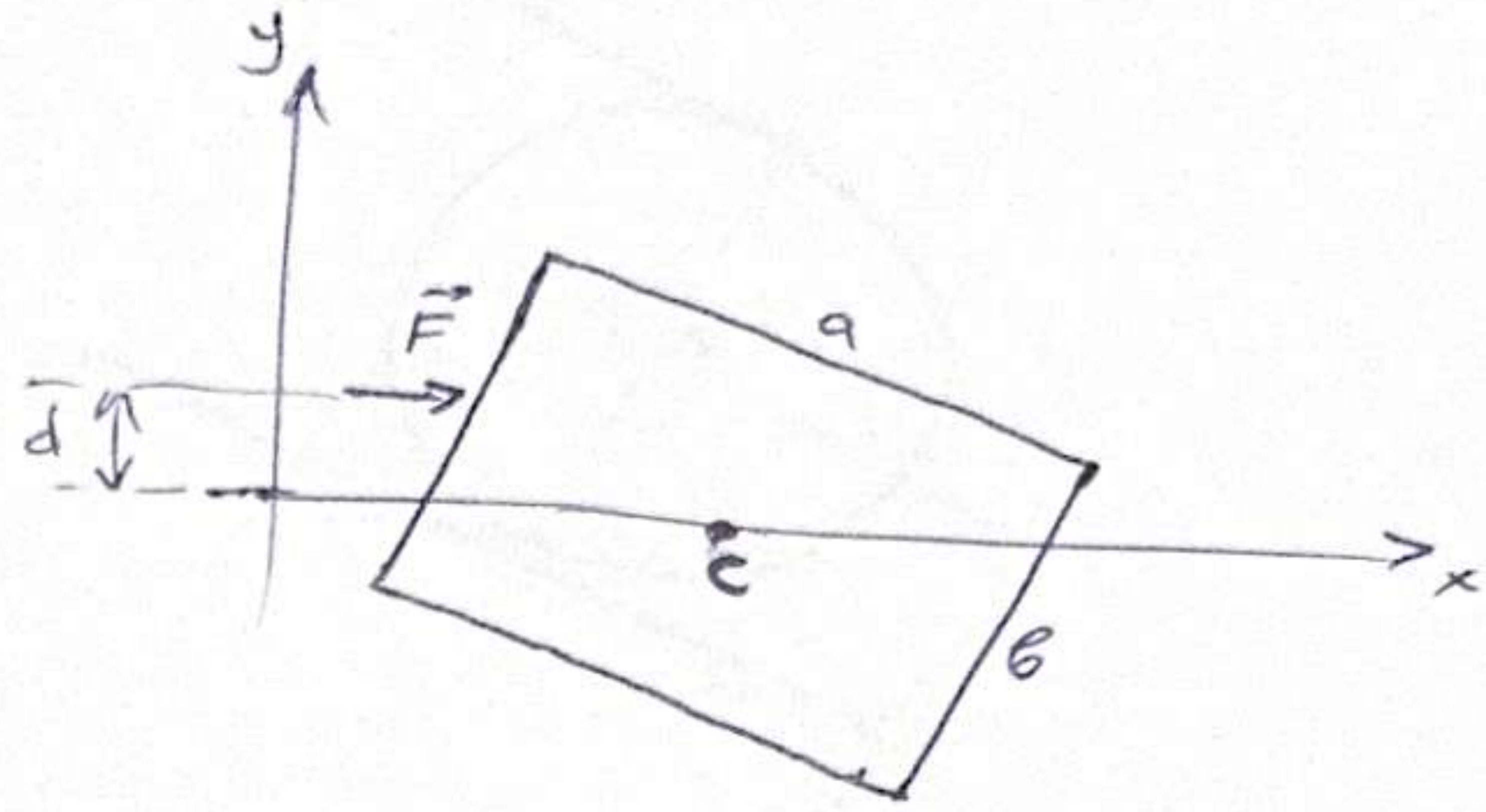
ploča: Q, $a \times b$, $a > b$

$\vec{F} \parallel Ox$, $a > 2d$, $b > 2d$

t_0: ploča miruje

položaj: koord. poč, $a \parallel Ox$

j-ne kretanja ?



$$m\vec{a}_c = \vec{F}$$

x: $m\ddot{x}_c = F$

$$\ddot{x}_c = \frac{F}{m} = \frac{F}{Q}\cdot g \quad \int$$

$$\dot{x}_c = \frac{F}{Q}\cdot g \cdot t + \cancel{C_1}^{\,0} \quad \int$$

$$x_c = \frac{1}{2}\frac{F}{Q} g t^2 + \cancel{C_2}^{\,0}$$

$$\boxed{x_c = \frac{1}{2}\frac{F}{Q} g t^2}$$

y: $\cancel{m}\ddot{y}_c = 0 \quad \int$

$$\dot{y}_c = \cancel{C_1}^{\,0} = 0 \quad \int$$

$$y_c = \cancel{C_2}^{\,0} = 0$$

$$\boxed{y_c = 0}$$

$$\frac{d\vec{L}_{C}}{dt} = \Sigma \vec{M}_{C}$$

$$\Sigma M_{C} = F\cdot d$$

$$L_{C} = J_c \cdot \omega = m\frac{a^2+b^2}{6}\omega$$

$$\frac{dL_{C}}{dt} = \frac{1}{6} m (a^2+b^2)\varepsilon$$

$$\frac{1}{6} m (a^2+b^2)\varepsilon = F\cdot d$$

$$\varepsilon = \frac{6\cdot F\cdot d}{m(a^2+b^2)} \quad \int$$

$$\omega = \frac{6\cdot F\cdot d}{m(a^2+b^2)} t + \cancel{C_1}^{\,0} \quad \int$$

$$\varphi = \frac{3Fd}{m(a^2+b^2)} t^2 + \cancel{C_2}^{\,0}$$

$$\boxed{\varphi = \frac{3Fdg}{Q(a^2+b^2)} t^2}$$

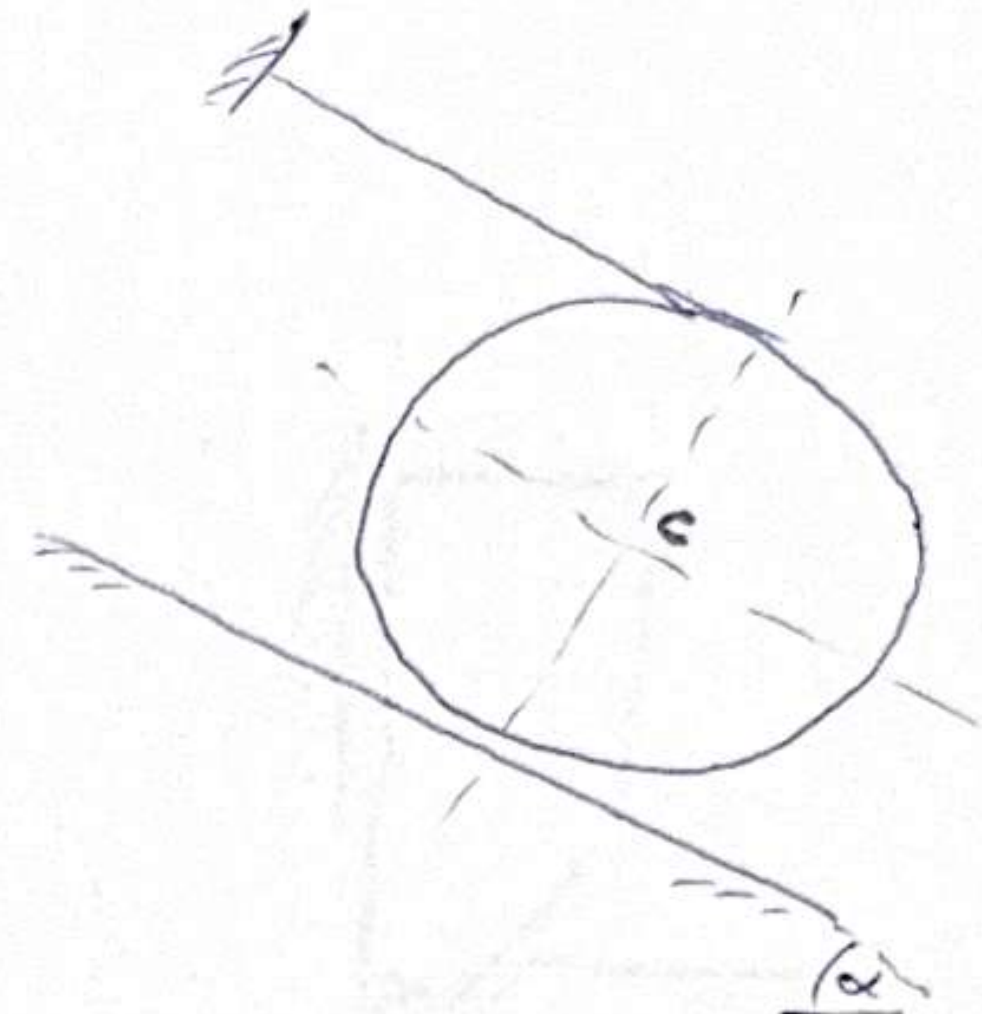
10. 38.

prsten: m, R

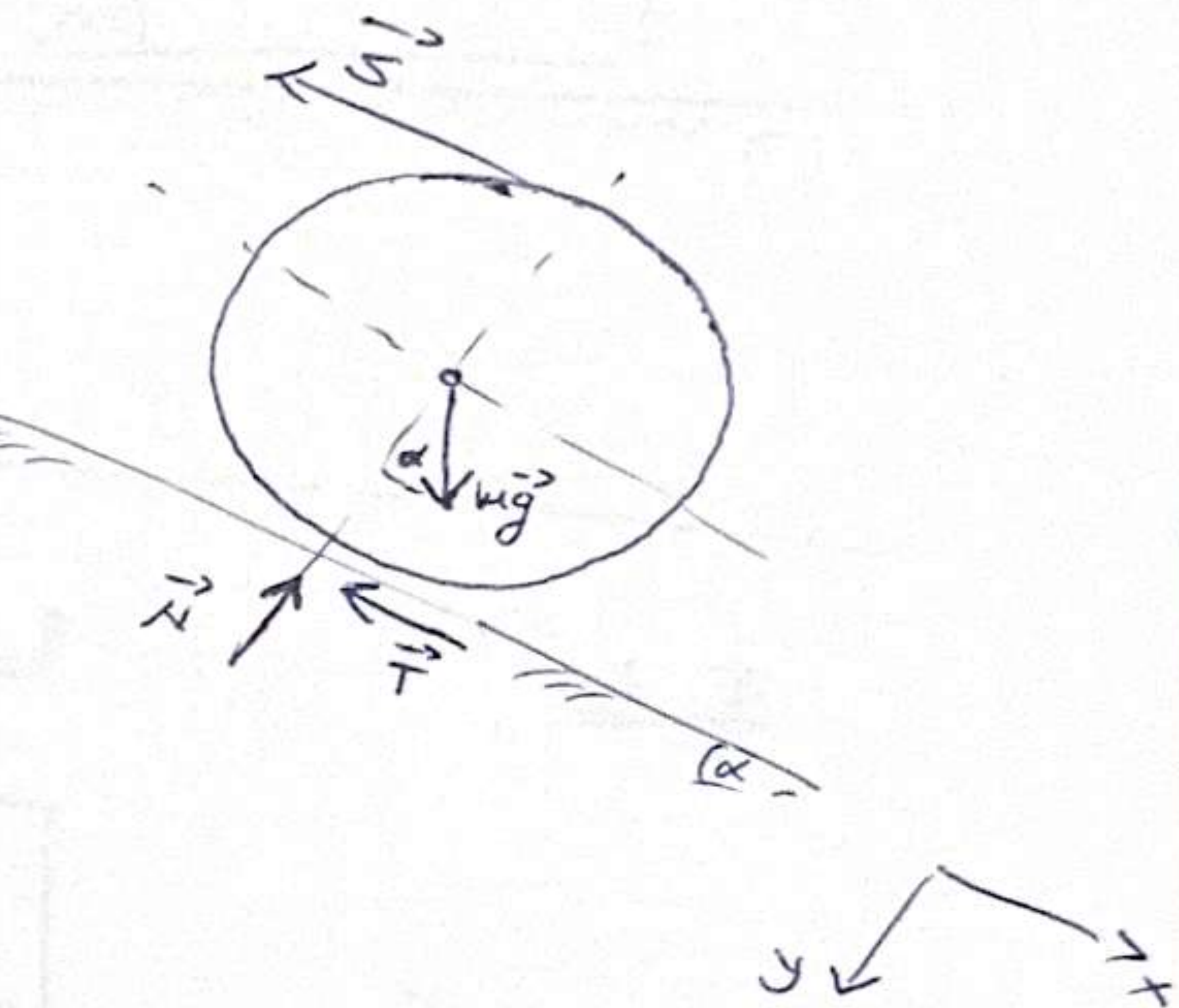
nagib α

μ - koef. trenja

a_c ? S?



$\underline{m\vec{a}_c = m\vec{g} + \vec{N} + \vec{T} + \vec{S}}$



y: $0 = mg - N$

$\underline{\underline{N = mg}}$

$\underline{\underline{T = \mu \cdot mg}}$

x: $m\ddot{x}_c = mg\sin\alpha - T - S$

$\underline{\underline{S = mg\sin\alpha - \mu \cdot mg\cos\alpha - m\ddot{x}_c}}$ (1)

$\underline{\frac{d\overleftarrow{L}_{C\tau}}{dt} = \Sigma \overleftarrow{M}_{C\tau}}$

$\Sigma M_{C\tau} = S \cdot R - T \cdot R = S \cdot R - \mu \cdot mg\cos\alpha \cdot R$

$L_{C\tau} = J_{C\tau} \cdot \omega = mR^2\omega$

$\frac{dL_{C\tau}}{dt} = mR^2\varepsilon = mR^{\cancel{2}}\frac{\ddot{x}_c}{\cancel{R}}$

$S \cdot \cancel{R} - \mu \cdot mg\cos\alpha \cdot \cancel{R} = m\cancel{R}\,\ddot{x}_c$

$\underline{\underline{S = m\ddot{x}_c + \mu \cdot mg\cos\alpha}}$ (2)

(1) = (2)

$mg\sin\alpha - \mu \cdot mg\cos\alpha - m\ddot{x}_c = m\ddot{x}_c + \mu mg\cos\alpha$

$2\cancel{m}\ddot{x}_c = \cancel{m}g(\sin\alpha - \mu\cos\alpha - \mu\cos\alpha)$

$$\boxed{\ddot{x}_c = \frac{1}{2}g(\sin\alpha - 2\mu\cos\alpha)}$$

(2) => $S = m \cdot \frac{1}{2}g(\sin\alpha - 2\mu\cos\alpha) + \mu mg\cos\alpha$

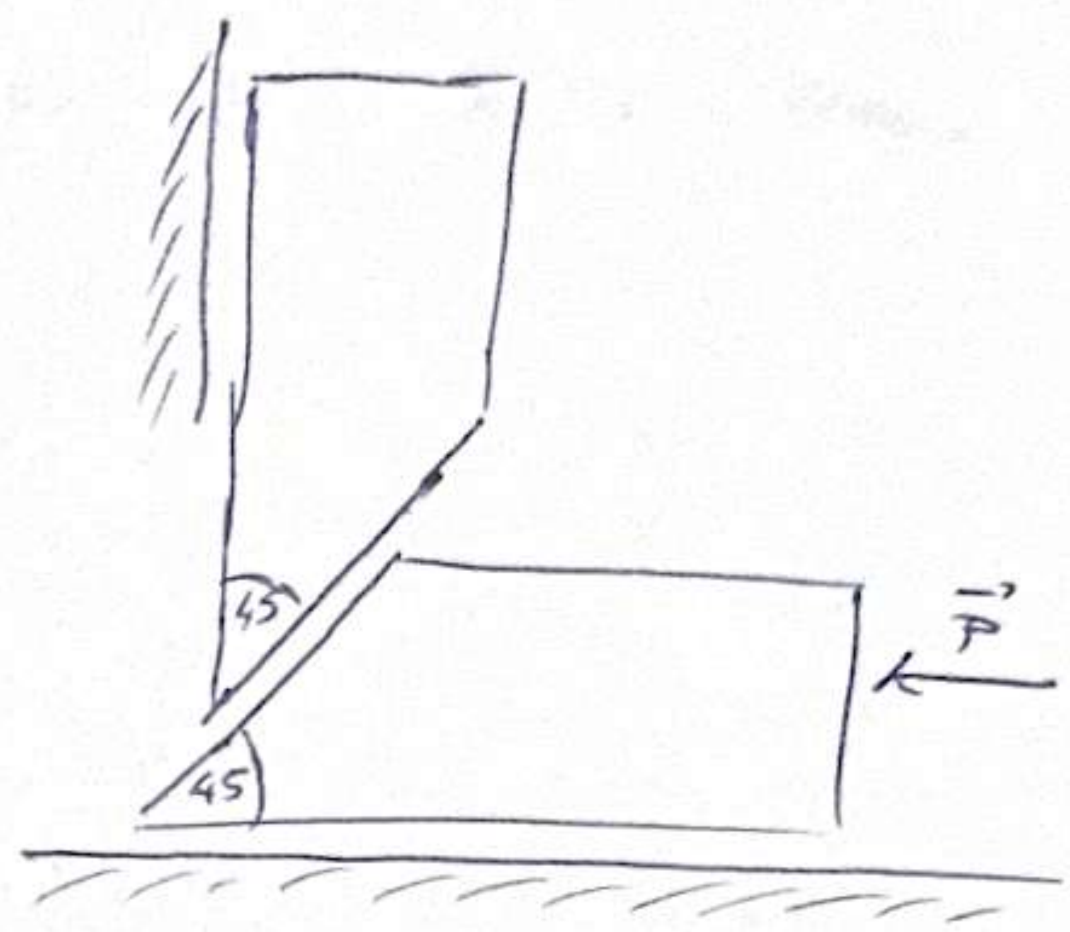
$S = \frac{1}{2}mg\sin\alpha - \cancel{\mu mg\cos\alpha} + \cancel{\mu mg\cos\alpha}$

$$\boxed{S = \frac{1}{2}mg\sin\alpha}$$

(10.39.)

A: m, 45°

B: m, 45°



1) na A deluje sila $\vec{P}$

to: sistem miruje

$\mu = \frac{1}{2}$

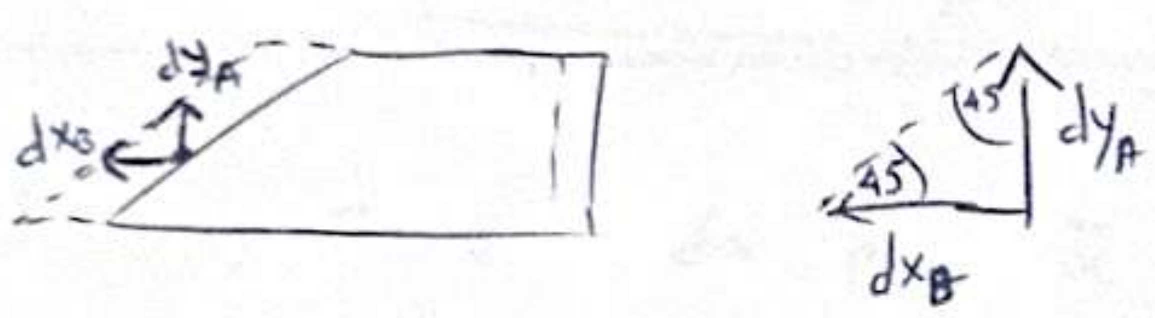
$P_{min} = ?$ – da bi se sistem pokrenuo u smeru dejstva sile P

2) ako je $P = 2 \cdot P_{min}$

$\mu_2 = \frac{3}{4}\mu$

$a_B = ?$

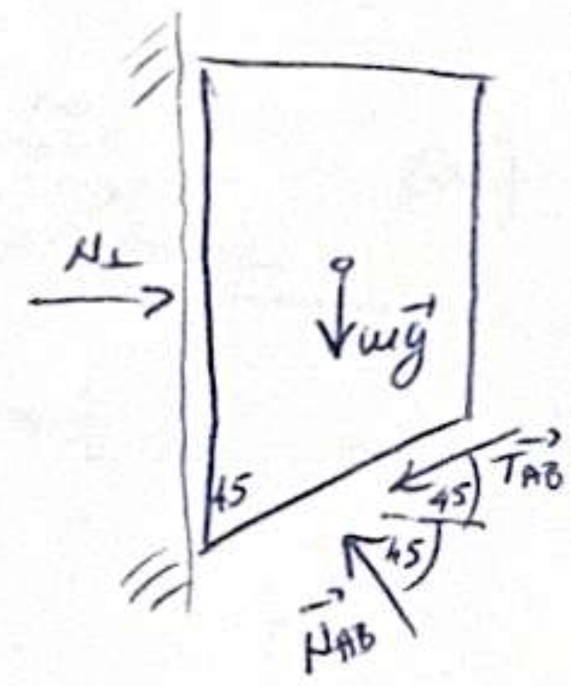
Kinematska veza:



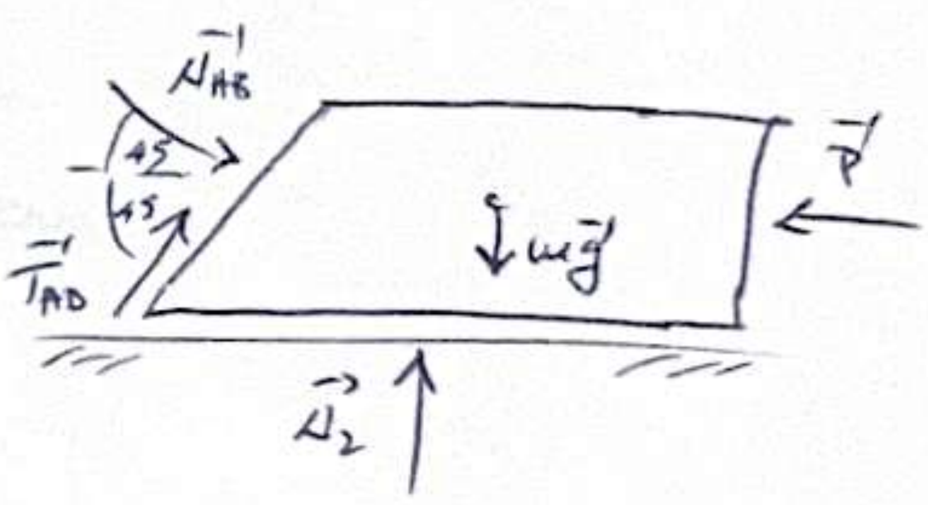
$dx_B = dy_A \quad / \frac{1}{dt} \cdot / \frac{d}{dt}$

$\ddot{x}_B = \ddot{y}_A = \ddot{x}$

telo A



telo B



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1) uslov: $\ddot{x} = 0$, $T_{AB} = T_{AB,max}$ (sistem je na granici da se pokrene), $T_{AB} = \mu \cdot N_{AB}$

Neću pisati $\ddot{x} = 0$ i $\mu = \frac{1}{2}$ sve do pred kraj, jer želim da dobijem jednačine koje važe i za 2)

$$m_A\vec{a}_A = m_A\vec{g} + \vec{N}_1 + \vec{N}_{AB} + \vec{T}_{AB}$$

x: $0 = N_1 - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$0 = N_1 - N_{AB}\frac{\sqrt{2}}{2} - \mu \cdot N_{AB} \cdot \frac{\sqrt{2}}{2}$

$0 = N_1 - \frac{\sqrt{2}}{2}N_{AB}(1+\mu) \quad (1)$

y: $m\ddot{y}_A = -mg + N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = -mg + N_{AB}\frac{\sqrt{2}}{2} - \mu \cdot N_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = -mg + \frac{\sqrt{2}}{2}N_{AB}(1-\mu) \quad (2)$

$$m_B\vec{a}_B = m_B\vec{g} + \vec{P} + \vec{N}_2 + \vec{N}_{AB} + \vec{T}_{AB}$$

y: $0 = -mg + N_2 - N_{AB}\frac{\sqrt{2}}{2} + T_{AB} \cdot \frac{\sqrt{2}}{2}$

$0 = -mg + N_2 - N_{AB}\frac{\sqrt{2}}{2} + \mu N_{AB}\frac{\sqrt{2}}{2}$

$0 = -mg + N_2 - \frac{\sqrt{2}}{2}N_{AB}(1-\mu) \quad (3)$

x: $m\ddot{x}_A = P - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = P - N_{AB}\frac{\sqrt{2}}{2} - \mu N_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = P - \frac{\sqrt{2}}{2}N_{AB}(1+\mu) \quad (4)$

I za a) i za б) su dovoljne j-ne (2) i (4), jer su to 2 j-ne i biće dve nepoznate: pod a) su to P i N_{AB}, jer je $\ddot{x}=0$ tad, a pod б) su to $\ddot{x}$ i N_{AB}.

$$(2) \Rightarrow \quad \frac{\sqrt{2}}{2}(1-\mu) N_{AB} = m\ddot{x} + mg$$

$$\underline{\underline{N_{AB} = \frac{(m\ddot{x}+mg)\sqrt{2}}{1-\mu}}}$$

$$(4) \Rightarrow \quad m\ddot{x} = P - \frac{\sqrt{2}}{2} N_{AB}(1+\mu)$$

$$m\ddot{x} = P - \frac{\sqrt{2}}{2}\,\frac{(m\ddot{x}+mg)\sqrt{2}}{(1-\mu)}\cdot(1+\mu)$$

$$m\ddot{x} = P - m\ddot{x}\cdot\frac{1+\mu}{1-\mu} - mg\cdot\frac{1+\mu}{1-\mu}$$

$$\underline{\underline{m\ddot{x}\left(1+\frac{1+\mu}{1-\mu}\right) = P - mg\cdot\frac{1+\mu}{1-\mu}}} \quad (*)$$

a) pustamo da je $\ddot{x}=0$, $\mu=\frac{1}{2}$, pa dobijamo:

$$(*) \Rightarrow \quad 0 = P_{min} - mg\cdot\frac{1+\frac{1}{2}}{1-\frac{1}{2}}$$

$$\boxed{P_{min} = 3mg}$$

б) pustamo da je $P = 2P_{min} = 6mg$, $\mu_1 = \frac{3}{4}\mu = \frac{3}{8}$, pa dobijamo:

$$(*) \Rightarrow \quad m\ddot{x}\left(1+\frac{1+\frac{3}{8}}{1-\frac{3}{8}}\right) = 6mg - mg\,\frac{1+\frac{3}{8}}{1-\frac{3}{8}}$$

$$m\ddot{x}\left(1+\frac{\frac{11}{8}}{\frac{5}{8}}\right) = 6mg - mg\,\frac{\frac{11}{8}}{\frac{5}{8}}$$

$$\frac{16}{5}m\ddot{x} = \frac{19}{5}mg$$

$$\boxed{\ddot{x} = \frac{19}{16}g}$$

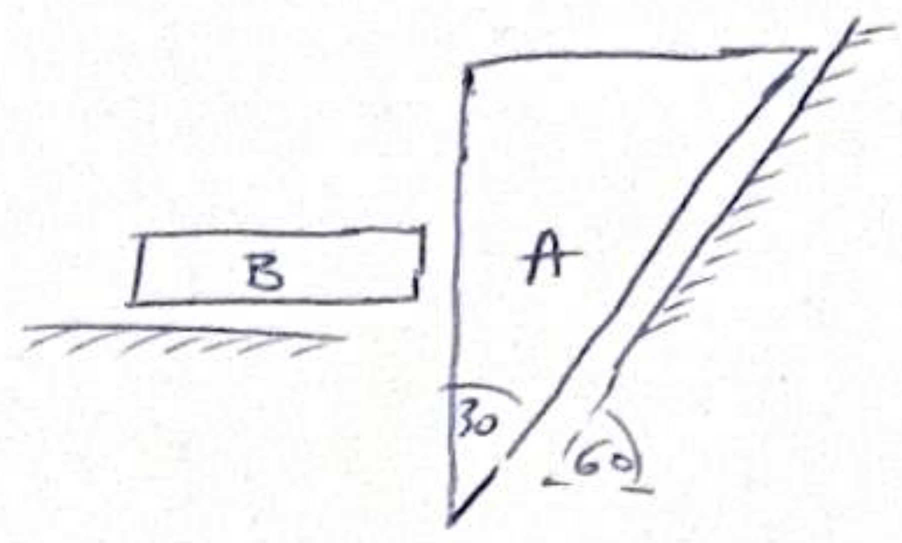
10.40

A: m

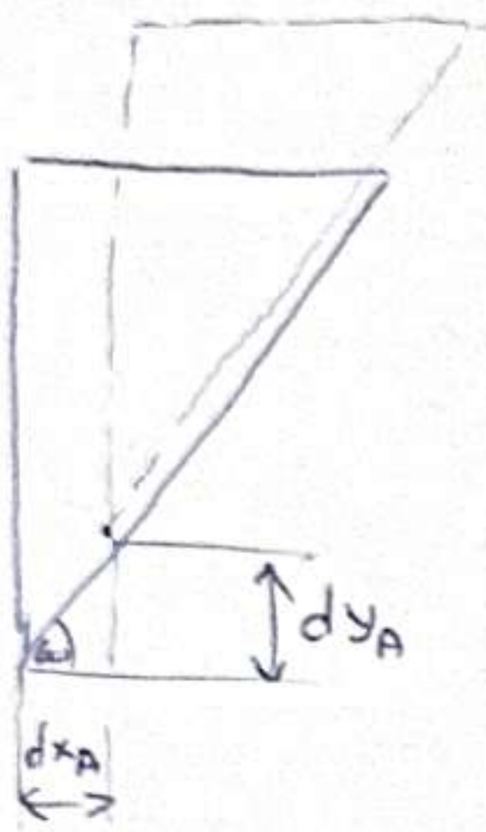
B: $2m$

$a_B = ?$

$N_B = ?$



kinematska veza:



$$\operatorname{tg} 60 = \frac{dy_A}{dx_A}$$

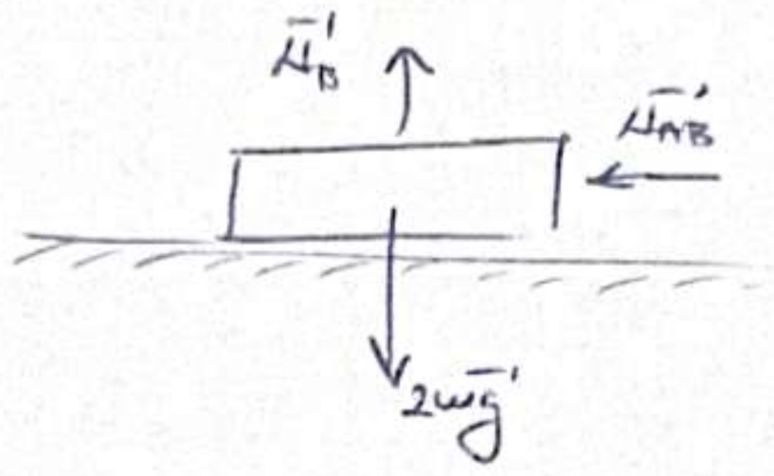
$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{dy_A}{dx_A}$$

$$dy_A = \sqrt{3}\, dx_A \quad / \frac{d}{dt} \quad / \frac{d}{dt}$$

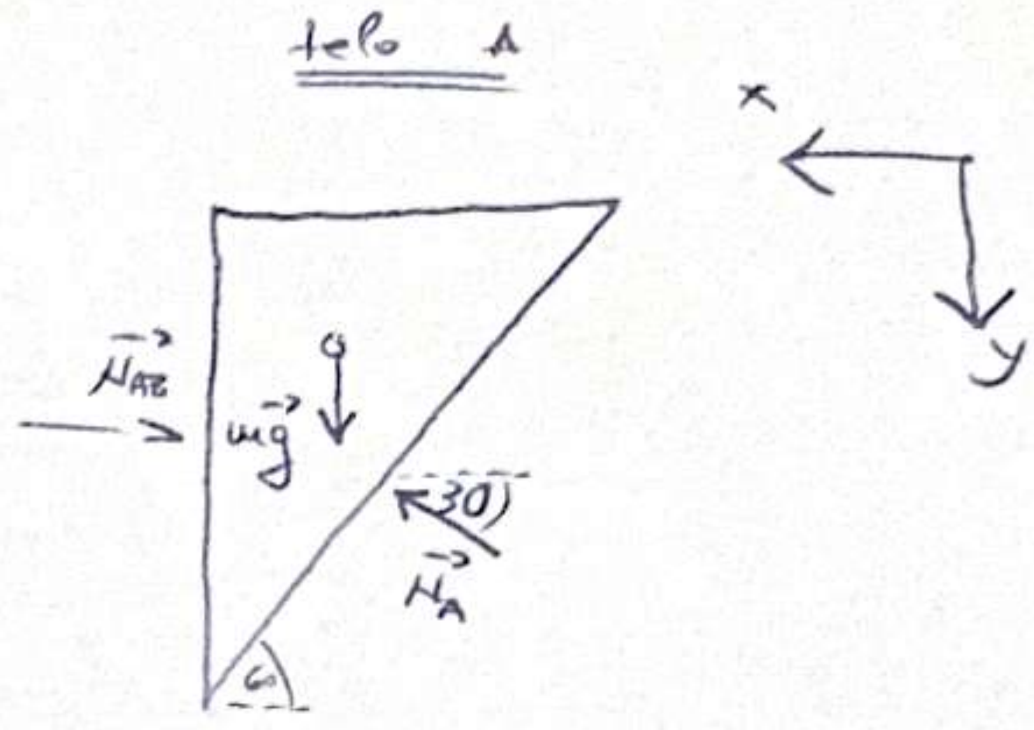
$$\underline{\underline{\ddot{y}_A = \sqrt{3}\, \ddot{x}_A = \sqrt{3}\, \ddot{x}}}$$

takođe važi i: $\underline{\underline{\ddot{x}_B = \ddot{x}_A = \ddot{x}}}$

telo B



telo A



$$m\vec{a}_A = m\vec{g} + \vec{N}_A + \vec{N}_{AB}$$

x: $m\ddot{x}_A = N_A \cos 30 - N_{AB}$

$$\underline{\underline{m\ddot{x} = \frac{\sqrt{3}}{2} N_A - N_{AB}}} \quad (1)$$

y: $m\ddot{y} = mg - N_A \sin 30$

$$\underline{\underline{\sqrt{3}\, m\ddot{x} = mg - \frac{1}{2} N_A}} \quad (2)$$

$$2m\vec{a}_B = m\vec{g} + \vec{N}_B + \vec{N}_{AB}$$

x: $2m\ddot{x}_B = N_{AB}$

$$\underline{\underline{2m\ddot{x} = N_{AB}}} \quad (3)$$

y: $0 = mg - N_B$

$$\underline{\underline{N_B = mg}}$$

(3) ubacimo u (1)

$$m\ddot{x} = \frac{\sqrt{3}}{2} N_A - 2m\ddot{x}$$

$$3m\ddot{x} = \frac{\sqrt{3}}{2} N_A$$

$$\underline{\underline{m\ddot{x} = \frac{\sqrt{3}}{6} N_A}} \quad \text{– ubacimo u (2)}$$

(2) $\Rightarrow$ $\sqrt{3}\,\frac{\sqrt{3}}{6} N_A = mg - \frac{1}{2} N_A$

$$\frac{1}{2} N_A = mg - \frac{1}{2} N_A$$

$$\underline{N_A = mg}$$

$\Rightarrow$ $m\ddot{x} = \frac{\sqrt{3}}{6} mg$

$$\boxed{\ddot{x} = \frac{\sqrt{3}}{6} g}$$

(1) $\Rightarrow$ $N_{AB} = 2m \cdot \frac{\sqrt{3}}{6} g$

$$\boxed{N_{AB} = \frac{\sqrt{3}}{3} mg}$$

(10.41)

A: G

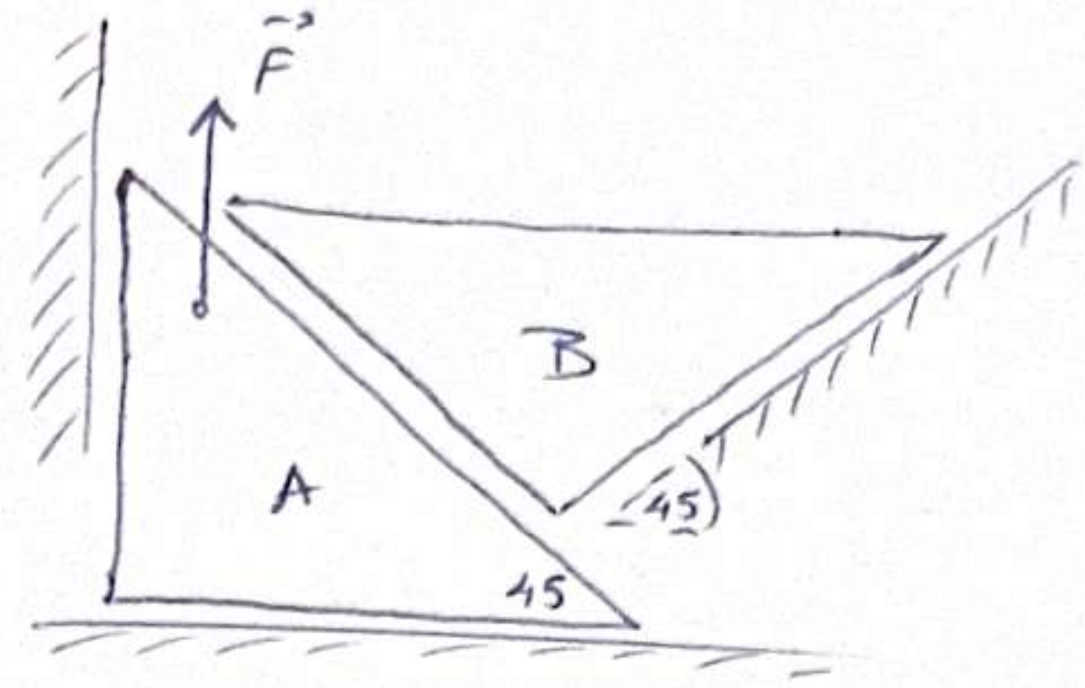
B: 2G

t_0: položaj na slici

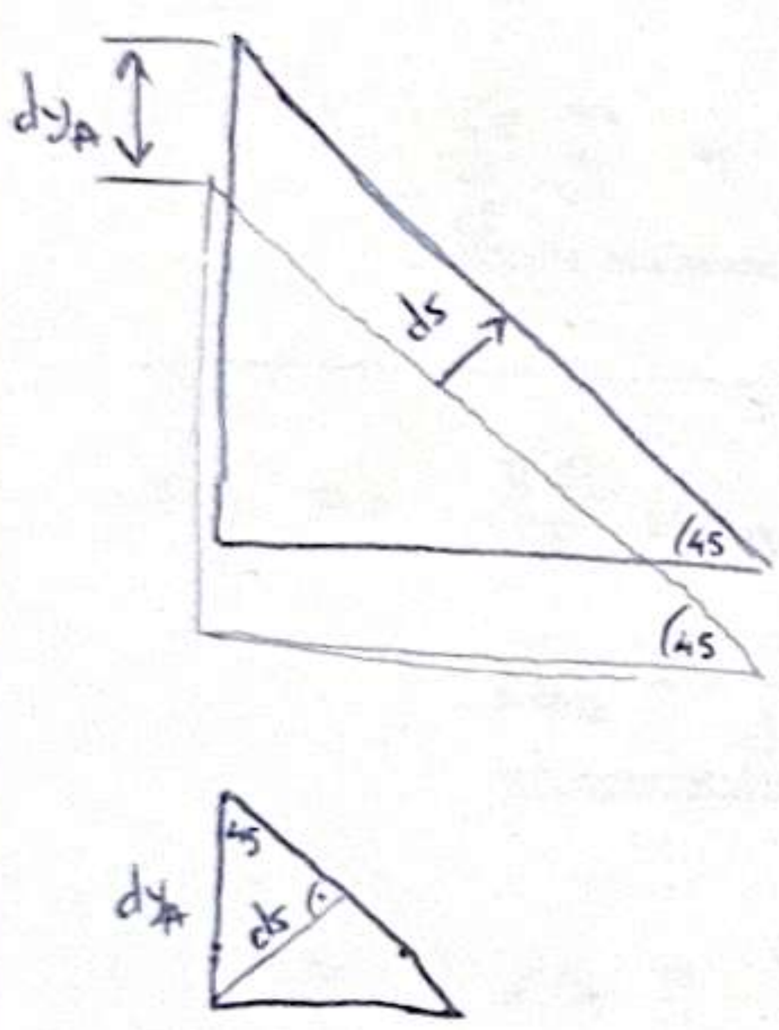
$F = 5G$

$\mu = 0{,}5$ - koef. trenja između prizmi

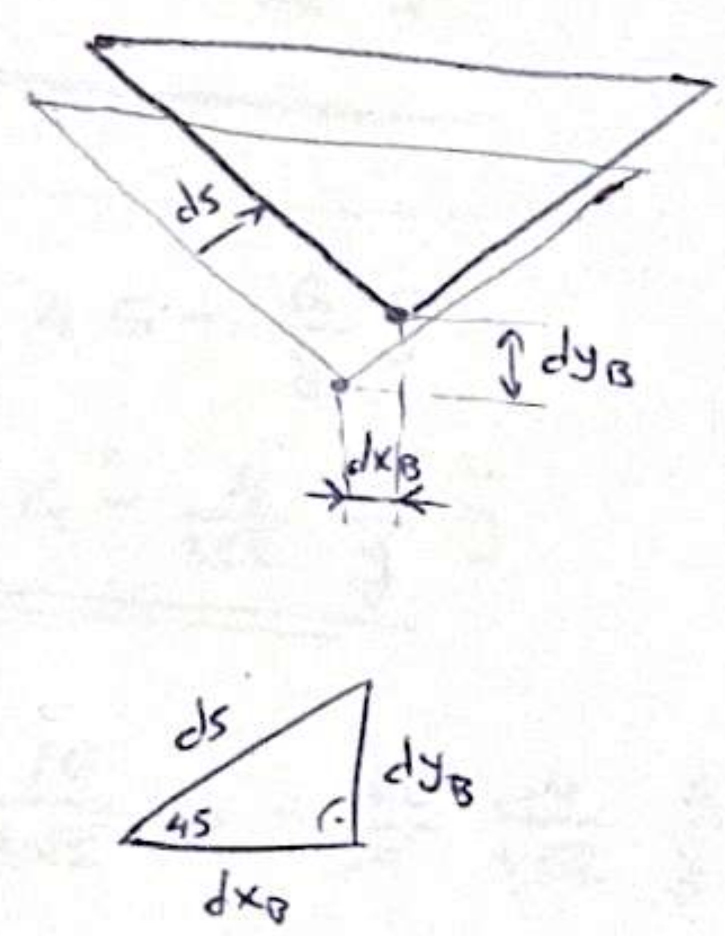
$a_A = ?$



geometrijska veza

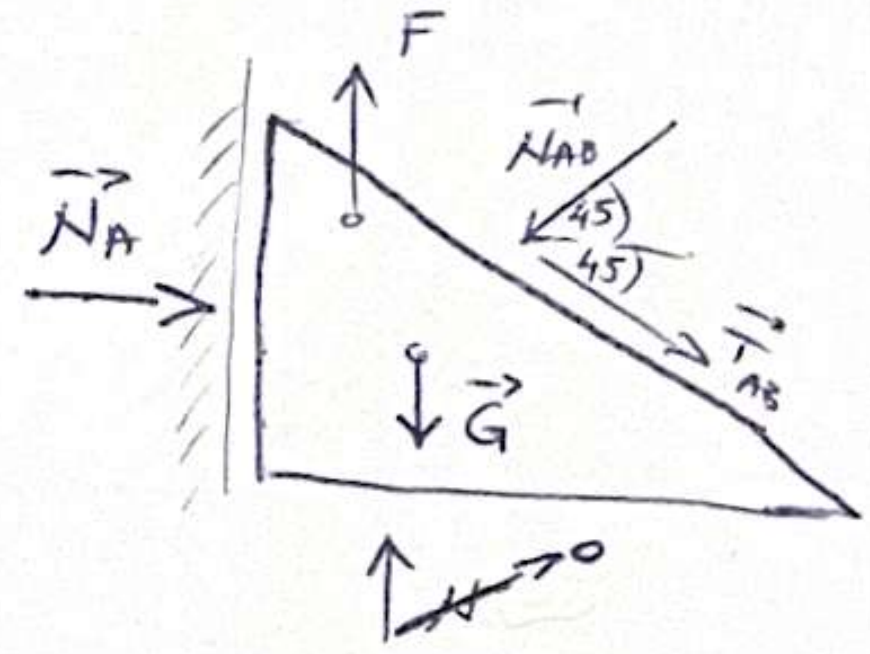


slika 1

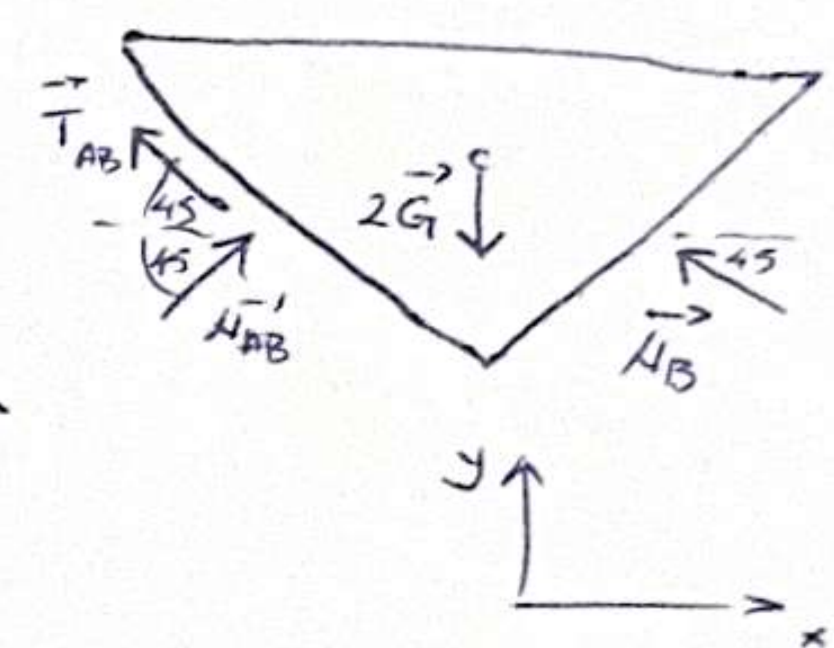


slika 2

telo A



telo B



Pošto ćemo sve izraziti preko y_A, reći ćemo da je: $\underline{\ddot{y}_A = \ddot{y}}$

