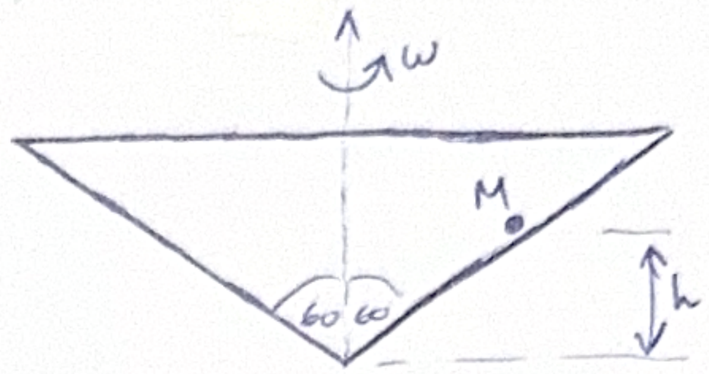
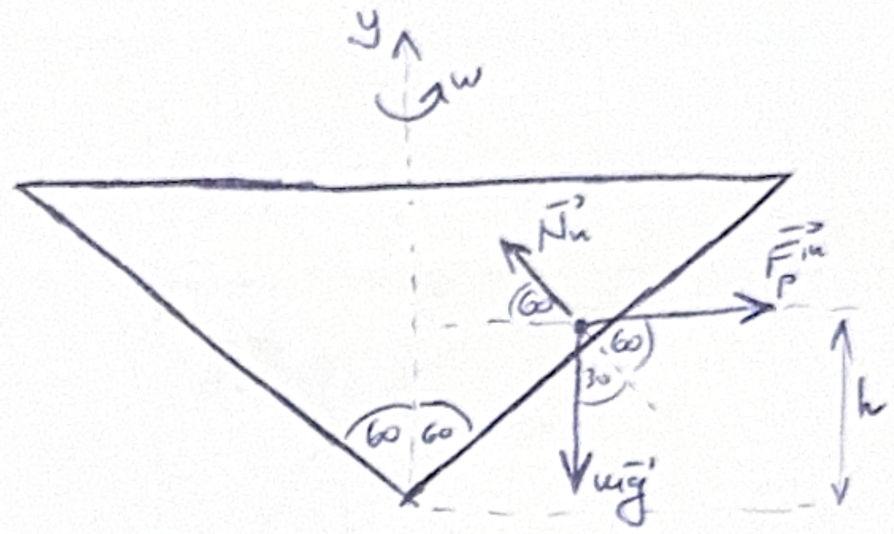
(6.1.) vertikalna ravan

$\omega = const.$

$\omega = ?$ – da ei tačka C.I.q u relativnoj ravnoteži na visini h



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n' + \vec{N}_6'$$



$$m\vec{g} = mg\cdot(-\vec{j})$$

$$\vec{F}_p^{in} = -m\left(\vec{a}_{\vec{o}} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})\right) = m\omega^2\cdot x\cdot(\vec{i}) =$$

$$= m\omega^2\frac{h}{\cos 60}\sin 60\cdot(\vec{i}) = m\omega^2\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}(\vec{i}) = m\omega^2\cdot h\sqrt{3}\cdot(\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega}\times\vec{v}_r = 0$$

$$\vec{N}_n' = N_n\cdot(\vec{n})$$

$$\vec{N}_6' = N_6\cdot(\vec{k})$$

t: $\quad 0 = -mg\sin 30 + F_p^{in}\cos 30 + 0 + 0 + 0$

$$0 = -\frac{1}{2}mg + m\omega^2 h\sqrt{3}\cdot\frac{\sqrt{3}}{2}$$

$$m\omega^2 h\cdot\frac{3}{2} = \frac{1}{2}mg$$

$$\boxed{\omega = \sqrt{\frac{g}{3h}}}$$

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(6.3.) poleu: m

$xy = a^2$

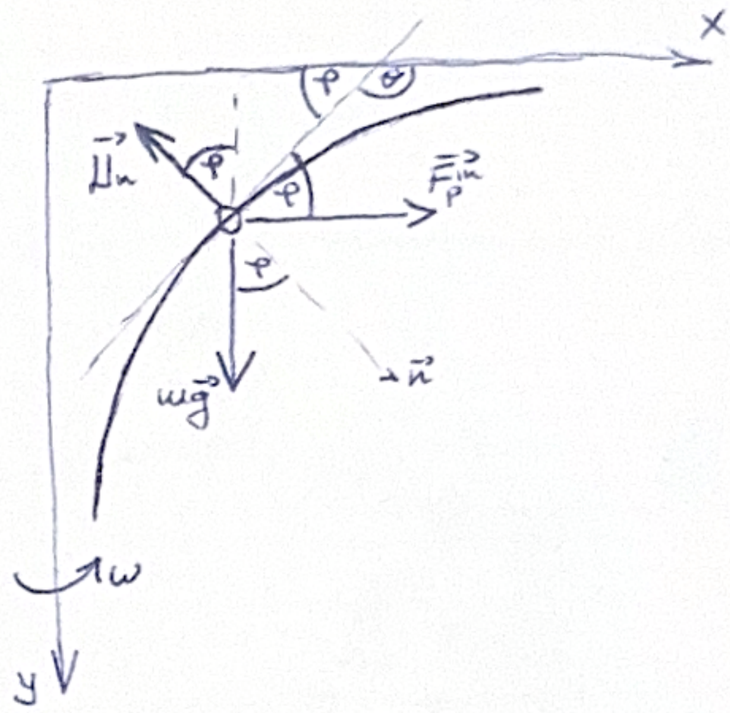
$\omega = const$

vertikalna osa . usmerena nagore

položaj relativne ravnoteže ?

reakcija žice u tom položaju ?

$y = \frac{1}{x} \cdot a^2$



$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$

$m\vec{g} = mg \cdot (\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \cancel{\vec{v}_r}^{\,0} = 0$

$\vec{N}_n = N_n \cdot (-\vec{n})$

$\vec{N}_6 = N_6 \cdot (\vec{k})$

t: $0 = -mg \sin\varphi + F_p^{in} \cos\varphi + 0 + 0$

$0 = -mg \frac{a^2}{\sqrt{x^4+a^4}} + m\omega^2 x \cdot \frac{x^2}{\sqrt{x^4+a^4}}$

$0 = -ga^2 + \omega^2 x^3$

$\boxed{x = \sqrt[3]{\frac{ga^2}{\omega^2}}}$

$y = \frac{1}{x} a^2 = \sqrt[3]{\frac{\omega^2}{ga^2}} \cdot a^2 = a\sqrt[3]{\frac{a\omega^2}{g}}$

$\boxed{y = a\sqrt[3]{\frac{a\omega^2}{g}}}$

$tg\theta = y' = -1 \cdot x^{-2} \cdot a^2 = -\frac{a^2}{x^2}$

$tg\varphi = tg(180 - \theta) = -tg\theta = \frac{a^2}{x^2}$

$\sin\varphi = \frac{tg\varphi}{\sqrt{1+tg^2\varphi}} = \frac{\frac{a^2}{x