Iz slike 1 zaključujemo:

$d_s = dy_A \cos 45 = \frac{\sqrt{2}}{2} dy_A$

Iz slike 2 zaključujemo:

$dy_B = ds \cdot \cos 45 = \frac{\sqrt{2}}{2} dy_A \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} dy_A$

$dx_B = ds \cdot \cos 45 = \frac{\sqrt{2}}{2} dy_A \frac{\sqrt{2}}{2} = \frac{1}{2} dy_A$

Kada prethodna dva izraza pomnožimo sa $\frac{1}{dt} \cdot \frac{d}{dt}$, dobijamo:

$$\underline{\ddot{y}_B = \frac{1}{2}\ddot{y}_A = \frac{1}{2}\ddot{y}}$$

$$\underline{\ddot{x}_B = \frac{1}{2}\ddot{y}_A = \frac{1}{2}\ddot{y}}$$

veza: $\underline{T_{AB} = \mu \cdot N_{AB} = \frac{1}{2} N_{AB}}$

pretpostavka: $\underline{N = 0}$ (ako se dobije $\ddot{y}_A < 0$, onda je $N \neq 0$, a sistem će mirovati)

$$\underline{\frac{2G}{g}\vec{a}_B = 2\vec{G} + \vec{N_B} + \vec{N_{AB}} + \vec{T_{AB}}}$$

x: $\frac{2G}{g}\ddot{x}_B = -2G + N_B\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{2} + T_{AB}\frac{\sqrt{2}}{2}$

$\frac{\cancel{2}G}{g}\frac{1}{\cancel{2}}\ddot{y} = -2G + N_B\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{4}$

$$\underline{\frac{G}{g}\ddot{y} = -2G + N_B\frac{\sqrt{2}}{2} + N_{AB}\cdot\frac{3\sqrt{2}}{4}} \quad (1)$$

y: $\frac{2G}{g}\ddot{y}_B = -N_B\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$\frac{2G}{g}\frac{1}{2}\ddot{y} = -N_B\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{2} - N_{AB}\frac{\sqrt{2}}{4}$

$$\underline{\frac{G}{g}\ddot{y} = -N_B\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{4}} \quad (2)$$

$$\frac{G}{g}\vec{a}_A = \vec{G} + \vec{F} + \vec{N}_A + \vec{N}_{AB} + \vec{T}_{AB}$$

$$x: \quad 0 = N_A - N_{AB}\frac{\sqrt{2}}{2} + T_{AB}\frac{\sqrt{2}}{2}$$

$$0 = N_A - N_{AB}\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{4}$$

$$0 = N_A - N_{AB}\frac{\sqrt{2}}{4} \qquad (3)$$

$$y: \quad \frac{G}{g}\ddot{y}_A = -G + F - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$$

$$\frac{G}{g}\ddot{y} = -G + 5G - N_{AB}\frac{\sqrt{2}}{2} - N_{AB}\frac{\sqrt{2}}{4}$$

$$\frac{G}{g}\ddot{y} = 4G - N_{AB}\frac{3\sqrt{2}}{4} \qquad (4)$$

$$(4) \Rightarrow \frac{3\sqrt{2}}{4}N_{AB} = 4G - \frac{G}{g}\ddot{y}$$

$$N_{AB} = \frac{16}{3\sqrt{2}}G - \frac{4}{3\sqrt{2}}\frac{G}{g}\ddot{y}$$ – ubacujemo u (3)

$$(3) \Rightarrow N_A = \frac{\cancel{\sqrt{2}}}{4}\frac{16}{3\cancel{\sqrt{2}}}G - \frac{\cancel{\sqrt{2}}}{\cancel{4}}\frac{\cancel{4}}{3\cancel{\sqrt{2}}}\frac{G}{g}\ddot{y}$$

$$N_A = \frac{4}{3}G - \frac{1}{3}\frac{G}{g}\ddot{y}$$ – ubacujemo u (2)

$$(2) \Rightarrow \frac{G}{g}\ddot{y} = -N_B\frac{\sqrt{2}}{2} + \frac{4G}{3} - \frac{1}{3}\frac{G}{g}\ddot{y}$$

$$N_B\frac{\sqrt{2}}{2} = \frac{4}{3}G - \frac{4}{3}\frac{G}{g}\ddot{y} \qquad /\cdot\sqrt{2}$$

$$N_B = \frac{4\sqrt{2}}{3}G - \frac{4\sqrt{2}}{3}\frac{G}{g}\ddot{y}$$

$$(1) \Rightarrow \frac{G}{g}\ddot{y} = -2G + \frac{\cancel{\sqrt{2}}}{\cancel{2}}\frac{4\cancel{\sqrt{2}}}{3} - \frac{\cancel{\sqrt{2}}}{\cancel{2}}\frac{4\cancel{\sqrt{2}}}{3}\frac{G}{g}\ddot{y} + 4G - \frac{G}{g}\ddot{y}$$

$$\frac{G}{g}\ddot{y}\left(1 + \frac{4}{3} + 1\right) = G\left(-2 + \frac{4}{3} + 4\right)$$

$$\cancel{\frac{10}{3}}\frac{G}{g}\ddot{y} = \cancel{\frac{10}{3}}\cancel{G}$$

$$\boxed{\ddot{y} = g}$$

(10.42.)

1: m, r

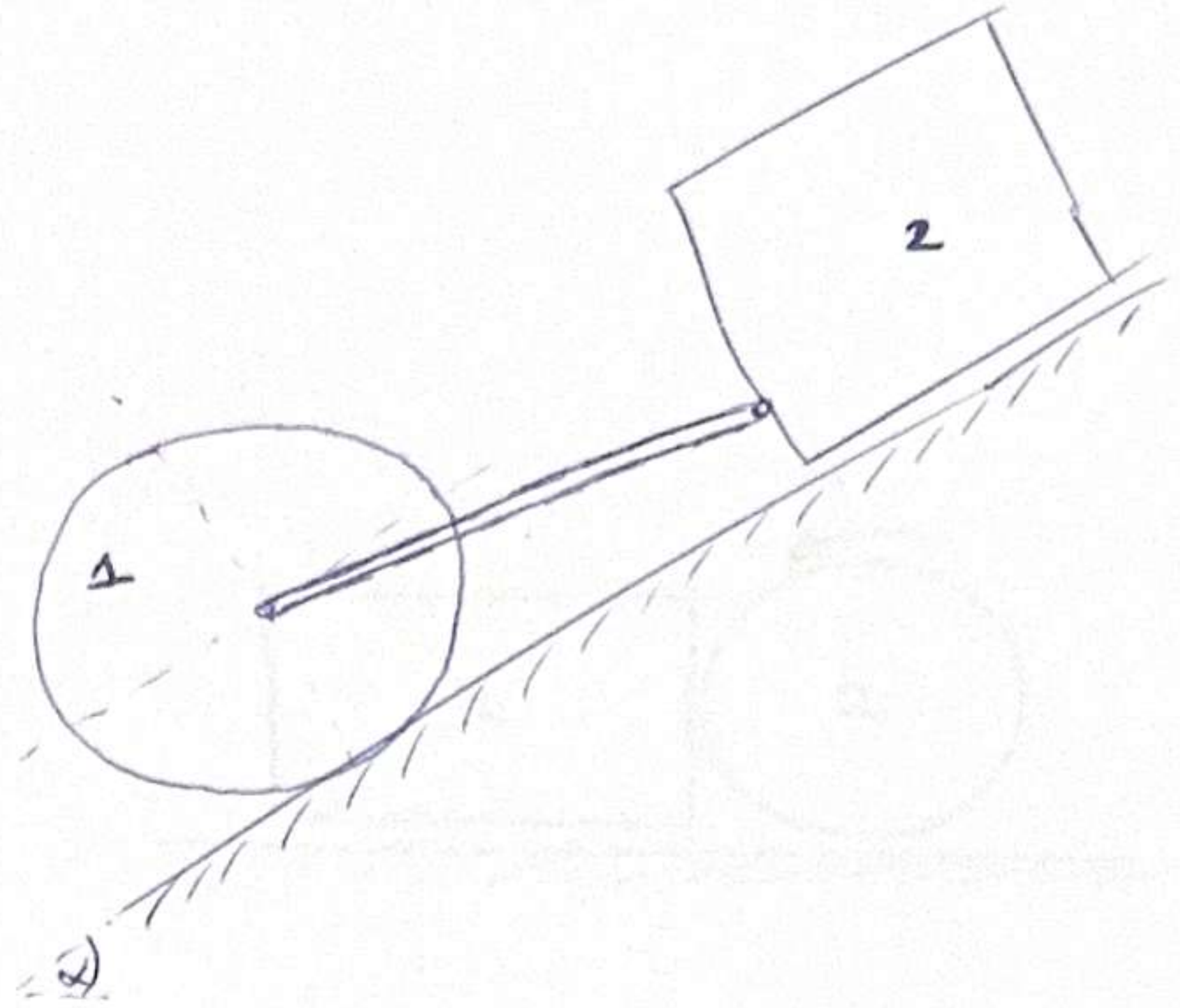
2: m

k - krak otpora kotrljanja

kotrljanje bez klizanja

μ - koef. trenja klizanja između tela 2 i tla

$\alpha = ?$ - da bi sila u štapu bila jednaka nuli



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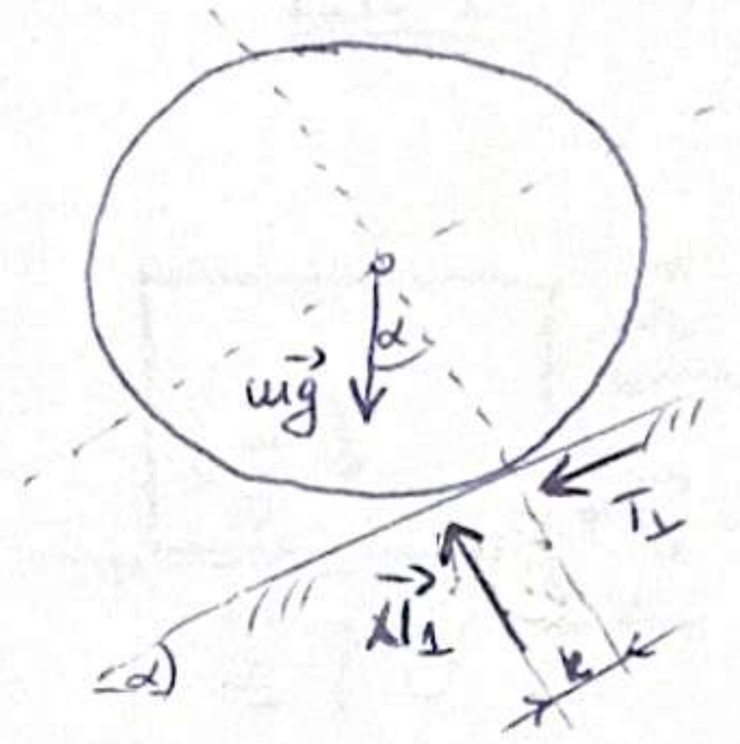
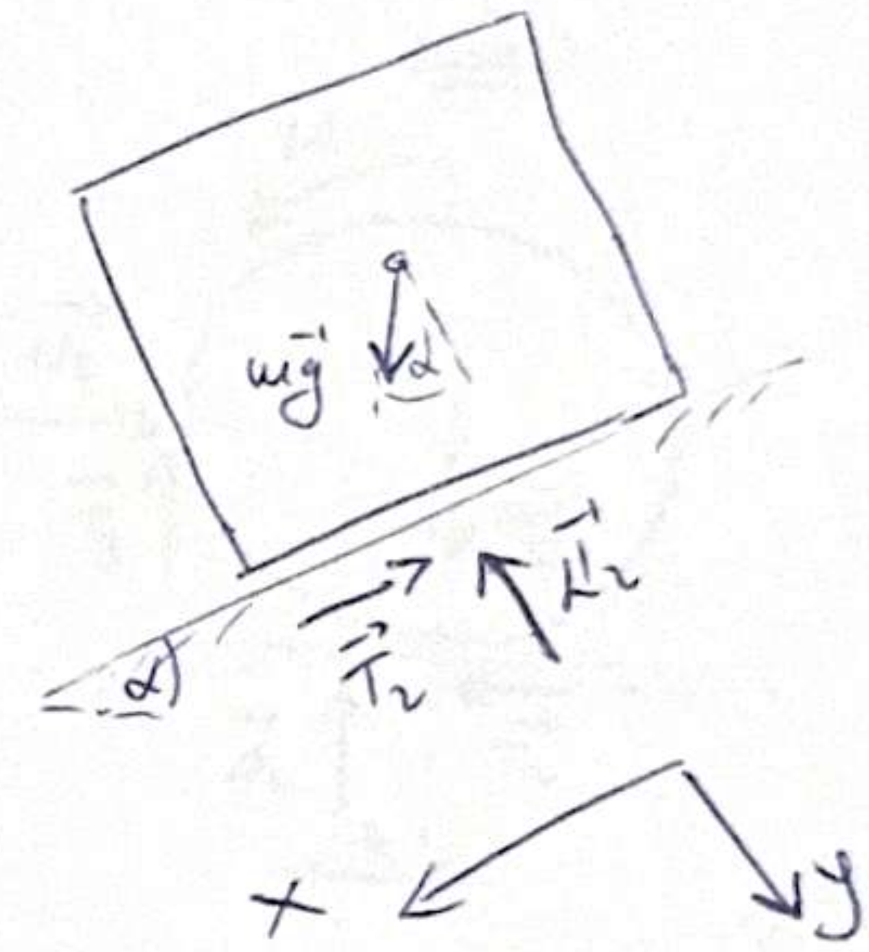
telo 1



telo 2



Kinematska veza: $\underline{\underline{\ddot{x}_1 = \ddot{x}_2 = \ddot{x}}}$

$$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{T}_1$$

y: $0 = mg\cos\alpha - N_1$

$$N_1 = mg\cos\alpha$$

x: $m\ddot{x} = mg\sin\alpha + T_1$ (1) $\begin{pmatrix} T_1 = ? \\ \ddot{x} = ? \\ \alpha = ? \end{pmatrix}$

$$\frac{d\overset{\curvearrowleft}{L}_{C_1}}{dt} = \sum \overset{\curvearrowleft}{M}_{C_1}$$

$$\sum M_{C_1} = -N_1 \cdot k - T_1 \cdot r = -mg\cos\alpha \cdot k - T_1 \cdot r$$

$$L_{C_1} = J_{C_1} \cdot \omega = \frac{1}{2} m r^2 \frac{\dot{x}}{r}$$

$$\frac{dL_{C_1}}{dt} = \frac{1}{2} m r \ddot{x}$$

$$\frac{1}{2} m r \ddot{x} = -mg\cos\alpha \cdot k - T_1 \cdot r \quad (2)$$

$$m\vec{a}_2 = m\vec{g} + \vec{N}_2 + \vec{T}_2$$

y: $0 = mg\cos\alpha - N_2$

$$N_2 = mg\cos\alpha$$

$$T_2 = \mu \cdot N_2 = \mu \cdot mg\cos\alpha$$

x: $m\ddot{x} = mg\sin\alpha - T_2$

$$m\ddot{x} = mg\sin\alpha - \mu mg\cos\alpha \quad (3)$$

(1) $\Rightarrow$ $T_1 = m\ddot{x} - mg\sin\alpha$

(2) $\Rightarrow$ $\frac{1}{2} m r \ddot{x} = -mg\cos\alpha \cdot k - m\ddot{x} \cdot r + mg\sin\alpha \cdot r$

$$\frac{3}{2} m r \ddot{x} = -mg\cos\alpha \cdot k + mg\sin\alpha \cdot r \quad (*)$$

(3) ubacimo u (*)

(*) $\Rightarrow$ $\frac{3}{2} r (mg\sin\alpha - \mu mg\cos\alpha) = -mg\cos\alpha \cdot k + mg\sin\alpha \cdot r$

$$\sin\alpha \left(\frac{3}{2} r \cdot mg - mg \cdot r\right) = \cos\alpha \left(-mg \cdot k + \frac{3}{2}\mu mg \cdot r\right)$$

$$\mathrm{tg}\,\alpha = \frac{\frac{3}{2}\mu mg \cdot r - mg \cdot k}{\frac{1}{2} mg\, r}$$

$$\boxed{\mathrm{tg}\,\alpha = 3\mu - 2\frac{k}{r}}$$

10.43.

1: m, klita bez trenja

2: m, r, kotrlja se bez klizanja

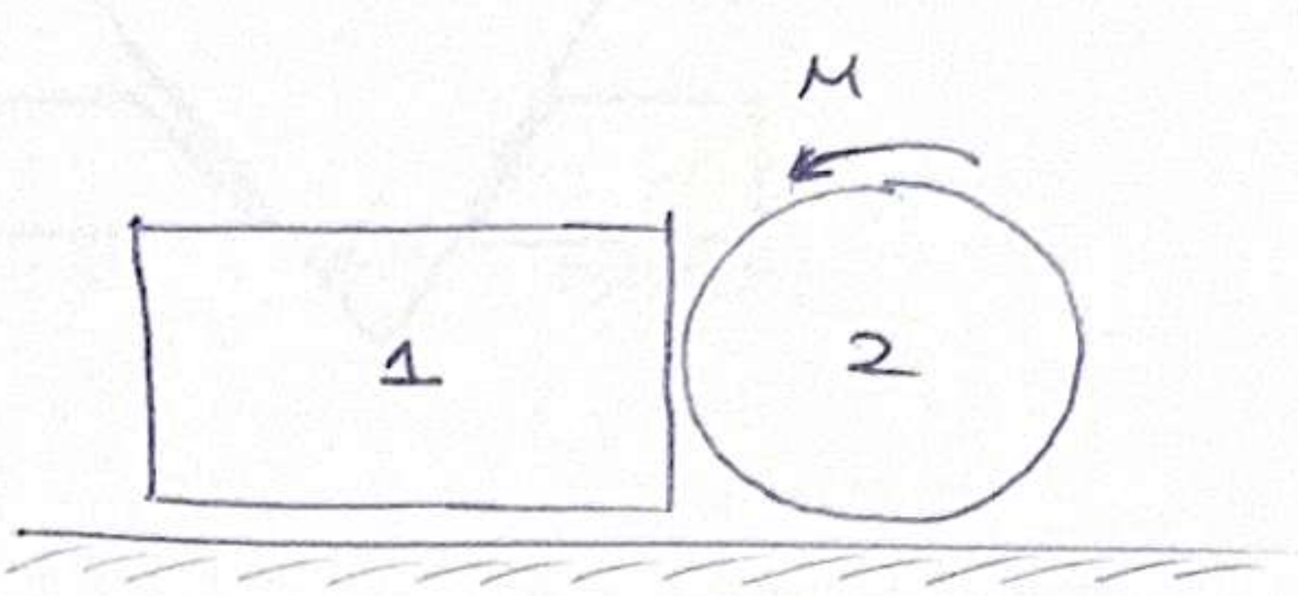
$\mu = \frac{1}{2}$ – koef. trenja između 1 i 2

$k = \frac{1}{10} r$ – krak otpora kotrljanju

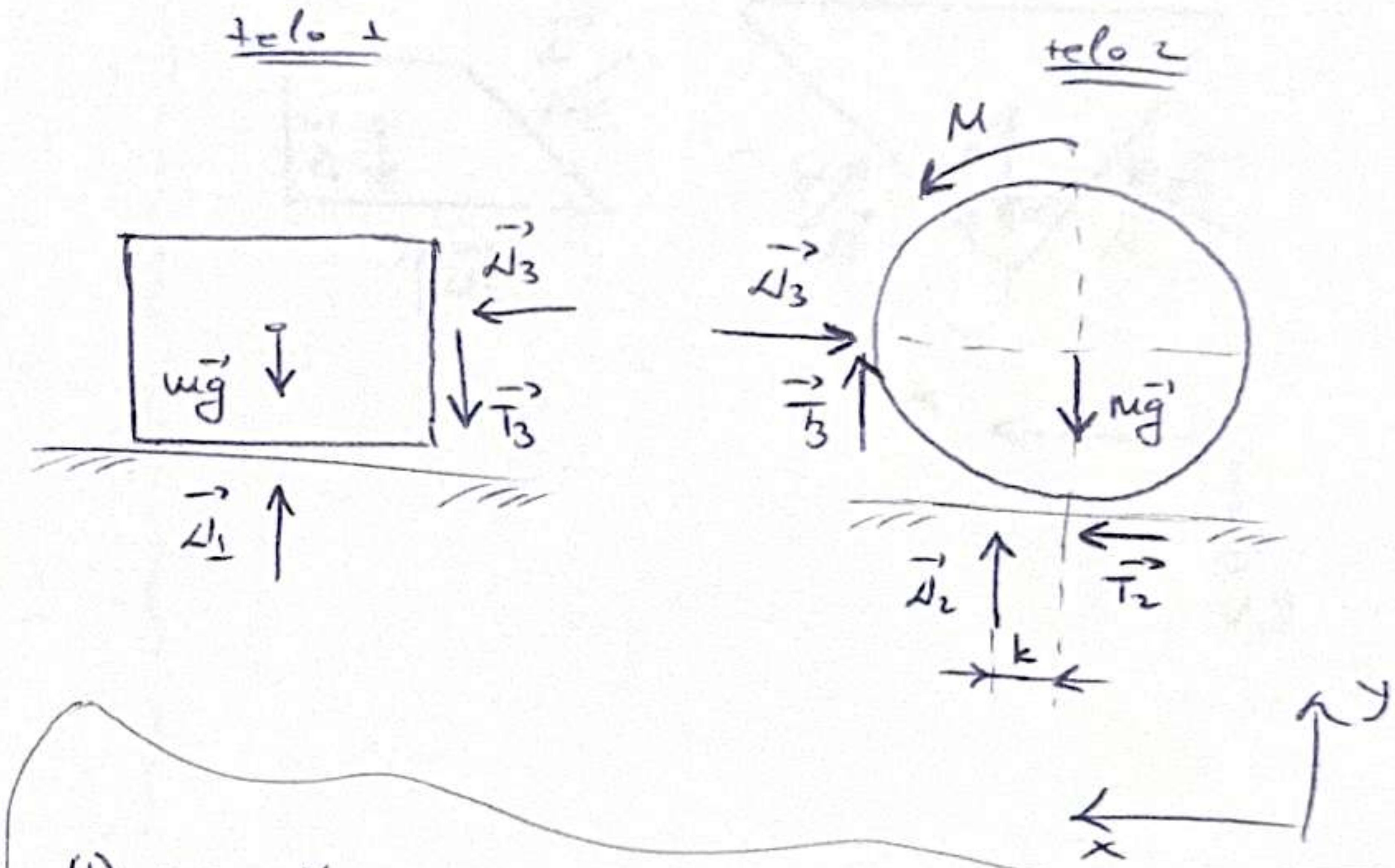
t_0: sistem miruje

M – moment

a_1? N_1? N_2? T_2?



kinematska veza: $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$



$T_2 = ?$, $T_3 = \mu \cdot N_3$

$m\vec{a}_2 = m\vec{g} + \vec{N}_2 + \vec{T}_2 + \vec{N}_3 + \vec{T}_3$

y: $0 = -mg + N_2 + \mu \cdot N_3$ (1) $\left(\begin{matrix} N_2 = ? \\ N_3 = ? \end{matrix}\right)$

x: $m\ddot{x} = T_2 - N_3$ (2) $\left(\begin{matrix} T_2 = ? \\ \ddot{x} = ? \end{matrix}\right)$

$$\frac{d\overleftarrow{L_{C}}}{dt} = \overleftarrow{\Sigma M_{C}}$$

$\Sigma M_{C} = M - T_3 \cdot r - N_2 \cdot k - T_2 \cdot r$

$\Sigma M_{C} = M - \mu \cdot N_3 \cdot r - \frac{1}{10} N_2 \cdot r - T_2 \cdot r$

$L_{C} = J_2 \omega = \frac{1}{2} m r^2 \cdot \frac{\dot{x}}{r}$

$\frac{dL_{C}}{dt} = \frac{1}{2} m r \ddot{x}$

$\frac{1}{2} m r \ddot{x} = M - \mu \cdot N_3 \cdot r - \frac{1}{10} N_2 \cdot r - T_2 \cdot r$ (3)

$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{N}_3 + \vec{T}_3$

y: $0 = -mg + N_1 - \mu \cdot N_3$ (4) $(N_1 = ?)$

x: $m\ddot{x} = N_3$ (5)

(1) $\Rightarrow$ $N_2 = mg - \mu N_3$

(4) $\Rightarrow$ $N_1 = mg + \mu \cdot N_3$

(5) $\Rightarrow$ $m\ddot{x} = N_3$

(2) $\Rightarrow$ $N_3 = T_2 - N_3$ $\Rightarrow$ $T_2 = 2 \cdot N_3$

(3) $\Rightarrow$ $\frac{1}{2} r \cdot N_3 = M - \mu \cdot r N_3 - \frac{1}{10} r (mg - \mu \cdot N_3) - 2 N_3 \cdot r$

$N_3 \left(\frac{1}{2} r + \mu r - \frac{1}{10} r \mu + 2r\right) = M - \frac{1}{10} mgr$

$N_3 \left(\frac{5}{2} r + \frac{1}{2} r - \frac{1}{20} r\right) = M - \frac{1}{10} mgr$

$N_3 = \dfrac{M - \frac{1}{10} mgr}{\frac{59}{20} r}$ $\Rightarrow$ $N_3 = \dfrac{2(10M - mgr)}{59 r}$

$T_2 = 2N_3$ $\Rightarrow$ $\boxed{T_2 = \dfrac{4(10M - mgr)}{59 r}}$

$N_1 = mg + \frac{1}{2} N_3 = \dfrac{59 mgr}{59 r} + \dfrac{10M - mgr}{59 r}$ $\Rightarrow$ $\boxed{N_1 = \dfrac{10M - 58mgr}{59r}}$

$N_2 = mg - \mu \cdot N_3 = \dfrac{59mgr}{59r} - \dfrac{10M - mgr}{59r}$ $\Rightarrow$ $\boxed{N_2 = \dfrac{60mgr - 10M}{59r}}$

$$\boxed{\ddot{x} = \frac{2(10M - mgr)}{59 mr}}$$

10.44.

klin: P, 2α

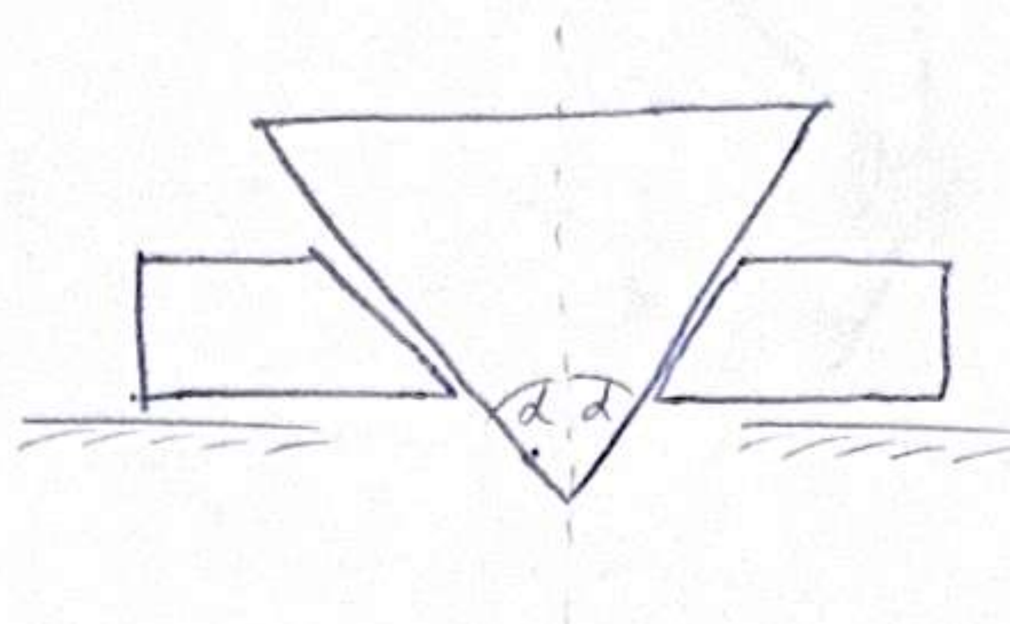
ploče: po P_1 svaka

t_0: mirovanje

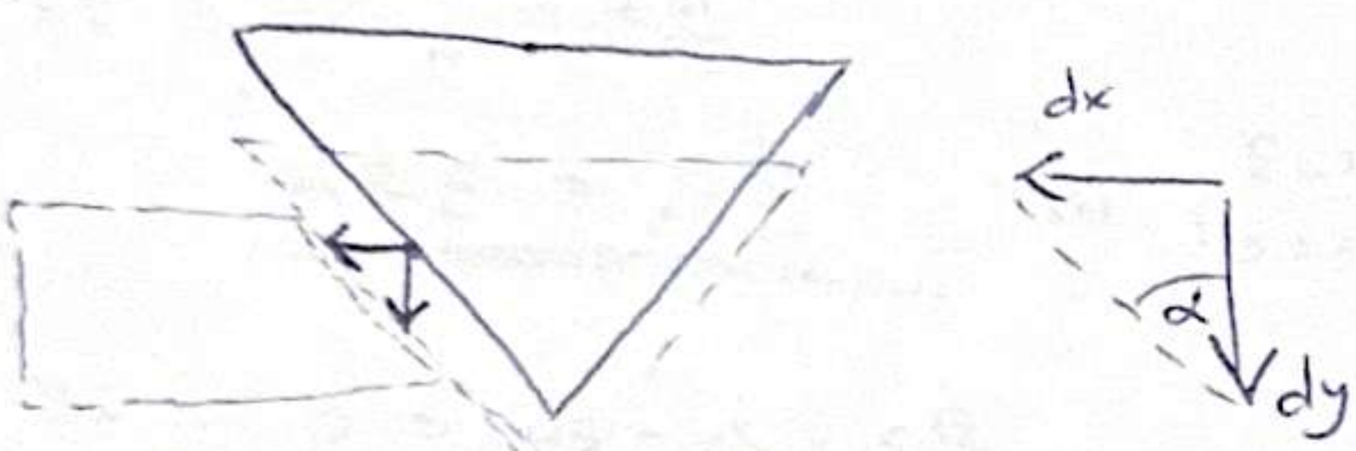
$x = ?$ – horiz. pomeranje ploča

$y = ?$ – vertikalno pomeranje ploča

$N = ?$ – pritisak klina na ploču



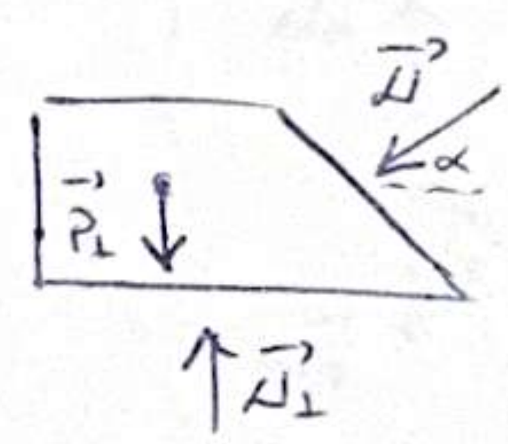
Kinematska veza:



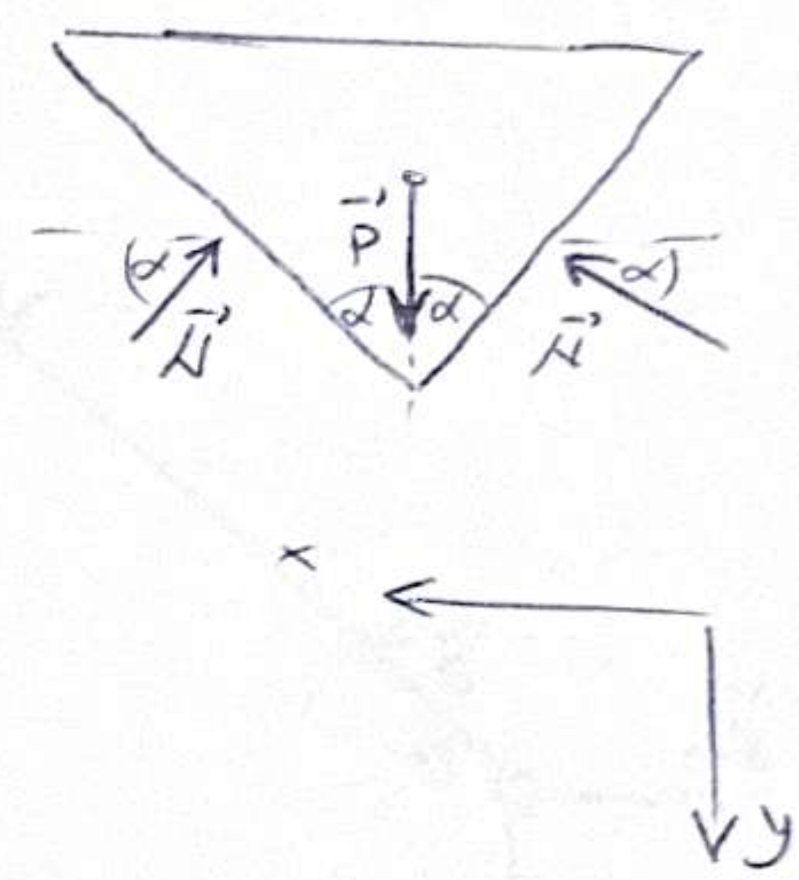
$$tg\alpha = \frac{dx}{dy} \Rightarrow dx = dy\, tg\alpha \quad / \cdot \frac{1}{dt} \quad / \frac{d}{dt}$$

$$\ddot{x} = \ddot{y}\, tg\alpha$$

telo 1



telo 2



$$m_2 \vec{a}_2 = \vec{P}' + \vec{N}' + \vec{N}'$$

y: $\frac{P}{g}\ddot{y} = P - 2N\sin\alpha \quad / \cdot \frac{g}{P}$

$$\ddot{y} = g - 2\frac{g}{P}N\sin\alpha \quad (1) \quad \begin{pmatrix} \ddot{y} = ? \\ N = ? \end{pmatrix}$$

x: —— (nebitno)

$$m_1 \vec{a}_1 = \vec{P}_1 + \vec{N}_1 + \vec{N}$$

y: $0 = P_1 - N_1 - N\sin\alpha$

$$N_1 = P_1 - N\sin\alpha \quad (2) \quad (N_1 = ?)$$

x: $\frac{P_1}{g}\ddot{x} = N\cos\alpha$

$$\frac{P_1}{g}\ddot{y}\, tg\alpha = N\cos\alpha \quad (3)$$

---

(1) ubacujemo u (3)

$$\frac{P_1}{g}\left(g - \frac{2g}{P}N\sin\alpha\right) tg\alpha = N\cos\alpha$$

$$P_1 tg\alpha - \frac{2P_1}{P}N\sin\alpha\, tg\alpha = N\cos\alpha$$

$$N\left(\cos\alpha + \frac{2P_1}{P}\sin\alpha\, tg\alpha\right) = P_1 tg\alpha$$

$$N = \frac{P_1 tg\alpha}{\cos\alpha + \frac{2P_1}{P}\sin\alpha\, tg\alpha} = \frac{P \cdot P_1 tg\alpha}{P\cos\alpha + 2P_1\sin\alpha\, tg\alpha}$$

$$\boxed{N = \frac{P \cdot P_1 tg\alpha}{\cos\alpha(P + 2P_1 tg^2\alpha)}}$$

vraćamo se u (3)

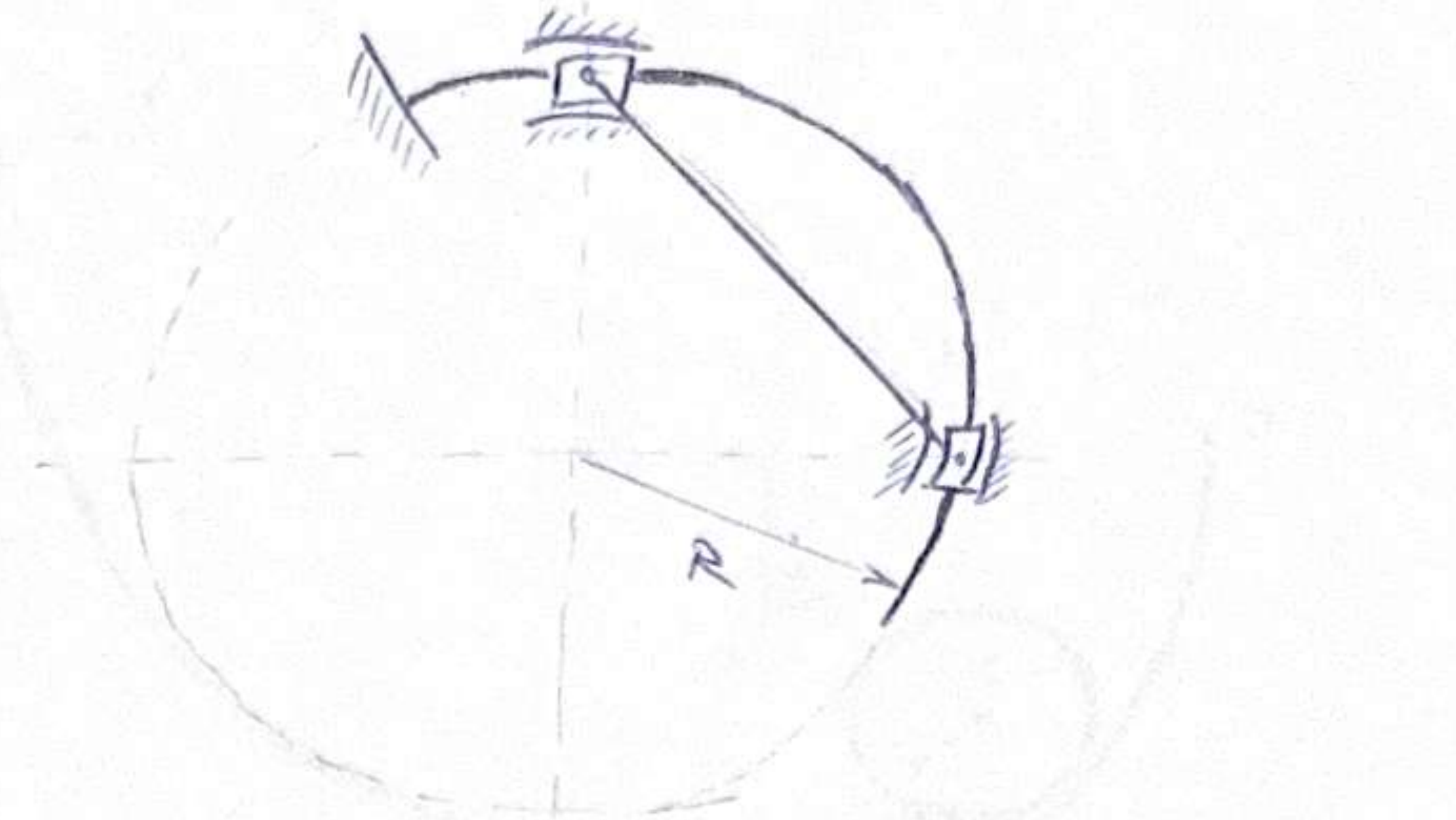
$$\frac{\cancel{P_1}}{g}\ddot{x} = \frac{P\,\cancel{P_1}\, tg\alpha}{\cancel{\cos\alpha}(P + 2P_1 tg^2\alpha)}\cancel{\cos\alpha} \quad / \cdot g \quad / \iint dt\,dt$$

$$\boxed{x = \frac{P g\, tg\alpha}{P + 2P_1 tg^2\alpha}\,\frac{t^2}{2}}$$

$$\frac{\cancel{P_1}}{g}\ddot{y}\,\cancel{tg\alpha} = \frac{P \cdot \cancel{P_1}\,\cancel{tg\alpha}}{\cancel{\cos\alpha}(P + 2P_1 tg^2\alpha)}\cancel{\cos\alpha} \quad / \cdot g \quad / \iint dt\,dt$$

$$\boxed{y = \frac{P g}{P + 2P_1 tg^2\alpha}\,\frac{t^2}{2}}$$

10.45



A: m

B: m

štap: $\varnothing$

t_0: položaj na slici
mirovanje

$N_A = N_A(t_0) = ?$

$N_B = N_B(t_0) = ?$

Posmatramo trenutak t_0:

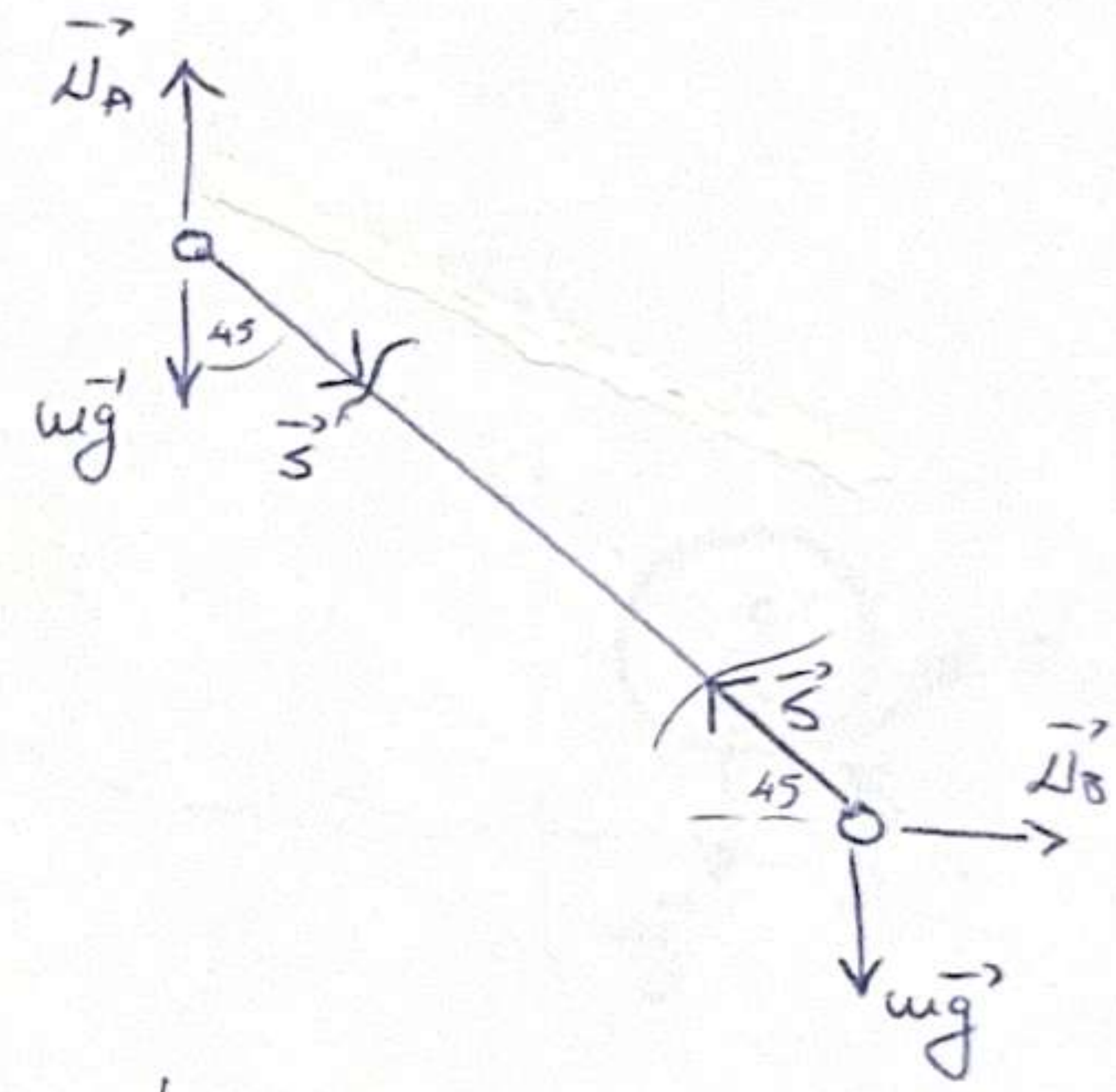
$$m\vec{a}_A = m\vec{g} + \vec{N}_A + \vec{S}$$

x: $m\ddot{x}_A = S\frac{\sqrt{2}}{2}$

$$mR\varepsilon = S\frac{\sqrt{2}}{2} \quad (1) \quad (\varepsilon = ?,\ S = ?)$$

y: $0 = mg - N_A + S\frac{\sqrt{2}}{2}$

$$N_A = mg + S\frac{\sqrt{2}}{2} \quad (2) \quad (N_A = ?)$$



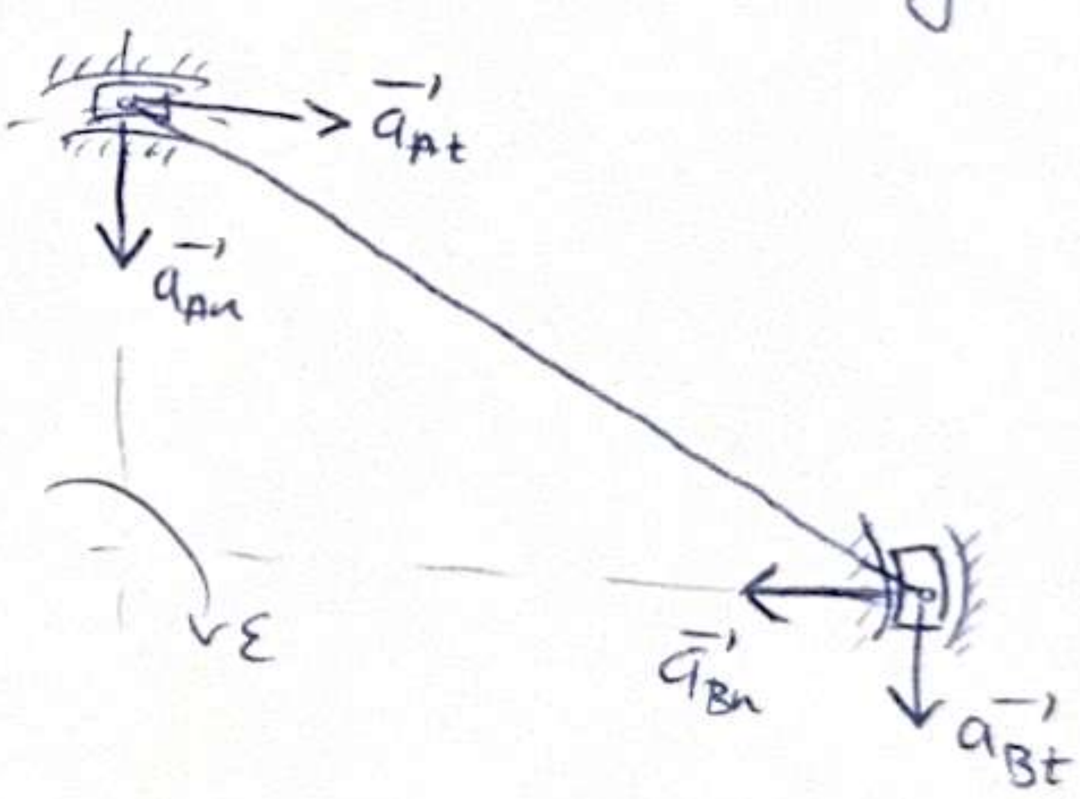
$$m\vec{a}_B = m\vec{g} + \vec{N}_B + \vec{S}$$

x: $0 = N_B - S\frac{\sqrt{2}}{2}$

$$N_B = S\frac{\sqrt{2}}{2} \quad (3) \quad (N_B = ?)$$

y: $m\ddot{y}_B = mg - S\frac{\sqrt{2}}{2}$

$$mR\varepsilon = mg - S\frac{\sqrt{2}}{2} \quad (4)$$



$\ddot{x}_A = a_{At} = \varepsilon R$

$\ddot{y}_A = a_{An} = \frac{v_A^2}{R} = 0$

$\ddot{x}_B = -a_{Bn} = -\frac{v_B^2}{R} = 0$

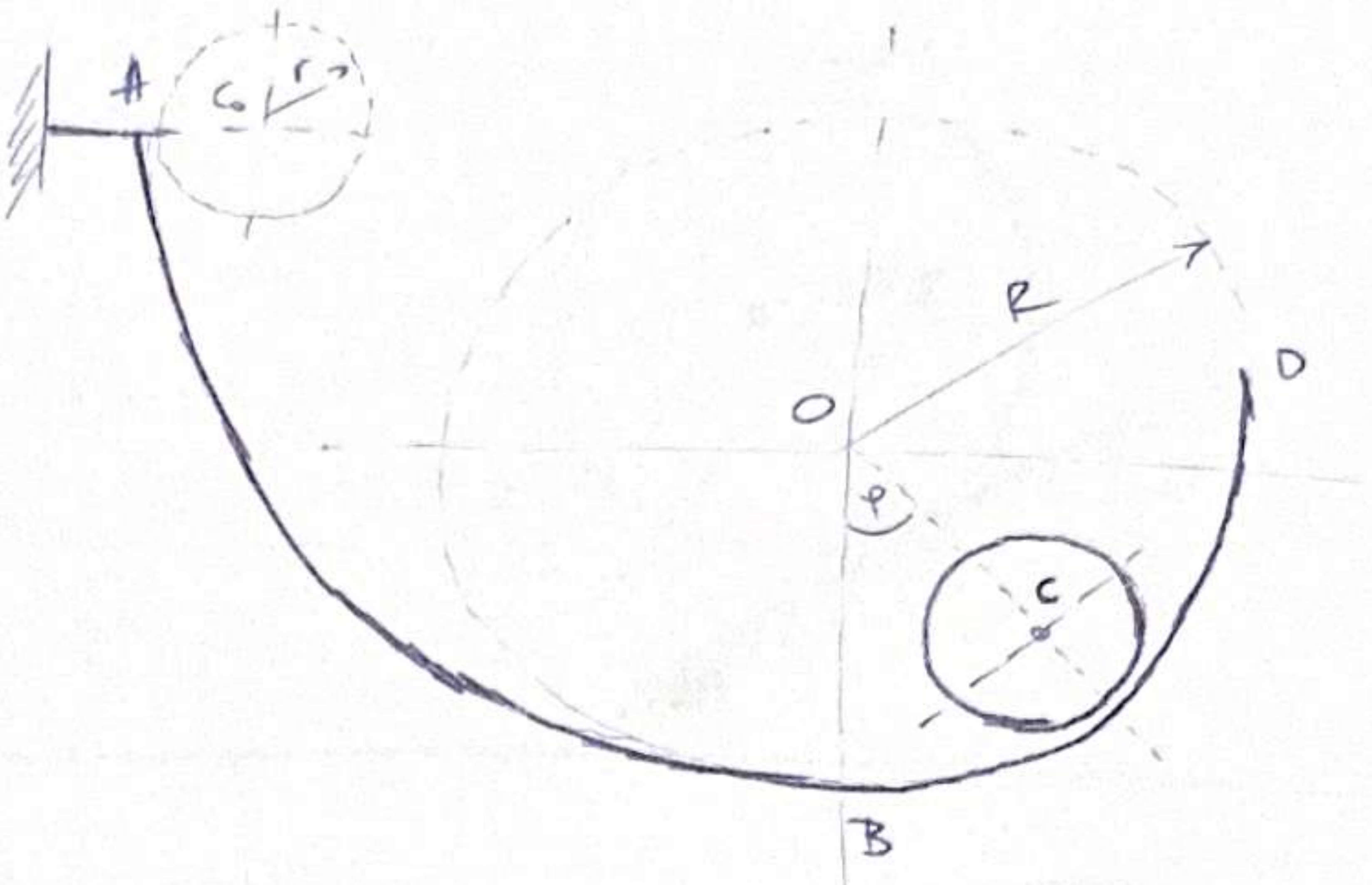
$\ddot{y}_B = a_{Bt} = \varepsilon R$

(1) = (4) $\Rightarrow$ $S\frac{\sqrt{2}}{2} = mg - S\frac{\sqrt{2}}{2}$ $\Rightarrow$ $S\sqrt{2} = mg$ $\Rightarrow$ $S = \frac{\sqrt{2}}{2}mg$

(2) $\Rightarrow$ $N_A = mg + mg\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}$ $\Rightarrow$ $\boxed{N_A = \frac{3}{2}mg}$

(3) $\Rightarrow$ $N_B = mg\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}$ $\Rightarrow$ $\boxed{N_B = \frac{1}{2}mg}$

10.46.



disk: m, r

kotrljanje bez klizanja

$t_0 = 0$: stanje mirovanja

$H_{C0} = 5r$

$R = 3r$

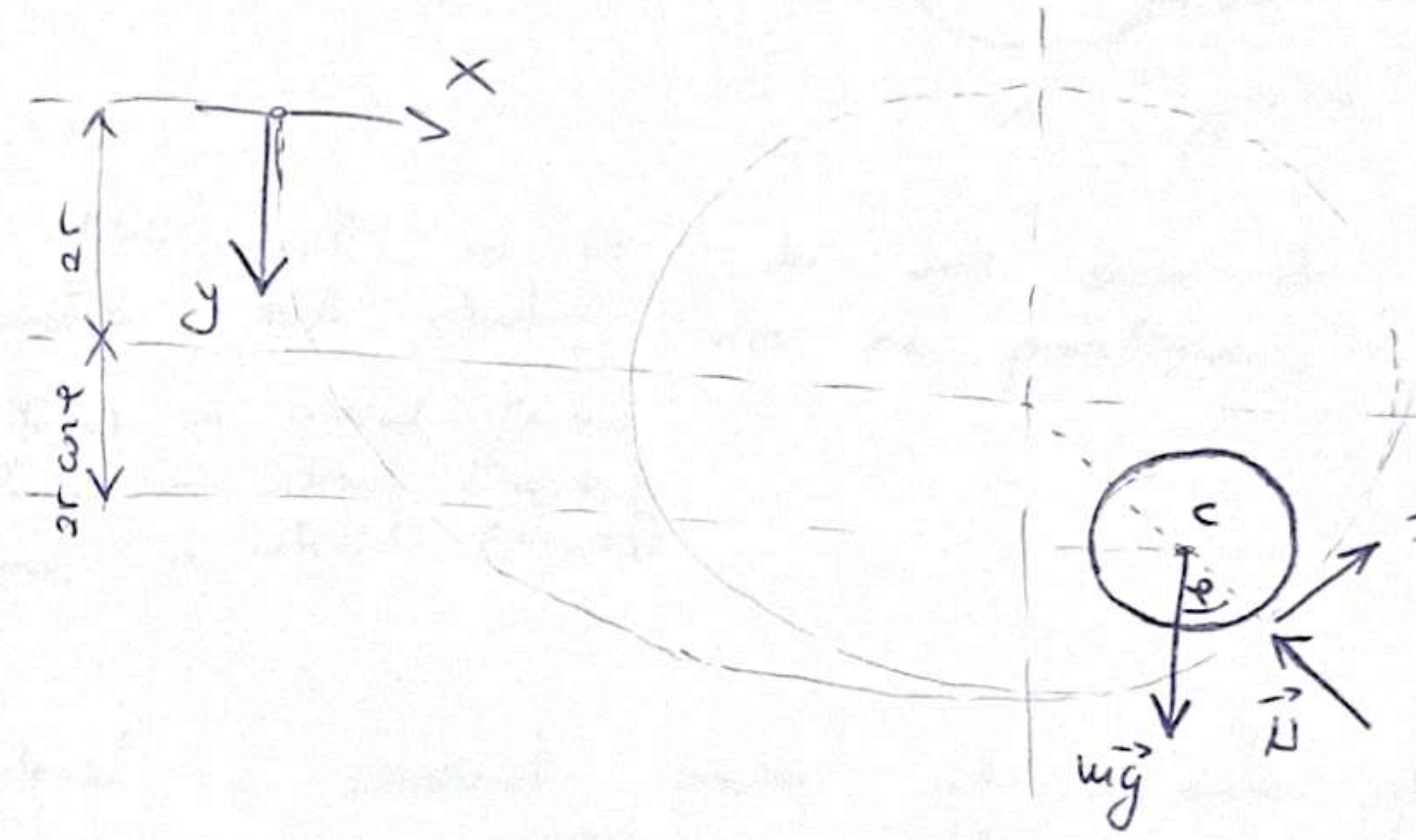
$N(\varphi) = ?$

$\underline{m\vec{a}_c = m\vec{g} + \vec{N} + \vec{T}}$

n: $m\dfrac{v^2}{2r} = -mg\cos\varphi + N \quad (1) \quad \left(\begin{matrix} v=? \\ N=? \end{matrix}\right)$

t: $m\ddot{s} = -mg\sin\varphi + T$

$mr\ddot{\varphi} = -mg\sin\varphi + T \quad (1)$



$\underline{T - T_0 = A}$

$T_0 = 0$

$T = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}mv_c^2 + \frac{1}{2}\frac{1}{2}mR^2\frac{v_c^2}{R^2} = \frac{3}{4}mv_c^2$

$A = \int m\vec{g}\,d\vec{r}_c = \int_0^{2r+2r\cos\varphi} mg\,dy_c = 2mgr(1+\cos\varphi)$

$\Rightarrow \frac{3}{4}\cancel{m}v_c^2 = 2\cancel{m}gr(1+\cos\varphi)$

$v_c^2 = \frac{8}{3}gr(1+\cos\varphi)$

$(1) \Rightarrow N = mg\cos\varphi + \frac{m}{2r}\frac{8gr}{3}(1+\cos\varphi) = mg\cos\varphi + \frac{4}{3}mg + \frac{4}{3}mg\cos\varphi$

$\boxed{N = \frac{1}{3}mg(4 + 7\cos\varphi)}$

$\underline{\dfrac{d\vec{L}_{C_T}}{dt} = \Sigma\vec{M}_{C_T}}$

$\Sigma M_{C_T} = -T\cdot r$

$L_{C_T} = J_{C_T}\cdot\omega = \frac{1}{2}mr^2\omega$

$\dfrac{dL_{C_T}}{dt} = \frac{1}{2}mr^2\varepsilon$

$\Rightarrow \frac{1}{2}mr^2\ddot{\varphi} = -T\cdot\cancel{r} \quad /\cdot 2$

$mr\ddot{\varphi} = -2T \quad (3)$

$(2) = (3)$

$-mg\sin\varphi + T = -2T$

$3T = mg\sin\varphi$

$\boxed{T = \frac{1}{3}mg\sin\varphi}$

10.47

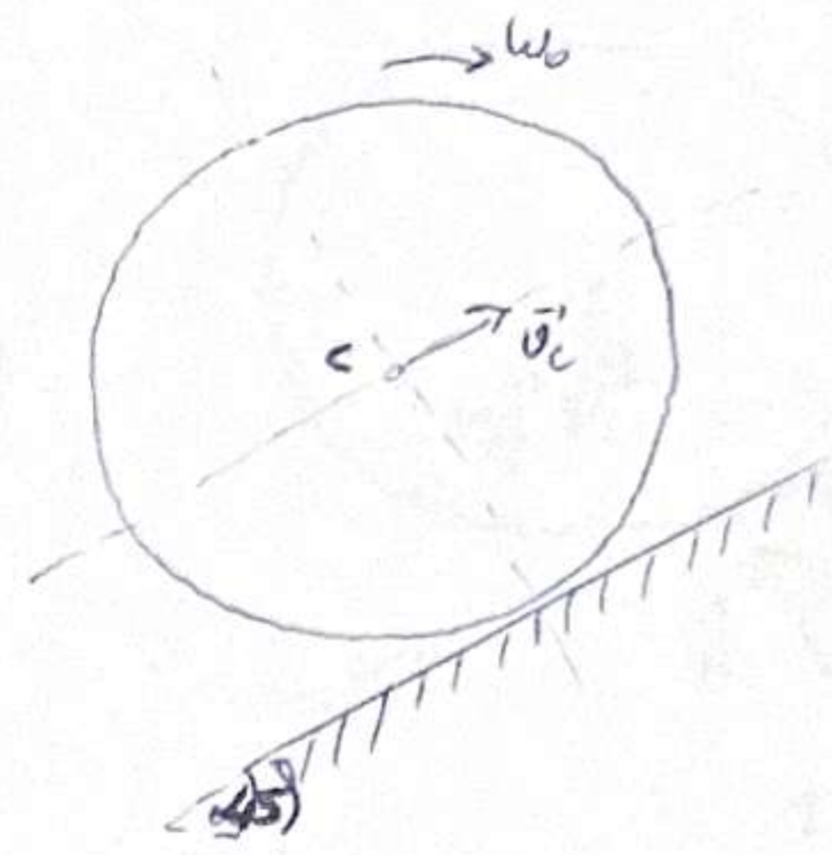
tanki cilindar: m, r

t_0: poč. ugaona brzina $\omega_0 = \sqrt{g/R}$

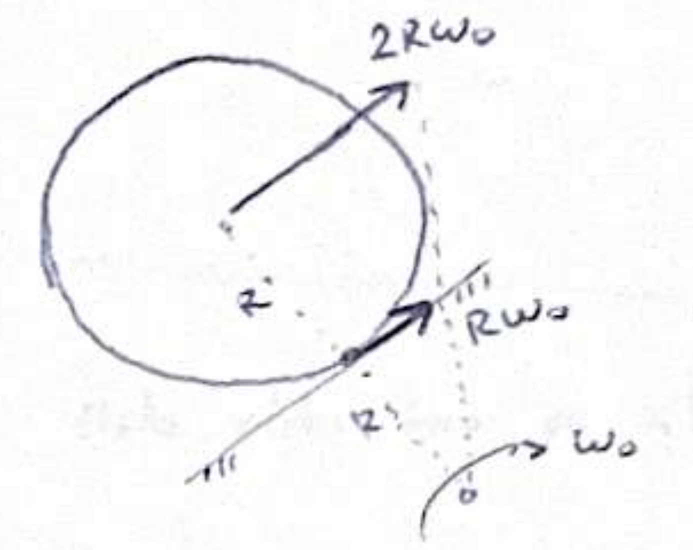
poč. brzina $v_C = 2R\omega_0$

$\mu = \frac{1}{3}$

$H_{max} = ?$



Da bi znali u koju stranu deluje sila $\vec{T}$, moramo znati u koju stranu disk proklizava, a da bi to znali, nacrtaćemo trenutak t_0 i vektor brzine: t_0



$\overline{CP_v} = \frac{v_C}{\omega_0}$

$\overline{CP_v} = \frac{2R\cancel{\omega_0}}{\cancel{\omega_0}}$

Sila $\vec{T}$ deluje suprotno od brzine $R\omega_0$, ali to će da važi samo do trenutka t_1 - a to je trenutak kada cilindar više ne proklizava.

Kretanje:
1) proklizavanje naviše ($T = T_{max}$) - interval $[0, t_1]$
2) bez proklizavanja ($T = [0, T_{max}]$) - interval $[t_1, t_2]$
3) proklizavanje naviše ($T = T_{max}$) - interval $[t_2, t_3]$

Sada ćemo analitički interval po interval, i izračunati pređeni put u svakom od njih, da bi na kraju dobili visinu penjanja. Treba primetiti da trenutak zaustavljanja (trenutak t_3) može da se desi i pre t_2, ili čak i t_1, pa bi u tom slučaju imali samo dva ili jedan interval kretanja.

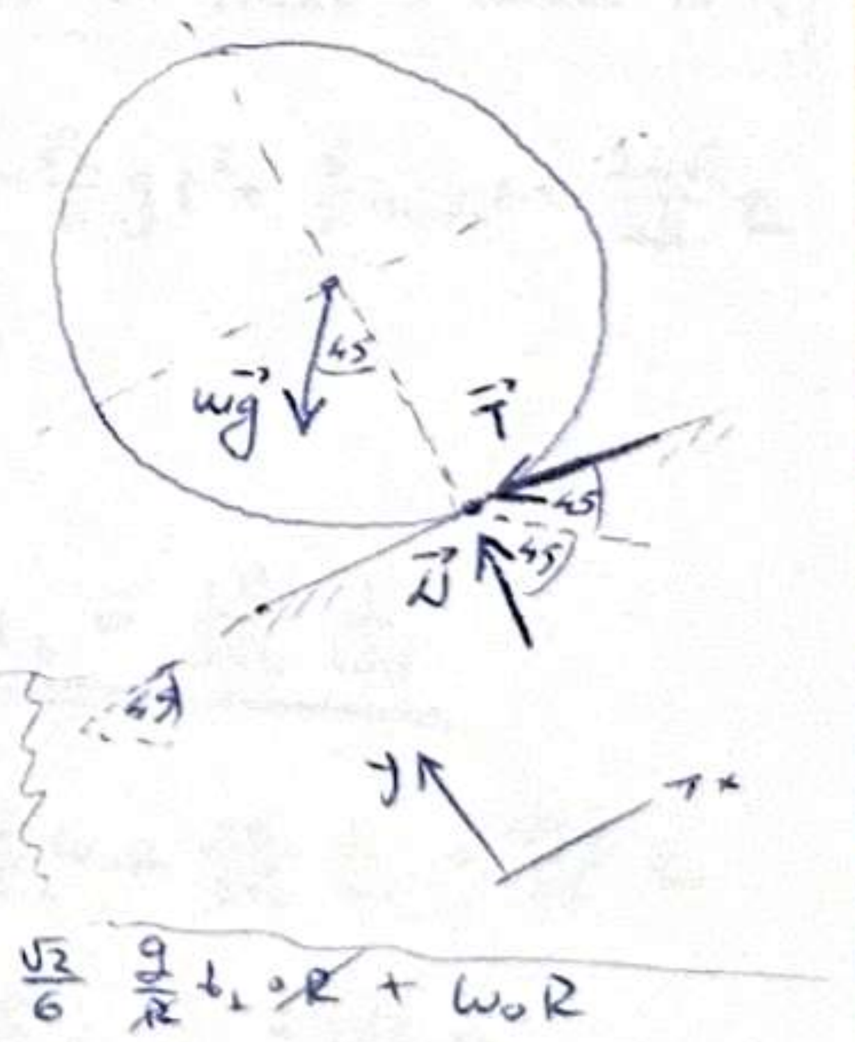
---

interval $t_0 - t_1$

opis kretanja: proklizavanje naviše, sve vreme deluje $T = T_{max}$, ne važi da je $v = r\omega$

cilj: naći $x(t_1)$?

uslov za nalazak t_1: $v(t_1) = R\omega(t_1)$



$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$

$y: \quad 0 = -mg\frac{\sqrt{2}}{2} + N$

$N = mg\frac{\sqrt{2}}{2}$

$T = \mu \cdot mg\frac{\sqrt{2}}{2} = mg\frac{\sqrt{2}}{6}$

$x: \quad m\ddot{x} = -mg\frac{\sqrt{2}}{2} - T$

$\cancel{m}\ddot{x} = -\cancel{m}g\frac{\sqrt{2}}{2} - \cancel{m}g\frac{\sqrt{2}}{6}$

$\ddot{x} = -\frac{2\sqrt{2}}{3}g \quad /\int dt$

$\dot{x} = -\frac{2\sqrt{2}}{3}gt + C_1 = -\frac{2\sqrt{2}}{3}gt + 2R\omega_0 \quad /\int dt$

$x = -\frac{\sqrt{2}}{3}gt^2 + 2R\omega_0 t + \cancel{C_2}^{\,0}, \quad x(t_1 = ?)$

$\frac{d\vec{L}_{C+}}{dt} = \sum \vec{M}_{C+}$

$\sum M_{C+} = T \cdot R = mgR\frac{\sqrt{2}}{6}$

$L_{C+} = J_{C+}\omega = mR^2\omega$

$\frac{dL_{C+}}{dt} = mR^2\varepsilon$

$\cancel{m}gR\frac{\sqrt{2}}{6} = \cancel{m}R^2\varepsilon$

$\varepsilon = \frac{g}{R}\frac{\sqrt{2}}{6} \quad /\int dt$

$\omega = \frac{g}{R}\frac{\sqrt{2}}{6}t + \omega_0$

za t_1 važi:

$\dot{x}(t_1) = R\omega(t_1)$

$-\frac{2\sqrt{2}}{3}gt_1 + 2R\omega_0 = \frac{\sqrt{2}}{6}\frac{g}{\cancel{R}}t_1 \cdot \cancel{R} + \omega_0 R$

$-\frac{2\sqrt{2}}{3}gt_1 - \frac{\sqrt{2}}{6}gt_1 = -\omega_0 R$

$\frac{5\sqrt{2}}{6}gt_1 = \omega_0 R \Rightarrow t_1 = \frac{6}{5\sqrt{2}}\frac{\omega_0 R}{g} = \frac{3\sqrt{2}}{5}\frac{\cancel{\omega_0}}{\omega_0^{\cancel{2}}}$

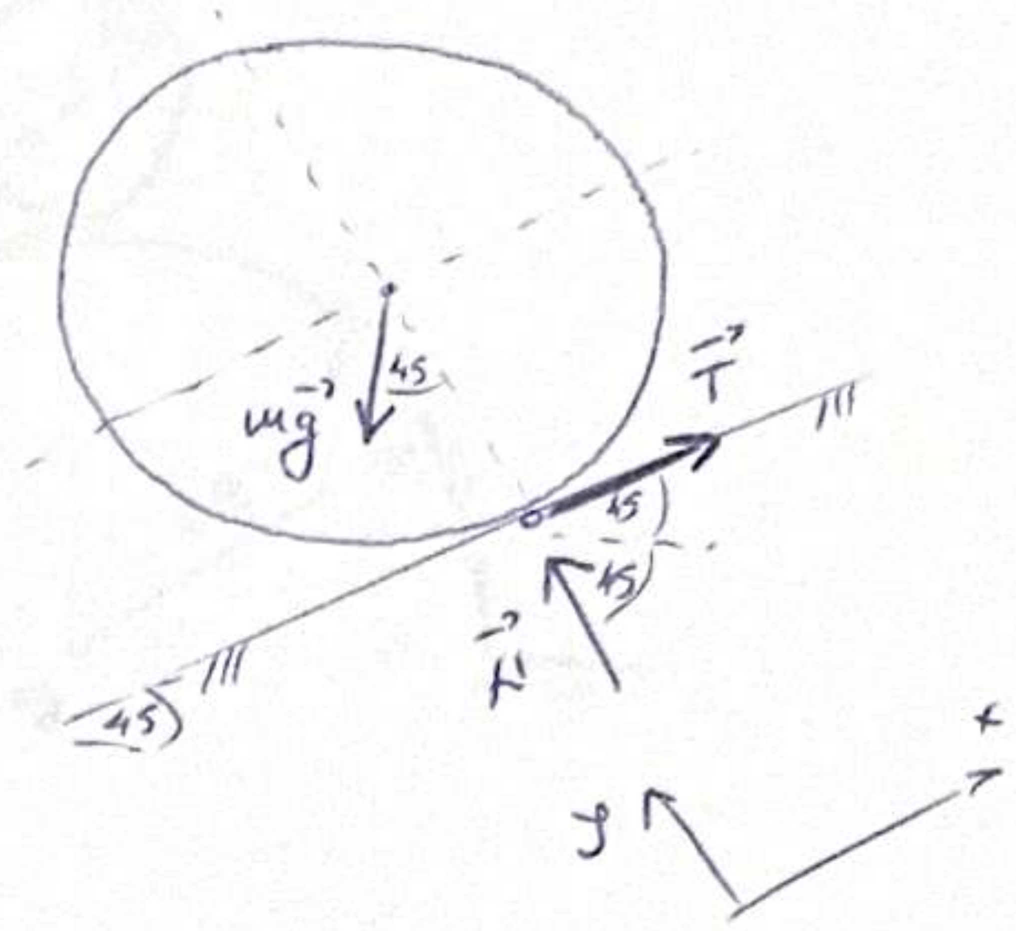
$x(t_1) = -\frac{\sqrt{2}}{3}g \cdot \frac{9 \cdot 2}{25}\frac{1}{\omega_0^2} + 2R\cancel{\omega_0} \cdot \frac{3\sqrt{2}}{5}\frac{1}{\cancel{\omega_0}}$

$x(t_1) = -\frac{6\sqrt{2}}{25}g\frac{R}{g} + \frac{6\sqrt{2}}{5}R$

$\boxed{x(t_1) = \frac{24\sqrt{2}}{25}R}$

interval $t_1 - t_2$

opis kretanja: sila $\vec{T}$ je svojim dejstvom učinila da više nema proklizavanja i ona je u trenutku t_1 jednaka nuli. Ona međutim vremenom raste, kako bi sprečila proklizavanje, sve dok ne dostigne max. vr. i taj trenutak ćemo obeležiti sa t_2 - nakon njega ponovo počinje proklizavanje, samo u suprotnu stranu



cilj: naći $x(t_2)$?

uslov za nalazak t_2: $T(t_2) = T_{max} = \mu \cdot N$

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$$

y: $0 = -mg\frac{\sqrt{2}}{2} + N$

$N = mg\frac{\sqrt{2}}{2}$

$T = ?$

x: $m\ddot{x} = -mg\frac{\sqrt{2}}{2} + T$ (1) $\left(\begin{matrix}\ddot{x}=? \\ T=?\end{matrix}\right)$

$$\frac{d\vec{L}_{C_1}}{dt} = \sum \vec{M}_{C_1}$$

$\sum M_{C_1} = -T \cdot R$

$L_{C_1} = J_{C_1} \cdot \omega = mR^{\cancel{2}}\frac{\dot{x}}{\cancel{R}}$

$\frac{dL_{C_1}}{dt} = mR\ddot{x}$

$m\cancel{R}\ddot{x} = -T \cdot \cancel{R}$

$m\ddot{x} = -T$ (2)

(1) = (2)

$-mg\frac{\sqrt{2}}{2} + T = -T$

$T = mg\frac{\sqrt{2}}{4} = \text{const.}$ - za sve vreme kretanja

s obzirom da je $T_{max} = \mu \cdot N = mg\frac{\sqrt{2}}{6}$, zaključujemo da se cilinder uopšte neće kretati bez proklizavanja, tako da odmah prelazimo na interval $t_2 - t_3$

$\left(x(t_2) = x(t_1) = \frac{24\sqrt{2}}{25}R \right)$

interval $t_2 - t_3$ (ista slika kao za interval $t_1 - t_2$)

cilj: naći $x(t_3)$

uslov za nalazak t_3: $\dot{x}(t_3) = 0$

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$$

y: $0 = -mg\frac{\sqrt{2}}{2} + N$

$N = mg\frac{\sqrt{2}}{2}$

$T = mg\frac{\sqrt{2}}{6}$

x: $m\ddot{x} = -mg\frac{\sqrt{2}}{2} + T = -mg\frac{\sqrt{2}}{2} + mg\frac{\sqrt{2}}{6} = -mg\frac{\sqrt{2}}{3}$ $/:m / \int dt$

$\dot{x} = -\frac{\sqrt{2}}{3}g + C_1 = -\frac{\sqrt{2}}{3}gt + \frac{6}{5}\omega_0 R$ $/\int$

$\boxed{C_1 = \dot{x}(t_1) = -\frac{2\sqrt{2}}{\cancel{3}}g \cdot \frac{\cancel{3}\sqrt{2}}{5}\frac{1}{\omega_0} + 2R\omega_0 = -\frac{4}{5}\omega_0^2 R \cdot \frac{1}{\omega_0} + 2R\omega_0}$

ovo smo našli tako što smo se vratili u interval $t_0 - t_1$

$x = -\frac{\sqrt{2}}{6}gt^2 + \frac{6}{5}\omega_0 Rt + C_2 = -\frac{\sqrt{2}}{6}gt^2 + \frac{6}{5}\omega_0 Rt + \frac{24\sqrt{2}}{25}R$

uslov za t_3: $\dot{x}(t_3) = 0$

$-\frac{\sqrt{2}}{3}gt_3 + \frac{6}{5}\omega_0 R = 0$

$+\frac{\sqrt{2}}{3}\omega_0^2 R t_3 = \frac{6}{5}\omega_0 R \Rightarrow t_3 = \frac{18}{5\sqrt{2}}\frac{1}{\omega_0}$

$x(t_3) = -\frac{\sqrt{2}}{6}g \cdot \frac{324}{50}\frac{1}{\omega_0^2} + \frac{6}{5}\omega_0 R\frac{18}{5\sqrt{2}}\frac{1}{\omega_0} + \frac{24\sqrt{2}}{25}R$

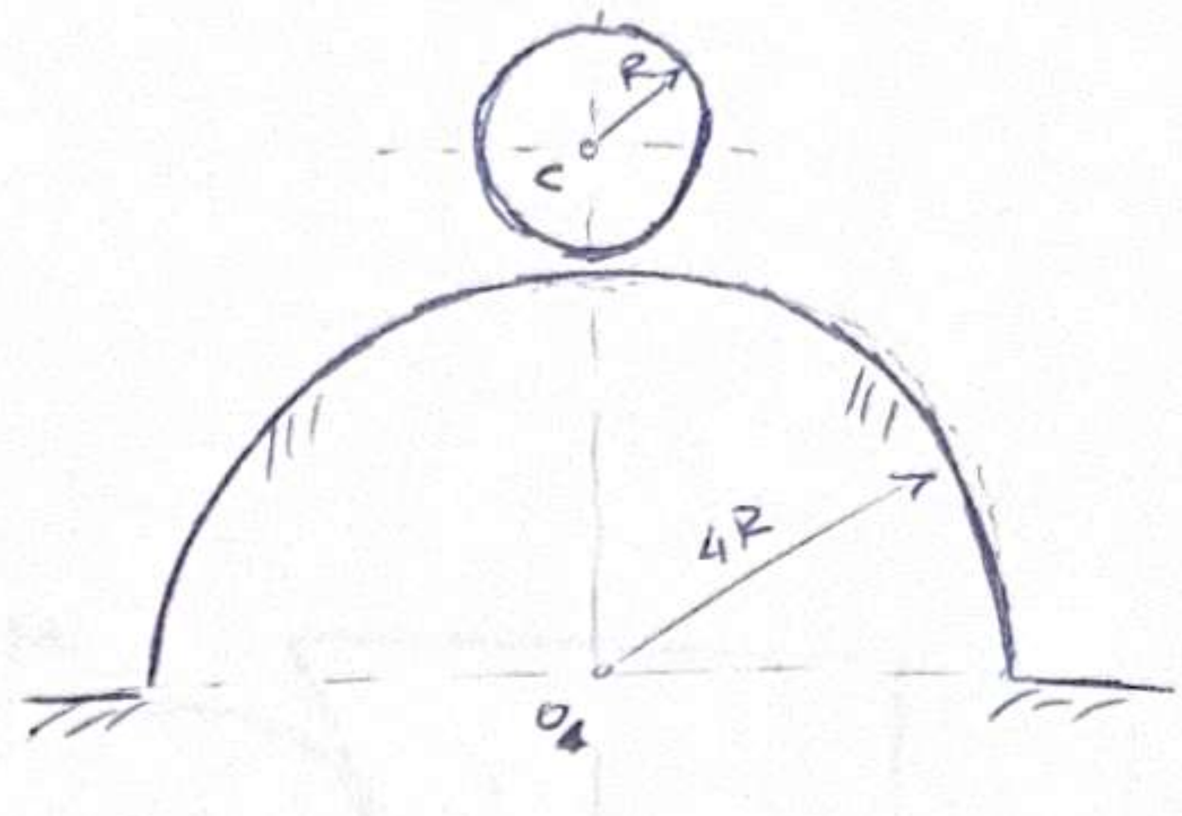
$x(t_3) = -\frac{27\sqrt{2}}{25}\cancel{\omega_0^2}R\frac{1}{\cancel{\omega_0^2}} + \frac{54\sqrt{2}}{25}R + \frac{24\sqrt{2}}{25}R$

$x(t_3) = \frac{51\sqrt{2}}{25}R$

$H = x(t_3) \cdot \frac{\sqrt{2}}{2}$

$$\boxed{H = \frac{51}{25}R}$$

10.48



tanki cilindar: m, R

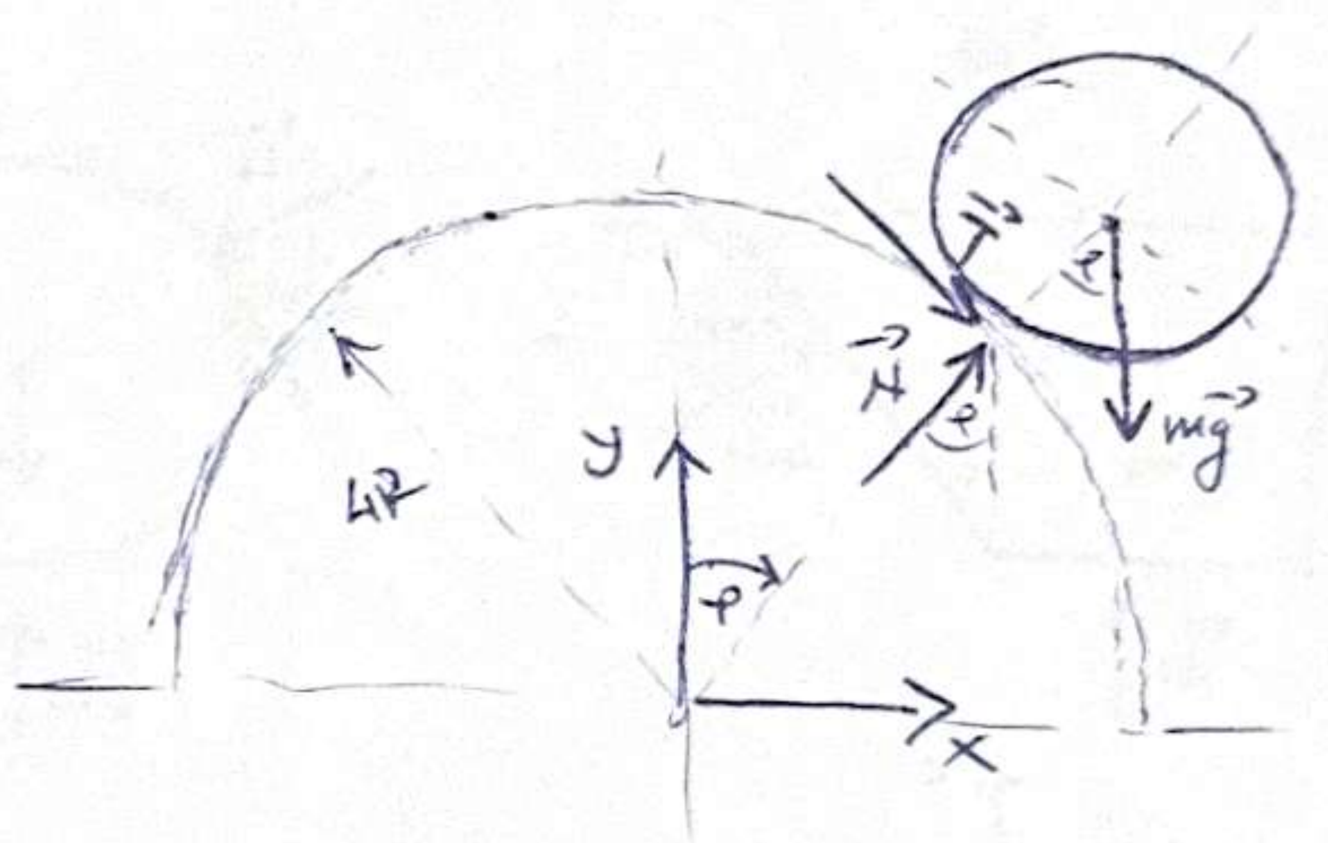
kotrljanje bez klizanja

t_0: cilindar u najvišem položaju
stanje mirovanja (zanemarljivo mala brzina)

odrediti položaj u kojem će doći do odvajanja cilindra od podloge?

---

ideja: naći $N(\varphi)$, pa onda postaviti uslov: $N(\varphi_1)=0$
(odatle ćemo naći φ_1)



$$m\vec a = m\vec g + \vec N + \vec T$$

n: $$m\cdot\frac{v^2}{5R} = mg\cos\varphi - N$$

$$N = -m\frac{v^2}{5R} + mg\cos\varphi \quad (1) \qquad (v(\varphi)=?)$$

t: ... (ne treba nam)

$$T - T_0 = A$$

$$T_0 = 0$$

$$T = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\frac{v^2}{R^2} = mv^2$$

$$A = \int m\vec g\, d\vec r_c = \int_{5R}^{5R\cos\varphi} -mg\,dy = 5mgR(1-\cos\varphi)$$

$$\cancel{m}v^2 = 5\cancel{m}gR(1-\cos\varphi) \quad (2)$$

(2) ubacimo u (1)

$$N = -m\frac{5gR(1-\cos\varphi)}{5R} + mg\cos\varphi$$

$$N = -mg + mg\cos\varphi + mg\cos\varphi$$

$$N = mg(2\cos\varphi - 1)$$

postavljamo uslov: $N(\varphi_1)=0$

$$mg(2\cos\varphi_1 - 1) = 0$$

$$\cos\varphi_1 = \frac{1}{2}$$

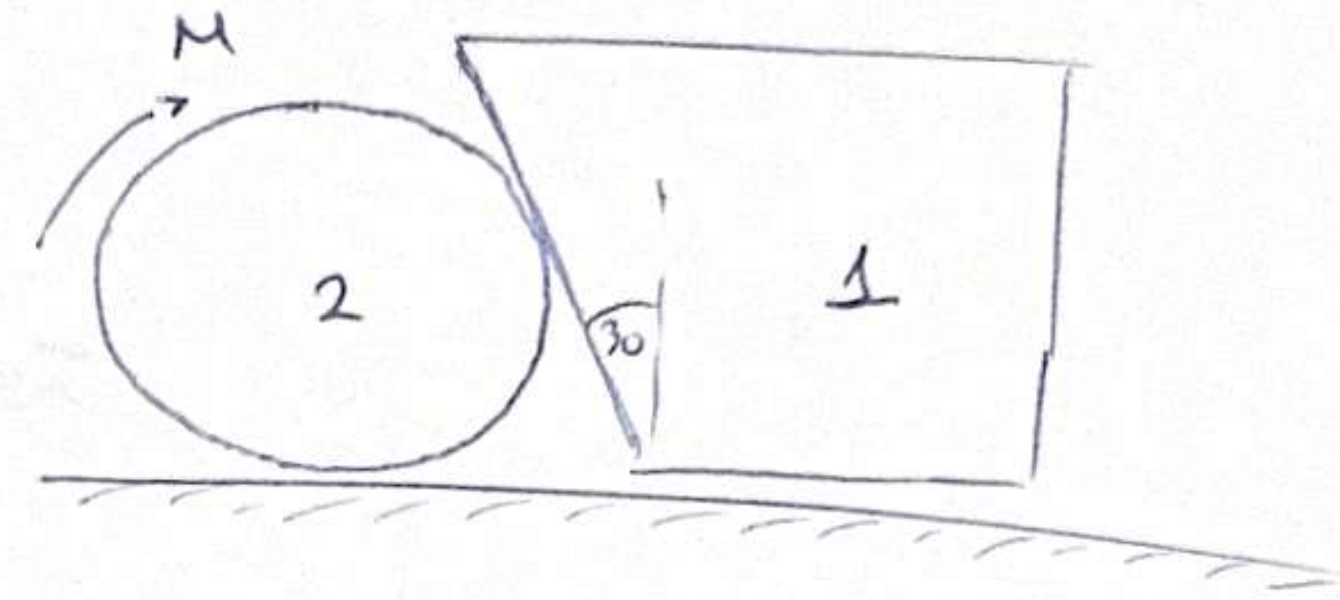
$$\boxed{\varphi_1 = 60^\circ}$$

10.49

2: $2m$, r,
kotrljanje bez klizanja

M, $\mu = \frac{\sqrt{3}}{4}$ - koef. trenja između prizme i valjka

1: m
kliza bez trenja



$\ddot{x}_1 = ?$ $\quad N_1 = ?$ $\quad N_2 = ?$ $\quad T_2 = ?$

$N_{12} = ?$ $\quad T_{12} = ?$

L: mirovanje

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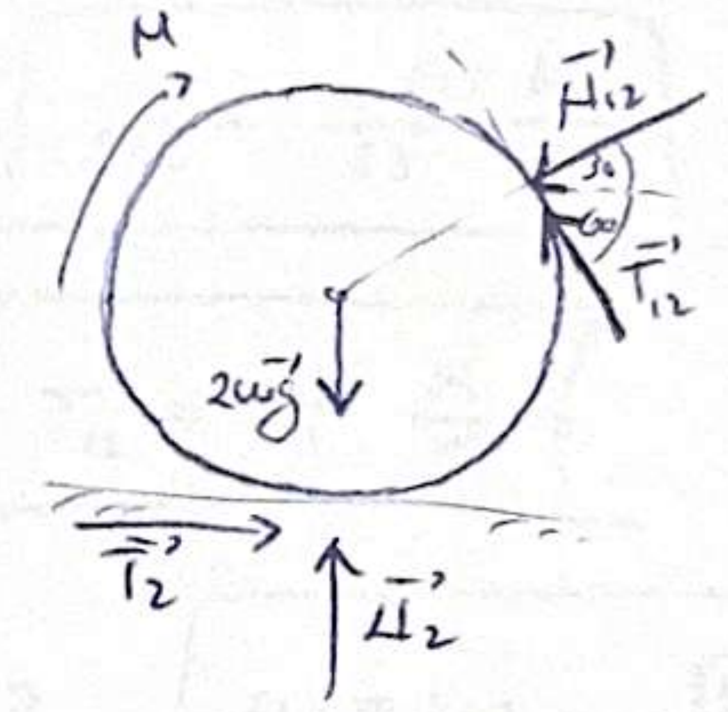
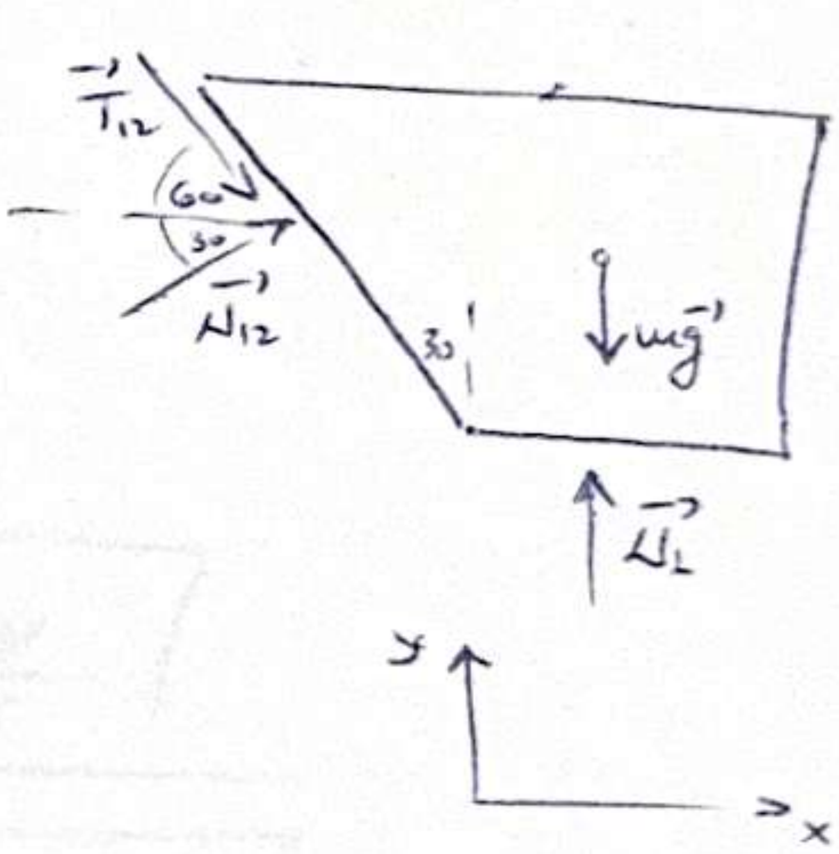
kinematska veza: $\quad \underline{\ddot{x}_1 = \ddot{x}_2 = \ddot{x}}$

veza: $\quad \underline{T_{12} = \mu \cdot N_{12} = \frac{\sqrt{3}}{4} N_{12}}$



telo 1



$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{N}_{12} + \vec{T}_{12}$

y: $\quad 0 = -mg + N_1 + N_{12} \sin 30 - T_{12} \cos 30$

$0 = -mg + N_1 + \frac{1}{2} N_{12} - \frac{\sqrt{3}}{4} \frac{\sqrt{3}}{2} N_{12}$

$\underline{0 = -mg + N_1 + \frac{1}{8} N_{12}}$ $\quad (1) \quad \left(\begin{matrix} N_1 = ? \\ N_{12} = ? \end{matrix}\right)$

x: $\quad m\ddot{x} = N_{12} \cos 30 + T_{12} \sin 30$

$m\ddot{x} = N_{12} \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} N_{12} \cdot \frac{1}{2}$

$\underline{m\ddot{x} = \frac{5\sqrt{3}}{8} N_{12}}$ $\quad (2) \quad (\ddot{x} = ?)$

$m\vec{a}_2 = 2m\vec{g} + \vec{N}_2 + \vec{T}_2 + \vec{N}_{12} + \vec{T}_{12}$

y: $\quad 0 = -2mg + N_2 - N_{12} \sin 30 + T_{12} \cos 30$

$0 = -2mg + N_2 - \frac{1}{2} N_{12} + \frac{\sqrt{3}}{4} N_{12} \cdot \frac{\sqrt{3}}{2}$

$\underline{0 = -2mg + N_2 - \frac{1}{8} N_{12}}$ $\quad (3) \quad (N_2 = ?)$

x: $\quad 2m\ddot{x} = T_2 - N_{12} \cos 30 - T_{12} \sin 30$

$2m\ddot{x} = T_2 - \frac{\sqrt{3}}{2} N_{12} - \frac{\sqrt{3}}{4} N_{12} \cdot \frac{1}{2}$

$\underline{2m\ddot{x} = T_2 - \frac{5\sqrt{3}}{8} N_{12}}$ $\quad (4)$

$\frac{d\vec{L}_{C_t}}{dt} = \Sigma \vec{M}_{C_t}$

$\Sigma M_{C_t} = M - T_2 \cdot R - T_{12} \cdot R = M - T_2 \cdot R - \frac{\sqrt{3}}{4} N_{12} \cdot R$

$L_{C_t} = J_{C_t} \cdot \omega = \frac{1}{2} 2mR^2 \omega = mR^{\cancel{2}} \frac{\dot{x}}{\cancel{R}}$

$\frac{dL_{C_t}}{dt} = mR\ddot{x}$

$\underline{mR\ddot{x} = M - T_2 \cdot R - \frac{\sqrt{3}}{4} N_{12} \cdot R}$ $\quad (5)$

$$(2) \Rightarrow N_{12} = \frac{8}{5\sqrt{3}} m\ddot{x}$$

$$(1) \Rightarrow N_1 = mg - \frac{1}{8}\frac{8}{5\sqrt{3}} m\ddot{x} = mg - \frac{1}{5\sqrt{3}} m\ddot{x}$$

$$(3) \Rightarrow N_2 = 2mg + \frac{1}{8}\frac{8}{5\sqrt{3}} m\ddot{x} = 2mg + \frac{1}{5\sqrt{3}} m\ddot{x}$$

$$(4) \Rightarrow T_2 = 2m\ddot{x} + \frac{5\cancel{\sqrt{3}}}{\cancel{8}} \cdot \frac{\cancel{8}}{5\cancel{\sqrt{3}}} m\ddot{x} = 3m\ddot{x}$$

$$(5) \Rightarrow M - 3m\ddot{x} \cdot R - \frac{\sqrt{3}}{4}\frac{8}{5\sqrt{3}} m\ddot{x} R = mR\ddot{x}$$

$$mR\ddot{x}\left(1 + 3 + \frac{2}{5}\right) = M$$

$$\boxed{\ddot{x} = \frac{5}{22}\frac{M}{mR}}$$

$$\Rightarrow N_{12} = \frac{8}{5\sqrt{3}} m \frac{5}{22}\frac{M}{mR} \Rightarrow \boxed{N_{12} = \frac{4\sqrt{3}}{33}\frac{M}{r}}$$

$$\Rightarrow T_{12} = \frac{\sqrt{3}}{4}\frac{4\sqrt{3}}{33}\frac{M}{r} \Rightarrow \boxed{T_{12} = \frac{1}{11}\frac{M}{r}}$$

$$\Rightarrow N_1 = mg - \frac{1}{5\sqrt{3}} m \frac{5}{22}\frac{M}{mR} \Rightarrow \boxed{N_1 = mg - \frac{\sqrt{3}}{66}\frac{M}{r}}$$

$$\Rightarrow N_2 = 2mg + \frac{1}{5\sqrt{3}} m \frac{5}{22}\frac{M}{mR} \Rightarrow \boxed{N_2 = 2mg + \frac{\sqrt{3}}{66}\frac{M}{R}}$$

$$\Rightarrow T_2 = 3m\frac{5}{22}\frac{M}{mR} \Rightarrow \boxed{T_2 = \frac{15}{22}\frac{M}{mR}}$$

10.50

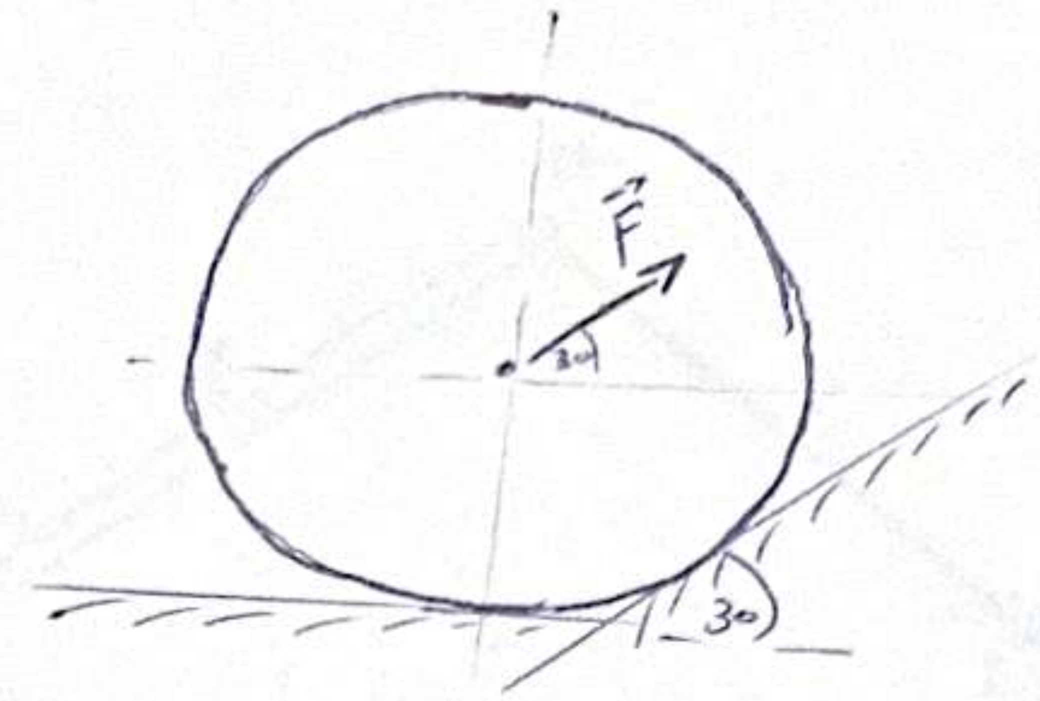
puni cilindar: m, R

$F = \frac{mg}{4k} t$

$\mu = \frac{\sqrt{3}}{3}$

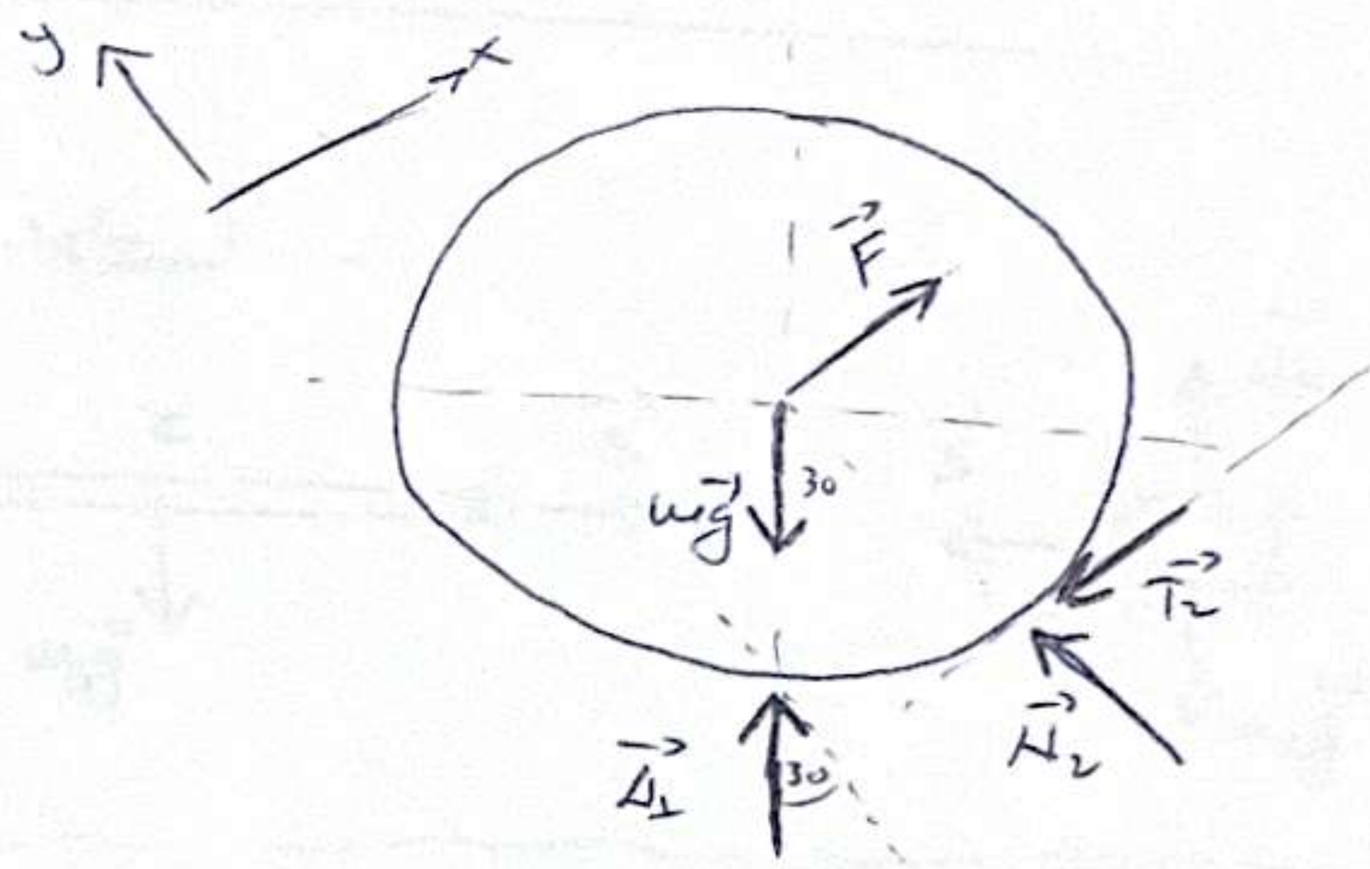
t_0: mirovanje

$t_2 = ?$ – trenutak u kojem dolazi do proklizavanja?



Kretanje:

1) $t_0 - t_1$: cilindar još uvek nije započeo kretanje
2) $t_1 - t_2$: kretanje uz strmu ravan bez proklizavanja



interval $t_0 - t_1$:

cilj: naći t_1?

uslov: $N_1(t_1) = 0$

$m\vec{a} = m\vec{g} + \vec{F} + \vec{N_1} + \vec{N_2} + \vec{T_2}$

x: $0 = -mg \sin 30 + F + N_1 \sin 30$

$0 = -\frac{1}{2} mg + \frac{mg}{4k} t + \frac{1}{2} N_1$

$N_1 = \frac{1}{2} mg - \frac{mg}{4k} t$

$N_1(t_1) = 0$

$\frac{1}{2} mg - \frac{mg}{4k} t_1 = 0$

$\frac{1}{2} = \frac{t_1}{4k}$

$t_1 = 2k$

interval $t_1 - t_2$

cilj: naći t_2

uslov: $T_2(t_2) = T_{2,max} = \mu \cdot N_2$, $N_1 = 0$

$m\vec{a} = m\vec{g} + \vec{F} + \vec{N_2} + \vec{T_2}$

y: $0 = -mg \cos 30 + N_2$

$N_2 = \frac{\sqrt{3}}{2} mg$

x: $m\ddot{x} = -mg \sin 30 + F - T_2$

$m\ddot{x} = -\frac{1}{2} mg + \frac{mg}{4k} t - T_2$ (1)

$\frac{d\vec{L}_{C_1}}{dt} = \Sigma \vec{M}_{C_1}$

$\Sigma M_{C_1} = T_2 \cdot R$

$L_{C_1} = J_{C_1} \cdot \omega = \frac{1}{2} mR^2 \omega = \frac{1}{2} mR^2 \frac{\dot{x}}{R}$

$\frac{dL_{C_1}}{dt} = \frac{1}{2} mR\ddot{x}$

$\frac{1}{2} mR\ddot{x} = + T_2 \cdot R$

$m\ddot{x} = 2T_2$ (2)

(1) = (2)

$2 \cdot T_2(t) = -\frac{1}{2} mg + \frac{mg}{4k} t - T_2(t)$

$T_2(t) = \frac{mg}{12k} t - \frac{1}{6} mg$

$T_2(t_2) = \frac{mg}{12k} t_2 - \frac{1}{6} mg$

$T_2(t_2) = \mu \cdot N_2 = \frac{\sqrt{3}}{3} \frac{\sqrt{3}}{2} mg = \frac{1}{2} mg$

$\Rightarrow \frac{mg}{12k} t_2 - \frac{1}{6} mg = \frac{1}{2} mg \quad / \cdot \frac{12k}{mg} \Rightarrow \boxed{t_2 = 8k}$

10.51.

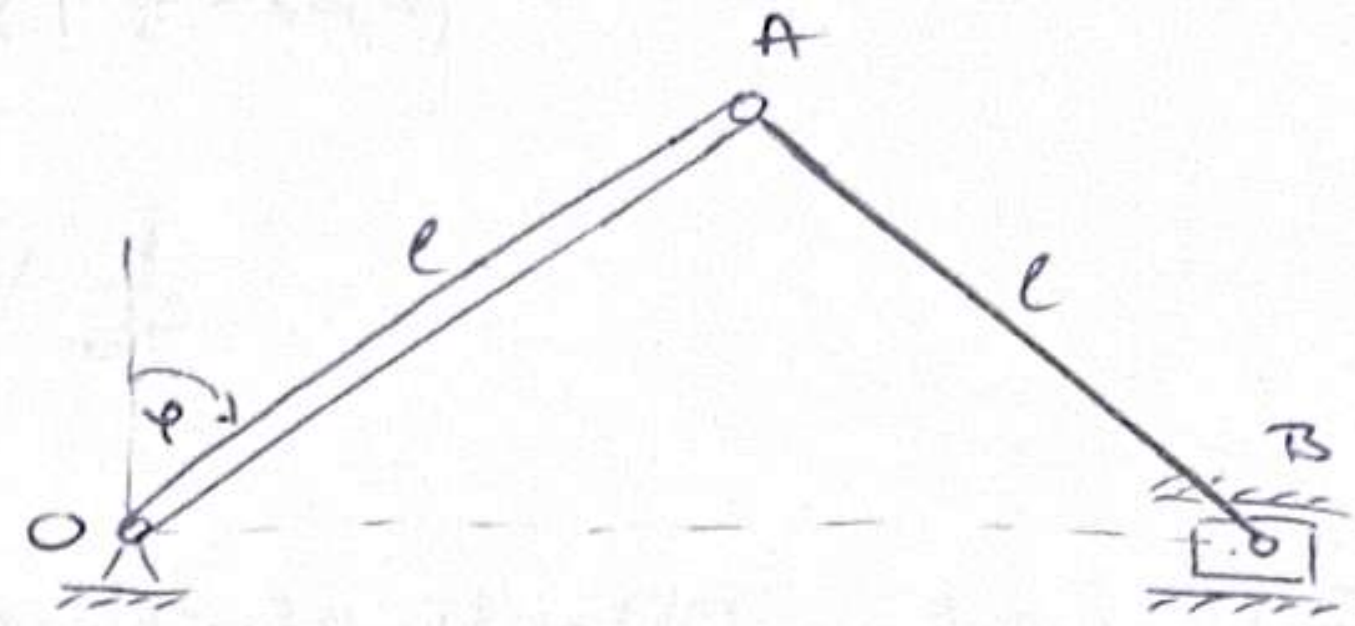
OA: $m_1,\ \ell$

AB: $\emptyset,\ \ell$

B: m_2

t_0: $\varphi_0 = 30°$

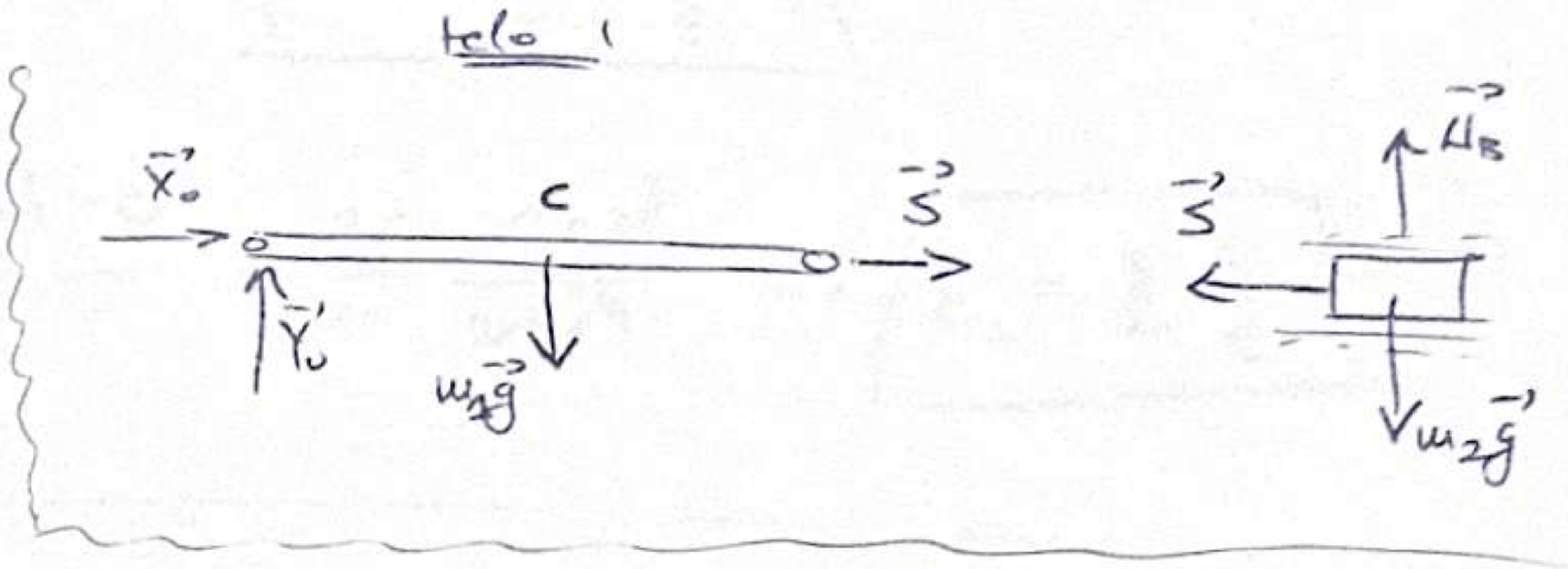
sistem miruje



$\vec{N}_0'(t_1) = ?$

t_1 – trenutak u kojem je $\varphi(t_1) = 90°$

Posmatramo sve za trenutak t_1:



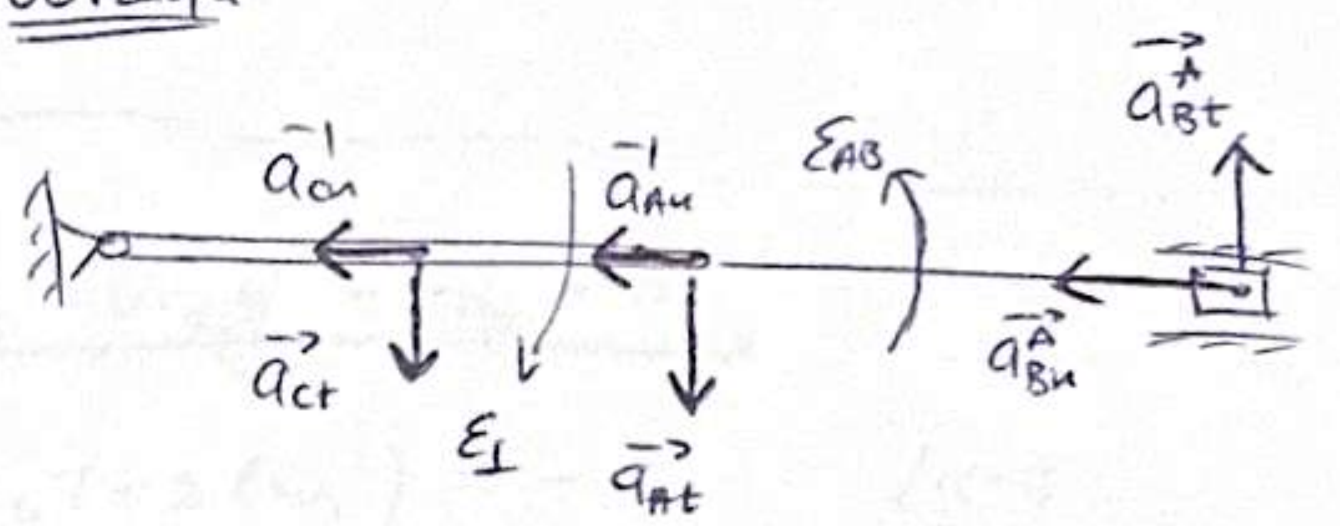
kinematske veze:

$$\vec{a}_C = \vec{a}_{Cn} + \vec{a}_{Ct}$$

x: $\ddot{x}_C = -a_{Cn} = -\omega_1^2 \cdot \frac{1}{2}\ell = -\frac{1}{2}\ell\omega_1^2$

y: $\ddot{y}_C = -a_{Ct} = -\varepsilon_1 \cdot \frac{1}{2}\ell = -\frac{1}{2}\ell\varepsilon_1$

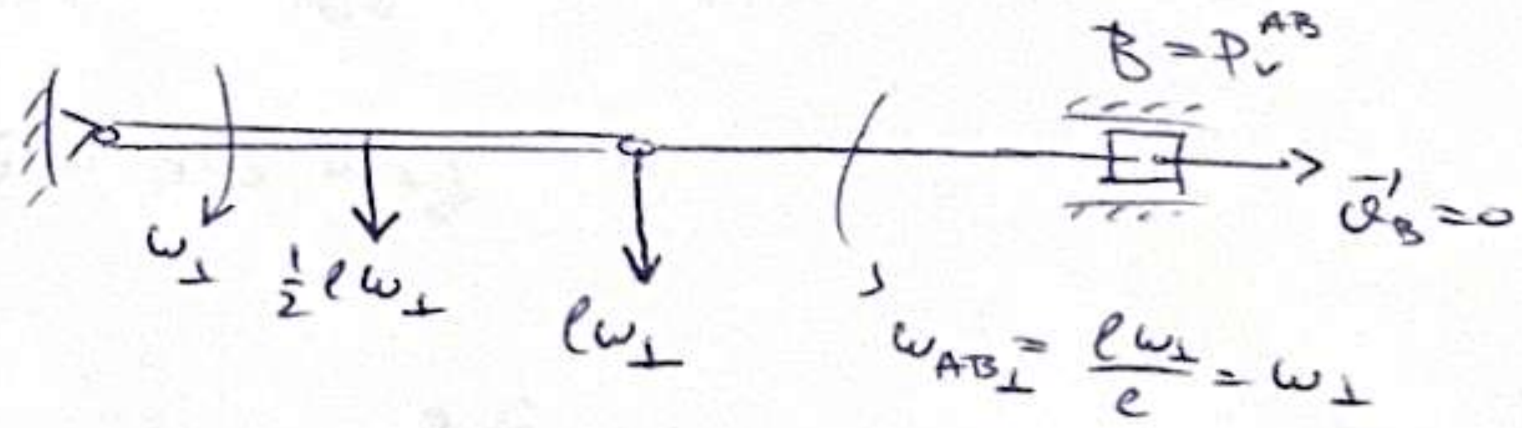
ubrzanja



$$\vec{a}_A = \vec{a}_{An} + \vec{a}_{At}$$

$a_{An} = \ell\omega_1^2$

$a_{At} = \ell\varepsilon_1$



$$\vec{a}_B = \vec{a}_{An} + \vec{a}_{At} + \vec{a}_{Bn}^A + \vec{a}_{Bt}^A$$

$a_{Bn}^A = \omega_{AB_1}^2 \cdot \ell = \ell\omega_1^2$

$a_{Bt}^A = \varepsilon_{AB} \cdot \ell$

y: $0 = 0 - a_{At} + 0 + a_{Bt}^A$

$a_{Bt}^A = a_{At}$

$\varepsilon_{AB} \cdot \cancel{\ell} = \varepsilon \cdot \cancel{\ell}$

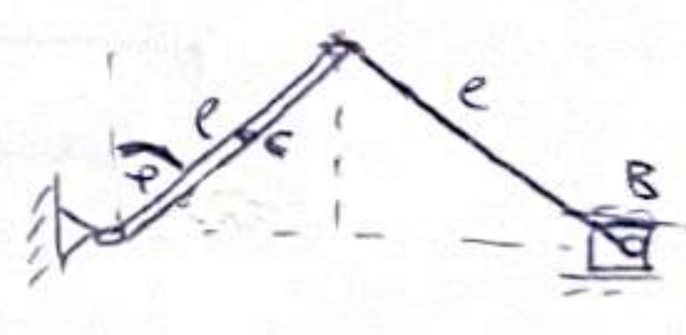
x: $\ddot{x}_B = -a_{An} + 0 - a_{Bn}^A + 0$

$\ddot{x}_B = -\ell\omega_1^2 - \ell\omega_1^2$

$\ddot{x}_B = -2\ell\omega_1^2$

$\ddot{y}_B = 0$

$T - T_0 = A$



$T_0 = 0$

$T = \left[\frac{1}{2}m_1 v_C^2 + \frac{1}{2}J_C\omega^2\right] + \left[\frac{1}{2}m_2 v_B^2 + 0\right]$

$x_C = \frac{1}{2}\ell\sin\varphi \Rightarrow \dot{x}_C = \frac{1}{2}\ell\omega\cos\varphi$

$y_C = \frac{1}{2}\ell\cos\varphi \Rightarrow \dot{y}_C = -\frac{1}{2}\ell\omega\sin\varphi$

$\Rightarrow v_C^2 = \dot{x}_C^2 + \dot{y}_C^2$

$x_B = 2\ell\sin\varphi \Rightarrow \dot{x}_B = 2\ell\omega\cos\varphi$

$y_B = 0 \Rightarrow \dot{y}_B = 0$

$\Rightarrow v_B^2 = 4\ell^2\omega^2\cos^2\varphi$

$T = \left[\frac{1}{2}m_1\frac{\ell^2\omega^2}{4} + \frac{1}{2}\frac{1}{12}m_1\ell^2\omega^2\right] + \left[\frac{1}{2}m_2 \cdot 4\ell^2\omega^2\cos^2\varphi\right]$

$T = \frac{1}{6}m_1\ell^2\omega^2 + 2m_2\ell^2\omega^2\cos^2\varphi$

$A = \int m_1\vec{g}\,d\vec{r}_C = \int_{\frac{1}{2}\ell\cos 30}^{\frac{1}{2}\ell\cos\varphi} -m_1 g\,dy = \frac{1}{2}m_1 g\ell\left(\frac{\sqrt{3}}{2} - \cos\varphi\right)$

$$\underline{T - T_0 = A}$$

$$\Rightarrow \frac{1}{6} m_1 l^2 \omega^2 + 2 m_2 l^2 \omega^2 \cos^2\varphi = \frac{1}{2} m_1 g l \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)$$

$$\omega = \sqrt{\frac{\frac{1}{2} m_1 g l \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)}{\frac{1}{6} m_1 l^2 + 2 m_2 l^2 \cos^2\varphi}} \quad / \frac{d}{dt}$$

$$\varepsilon = \frac{1}{2 \cdot \sqrt{\ldots}} \cdot \frac{\frac{1}{2} m_1 g l \cdot \omega \sin\varphi \left(\frac{1}{6} m_1 l^2 + 2 m_2 l^2 \cos^2\varphi\right) - \left(\frac{1}{2} m_1 g l \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)\right) \cdot \left(-2 m_2 l^2 \cdot 2 \cos\varphi \cdot \sin\varphi \cdot \omega\right)}{\left(\frac{1}{6} m_1 l^2 + 2 m_2 l^2 \cos^2\varphi\right)^2}$$

$$\omega_1 = \omega(\varphi = 90) = \sqrt{\frac{\frac{1}{2} m_1 g l \cdot \frac{\sqrt{3}}{2}}{\frac{1}{6} m_1 l^2}} \Rightarrow \boxed{\omega_1 = \frac{3\sqrt{3}}{2} \frac{g}{l}}$$

$$\varepsilon_1 = \varepsilon(\varphi = 90) = \frac{1}{2} \frac{\frac{1}{2} m_1 g l \cdot \left(-\frac{1}{6} m_1 l^2 + 0\right) - 0}{\left(\frac{1}{6} m_1 l^2 + 0\right)^2} = \frac{36}{24} \frac{m_1^2 g l^3}{m_1^2 l^4} \Rightarrow \boxed{\varepsilon_1 = \frac{3}{2} \frac{g}{l}}$$

$$\underline{m_1 \vec{a}_c = m_1 \vec{g} + \vec{X}_0 + \vec{Y}_0 + \vec{S}}$$

$x:\quad m_1 \cdot \left(-\frac{1}{2} l \omega_1^2\right) = X_0 + S$

$$-\frac{1}{2} m_1 l \frac{3\sqrt{3} g}{2 l} = X_0 + S$$

$$-\frac{3\sqrt{3}}{4} m_1 g = X_0 + S \quad (1)$$

$y:\quad m_1 \left(-\frac{1}{2} l \varepsilon_1\right) = -m_1 g + Y_0$

$$-\frac{1}{2} m_1 l \cdot \frac{3g}{2l} = -m_1 g + Y_0$$

$$-\frac{3}{4} m_1 g = -m_1 g + Y_0$$

$$\boxed{Y_0 = \frac{1}{4} m_1 g}$$

$$\underline{m_2 \vec{a}_B = m_2 \vec{g} + \vec{N}_B + \vec{S}}$$

$x:\quad m_2 \left(-2 l \omega_1^2\right) = -S \quad / \cdot (-1)$

$$2 m_2 l \cdot \frac{3\sqrt{3} g}{2 l} = S$$

$$S = 3\sqrt{3}\, m_2 g$$

$$(1) \Rightarrow X_0 = -\frac{3\sqrt{3}}{4} m_1 g - 3\sqrt{3}\, m_2 g$$

$$\boxed{X_0 = -\frac{3\sqrt{3}}{4} g (m_1 + 4 m_2)}$$

$$\underline{\frac{d\vec{L}_{C_1}}{dt} = \sum \vec{M}_{C_1}}$$

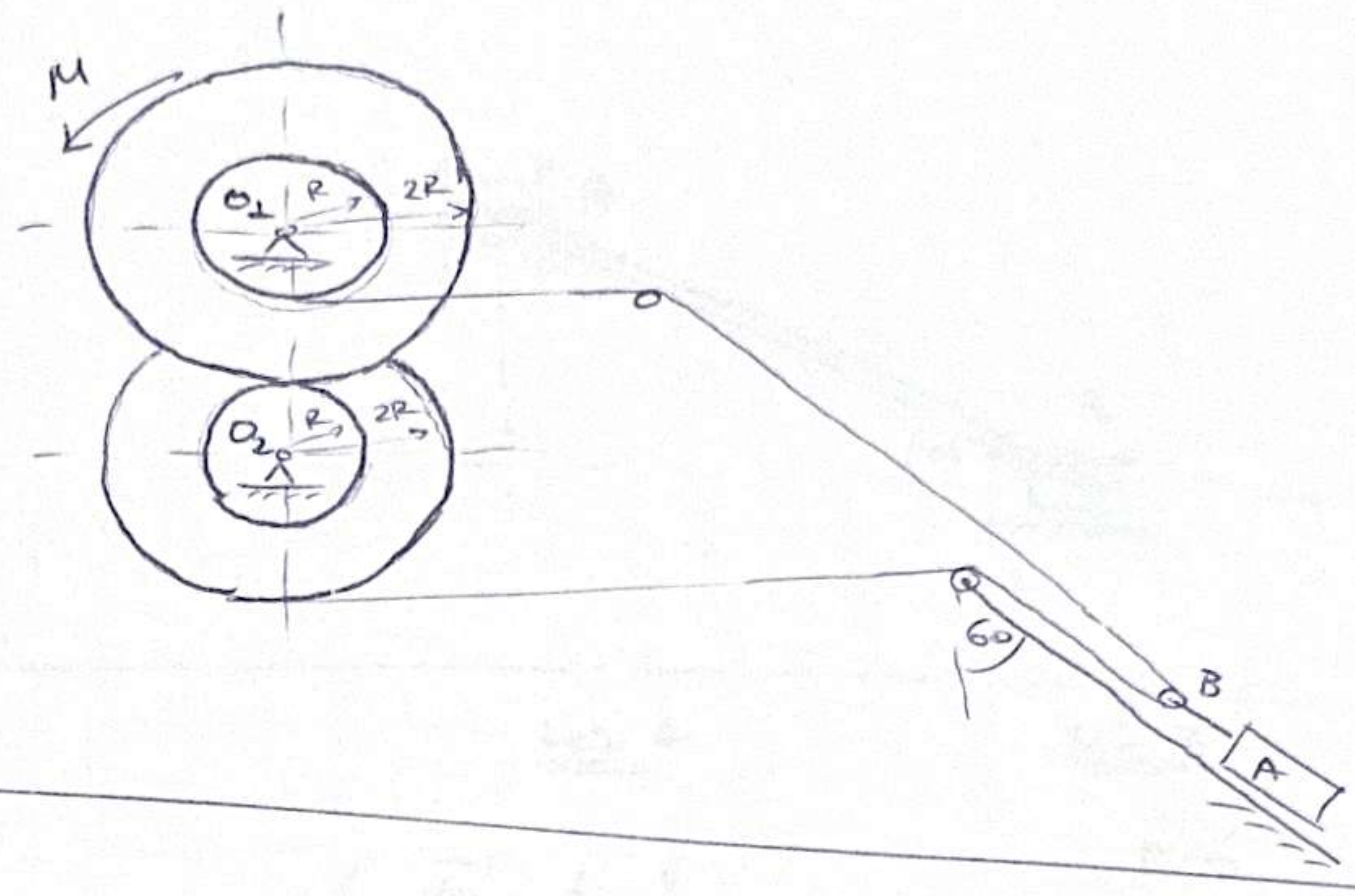
... (ne dobijam više)

10.52.

A: m, podiže se

župčanici, koturovi, užad: $\emptyset$

$M = ?$ – da li $S_{AB} = mg$



$S_{AB} = mg$ – uslov zadatka

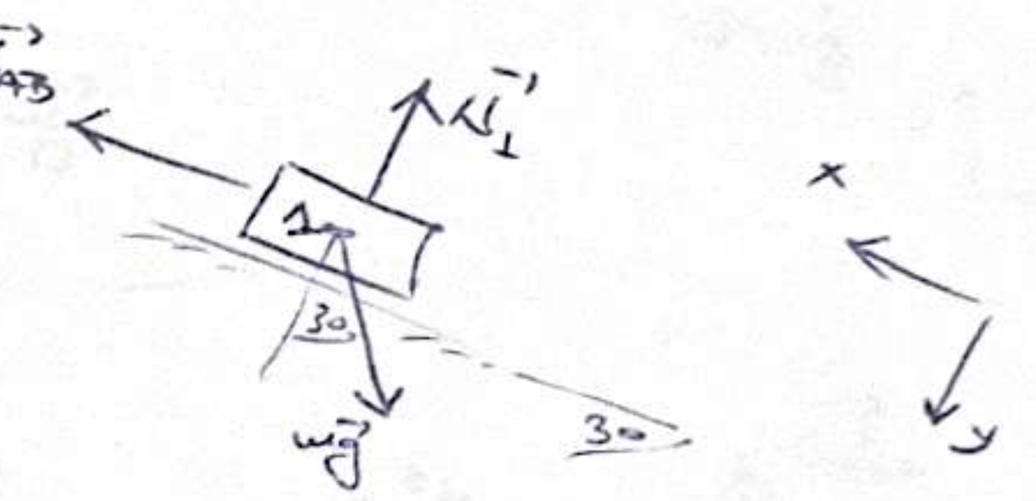
$$m\vec{a}_1 = m\vec{g} + \vec{N} + \vec{S}_{AB}$$

y: --- (nepotrebno)

x: $m\ddot{x}_1 = -mg\sin 30 + S_{AB}$

$\cancel{m}\ddot{x}_1 = -\frac{1}{2}\cancel{m}g + \cancel{m}g$

$$\ddot{x}_1 = \frac{1}{2}mg$$

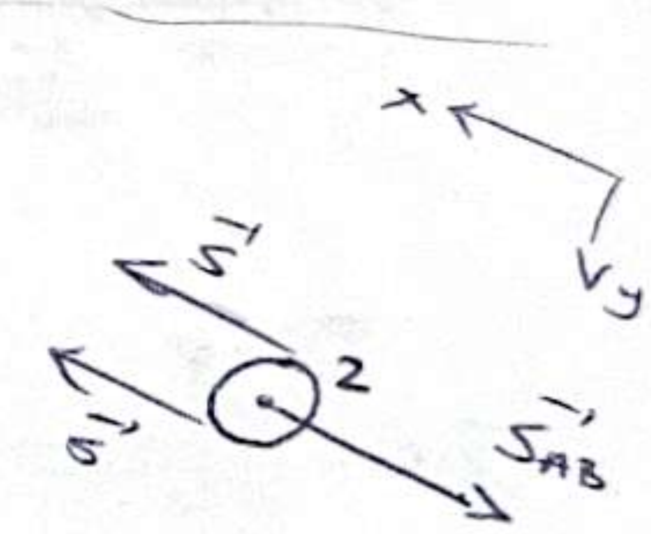


$$m\vec{a}_2 = \vec{S}_{AB} + \vec{S} + \vec{S}$$

x: $0 = -S_{AB} + 2S$

$S = \frac{1}{2}S_{AB}$

$$S = \frac{1}{2}mg$$



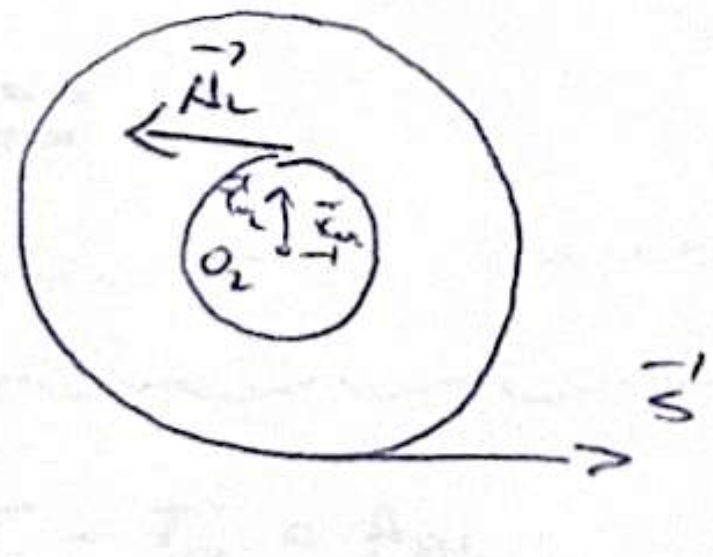
$$\frac{d\vec{L}_{O_2 z}}{dt} = \sum \vec{M}_{O_2 z}$$

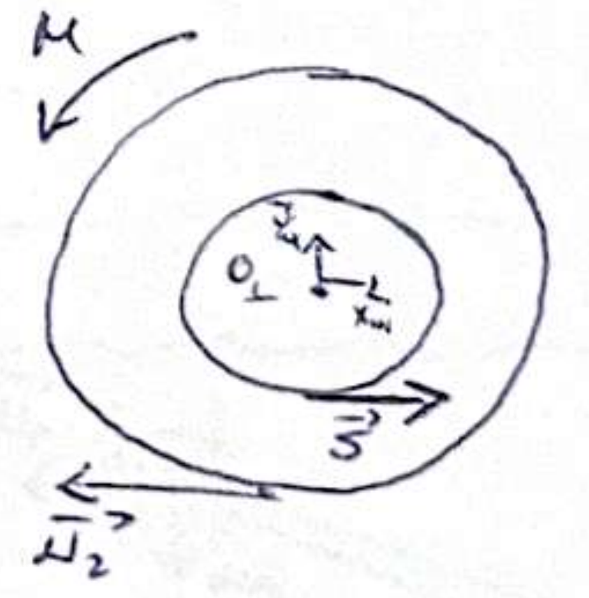
$\sum M_{O_2 z} = N_2 \cdot R - S \cdot 2R$

$L_{O_2 z} = 0 \Rightarrow \frac{dL_{O_2 z}}{dt} = 0$

$0 = N_2 \cdot R - \frac{1}{2}mg \cdot 2R$

$$N_2 = mgR$$





$$\frac{d\vec{L}_{O_1 z}}{dt} = \sum \vec{M}_{O_1 z}$$

$\sum M_{O_1 z} = M + S \cdot R - N_2 \cdot 2R$

$L_{O_1 z} = 0 \Rightarrow \frac{dL_{O_1 z}}{dt} = 0$

$0 = M + \frac{1}{2}mgR - 2mgR$

$$\boxed{M = \frac{3}{2}mgR}$$

10.53.

A: m

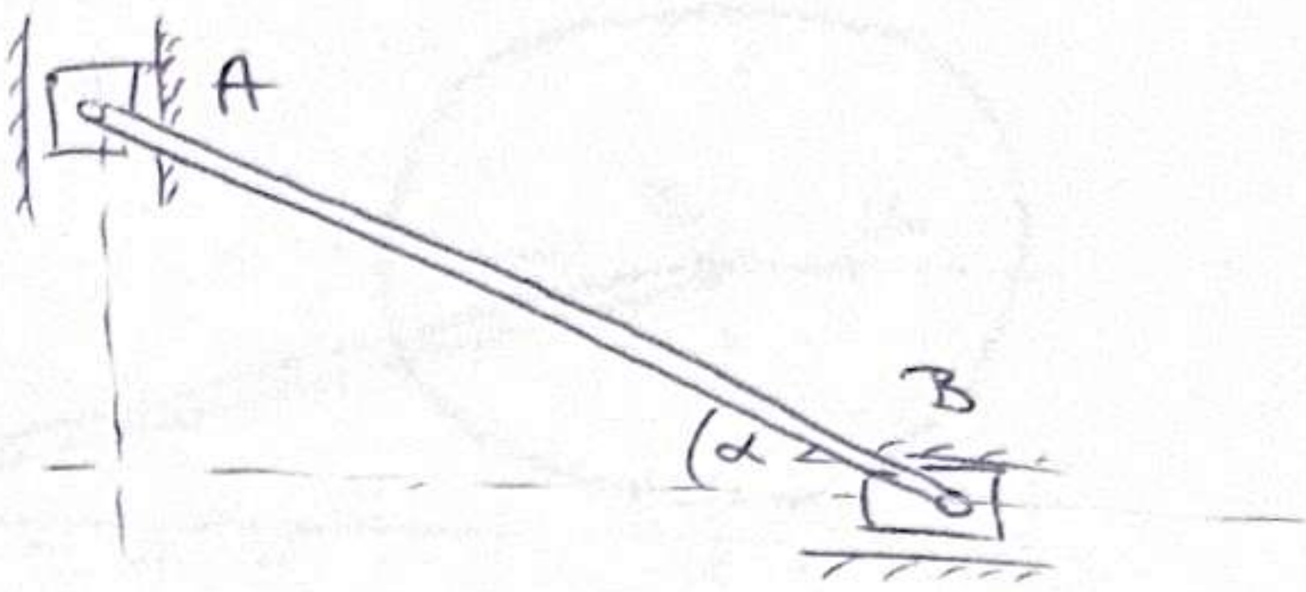
B: m

AB: $\emptyset$, l

t_0: sistem miruje

$\alpha_0 = 30°$

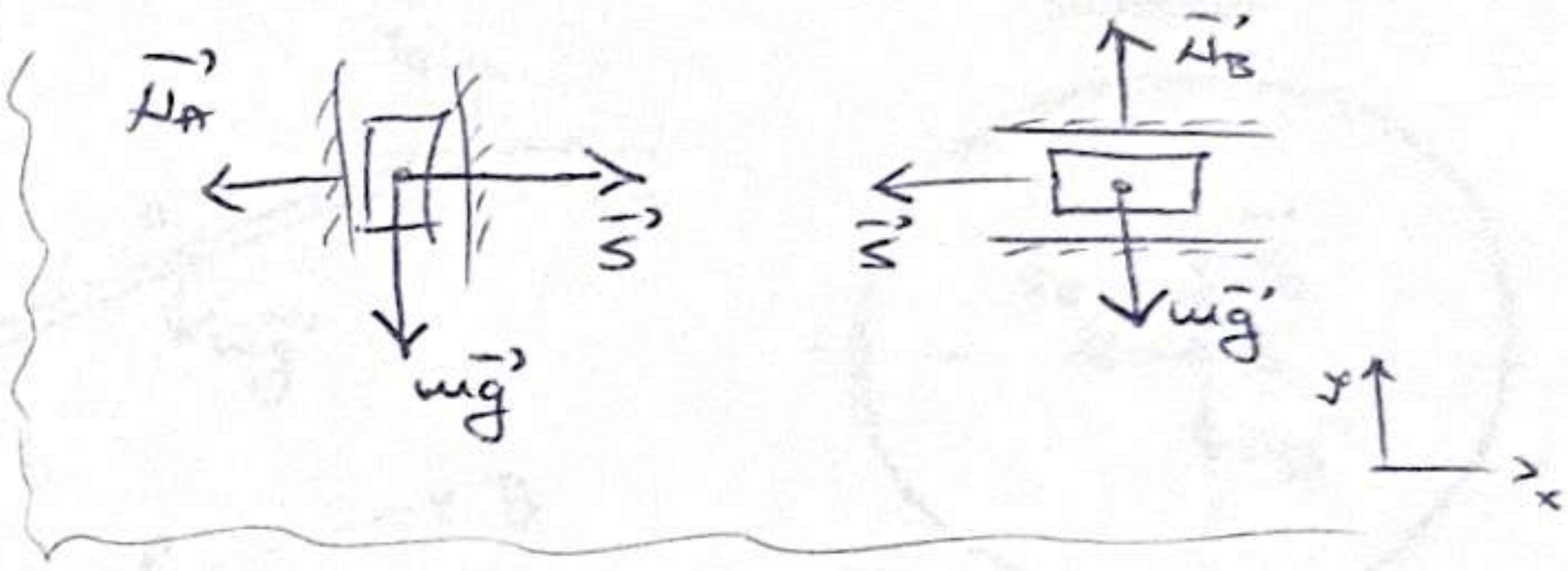
$S(t_1) = ?$, t_1 - AB je horizontalan



Posmotramo sve za trenutak t_1:

telo A

telo B



kinematska veza:

$\vec{a}_{A1} = ?$

x: $\ddot{x}_{A1} = 0$

y: $\ddot{y}_{A1} = -a_{A1} = ?$

$\vec{a}_{B1} = \vec{a}_{A1} + \vec{a}^{A}_{Bn1} + \vec{a}^{A}_{Bt1}$

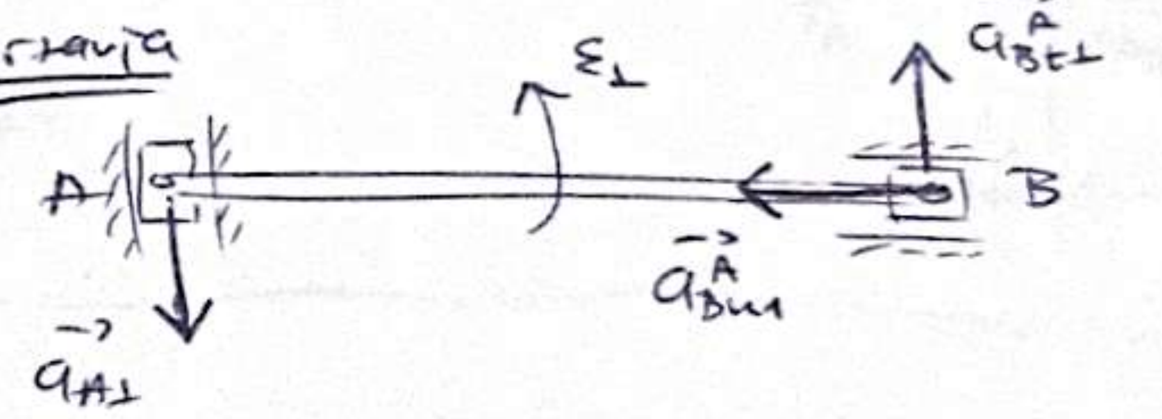
x: $\ddot{x}_{B1} = 0 - a^{A}_{Bn1} + 0$

$\ddot{x}_{B1} = -\omega_1^2 l$, $\ddot{y}_{B1} = 0$

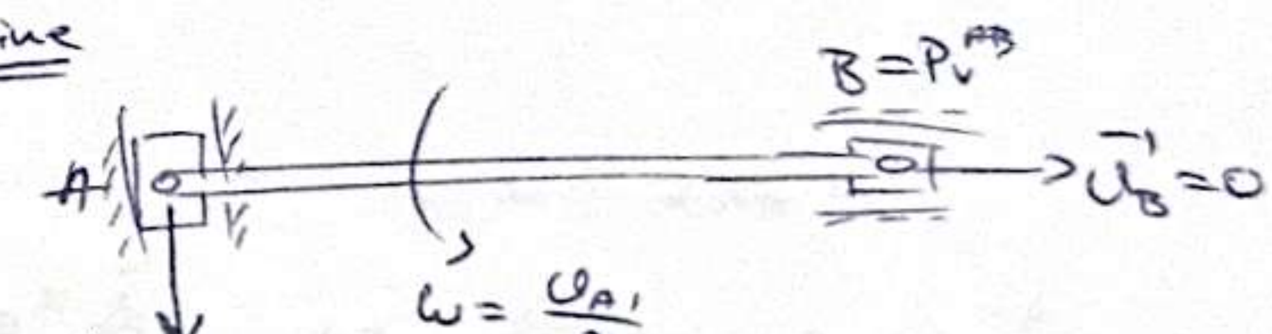
y: $0 = -a_{A1} + 0 + a^{A}_{Bt1}$

$\ddot{y}_{A1} = -\varepsilon_1 \cdot l$, $\ddot{x}_{A1} = 0$

Ubrzanja



Brzine



$m\vec{a}_{A1} = m\vec{g} + \vec{N}_A + \vec{S}$

x: $0 = S - N_A$

$S = N_A$ (1)

y: $m\ddot{y}_{A1} = -mg$ $/\cdot(-1)$

$-\varepsilon_1 \cdot l = -g$

$\varepsilon_1 = \frac{g}{l}$

$m\vec{a}_{B1} = m\vec{g}' + \vec{N}_B + \vec{S}'$

x: $m\ddot{x}_{B1} = -S$

$-m\omega_1^2 l = -S$

$S = ml\omega_1^2$ (2)

cilj: naći ω_1!

$T_1 - T_0 = A_{01}$

$T_0 = 0$

$T_1 = [\frac{1}{2}m v_{A1}^2 + 0] + [\frac{1}{2}v_{B1}^2 \cdot m + 0] =$

$T_1 = \frac{1}{2}ml^2\omega_1^2$ [redacted] $+ 0 = \frac{1}{2}ml^2\omega_1^2$

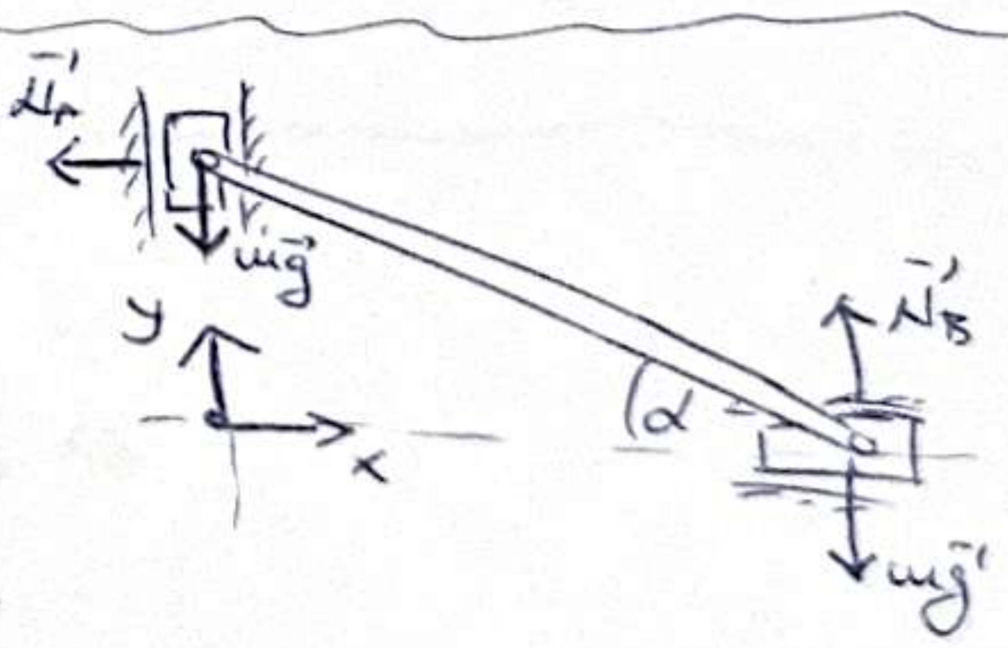
$A = \int m\vec{g}\, d\vec{r}_A = \int_{l\sin 30}^{0} -mg\, dy_A = \frac{1}{2}mgl$

$\frac{1}{2}ml^2\omega_1^2 = \frac{1}{2}mgl$

$\omega_1^2 = \frac{g}{l}$

(2) $\Rightarrow$ $S = ml\frac{g}{l}$

$\boxed{S = mg}$



(10.54.)

disk: m, R

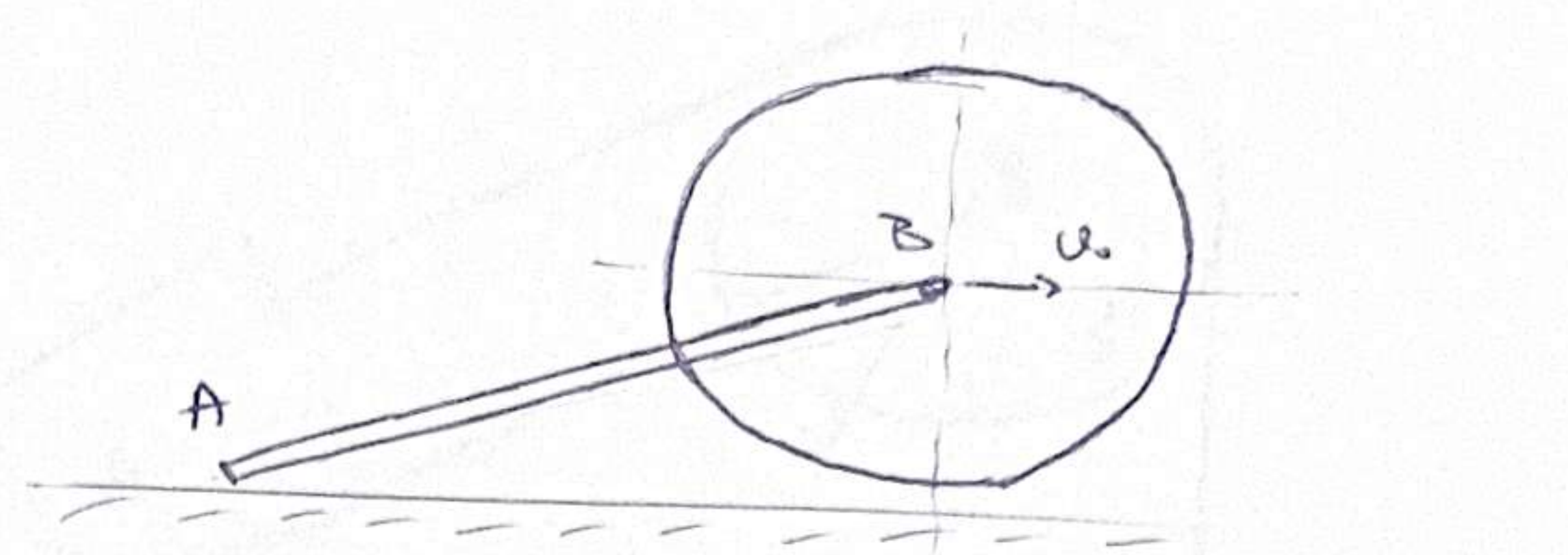
AB: m

$\mu = 0{,}1$

$k = \frac{1}{10} R$

t_0: $v_c(t_0) = v_0$

$s = ?$ - pređeni put diska do zaustavljanja



veza: $T_A = \mu \cdot N_A = \frac{1}{10} N_A$

kinematske veze: $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

$m\vec{a}_1 = m\vec{g} + \vec{N}_A + \vec{T}_A + \vec{X}_B + \vec{Y}_B$

x: $m\ddot{x} = -T_A + X_B$

$m\ddot{x} = -\frac{1}{10} N_A + X_B \quad (1)$

y: $0 = N_A - mg + Y_B \quad (2)$

$\frac{d\vec{L}_{1z}}{dt} = \sum \vec{M}_{1z}$

$\sum M_{1z} = N_A \cdot \frac{1}{2} h + T_A \cdot \frac{1}{2} R + X_B \cdot \frac{1}{2} R - Y_B \cdot \frac{1}{2} h$

$L_{1z} = 0 \Rightarrow \frac{dL_{1z}}{dt} = 0$

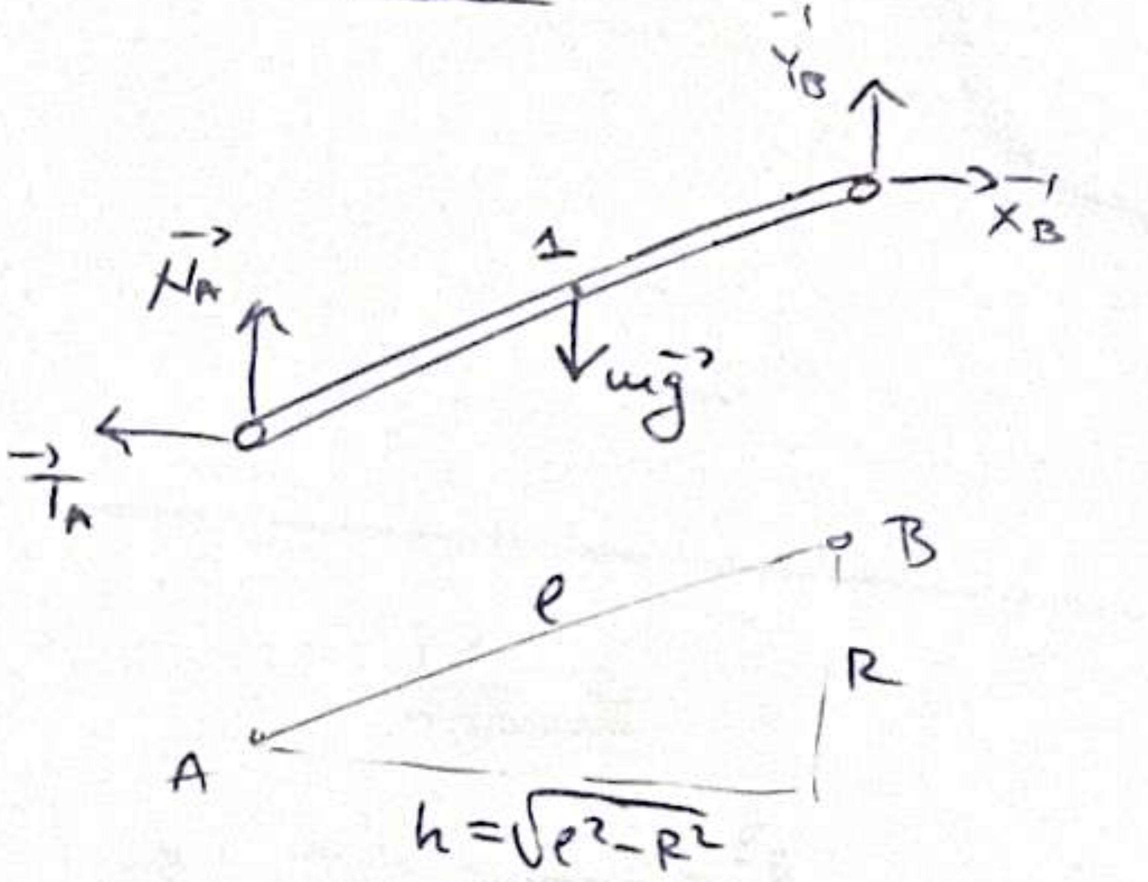
$0 = \frac{1}{2}\left(N_A \cdot h + \frac{1}{10} N_A \cdot R + X_B \cdot R - Y_B \cdot h\right) \quad (3)$

$m\vec{a}_2 = m\vec{g} + \vec{N}_B + \vec{T}_B + \vec{X}_B + \vec{Y}_B$

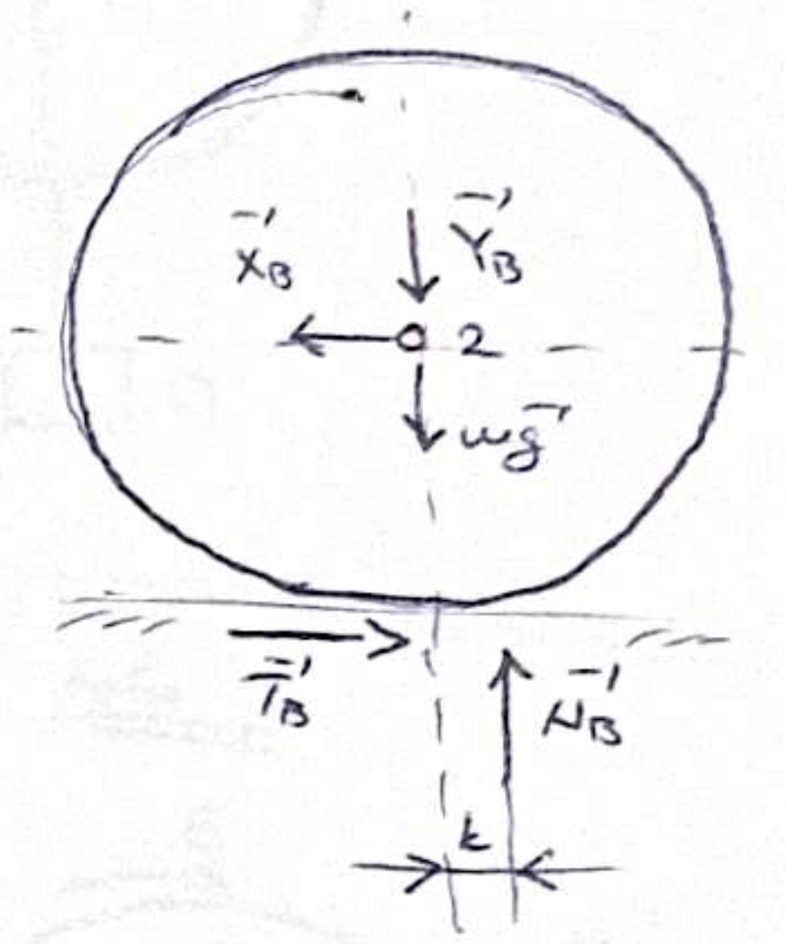
x: $m\ddot{x} = -X_B + T_B \quad (4)$

y: $0 = -mg - Y_B + N_B \quad (5)$

telo 1



telo 2



$\frac{d\vec{L}_{2z}}{dt} = \sum \vec{M}_{2z}$

$\sum M_{2z} = -T_B \cdot R - N_B \cdot k = -T_B \cdot R - \frac{1}{10} N_B R$

$L_{2z} = J_2 \omega = \frac{1}{2} m R^{\cancel{2}} \frac{\dot{x}}{\cancel{R}} \Rightarrow \frac{dL_{2z}}{dt} = \frac{1}{2} m R \ddot{x}$

$\frac{1}{2} m R \ddot{x} = -T_B \cdot R - \frac{1}{10} N_B \cdot R \quad / \cdot \frac{2}{R}$

$m\ddot{x} = -2T_B - \frac{1}{5} N_B \quad (6)$

$(4) \Rightarrow T_B = m\ddot{x} + X_B$

$(5) \Rightarrow N_B = mg + Y_B$

$(1) \Rightarrow X_B = m\ddot{x} + \frac{1}{10} N_A$

$(2) \Rightarrow N_A = mg - Y_B$

$(6) \Rightarrow m\ddot{x} = -2\left(m\ddot{x} + m\ddot{x} + \frac{1}{10}(mg - Y_B)\right) - \frac{1}{5}(mg + Y_B)$

$m\ddot{x} = -4m\ddot{x} - \frac{1}{5} mg + \cancel{\frac{1}{5} Y_B} - \frac{1}{5} mg - \cancel{\frac{1}{5} Y_B}$

$5m\ddot{x} = -\frac{2}{5} mg \quad / : 5m \quad / \int dt$

$\dot{x} = -\frac{2}{25} g t + v_0 \quad / \int dt$

$x = -\frac{1}{25} g t^2 + v_0 t$

$\dot{x}(t_1) = 0 \Rightarrow -\frac{2}{25} g t_1 + v_0 = 0 \Rightarrow t_1 = \frac{25}{2} \frac{v_0}{g}$

$s = x(t_1) = -\frac{1}{\cancel{25}} \cancel{g} \frac{25^{\cancel{2}}}{4} \frac{v_0^2}{g^{\cancel{2}}} + v_0 \cdot \frac{25}{2} \frac{v_0}{g} = \frac{v_0^2}{g}\left(-\frac{25}{4} + \frac{25}{2}\right)$

$$\boxed{s = \frac{25}{4} \frac{v_0^2}{g}}$$

(10. 55)

A: G_1, μ

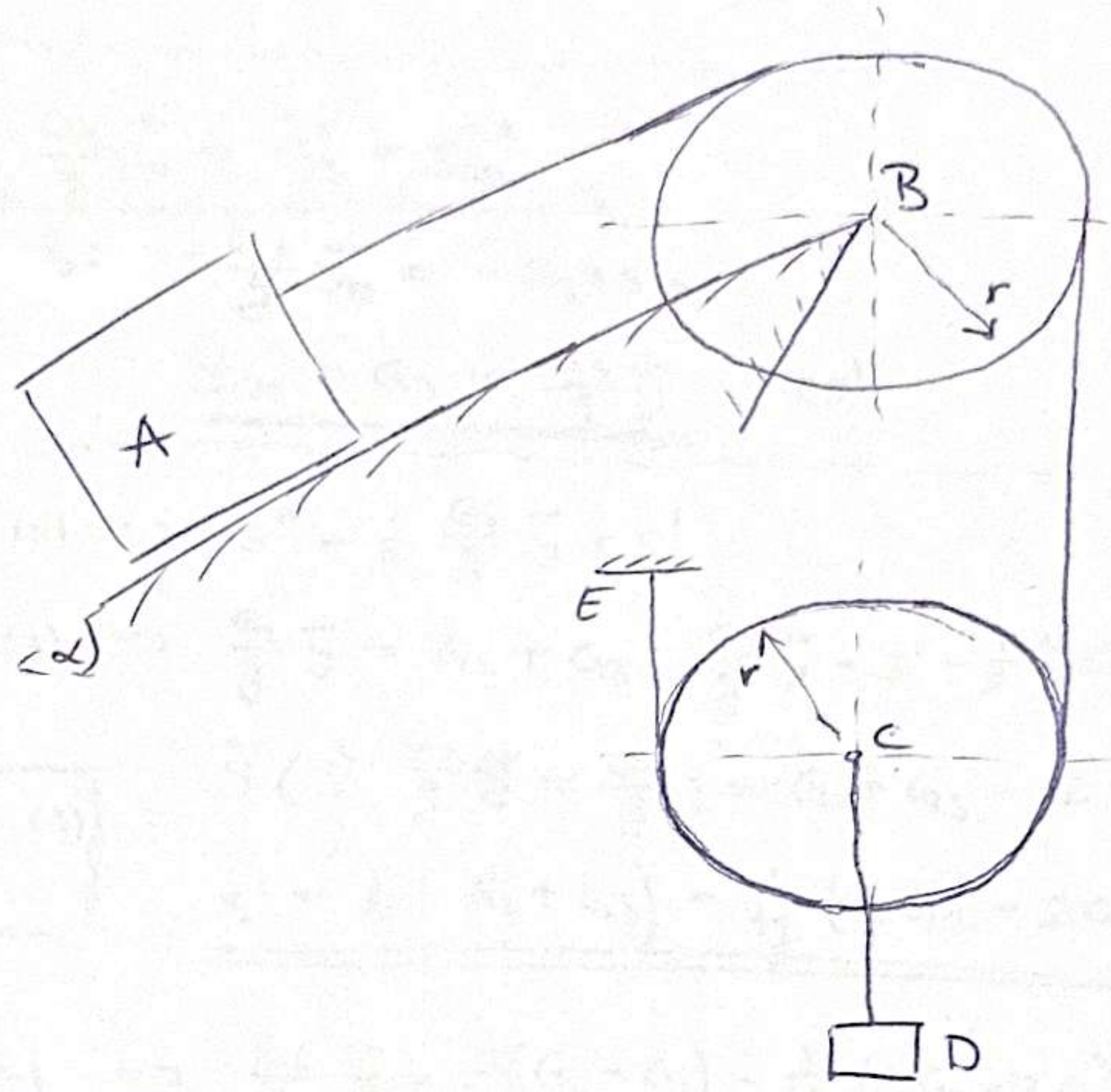
B: G_2, μ

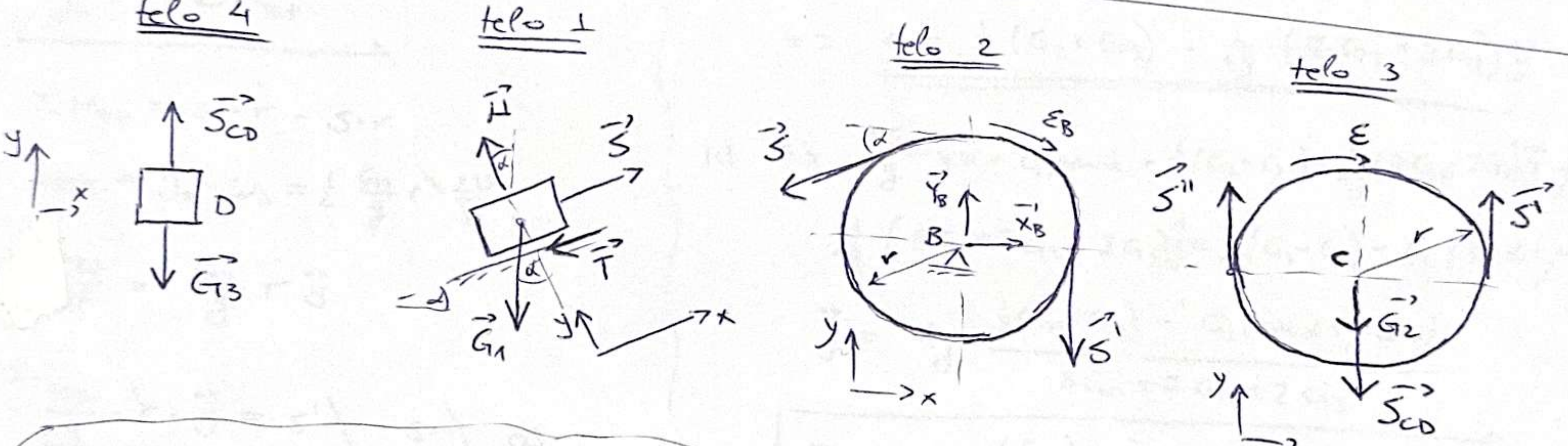
C: G_2, μ

D: G_3

$a_D = ?$ $\quad S_{AB} = ?$

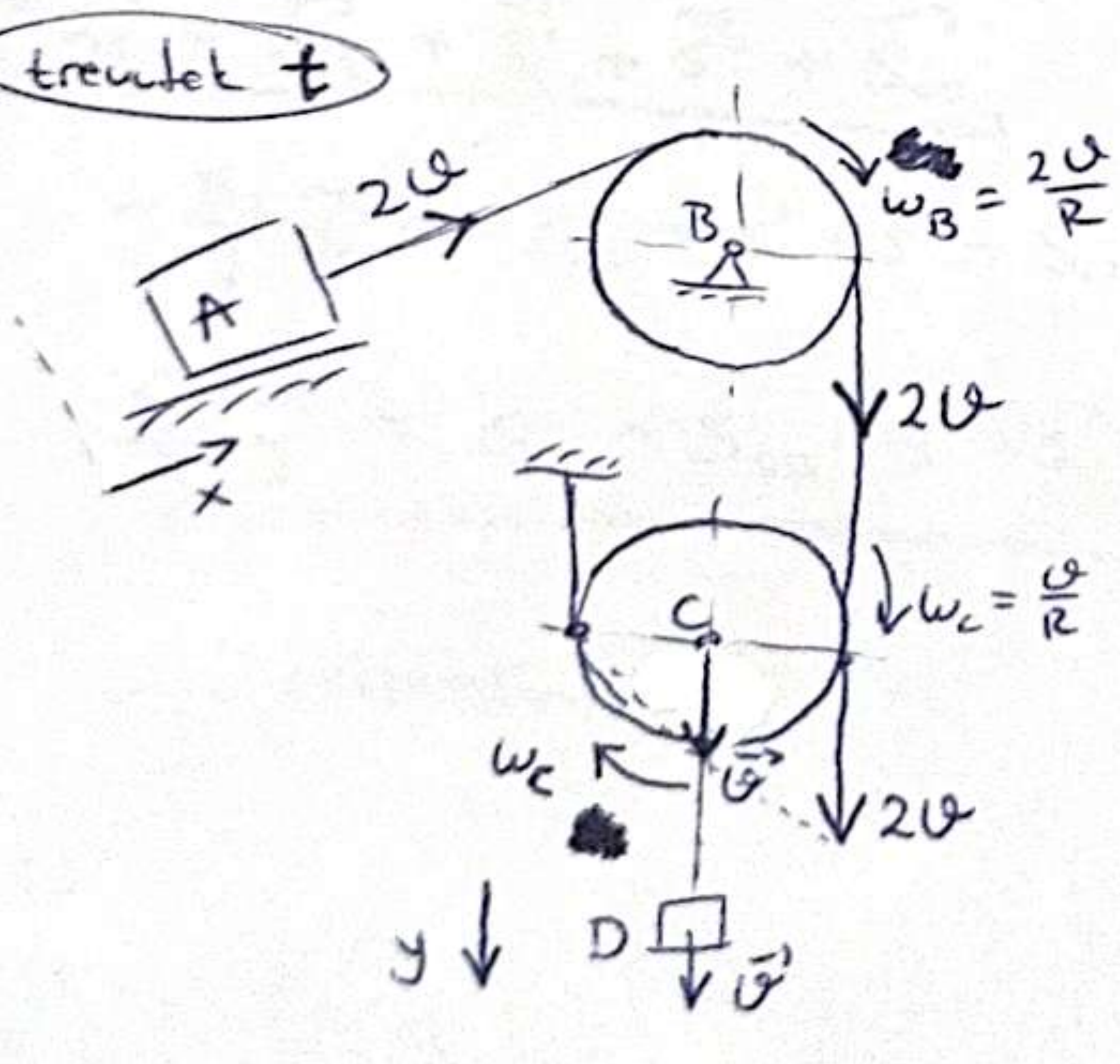
(teret se kreće uz strmu ravan)





kinematske veze:

trenutak t



(posmatramo opšti trenutak t, da bi mogli da diferenciramo)

izrazićemo sve preko brzine (i ubrzanja) tačke D:

neka je $\dot{y}_D = v_D$, i $\ddot{y}_D = \frac{dv}{dt} = \ddot{y}$,

pa odatle dobijamo:

$\dot{x}_A = 2v \Rightarrow \ddot{x}_A = 2\ddot{y}$

$\omega_B = \frac{2v}{r} \Rightarrow \varepsilon_B = \frac{2\ddot{y}}{r}$

$\omega_C = \frac{2v}{2r} \Rightarrow \varepsilon_C = \frac{\ddot{y}}{r}$

$$M\vec{a}_A = \vec{G}_1 + \vec{S} + \vec{N} + \vec{T}$$

y: $0 = -G_1\cos\alpha + N$

$N = G_1\cos\alpha$

$T = \mu_0 G_1\cos\alpha$

x: $\frac{G_1}{g}\ddot{x}_A = -G_1\sin\alpha + S - T$

$$\frac{2G_1}{g}\ddot{y} = -G_1\sin\alpha + S - \mu G_1\cos\alpha \quad (1)$$

$$\frac{d\vec{L}_{Bz}}{dt} = \sum \vec{M}_{Bz}$$

$\sum M_{Bz} = S'\cdot r - S\cdot r$

$L_{Bz} = J_B\cdot\omega_B = \frac{1}{2}\frac{G_2}{g}r^2\frac{2v}{r}$

$\frac{dL_{Bz}}{dt} = \frac{G_2}{g}r\ddot{y}$

$$\Rightarrow \frac{G_2}{g}\cdot\cancel{r}\cdot\ddot{y} = S'\cdot\cancel{r} - S\cdot\cancel{r} \quad (2)$$

$$\frac{G_2}{g}\vec{a}_C = \vec{G}_2 + \vec{S'} + \vec{S''} + \vec{S}_{CD}$$

y: $-\frac{G_2}{g}\ddot{y}_C = -G_2 + S' + S'' - S_{CD}$

$$\frac{G_2}{g}\ddot{y} = G_2 + S_{CD} - S' - S'' \quad (3)$$

x: – – (nepotreban)

$$\frac{dL_{Cz}}{dt} = \sum M_{Cz}$$

$\sum M_{Cz} = (S'' - S')\cdot r$

$L_{Cz} = J_C\omega_C = \frac{1}{2}\frac{G_2}{g}r^2\frac{v}{r}$

$\frac{dL_{Cz}}{dt} = \frac{1}{2}\frac{G_2}{g}r\ddot{y}$

$$\Rightarrow \frac{1}{2}\frac{G_2}{g}\cancel{r}\ddot{y} = (S'' - S')\cdot\cancel{r} \quad (4)$$

$$\frac{G_3}{g}\vec{a}_D = \vec{G}_3 + \vec{S}_{CD}$$

y: $-\frac{G_3}{g}\ddot{y}_D = -G_3 + S_{CD}$

$$S_{CD} = G_3 - \frac{G_3}{g}\ddot{y} \quad (5)$$

(4) $\Rightarrow$ $S'' = \frac{1}{2}\frac{G_2}{g}\ddot{y} + S'$

(3) $\Rightarrow$ $\frac{G_2}{g}\ddot{y} = G_2 + G_3 - \frac{G_3}{g}\ddot{y} - S' - \frac{1}{2}\frac{G_2}{g}\ddot{y} - S'$

$\ddot{y}\left(\frac{3}{2}\frac{G_2}{g} + \frac{G_3}{g}\right) = G_2 + G_3 - 2S'$

$$S' = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(3G_2 + 2G_3)\ddot{y}$$

(2) $\Rightarrow$ $\frac{G_2}{g}\ddot{y} = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(3G_2 + 2G_3)\ddot{y} - S$

$$\Rightarrow S = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(7G_2 + 2G_3)\ddot{y}$$

(1) $\Rightarrow$ $\frac{2G_1}{g}\ddot{y} = -G_1\sin\alpha + \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(7G_2 + 2G_3)\ddot{y} - \mu G_1\cos\alpha$

$\frac{1}{4g}(8G_1 + 7G_2 + 2G_3)\ddot{y} = \frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)$

$\ddot{y} = 4g\frac{\frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$

$$\boxed{\ddot{y} = 2g\frac{(G_2 + G_3) - 2G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}}$$

↑ ovo je dobro (kao u zbirci)

$\Rightarrow S = \frac{1}{2}(G_2 + G_3) - \frac{1}{\cancel{4g}}(7G_2 + 2G_3)\cdot\cancel{4g}\frac{\frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$

$$\boxed{S_{AB} = S = \frac{1}{2}(G_2 + G_3) - \frac{1}{2}(7G_2 + 2G_3)\frac{(G_2 + G_3) - 2G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}}$$

↑ ne poklapa se baš sa rešenjem, možda samo nisam sredio?

10.56.

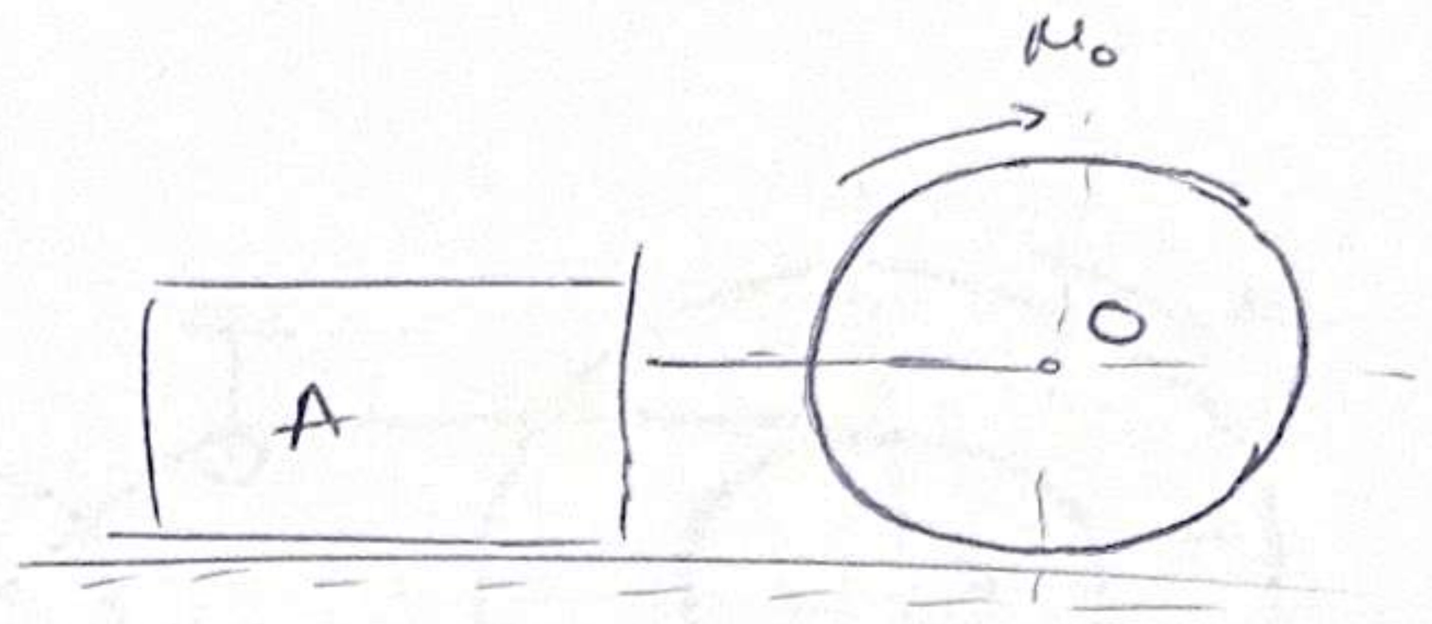
O: G_1, R, bez klizanja (kotrljanje), k - krak otpora kotrljanju, cilindar

A: G_2, μ - koef. trenja klizanja

M_o - moment

t_0: sistem je mirovao

$a_A = ?$ $\quad S = ?$



Kinematska veza: $\ddot{x}_O = \ddot{x}_A = \ddot{x}$

$$\frac{G_1}{g}\vec{a}_1 = \vec{G}_1 + \vec{N}_1 + \vec{T}_1 + \vec{S}$$

y: $0 = -G_1 + N_1$

$N_1 = G$

x: $\frac{G_1}{g}\ddot{x} = T_1 - S \quad (1)$

$$\frac{d\vec{L}_{1z}}{dt} = \Sigma \vec{M}_{1z}$$

$\Sigma M_{1z} = M_o - T_1 \cdot R - N_1 \cdot k$

$L_{1z} = J_1 \cdot \omega = \frac{G_1}{g} r^2 \frac{\dot{x}}{R}$

$\frac{dL_{1z}}{dt} = \frac{G_1}{g} r \ddot{x}$

$\frac{G_1}{g} R \ddot{x} = M_o - T_1 \cdot R - G_1 \cdot k \quad (2)$

$$\frac{G_2}{g}\vec{a}_2 = \vec{G}_2 + \vec{N}_2 + \vec{T}_2 + \vec{S}$$

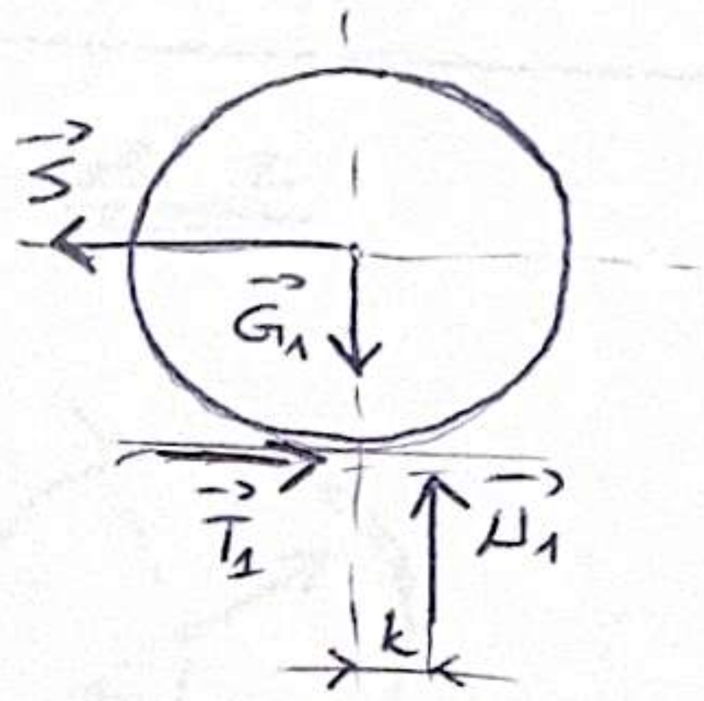
y: $0 = -G_2 + N_2$

$N_2 = G_2$

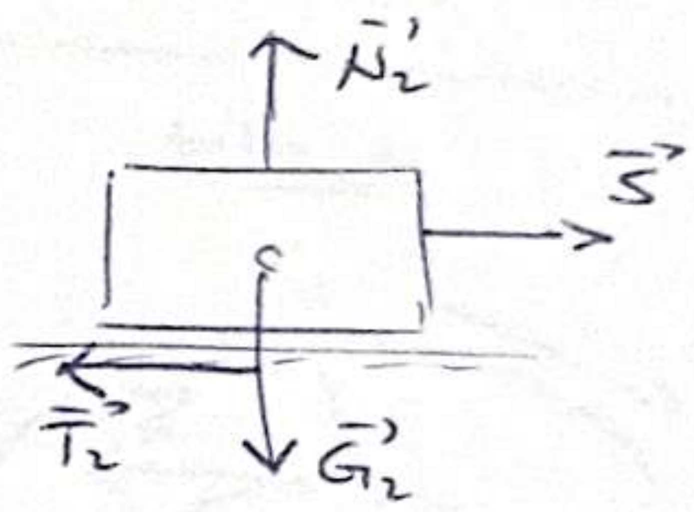
$T_2 = \mu \cdot G_2$

x: $\frac{G_2}{g}\ddot{x} = -T_2 + S$

$\frac{G_2}{g}\ddot{x} = S - \mu G_2 \quad (3)$



telo 2



(3) $\Rightarrow$ $S = \frac{G_2}{g}\ddot{x} + \mu G_2$

(1) $\Rightarrow$ $T_1 = \frac{G_1}{g}\ddot{x} + \frac{G_2}{g}\ddot{x} + \mu G_2$

(2) $\Rightarrow$ $\frac{G_1}{g} R \ddot{x} = M_o - \frac{G_1}{g} R \ddot{x} - \frac{G_2}{g} R \ddot{x} - \mu G_2 R - G_1 k$

$\frac{1}{g} r \ddot{x} (2G_1 + G_2) = M_o - \mu G_2 R - G_1 k$

$$\boxed{\ddot{x} = \frac{g}{R} \frac{M_o - \mu G_2 R - G_1 k}{2G_1 + G_2}}$$

ovo se poklapa s rešenjem

$\Rightarrow$ $S = \frac{G_2}{g} \frac{g}{R} \frac{M_o - \mu G_2 R - G_1 k}{2G_1 + G_2} + \mu G_2$

$$\boxed{S = \frac{G_2}{R} \frac{M_o - \mu G_2 R - G_1 k}{2G_1 + G_2} + \mu G_2}$$

ovo se nešto ne poklapa, možda jer nisam sredio?

10.57.

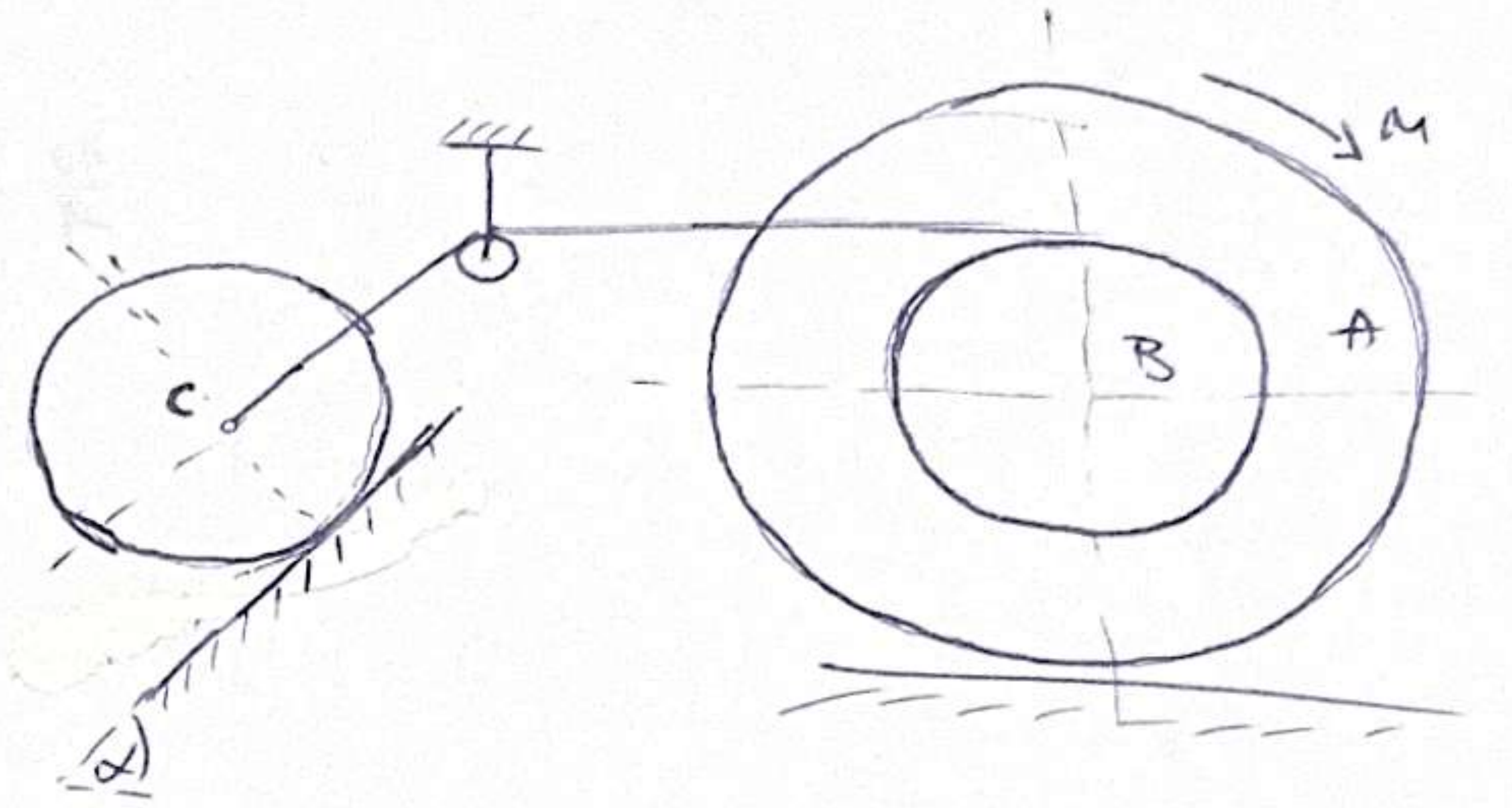
dva koaksijalna cilindra kruto spojena:

$A: \; 2R$
$B: \; R$ $\Big\}$ $G, \; i = R\sqrt{2}$

$C: \; R, \; G$

$M, \; \alpha$

$S = ?$



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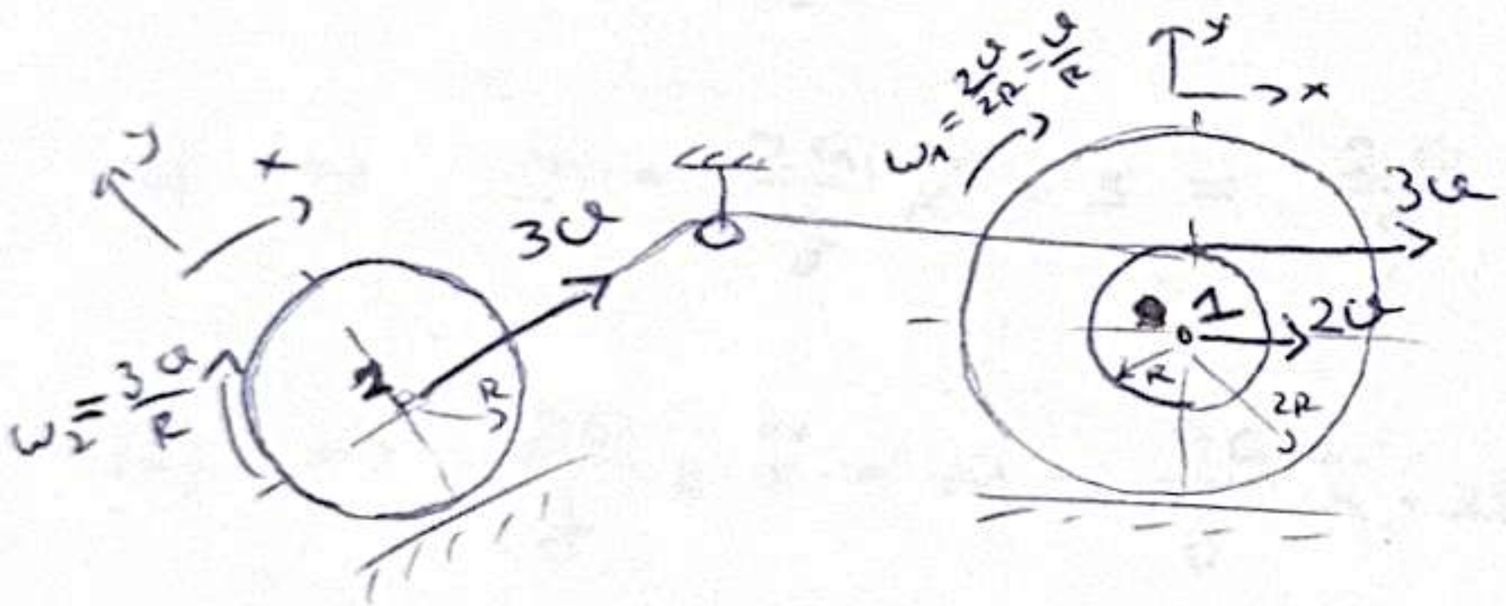
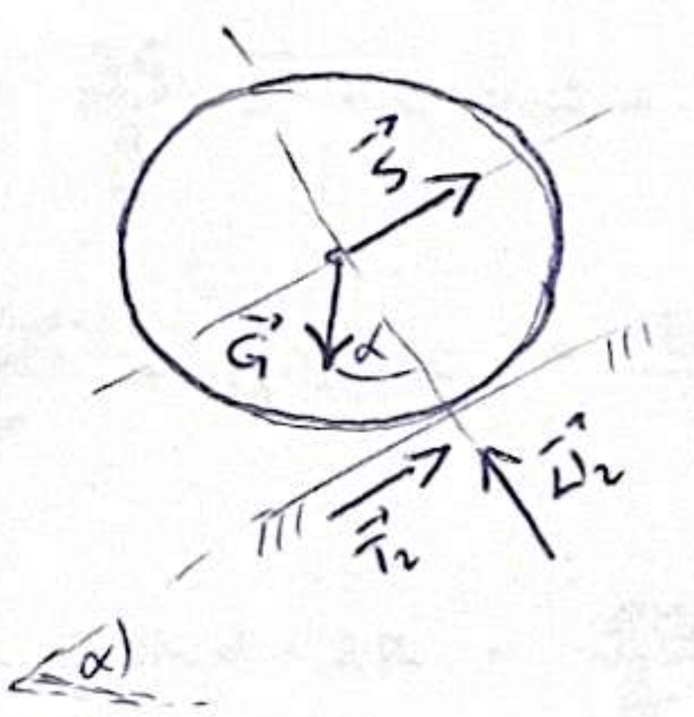
kinematska veza:

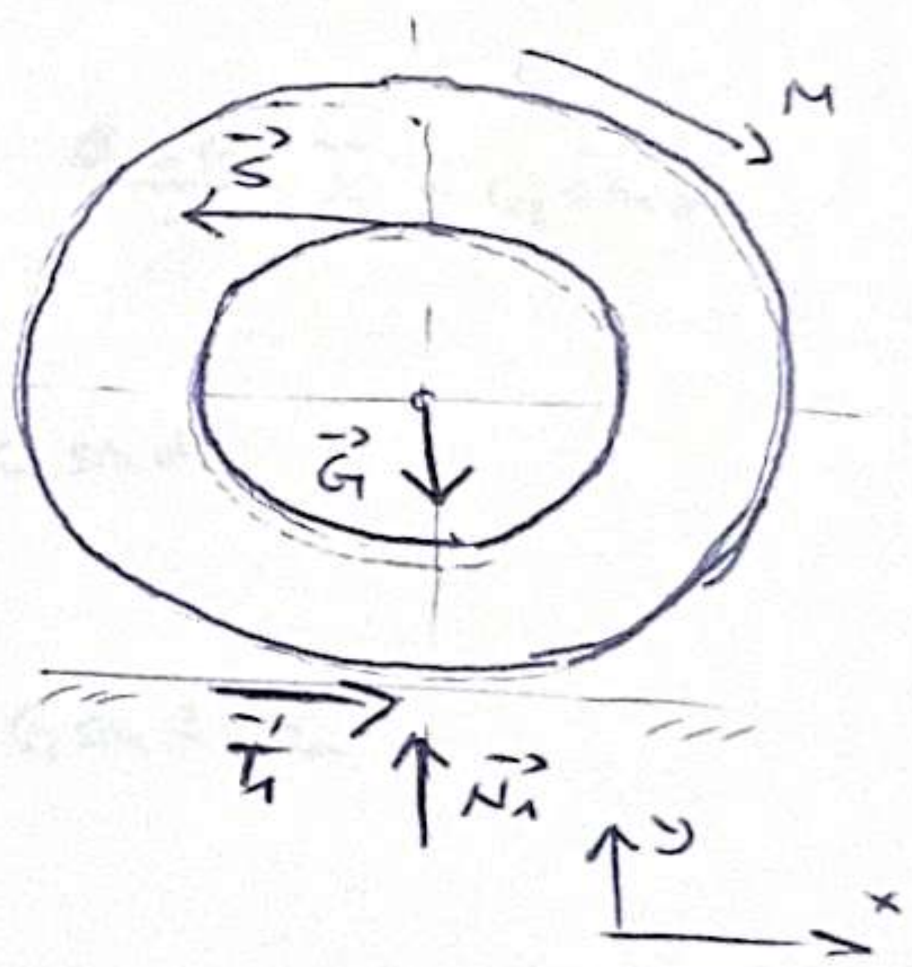
posmatramo opšti trenutak t
(da bi mogli da diferenciramo)



telo 2



telo 1



$\dot{x}_1 = 2v, \quad \ddot{x}_1 = 2\ddot{x} \; (=?)$

$\dot{x}_2 = 3v, \quad \ddot{x}_2 = 3\ddot{x}$

$\omega_1 = \frac{v}{R}, \quad \varepsilon_1 = \frac{\dot{x}}{R}$

$\omega_2 = \frac{v}{R}, \quad \varepsilon_2 = \frac{\dot{x}}{R}$

$$\frac{G}{g}\vec{a}_2 = \vec{G} + \vec{N}_2 + \vec{T}_2 + \vec{S}$$

$y: \quad 0 = -G\cos\alpha + N_2$

$N_2 = G\cos\alpha$

$x: \quad \frac{G}{g}\ddot{x}_2 = -G\sin\alpha + T_2 + S$

$\frac{G}{g}\cdot 3\ddot{x} = -G\sin\alpha + T_2 + S \quad (1)$

$$\frac{d\vec{L}_{2T}}{dt} = \Sigma\vec{M}_{2T}$$

$\Sigma M_{2T} = -T_2 \cdot R$

$L_{2T} = J_2 \cdot \omega_2 = \frac{G}{g}R^2 \cdot \frac{3v}{R}$

$\frac{dL_{2T}}{dt} = \frac{3G}{g}R\ddot{x}$

$$\frac{d\vec{L}_{2T}}{dt} = \Sigma\vec{M}_{2T}$$

$(2) \quad \frac{3G}{g}R\ddot{x} = -T_2 \cdot R$

$$\frac{G}{g}\vec{a}_1 = \vec{G} + \vec{N}_1 + \vec{T}_1 + \vec{S}$$

$y: \quad 0 = -G + N_1$

$N_1 = G$

$x: \quad \frac{G_1}{g}\ddot{x}_1 = T_1 - S$

$\frac{G_1}{g}\cdot 2\ddot{x} = T_1 - S \quad (3)$

$$\frac{d\overset{\curvearrowright}{L}_{1z}}{dt} = \Sigma \overset{\curvearrowright}{M}_{1z}$$

$$\Sigma M_{1z} = M - T_1 \cdot 2R - S \cdot R$$

$$L_{1z} = J_1 \cdot \omega_1 = r_0^2 \cdot \frac{G}{g} \cdot \frac{v}{R} = R^2 \cdot 2 \cdot \frac{G}{g} \cdot \frac{v}{R}$$

$$\frac{dL_{1z}}{dt} = \frac{2G}{g} R \ddot{x}$$

$$\Rightarrow \frac{2G}{g} R \ddot{x} = M - T_1 \cdot 2R - S \cdot R \qquad (4)$$

$$(2) \Rightarrow \quad T_2 = -\frac{3G}{g}\ddot{x}$$

$$(1) \Rightarrow \quad S = \frac{3G}{g}\ddot{x} + G\sin\alpha - T_2 = \frac{3G}{g}\ddot{x} + G\sin\alpha + \frac{3G}{g}\ddot{x} = \frac{6G}{g}\ddot{x} + G\sin\alpha$$

$$(3) \Rightarrow \quad T_1 = \frac{2G_1}{g}\ddot{x} + S = \frac{2G_1}{g}\ddot{x} + \frac{6G_1}{g}\ddot{x} + G\sin\alpha = \frac{8G_1}{g}\ddot{x} + G\sin\alpha$$

$$(4) \Rightarrow \quad \frac{2G}{g} R\ddot{x} = M - \frac{8G}{g}\ddot{x} \cdot 2R - G\sin\alpha \cdot 2R - \frac{6G}{g}\ddot{x}R - G\sin\alpha \cdot R$$

$$\frac{G}{g} R \ddot{x} \cdot 24 = M - 3GR\sin\alpha$$

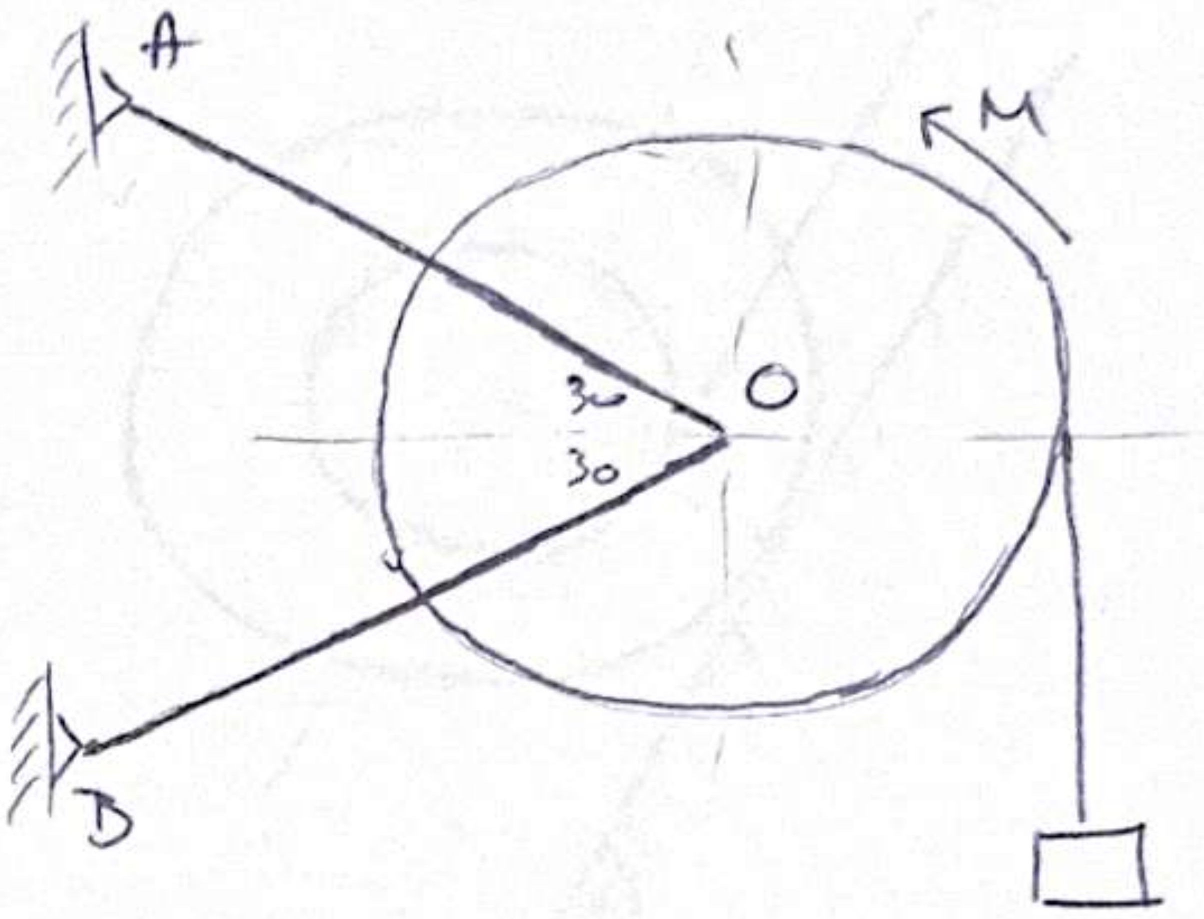
$$\ddot{x} = \frac{1}{24}\frac{g}{G}\left(\frac{M}{R} - 3G\sin\alpha\right)$$

$$(1) \Rightarrow \quad S = \frac{6G}{g} \cdot \frac{1}{24}\frac{g}{G}\left(\frac{M}{R} - 3G\sin\alpha\right) + G\sin\alpha = \frac{1}{4}\frac{M}{R} - \frac{3}{4}G\sin\alpha + G\sin\alpha$$

$$\boxed{S = \frac{1}{4}\frac{M}{R} + \frac{1}{4}G\sin\alpha}$$

(10. 58.)

$AO: \emptyset$
$BO: \emptyset$ } isth dužina



dobot: $G_2,\ r$

$M = 4rG_1$

teret: $G_1 = 3G_2$

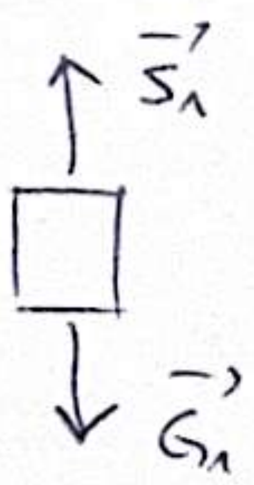
sile u štapovima ?

$$\underbrace{\frac{G_1}{g}\vec{a}_1' = \vec{G}_1 + \vec{S}_1'}$$

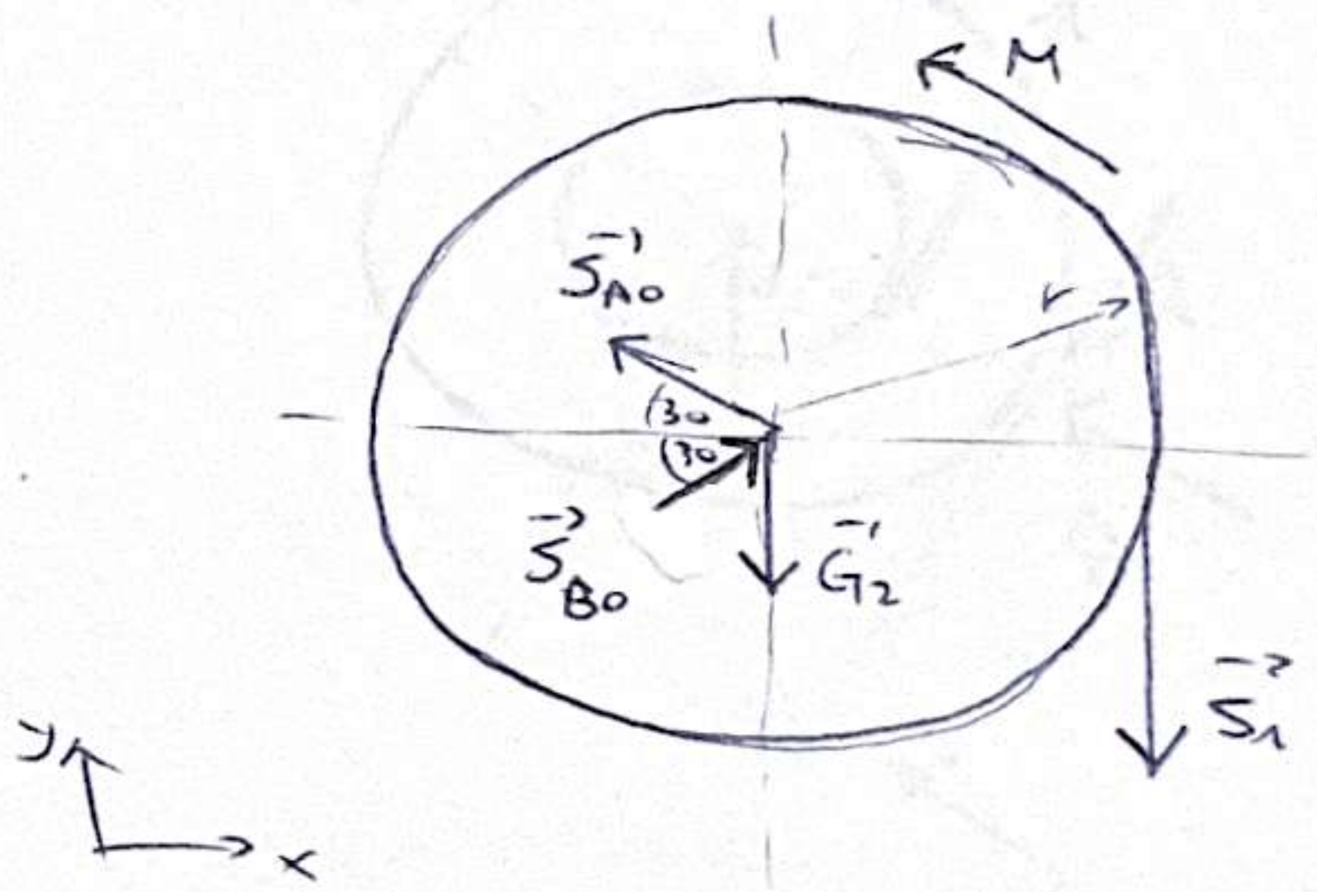
x: ______ (vertikalno)

y: $\underline{\frac{3G_2}{g}\ddot{y} = -3G_2 + S_1}$ (1)

telo 1



telo 2



$$\underbrace{\frac{G_2}{g}\vec{a}_2' = \vec{G}_2 + \vec{S}_1' + \vec{S}_{A0}' + \vec{S}_{B0}'}$$

x: $0 = -S_{A0}\cancel{\cos 30} + S_{B0}\cancel{\cos 30}$

$\underline{S_{A0} = S_{B0} = S}$

y: $0 = -G_2 + S\cdot\sin 30 + S\sin 30 - S_1$

$\underline{0 = -G_2 + S - S_1}$ (2)

$$\underbrace{\frac{d\overset{\frown}{L}_{2z}}{dt} = \Sigma \overset{\frown}{M}_{2z}}$$

$\Sigma M_{2z} = M - S_1\cdot R$

$L_{2z} = J_2\cdot\omega_2 = \frac{1}{2}\frac{G_1}{g}R^2\omega_2$

$\frac{dL_{2z}}{dt} = \frac{1}{2}\frac{G_2}{g}R^2\varepsilon_2 = \frac{1}{2}\frac{G_2}{g}R^{\cancel{2}}\frac{\ddot{y}}{\cancel{R}}$

$\underline{\frac{1}{2}\frac{G_2}{g}R\ddot{y} = M - S_1\cdot R}$ (3)

(1) $\Rightarrow$ $S_1 = \frac{3G_2}{g}\ddot{y} + 3G_2$

(2) $\Rightarrow$ $\frac{1}{2}\frac{G_2}{g}R\ddot{y} = M - \frac{3G_1}{g}\ddot{y}\cdot R - 3G_2\cdot R$

$\frac{7}{2}\frac{G_2}{g}R\ddot{y} = M - 3G_2R$

$\frac{G_2}{g}\ddot{y} = \frac{2}{7}\frac{1}{R}(M - 3G_2R)$

(1) $\Rightarrow$ $S_1 = 3\cdot\frac{2}{7}\cdot\frac{1}{R}(M - 3G_2R) + 3G_2$

$S_1 = \frac{6}{7}\frac{M}{R} - \frac{18}{7}G_2 + 3G_2$

$S_1 = \frac{6}{7}\frac{M}{R} + \frac{3}{7}G_2$

(2) $\Rightarrow$ $S = G_2 + \frac{6}{7}\frac{M}{R} + \frac{3}{7}G_2$

$S = \frac{10}{7}G_2 + \frac{6}{7}\frac{M}{R}$

$S = \frac{10}{7}G_2 + \frac{6}{7}\cdot\frac{4\cancel{R}\cdot 3G_2}{\cancel{R}}$

$\boxed{S = S_{A0} = S_{B0} = \frac{82}{7}G_2}$

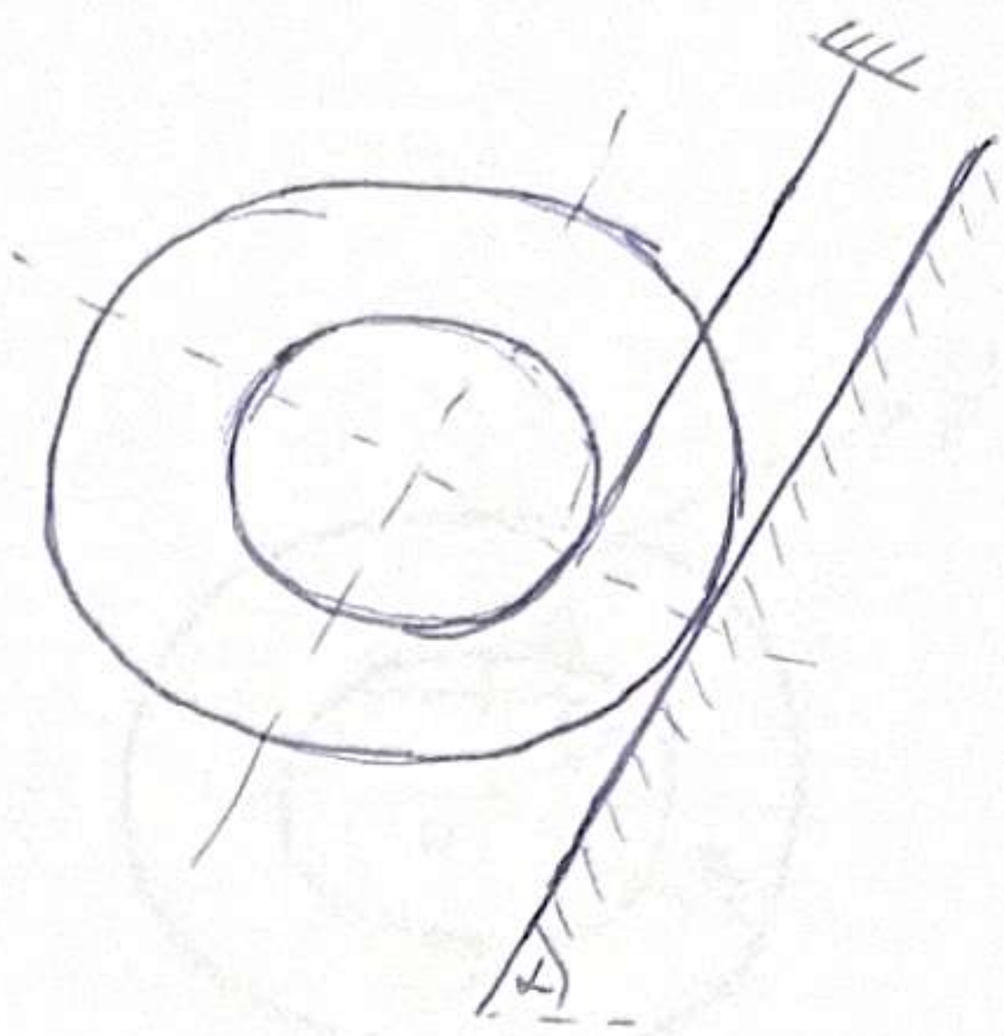
(10.61.)

$\alpha,\ \mu$

koaks. cilindar: $m,\ R,\ 2R,\ i_0 = R$

t_0: mirovanje

$S = ?$



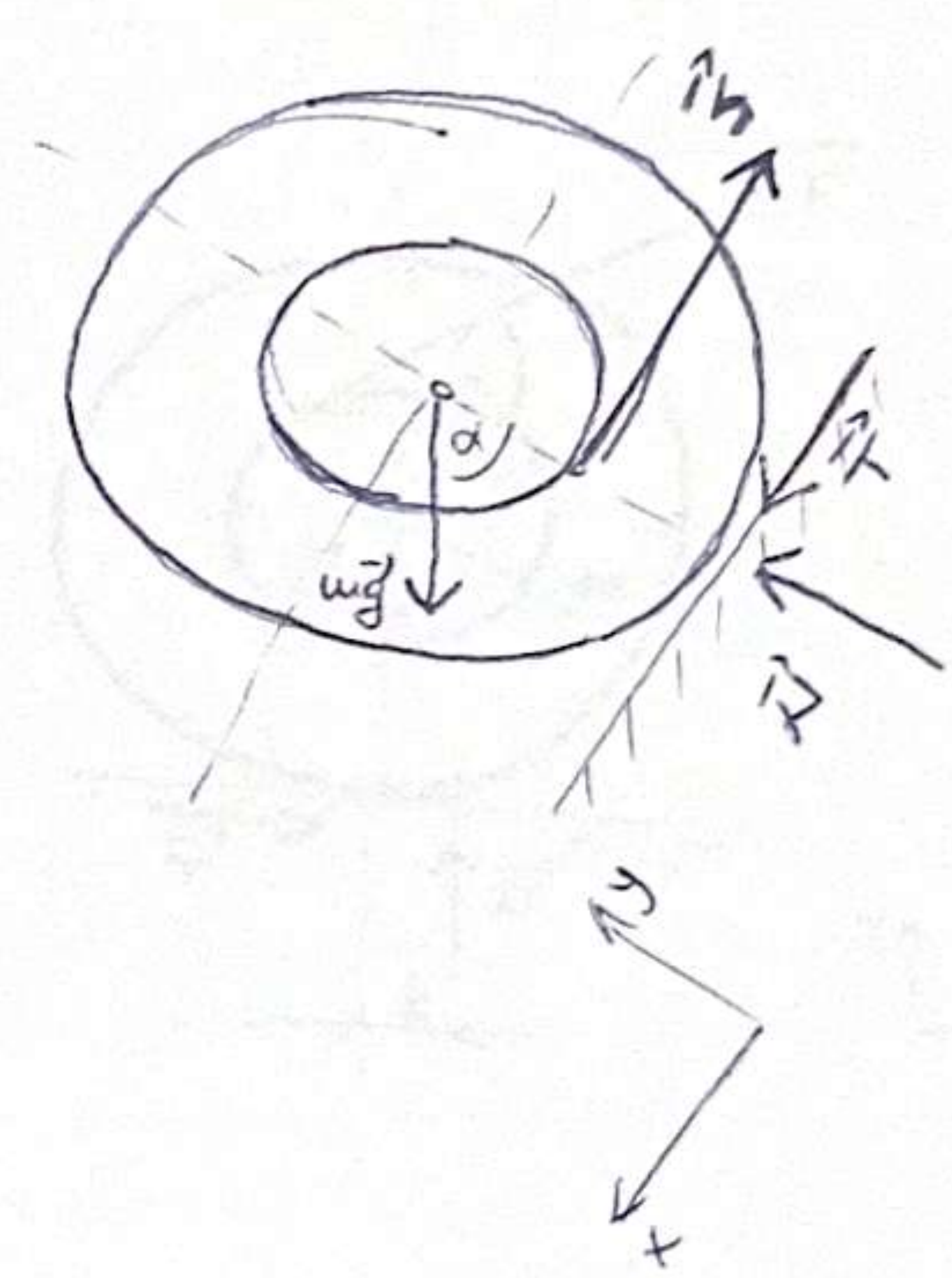
$$\underline{m\vec{a} = m\vec{g} + \vec{N} + \vec{T} + \vec{S}}$$

y: $0 = -mg\cos\alpha + N$

$\underline{\underline{N = mg\cos\alpha}}$

x: $m\ddot{x} = mg\sin\alpha + T - S$

$\underline{\underline{m\ddot{x} = mg\sin\alpha + \mu mg\cos\alpha - S}}$ (1)



$$\underline{\frac{d\overset{\curvearrowleft}{L}_{C\zeta}}{dt} = \sum \overset{\curvearrowleft}{M}_{C\zeta}}$$

$\sum M_{C\zeta} = S \cdot R - T \cdot 2R = S \cdot R - \mu \cdot mg\cos\alpha \cdot 2R$

$L_{C\zeta} = J_C \cdot \omega = R^2 \cdot m \cdot \omega$

$\frac{dL_{C\zeta}}{dt} = mR^2\varepsilon$

$mR^{\cancel{2}}\varepsilon = S \cdot \cancel{R} - 2\mu mg\cos\alpha \cdot \cancel{R}$

$\underline{\underline{mR\varepsilon = S - 2\mu mg\cos\alpha}}$ (2)

(1) $\Rightarrow$ $\underline{\underline{mR\varepsilon = mg\sin\alpha + \mu mg\cos\alpha - S}}$ (*)

(*) = (2)

$mg\sin\alpha + \mu mg\cos\alpha - S = S - 2\mu mg\cos\alpha$

$2S = mg\sin\alpha + 3\mu mg\cos\alpha$

$$\boxed{S = \frac{1}{2} mg(\sin\alpha + 3\mu\cos\alpha)}$$

10. 62.

$F = const.$

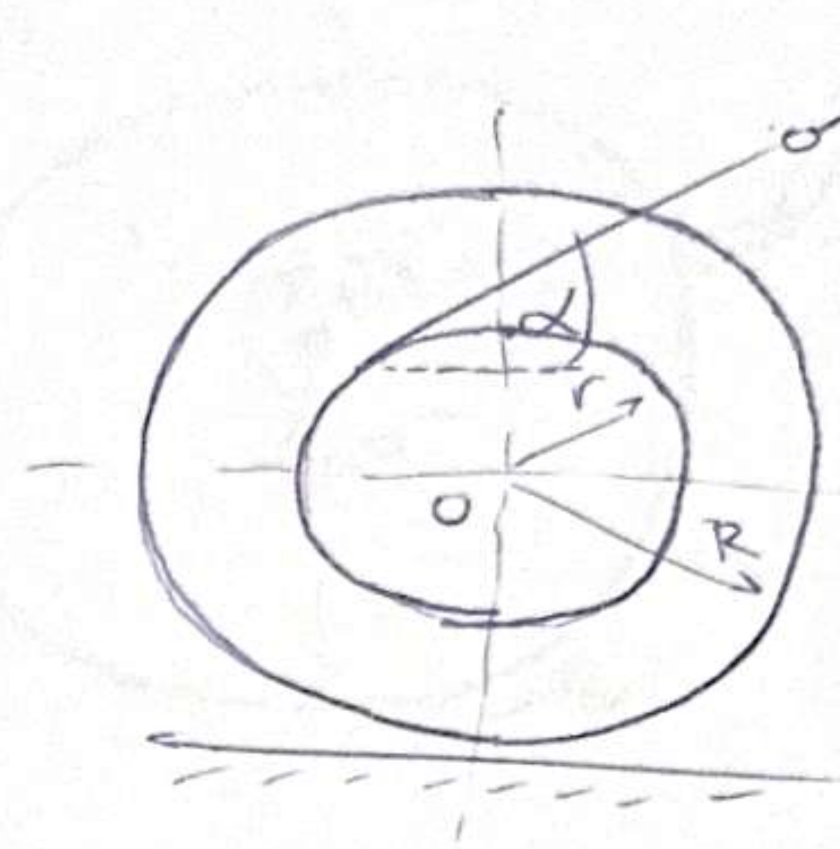
α, k

kooks. cilindri: G, r, R, i

kotrljanje bez klizanja

t_0: sistem miruje

v_0? s?

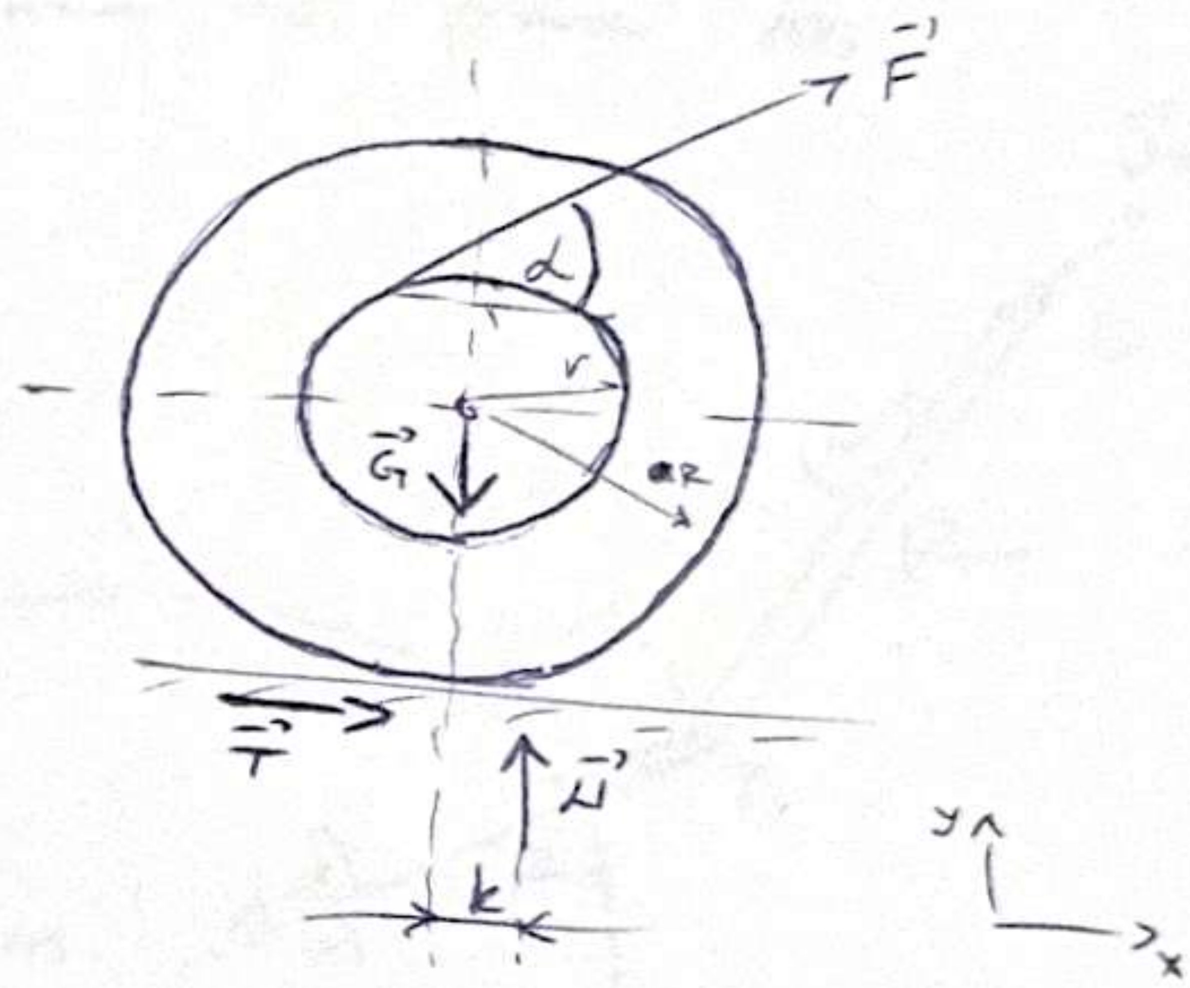


$$\frac{G}{g}\vec{a} = \vec{G} + \vec{F} + \vec{N} + \vec{T}$$

x: $\frac{G}{g}\ddot{x} = F\cos\alpha + T$ (1)

y: $0 = -G + F\sin\alpha + N$

$N = G - F\sin\alpha$



$$\frac{d\vec{L}_{Oz}}{dt} = \sum \vec{M}_{Oz}$$

$\sum M_{Oz} = F\cdot r - T\cdot R - N\cdot k = F\cdot r - T\cdot R - G\cdot k + F\sin\alpha\cdot k$

$L_{Oz} = R\cdot\frac{G}{g}v_0 + J_0\omega = R\cdot\frac{G}{g}R\omega + i^2\frac{G}{g}\omega = \frac{G}{g}(R^2+i^2)\omega$

$\frac{dL_{Oz}}{dt} = \frac{G}{g}(R^2+i^2)\varepsilon$

$\frac{G}{g}(R^2+i^2)\varepsilon = F\cdot r - T\cdot R - G\cdot k + F\sin\alpha\cdot k$ (2)

(1) => $\frac{G}{g}R\varepsilon = F\cos\alpha + T$ => $T = \frac{G}{g}R\varepsilon - F\cos\alpha$

(2) => $\frac{G}{g}(R^2+i^2)\varepsilon = F\cdot r - \frac{G}{g}R\varepsilon\cdot R + F\cos\alpha\cdot R - G\cdot k + F\sin\alpha\cdot k$

$\frac{G}{g}(2R^2+i^2)\varepsilon = F\cdot r + F\cos\alpha\cdot R - G\cdot k + F\sin\alpha\cdot k$

$\frac{G}{g}(2R^2+i^2)\frac{\ddot{x}}{R} = F(r + R\cos\alpha + k\sin\alpha) - G\cdot k$

$\ddot{x} = \frac{gR(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{G(2R^2+i^2)}$ $/ \int dt$

$$\boxed{v = \dot{x} = \frac{gRt(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{G(2R^2+i^2)}}$$ $/ \int dt$

$$\boxed{s = x = \frac{gRt^2(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{2G(2R^2+i^2)}}$$

u rešenju je (R^2+i^2) umesto $(2R^2+i^2)$

(10.63.)

disk: J

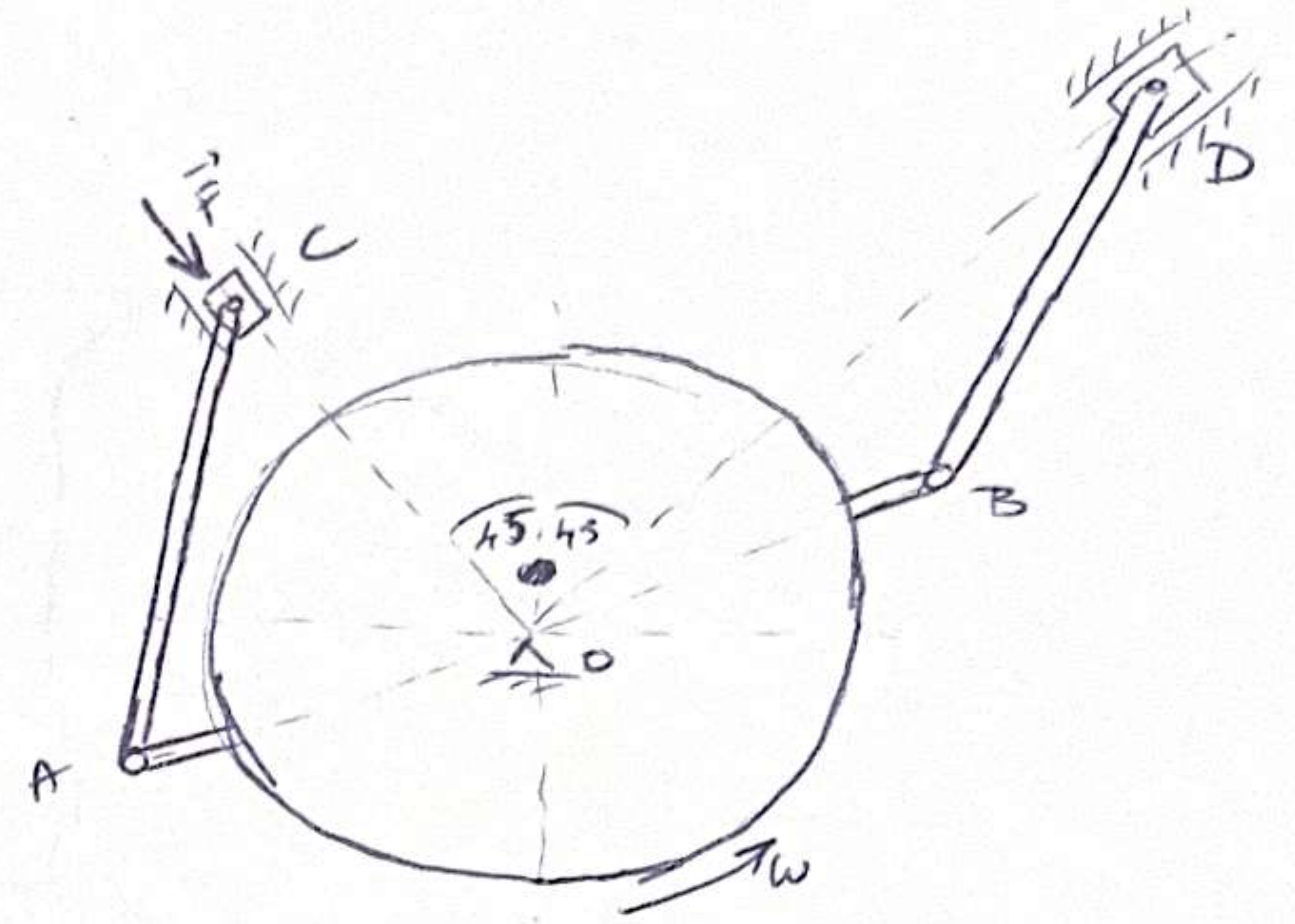
štapovi: $\emptyset$

klizači: m

$\overline{OA} = \overline{AC} = \overline{OB} = \overline{BD} = R$

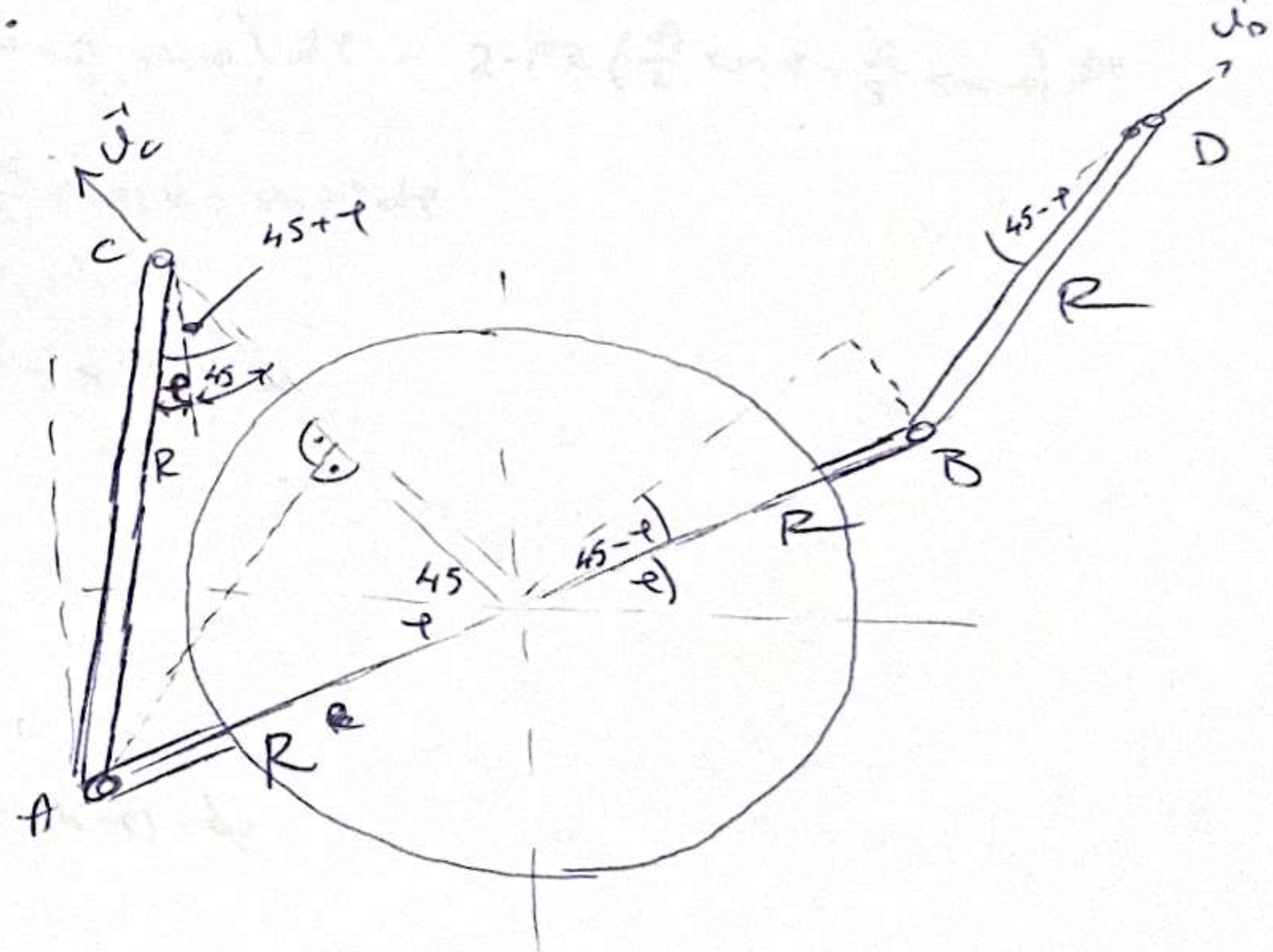
$F(\varphi) = ?$

uslov: $\omega = const.$



Da bi pojednostavili sliku, ugao φ ćemo da merimo od pravca AB kada je on bio horizontalan.

$$\underline{\frac{dT}{dt} = \frac{dA}{dt}}$$



$$T = \left[\frac{1}{2}m v_C^2 + 0\right] + \left[\frac{1}{2}m v_D^2 + 0\right] + \left[0 + \frac{1}{2}J\omega^2\right]$$

$$r_C = \overline{OC} = 2\cdot R\cos(\varphi+45)$$

$$v_C = \frac{d(\overline{OC})}{dt} = -2R\omega\sin(\varphi+45)$$

$$r_D = \overline{OD} = 2R\cos(45-\varphi)$$

$$v_D = \frac{d(\overline{OD})}{dt} = +2R\omega\sin(45-\varphi)$$

$$T = \left[\frac{1}{2}m\cdot 4R^2\sin^2(\varphi+45)\cdot\omega^2\right] + \left[\frac{1}{2}m4R^2\sin^2(45-\varphi)\omega^2\right] + \left[\frac{1}{2}J\omega^2\right]$$

$$T = 2mR^2\omega^2\left(\sin^2(\varphi+45) + \sin^2(45-\varphi)\right) + \frac{1}{2}J\omega^2$$

$$T = 2mR^2\omega^2\left((\sin\varphi\cos45 + \cos\varphi\sin45)^2 + (\sin45\cos\varphi - \sin\varphi\cos45)^2\right) + \frac{1}{2}J\omega^2$$

$$T = 2mR^2\omega^2\left(\left(\frac{1}{2}\sin^2\varphi + \frac{1}{2}\cos^2\varphi + \cancel{\sin\varphi\cos\varphi}\right) + \left(\frac{1}{2}\sin^2\varphi + \frac{1}{2}\cos^2\varphi - \cancel{\sin\varphi\cos\varphi}\right)\right) + \frac{1}{2}J\omega^2$$

$$T = 2mR^2\omega^2(\sin^2\varphi + \cos^2\varphi) + \frac{1}{2}J\omega^2$$

$$T = 2mR^2\omega^2 + \frac{1}{2}J\omega^2$$

$$\frac{dT}{dt} = 0 \qquad (\text{jer je } \omega = const.)$$

$$\underline{d'A} = -mg\,dy_C - mg\,dy_D - F\cdot dr_D$$

$$\frac{dy_C}{dt} = \dot{y}_C = v_C\frac{\sqrt{2}}{2} = -\sqrt{2}\,R\sin(\varphi+45)\frac{d\varphi}{dt}$$

$$\frac{dy_D}{dt} = \dot{y}_D = v_D\frac{\sqrt{2}}{2} = \sqrt{2}\,R\sin(45-\varphi)\frac{d\varphi}{dt}$$

$$\frac{dr_D}{dt} = v_D = 2R\sin(45-\varphi)\frac{d\varphi}{dt}$$

$$d'A = mg\cdot\sqrt{2}R\sin(\varphi+45)\,d\varphi - mg\sqrt{2}\cdot R\sin(45-\varphi)\,d\varphi - F\cdot 2R\sin(45-\varphi)\,d\varphi$$

$$d'A = \sqrt{2}\,mgR\,(\sin(\varphi+45)-\sin(45-\varphi))\,d\varphi - 2\cdot F\cdot R\cdot\sin(45-\varphi)\,d\varphi$$

$$d'A = \sqrt{2}\,mgR\left(\sin\varphi\frac{\sqrt{2}}{2}+\cos\varphi\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\cos\varphi+\frac{\sqrt{2}}{2}\sin\varphi\right)d\varphi - 2\cdot FR\left(\frac{\sqrt{2}}{2}\cos\varphi-\frac{\sqrt{2}}{2}\sin\varphi\right)d\varphi$$

$$d'A = \cancel{\sqrt{2}}\,mgR\sin\varphi\frac{\cancel{\sqrt{2}}}{\cancel{2}}\cdot 2\,d\varphi - 2\cdot F\cdot R\frac{\sqrt{2}}{2}(\cos\varphi-\sin\varphi)\,d\varphi$$

$$\frac{d'A}{dt} = 2mgR\sin\varphi\cdot\omega - \sqrt{2}FR(\cos\varphi-\sin\varphi)\,\omega$$

$$\frac{dT}{dt} = \frac{d'A}{dt}$$

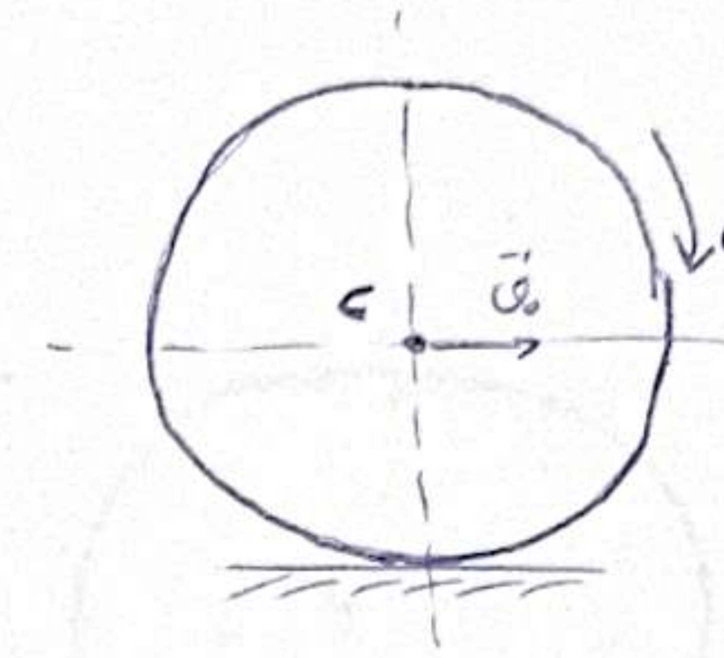
$$0 = 2mg\cancel{R}\sin\varphi\cdot\cancel{\omega} - \sqrt{2}F\cancel{R}(\cos\varphi-\sin\varphi)\cdot\cancel{\omega}$$

$$F = \frac{2\,mg\sin\varphi}{\sqrt{2}(\cos\varphi-\sin\varphi)}$$

$$\boxed{F = \frac{\sqrt{2}\,mg\sin\varphi}{\cos\varphi-\sin\varphi}}$$

ujina je u rešenju ~~...~~ $-(\cos\varphi+\sin\varphi)$ umesto mog $+(\cos\varphi-\sin\varphi)$

(10.64.)



C: P, r

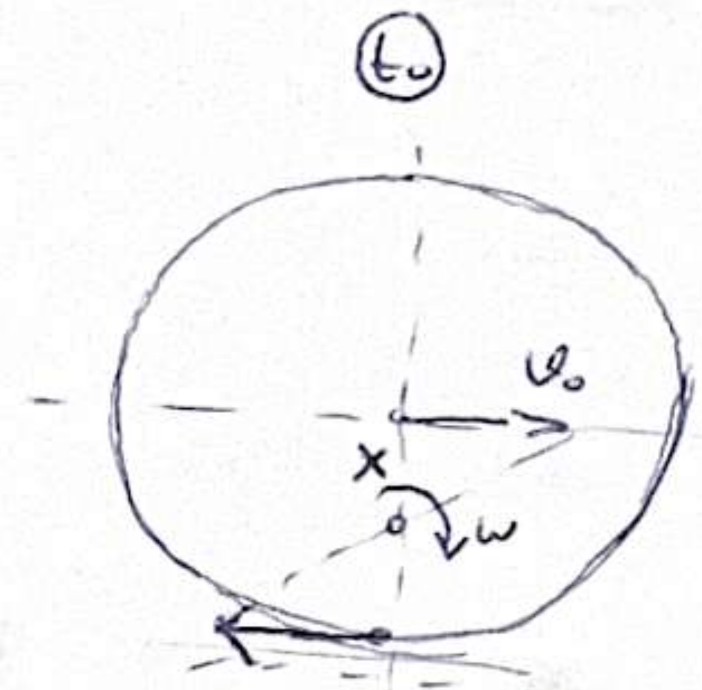
t_0: v_0, ω_0, gde je $r\omega_0 > v_0$

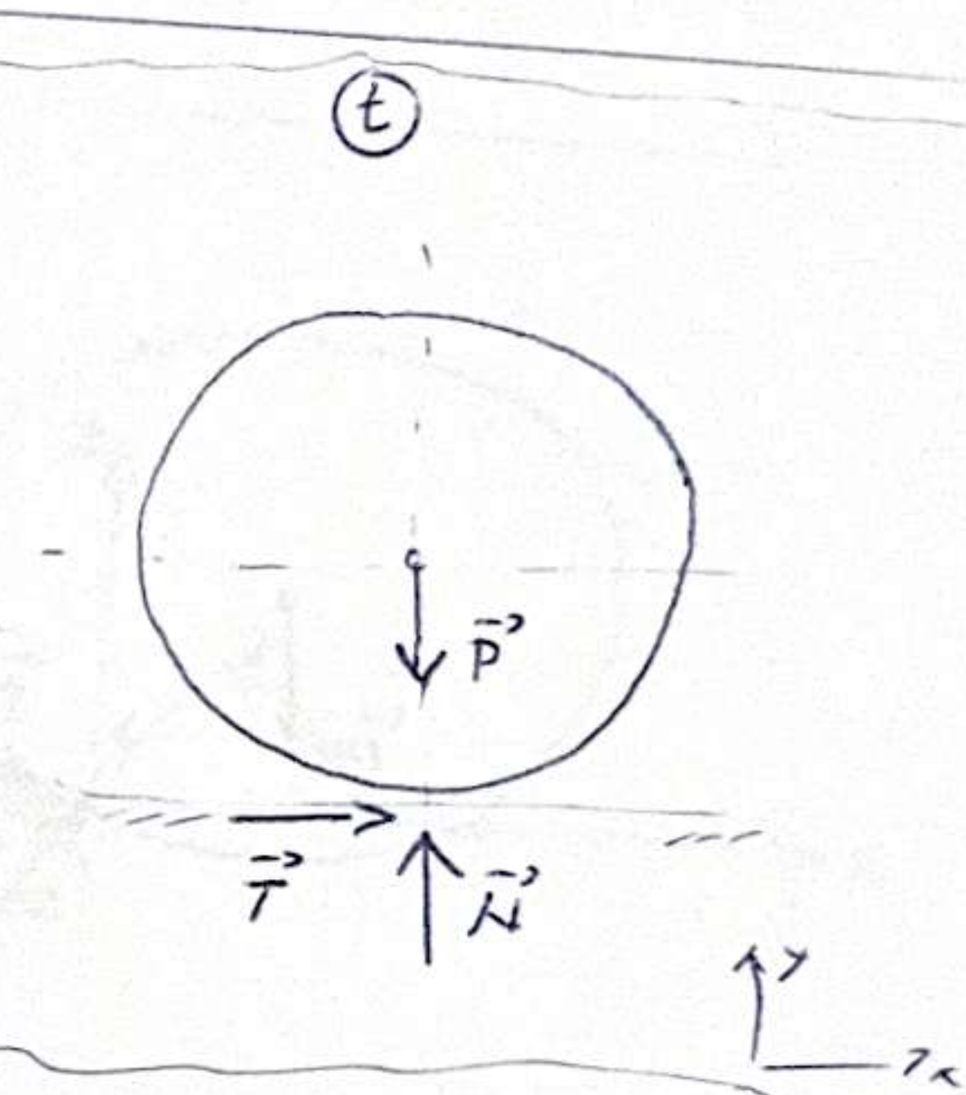
μ

$t_K = ?$ – trenutak kada će centar cilindra C početi da se okreće sa konst. brzinom

$v_C(t_K) = ?$

Da bi znali u kom smeru deluje sila $\vec{T}$, moramo znati u koju stranu proklizava cilindar, pa zato najpre posmatramo trenutak t_0:





$$\frac{P}{g}\vec{a} = \vec{P} + \vec{N} + \vec{T}$$

y: $0 = P - N$

$\underline{N = P}$

x: $\frac{P}{g}\ddot{x} = T$

$$\frac{d\vec{L}_{C1}}{dt} = \Sigma \vec{M}_{C1}$$

$\Sigma M_{C1} = -T \cdot r$

$L_{C1} = J_C \cdot \omega = \frac{P}{g} r^2 \cdot \omega$

$\frac{dL_{C1}}{dt} = \frac{P}{g} r^2 \varepsilon$

$\frac{P}{g} r^2 \varepsilon = -$ (2)

(1) $\Rightarrow$ $\frac{P}{g}\ddot{x} = \mu \cdot P$ $\Rightarrow$ $\underline{\dot{x} = \mu g t + v_0}$

(2) $\Rightarrow$ $\frac{P}{g} r\varepsilon = -\mu P$ $\Rightarrow$ $\underline{\omega = -\frac{\mu g}{r} t + \omega_0}$

$\dot{x}(t_1) = r\omega(t_1)$

$\mu \cdot g \cdot t_1 + v_0 = -\mu g \cdot t_1 + \omega_0 r$

$2\mu g t_1 = r\omega_0 - v_0$

$$\boxed{t_1 = \frac{\omega_0 r - v_0}{2\mu g}}$$

$v(t_1) = \dot{x}(t_1) = \cancel{\mu g} \cdot \frac{\omega_0 r - v_0}{2\cancel{\mu g}} + v_0$

$$\boxed{v(t_1) = \frac{1}{2}(v_0 + \omega_0 r)}$$

za t_1 važi: $\ddot{x}(t_1) = 0$

$\Rightarrow$ $T(t_1) = 0$ $\Rightarrow$ eliminisano je proklizavanje

$\Rightarrow$ $\underline{\dot{x}(t_1) = r\omega(t_1)}$

(10.66.)

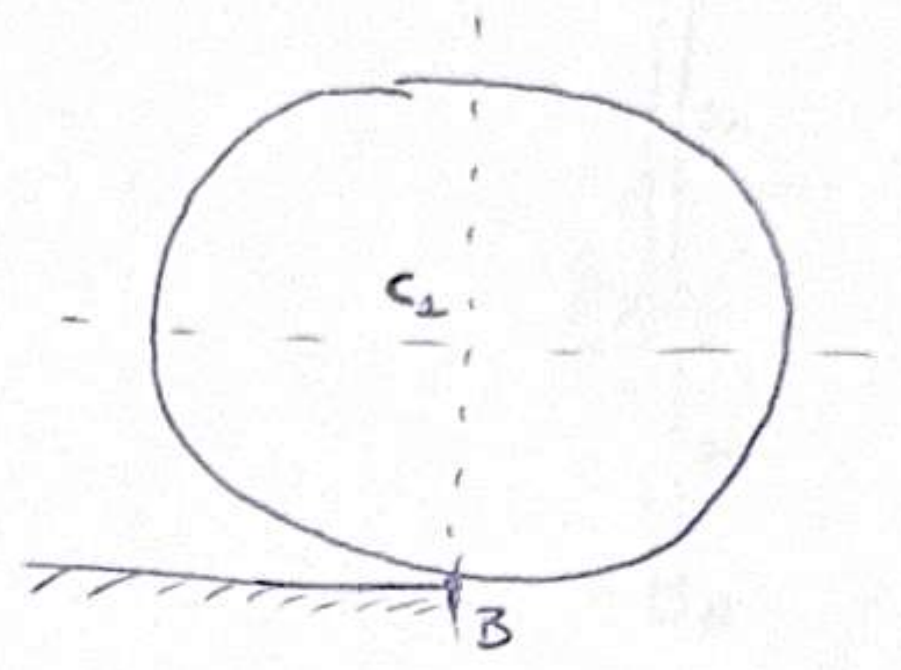
cilindar C: R

t_0: zanemarljivo mala brzina

centar pada s ivice
bez proklizavanja

t_1: odvaja se od podloge

$\alpha_1 = ?$ $\omega_1 = ?$



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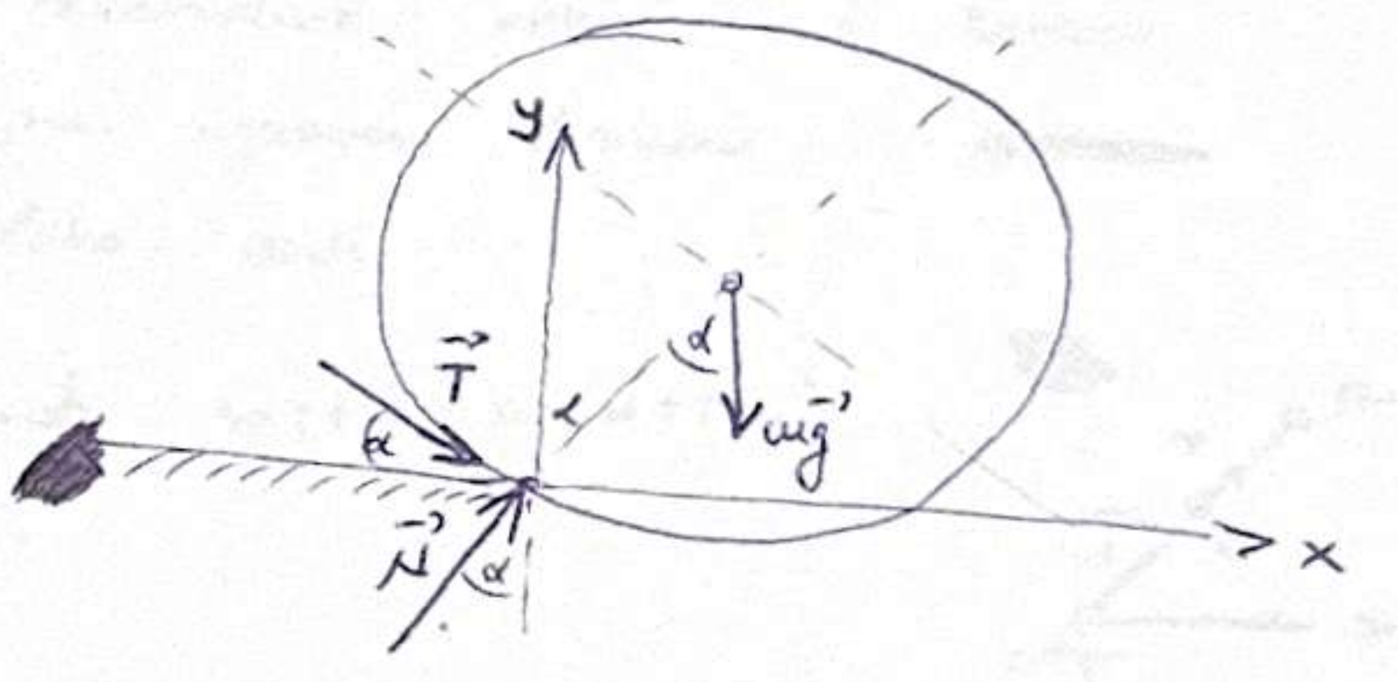
trenutak t_1: $N(t_1) = 0$, $T(t_1) = 0$
(odvajanje od podloge)



ideja: izračunati N u f-ji α/ω,
pustiti da je $N_1 = 0$;
odatle naći α_1/ω_1
(moramo naći vezu između α_1 i ω_1,
da kad nađemo jedno da mogu naći i drugo)

$$T - T_0 = A$$

$T_0 = 0$

$T = \frac{1}{2} m v^2 + \frac{1}{2} J\omega^2 = \frac{1}{2} mR^2\omega^2 + \frac{1}{2} mR^2\omega^2 = mR^2\omega^2$ (Rcrd)

$A = \int_R -mg\, dy_c = mgR(1-\cos\alpha)$

$mR^2\omega^2 = mgR(1-\cos\alpha)$

$$\omega^2 = \frac{g(1-\cos\alpha)}{R}$$

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$$

t: — — (ne treba)

n: $m\frac{v^2}{R} = mg\cos\alpha - N$

$N = -m\frac{R^2\omega^2}{R} + mg\cos\alpha = mg\cos\alpha - mR\cdot\frac{g(1-\cos\alpha)}{R} = mg\cos\alpha - mg + mg\cos\alpha = mg(2\cos\alpha - 1)$

$N_1 = mg(2\cos\alpha_1 - 1) = 0 \Rightarrow 2\cos\alpha_1 = 1 \Rightarrow \cos\alpha_1 = \frac{1}{2} \Rightarrow \boxed{\alpha_1 = 60^\circ}$

$\omega_1^2 = \frac{g(1-\cos\alpha_1)}{R} = \frac{g(1-\frac{1}{2})}{R} = \frac{\frac{1}{2}g}{R} = \frac{g}{2R} \Rightarrow \boxed{\omega_1 = \frac{g}{2R}}$

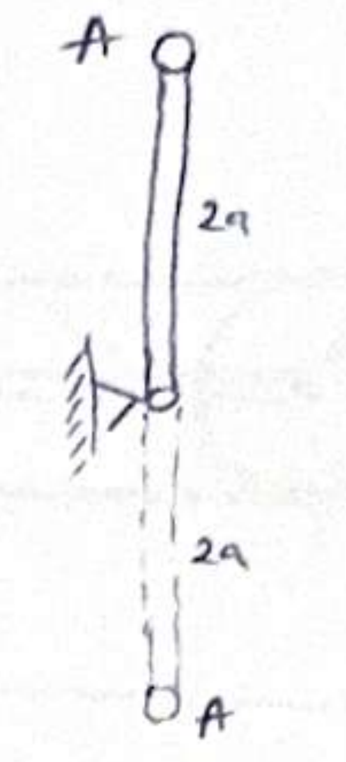
(10. 67.)

štap: $M = 3m,\ 2a$

A: m

t_0: najviši vertikalni položaj
zanemarljivo mala poč. brzina

t_1: najniži vertikalni položaj
uklanja se zglob C

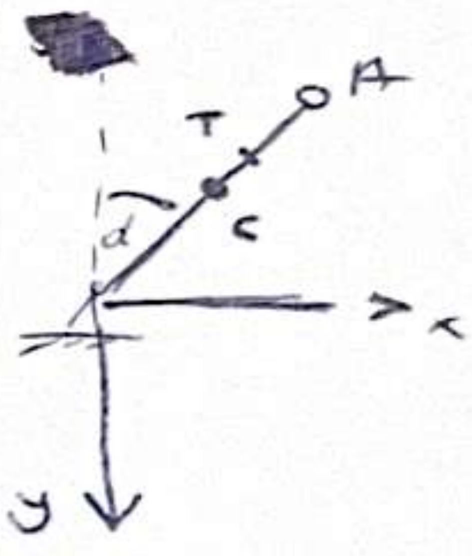


odrediti j-ne kretanja štapa nakon t_1 ?

---

ideja: – da nađemo $\omega(t_1)$ i da onda bukv resetujemo vreme i kažemo da je štap započeo kretanje tom poč. ugaonom brzinom i poč. brzinom centra koju ćemo lako naći

– najbolje je naći $\omega(\alpha)$, jer je onda $\omega(t_1) = \omega(\alpha = 180^\circ)$



$\underline{T_1 - T_0 = A_{01}}$

$T_0 = 0$

$T_1 = \left[\frac{1}{2} 3m\, v_{C1}^2 + \frac{1}{2} J_C \omega_{01}^2\right] + \left[\frac{1}{2} m v_{A1}^2 + 0\right] =$

$T_1 = \left[\frac{3}{2} m R^2 \omega_1^2 + \frac{1}{2}\frac{1}{12} 4a^2 \cdot 3m \cdot \omega_1^2\right] + \left[\frac{1}{2} m \cdot 4a^2 \cdot \omega_1^2\right] =$

$T_1 = 4 m R^2 \omega_1^2$

$A_{01} = \int_{-a}^{a} 3mg\, dy_C + \int_{-2a}^{2a} mg\, dy_A = 3mg \cdot 2a + mg \cdot 4a = 10\, mga$

$4m x_T = 3m x_C + m x_A$

$4m x_T = 3m x_C + 2m x_C$

$4m x_T = 5m x_C$

$\underline{\underline{x_T = \frac{5}{4} x_C}}$ $\quad \underline{\underline{y_T = \frac{5}{4} y_C}}$

(na sličan način se dobije)

$4 m a \omega_1^2 = 10 g$ $\Rightarrow$ $\omega_1^2 = \frac{5}{2}\frac{g}{a}$ $\Rightarrow$ $\underline{\underline{\omega_1 = \sqrt{\frac{5}{2}\frac{g}{a}}}}$, $\underline{\underline{\dot{x}_{C1} = -\sqrt{\frac{5}{2} ga}}}$, $\underline{\underline{\dot{y}_{C1} = 0}}$

(ovo su poč. uslovi za novo (slobodno) kretanje štapa)

$\underline{4m \vec{a}_T = m\vec{g} + 3m\vec{g}}$

x: $4m \ddot{x}_T = 0$ $\quad // \int dt$

$\dot{x}_T = const = -\frac{5}{4}\sqrt{\frac{5}{2} ga}$ $\quad // \int dt$

$\boxed{x_T = -\frac{5}{4}\sqrt{\frac{5}{2} ga} \cdot t}$

y: $4m \ddot{y}_T = -mg - 3mg$

$\ddot{y}_T = -g$ $\quad // \int dt$

$\dot{y}_T = -gt$ $\quad // \int dt$

$\boxed{y_T = -\frac{1}{2} g t^2 - \frac{5}{4} a}$

$\underline{\frac{d\vec{L}_{Tz}}{dt} = \sum \vec{M}_{Tz}}$

$\sum M_{Tz} = -mg \cdot \frac{3}{4} a \sin\varphi + 3mg \cdot \frac{1}{4} a \sin\varphi = 0$

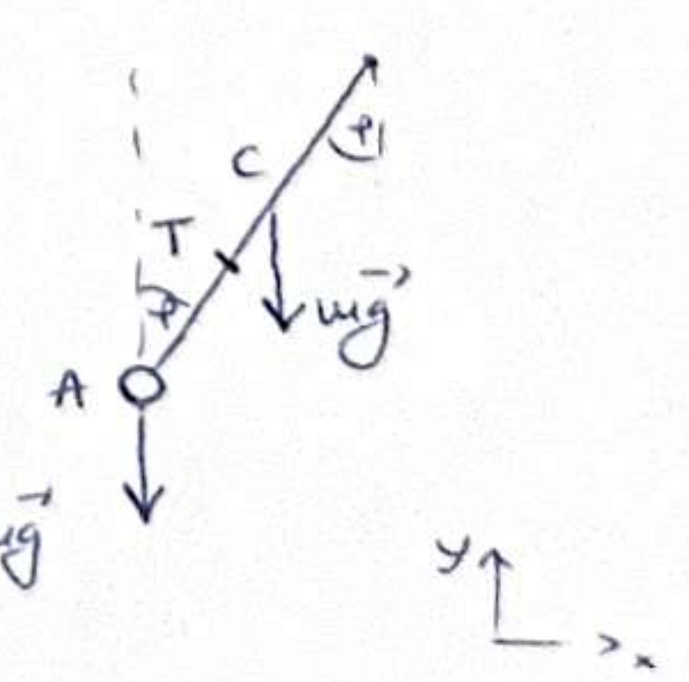
$L_{CT} = J\omega = \dots$

$\frac{dL_{CT}}{dt} = J\varepsilon = \dots$

$J\varepsilon = 0$ $\quad // \int dt$

$\omega = \omega_{nul.} = \sqrt{\frac{5}{2}\frac{g}{a}}$ $\quad // \int dt$

$\boxed{\varphi = \sqrt{\frac{5}{2}\frac{g}{a}}\, t}$



10.68.

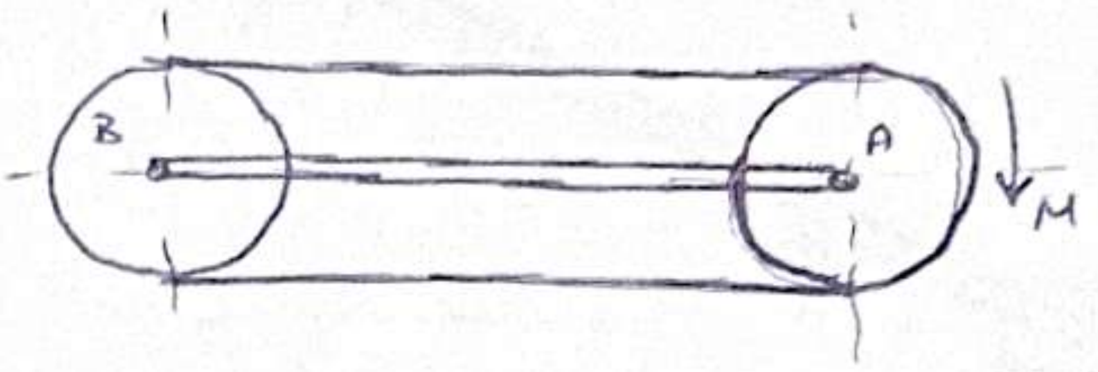
gusenica: m, zanemarljiva debljina

diskovi: m, r

AB : $2m$, e

$M = const.$

$\varepsilon = ?$



$$\frac{dT}{dt} = \frac{d'A}{dt}$$

Brzina donjeg dela gusenice je jednaka nuli

Brzina gornjeg dela gusenice

$$T = 2\left[\frac{1}{2}m v_A^2 + \frac{1}{2}J\omega^2\right]_D + \left[\frac{1}{2}2m v_{AB}^2 + 0\right]_{AB} + \left[\frac{1}{2}\cdot\frac{1}{2}m\cdot 0 + 0\right] + \left[\frac{1}{2}\cdot\frac{1}{2}m\, v_{GG}^2 + 0\right]$$

$$T = \left[mR^2\omega^2 + \frac{1}{2}mR^2\omega^2\right] + \left[mR^2\omega^2\right] + \left[\frac{1}{4}m(2R\omega)^2\right]$$

$$T = \frac{7}{2}mR^2\omega^2$$

$$\frac{dT}{dt} = 7mR^2\omega\varepsilon$$

$$d'A = M\,d\varphi$$

$$\frac{d'A}{dt} = M\omega$$

$$7mR^2\omega\varepsilon = M\omega$$

$$\boxed{\varepsilon = \frac{M}{7mR^2}}$$

10.70.

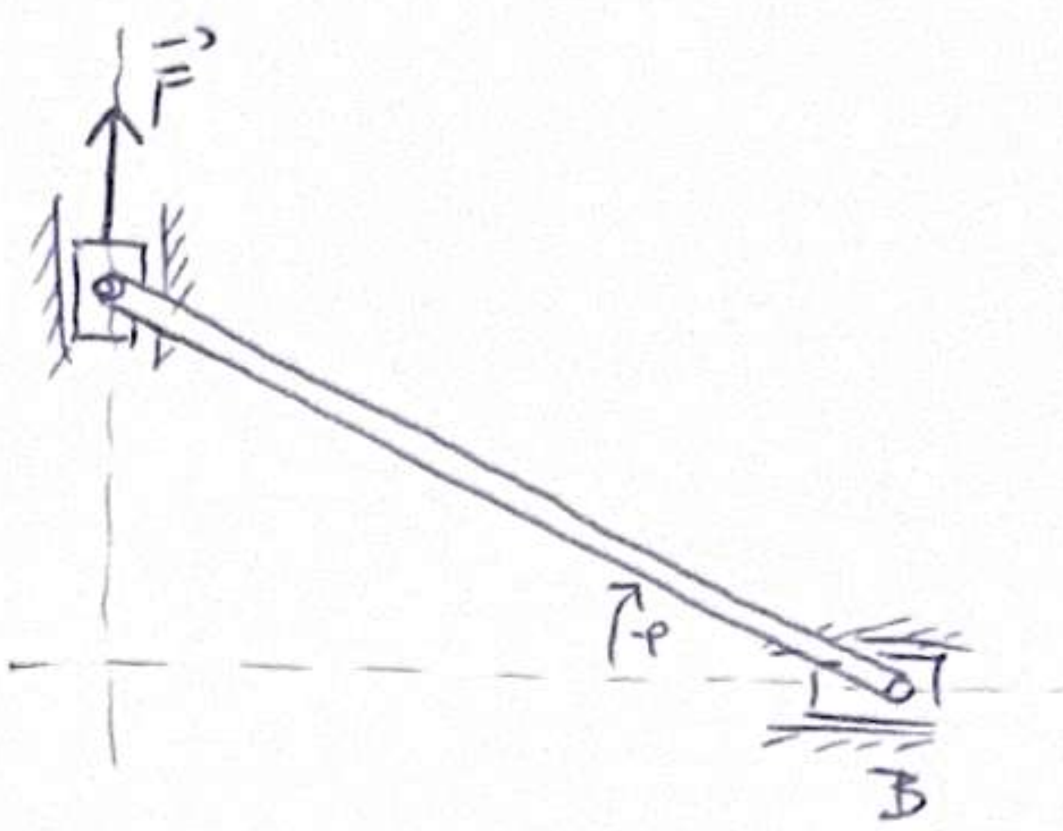
AB: $m, 2l$

A: $\varnothing$

$F = const. = ?$

t_0: mirovanje u horiz. položaju

uslov: kraj B se odvaja od podloge pod uglom $\alpha = 30°$



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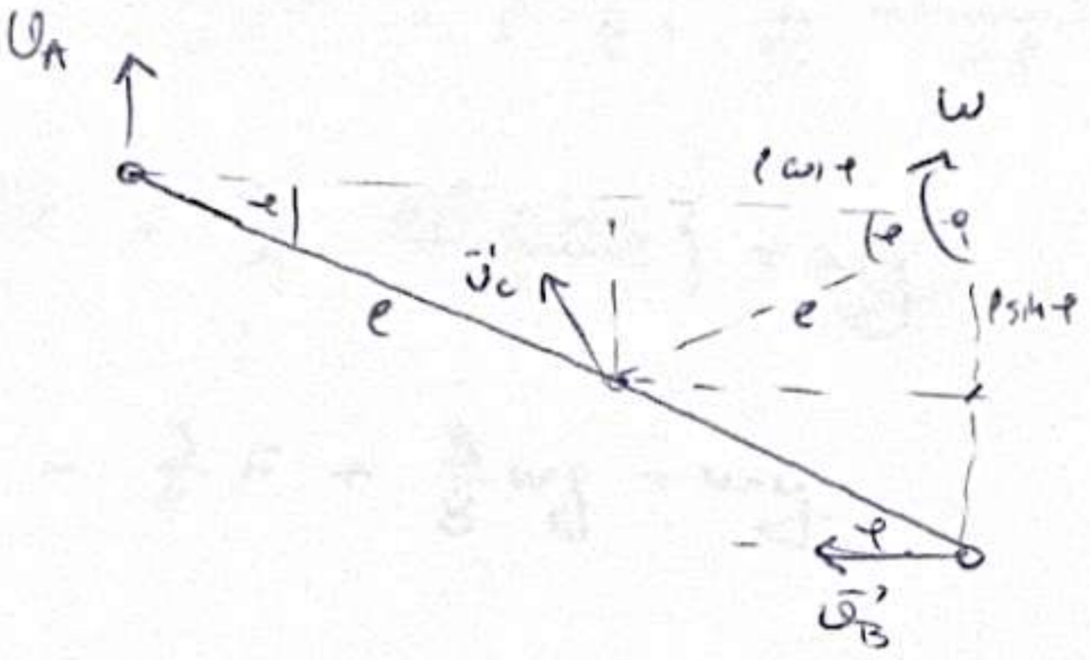
kinematska veza:

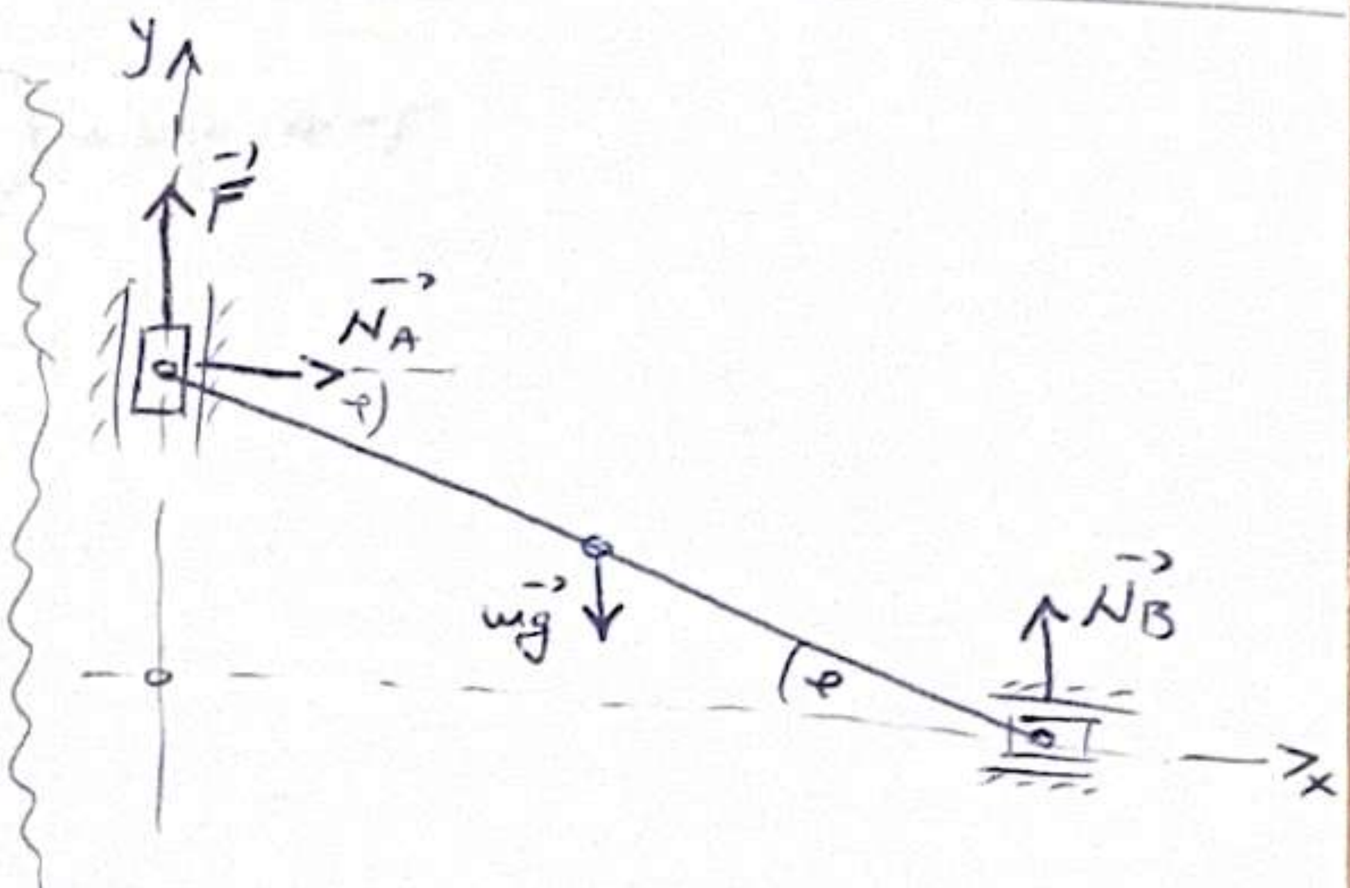
$\dot{x}_c = l\sin\varphi\cdot\omega$

$\dot{y}_c = l\cos\varphi\cdot\omega$

$\ddot{x}_c = l\cos\varphi\cdot\omega^2 + l\sin\varphi\cdot\varepsilon$

$\ddot{y}_c = -l\sin\varphi\cdot\omega^2 + l\cos\varphi\cdot\varepsilon$





ideja:
- izraziti N_B u f-iji φ i F kao jedinih nepoznatih, zatim reći da je $N_1 = 0$, $\varphi_1 = 30$, pa odatle naći F (stoga N_B i φ neće tretirati kao nepoznate)
- za sad imamo 3-jne i 4 nepoznate $(\omega, \varepsilon, F, N_A)$
- preko promene kin. energije možemo izračunati ω i ε u f-iji ugla φ

$T - T_0 = A$

$T_0 = 0$

$T = \frac{1}{2} m v_c^2 + \frac{1}{2} J\omega^2 = \frac{1}{2} m(l^2\omega^2\sin^2\varphi + l^2\omega^2\cos^2\varphi) + \frac{1}{2}\frac{1}{12} m 4l^2\omega^2 =$

$T = \frac{1}{2} m l^2\omega^2 + \frac{1}{6} m l^2\omega^2 = \frac{2}{3} m l^2\omega^2$

$A = \int_0^{l\sin\varphi} -mg\,dy_c + \int_0^{2l\sin\varphi} F\,dy_A = -mgl\sin\varphi + 2Fl\sin\varphi$

$\frac{2}{3} m l^{\cancel{2}}\omega^2 = 2F\cancel{l}\sin\varphi - mg\cancel{l}\sin\varphi$

$\omega = \sqrt{\frac{3}{2}\frac{2F-mg}{ml}\sin\varphi} \quad \Rightarrow \quad \underline{\underline{\omega_1 = \sqrt{\frac{3}{4}\cdot\frac{2F-mg}{ml}}}}$

$\varepsilon = \frac{1}{2\sqrt{\ldots}}\cdot\frac{3}{2}\frac{2F-mg}{ml}\cos\varphi\cdot\dot{\varphi}$

$\varepsilon = \frac{3}{4}\frac{2F-mg}{ml}\cos\varphi \quad \Rightarrow \quad \underline{\underline{\varepsilon_1 = \frac{3\sqrt{3}}{8}\frac{2F-mg}{ml}}}$

$$m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N}_A + \vec{N}_B$$

$$y: \quad m\ddot{y}_c = -mg + F + N_B$$

$$m(l\cos\varphi\,\varepsilon - l\sin\varphi\,\omega^2) = -mg + F + N_B$$

$$N_B = m(l\cos\varphi\cdot\varepsilon - l\sin\varphi\cdot\omega^2) + mg - F$$

pustamo trenutak t_1 gde je: $N_{B1}=0, \quad \varphi_1 = 30, \quad \varepsilon_1 = \frac{3\sqrt{3}}{8}\,\frac{2F-mg}{ml}, \quad \omega_1 = \sqrt{\frac{3}{4}\,\frac{2F-mg}{ml}}$

$$0 = m\left(l\cdot\frac{\sqrt{3}}{2}\cdot\frac{3\sqrt{3}}{8}\,\frac{2F-mg}{ml} - l\cdot\frac{1}{2}\cdot\frac{3}{4}\,\frac{2F-mg}{ml}\right) + mg - F$$

$$F = \cancel{m}\left(\frac{9}{16}\,\frac{2F-mg}{\cancel{m}} - \frac{3}{8}\,\frac{2F-mg}{\cancel{m}}\right) + mg$$

$$F = \frac{9}{8}F - \frac{9}{16}mg - \frac{3}{4}F + \frac{3}{8}mg + mg$$

$$F\left(1 - \frac{9}{8} + \frac{3}{4}\right) = mg\left(-\frac{9}{16} + \frac{3}{8} + 1\right)$$

$$F\cdot\frac{5}{8} = mg\cdot\frac{13}{16} \qquad /\cdot\frac{8}{5}$$

$$\boxed{F = \frac{13}{10}mg}$$

10.71

A: m

B: m

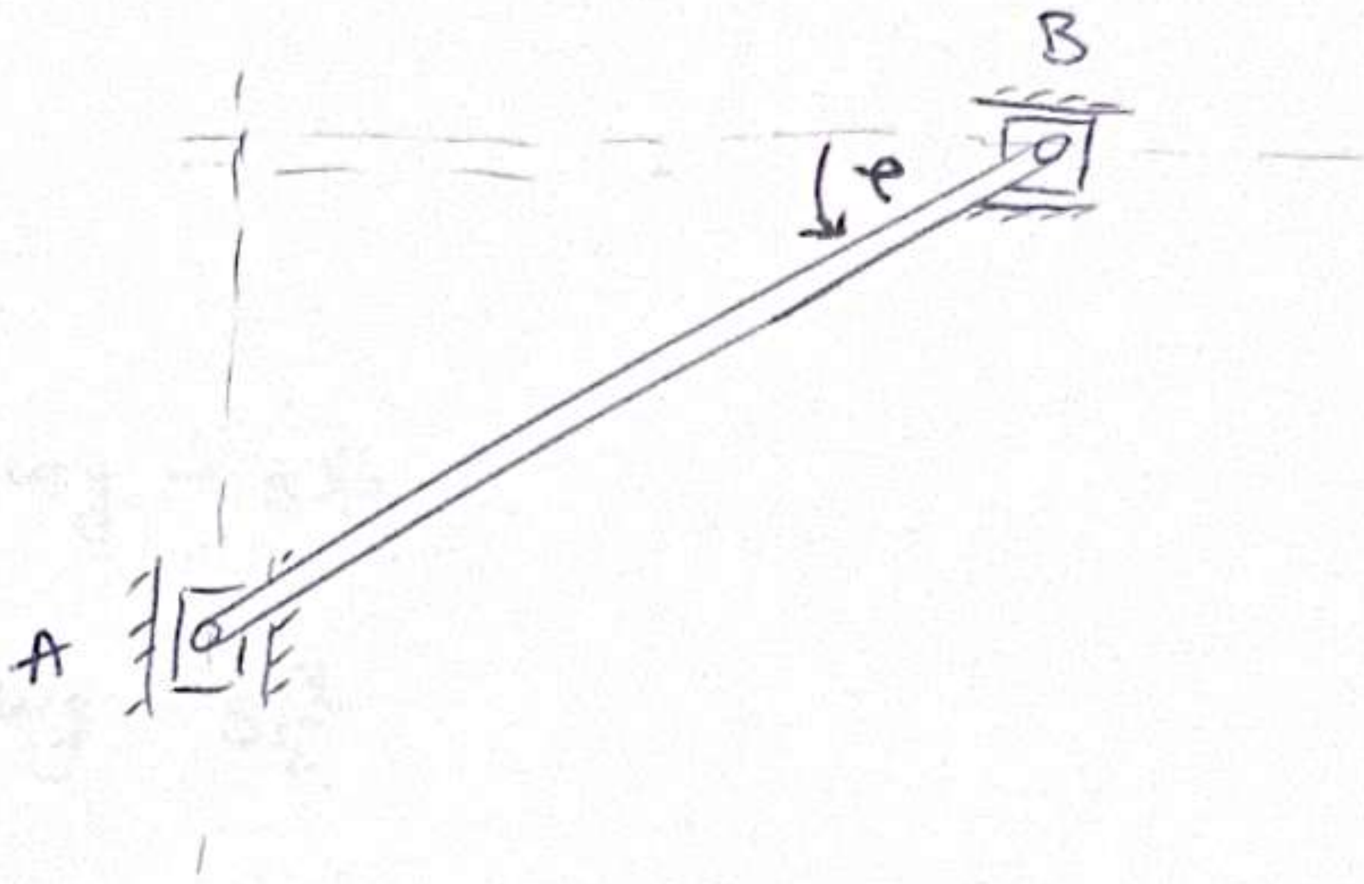
AB: $\emptyset$, l

t_0: $\varphi_0 = 0$

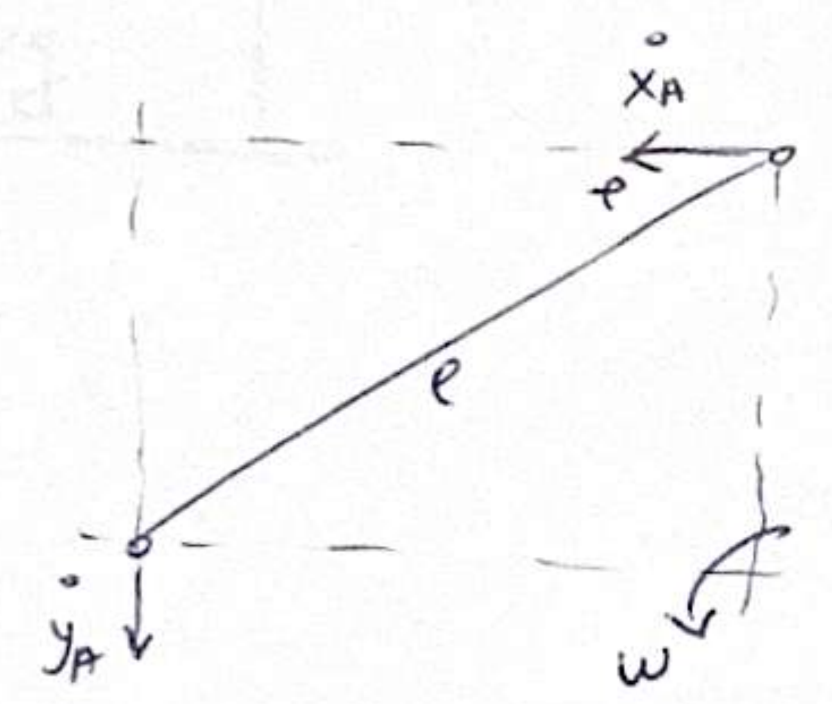
mirovanje

t_1: $\varphi_1 = 45°$

$S(t_1) = ?$

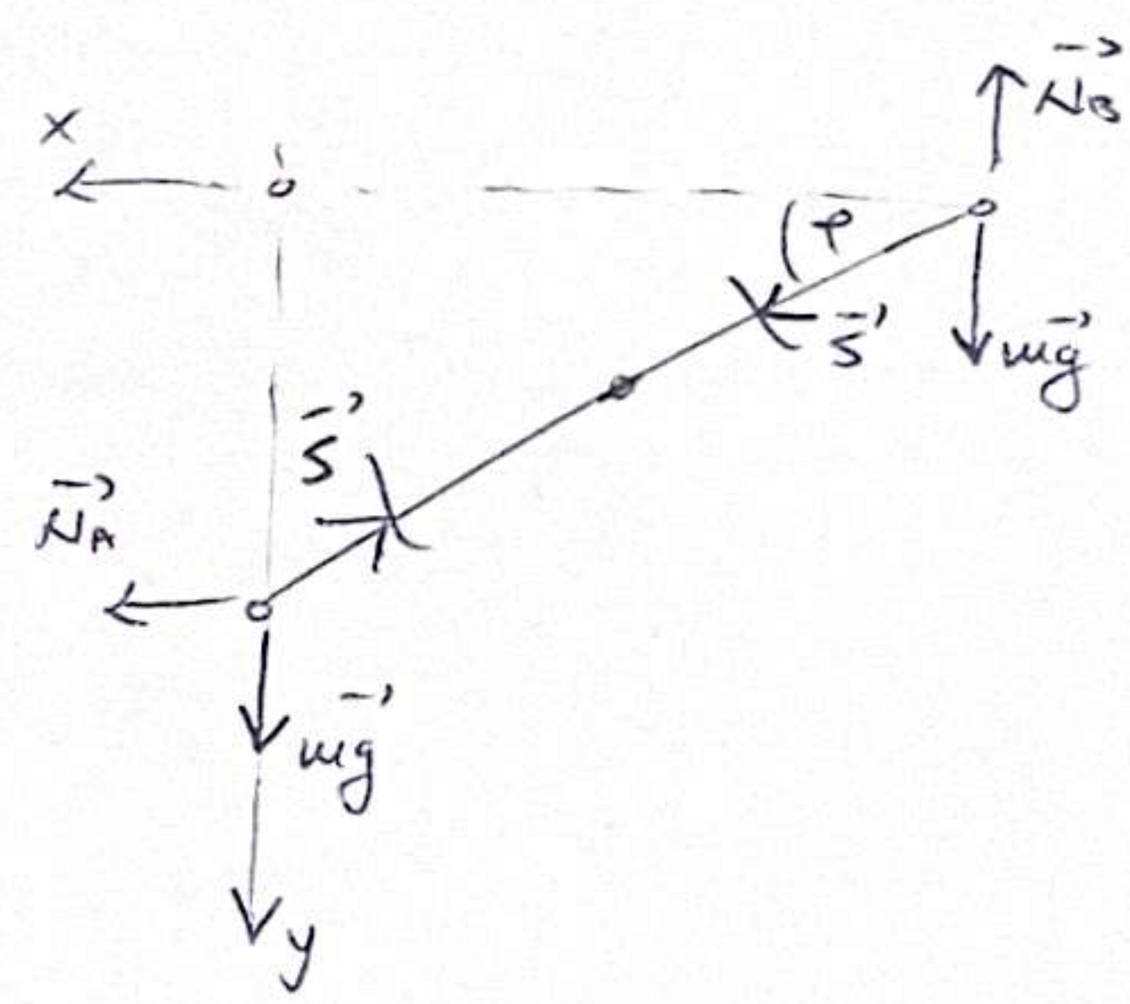


kinematska veza:



$$\dot{x}_A = l \sin\varphi \cdot \omega$$

$$\dot{y}_A = l \cos\varphi \cdot \omega$$



$$\ddot{x}_A = l\omega^2 \cos\varphi + l\varepsilon \sin\varphi$$

$$\ddot{y}_A = -l\omega^2 \sin\varphi + l\varepsilon \cos\varphi$$

ideja: - imamo 4 j-ne i 5 nepoznatih $(\omega, \varepsilon, N_A, N_B, S)$

- dopisaćemo još jednu j-nu za kinetičku energiju - to ćemo naći ω i ε

$T - T_0 = A$

$T_0 = 0$

$$T = \left[\tfrac{1}{2} m v_A^2 + 0\right] + \left[\tfrac{1}{2} m v_B^2 + 0\right] = \tfrac{1}{2} m l^2 \omega^2 \sin^2\varphi + \tfrac{1}{2} m l^2 \omega^2 \cos^2\varphi = \tfrac{1}{2} m l^2 \omega^2$$

$$A = \int_0^{l\sin\varphi} mg\, dy_A = mgl \sin\varphi$$

$$\tfrac{1}{2} m l^2 \omega^2 = mgl \sin\varphi$$

$$\omega = \sqrt{\frac{2g}{l} \sin\varphi} \quad \Rightarrow \quad \omega_1 = \sqrt{\frac{2g}{l} \frac{\sqrt{2}}{2}} = \sqrt{\frac{\sqrt{2} g}{l}}$$

$$\varepsilon = \frac{1}{2\sqrt{\dots}} \cdot \frac{2g}{l} \cos\varphi \cdot \omega$$

$$\varepsilon = \frac{g}{l} \cos\varphi \quad \Rightarrow \quad \varepsilon_1 = \frac{g}{l} \frac{\sqrt{2}}{2}$$

$$m\vec{a}_{A1} = m\vec{g} + \vec{N}_{A1} + \vec{S}$$

x:

y: $m\ddot{y}_{A1} = mg - S\sin 45$

$$m(-l\sin\varphi_1\,\omega_1^2 + l\cos\varphi_1\cdot\varepsilon_1) = mg - S\frac{\sqrt{2}}{2}$$

$$m\left(-l\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}\,g}{l} + l\frac{\sqrt{2}}{2}\,\frac{\sqrt{2}}{2}\,\frac{g}{l}\right) = mg - S\frac{\sqrt{2}}{2}$$

$$-\frac{1}{2}mg = mg - S\frac{\sqrt{2}}{2}$$

$$S\frac{\sqrt{2}}{2} = \frac{3}{2}mg$$

$$\boxed{S = \frac{3\sqrt{2}}{2}mg}$$

10.72

1: $m, \quad R, \quad 2R, \quad i_0 = R\sqrt{2}$

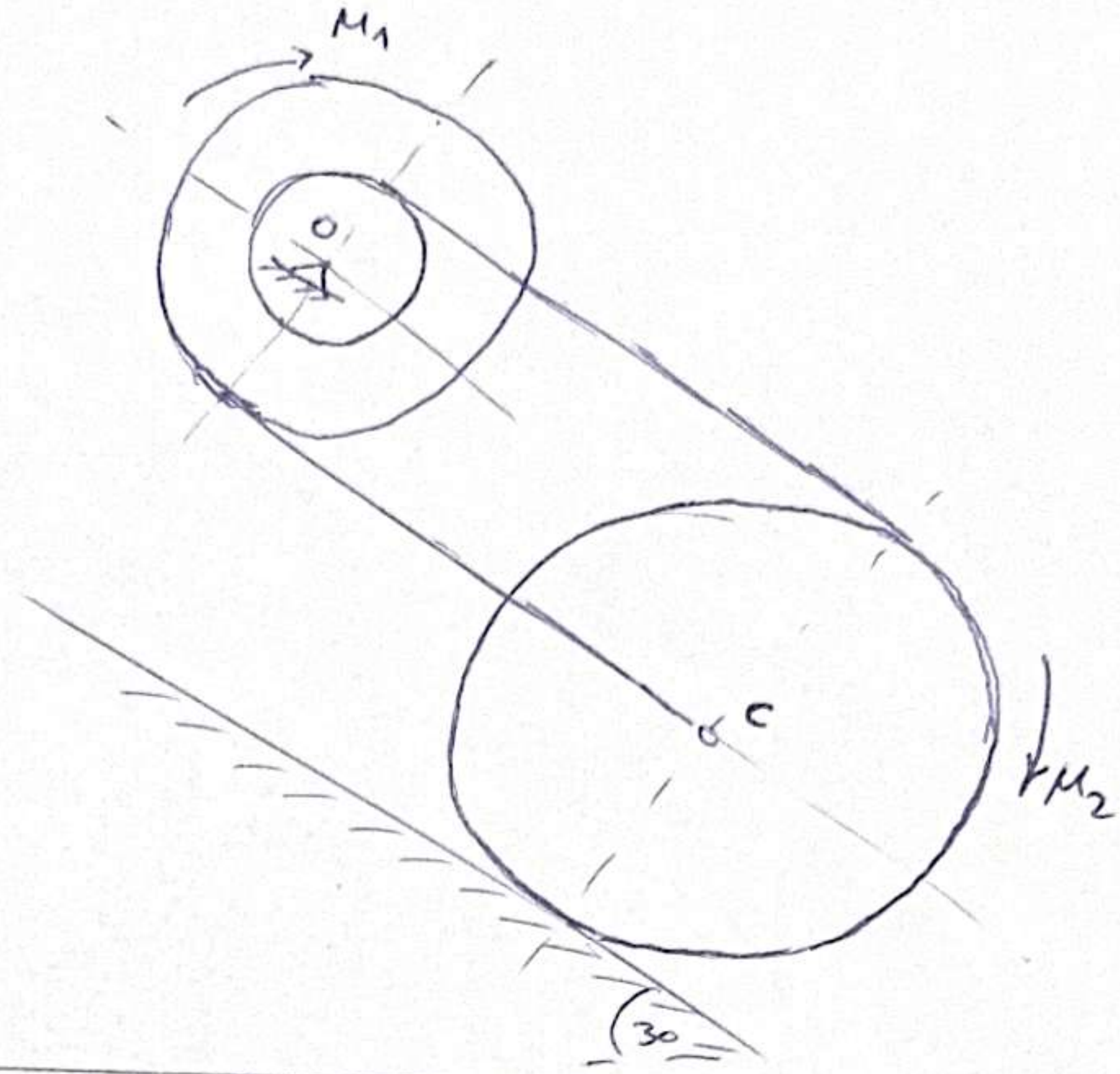
2: $m, \quad 3R$

$\alpha = 30^\circ \quad , \quad \mu = \frac{\sqrt{3}}{3}$

$M_1 = const. = ?$

$M_2 = const. = ?$

$S_1 = S_2 = mg$



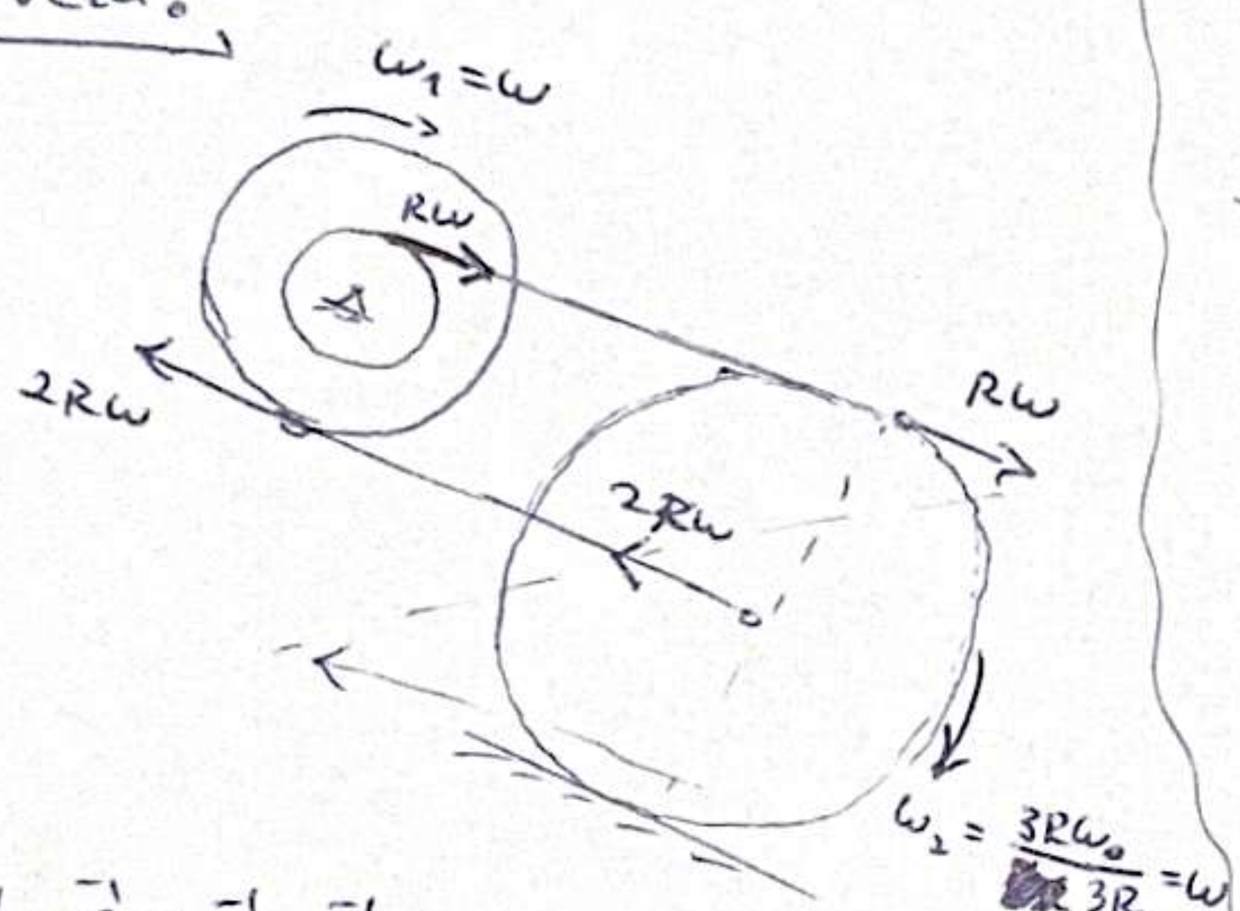
kinematska veza:

$\omega_1 = \omega, \quad \underline{\varepsilon_1 = \varepsilon}$

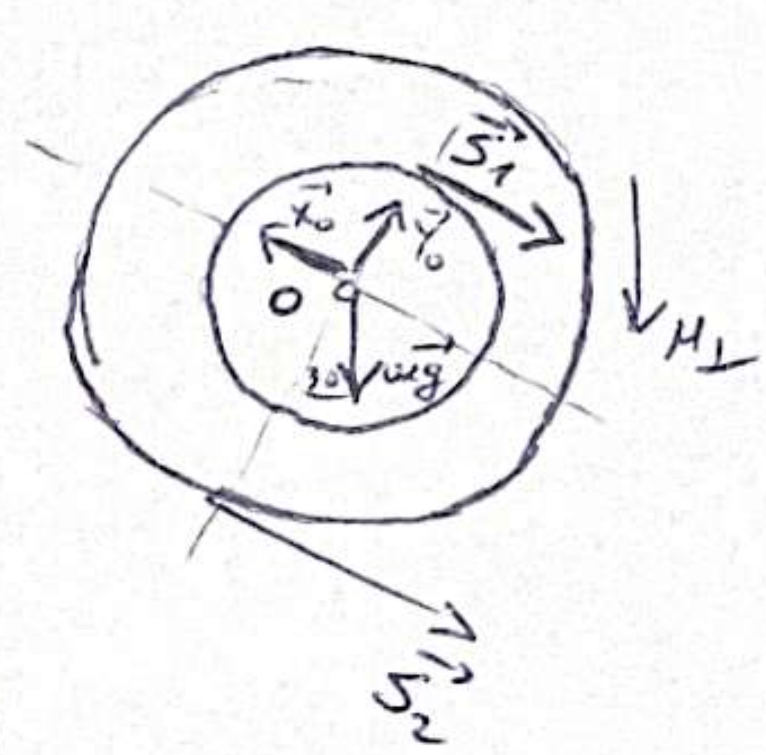
$\omega_2 = \omega, \quad \underline{\varepsilon_2 = \varepsilon}$

$\dot{x}_2 = -2R\omega$

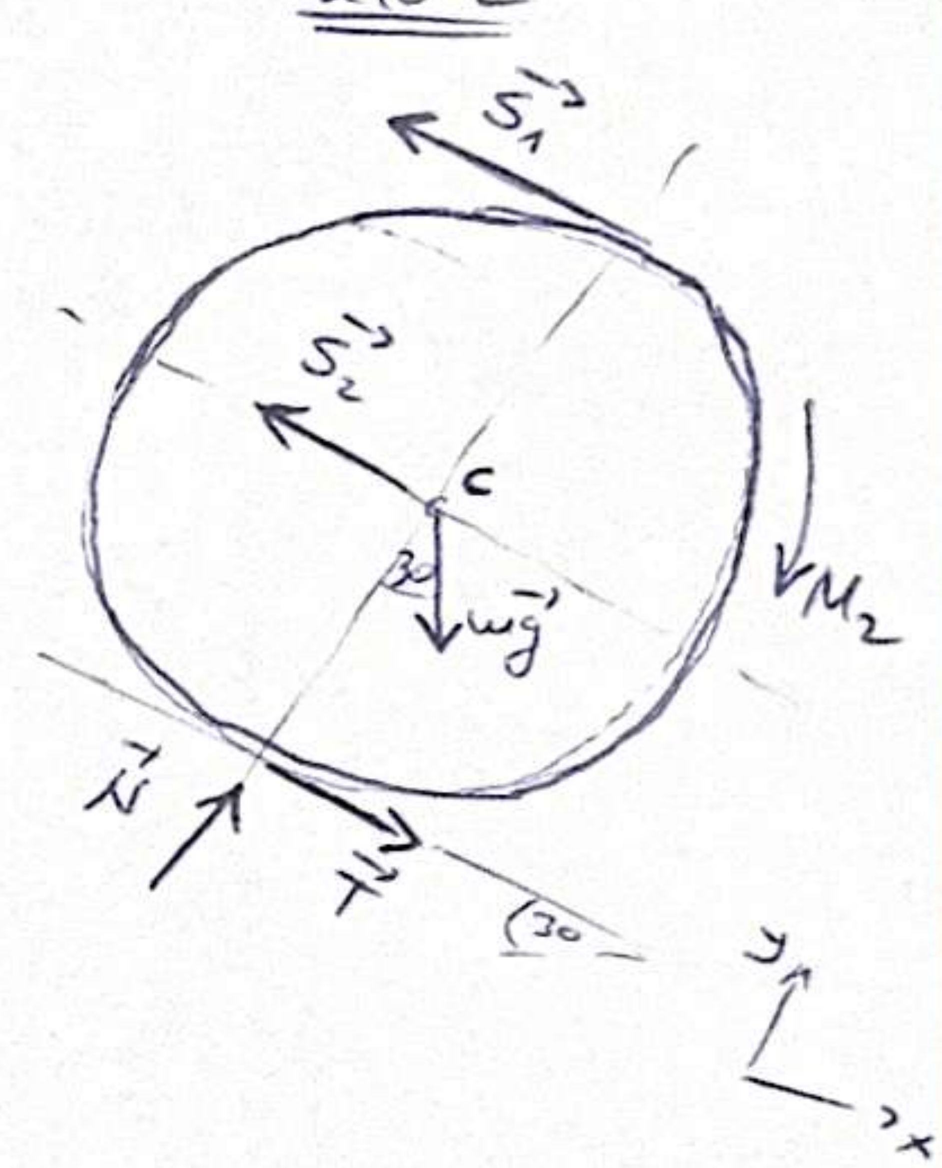
$\ddot{x}_2 = -2R\varepsilon$



telo 1



telo 2



$m\vec{a}_1 = m\vec{g} + \vec{S}_1 + \vec{S}_2 + \vec{X}_0 + \vec{Y}_0$

x: — (nepotrebno) (jer ne treba ni X_0)

y: —— (nepotrebno) (jer ne treba ni Y_0)

$\frac{d\vec{L}_{Oz}}{dt} = \sum \vec{M}_{Oz}$

$\sum M_{Oz} = M_1 + S_1 \cdot R - S_2 \cdot 2R$

$\sum M_{Oz} = M_1 - mgR$

$L_{Oz} = J_{Oz} \cdot \omega_1 = m \cdot R^2 \cdot 2 \cdot \omega$

$\frac{dL_{Oz}}{dt} = 2mR^2\varepsilon$

$M_1 - mgR = 2mR^2\varepsilon$

$m\vec{a}_2 = m\vec{g} + \vec{S}_1 + \vec{S}_2 + \vec{N} + \vec{T}$

y: $0 = -mg\cos 30 + N$

$N = \frac{\sqrt{3}}{2}mg \quad , \quad T = \mu \cdot N = \frac{1}{2}mg$

x: $m\ddot{x}_2 = mg\sin 30 - S_1 - S_2 + T$

$-2mR\varepsilon = \cancel{\frac{1}{2}mg} - 2mg + \cancel{\frac{1}{2}mg}$

$\varepsilon = \frac{1}{2}\frac{g}{R}$

(1) => $M_1 = mgR + 2mR^2 \cdot \frac{1}{2}\frac{g}{R} = 2mgR$

$\boxed{M_1 = 2mgR}$

$\frac{d\vec{L}_{Cz}}{dt} = \sum \vec{M}_{Cz}$

$\sum M_{Cz} = M_2 - S_1 \cdot 3R - T \cdot 3R = M_2 - 3mgR - \frac{1}{2} \cdot 3mgR = M_2 - \frac{9}{2}mgR$

$L_{Cz} = J_{Cz} \cdot \omega_2 = \frac{1}{2}m \cdot 9R^2\omega$

$\frac{dL_{Cz}}{dt} = \frac{9}{2}mR^2\varepsilon = \frac{9}{2}mR^2 \cdot \frac{1}{2}\frac{g}{R} = \frac{9}{4}mgR$

$M_2 - \frac{9}{2}mgR = \frac{9}{4}mgR$

$\boxed{M_2 = \frac{27}{4}mgR}$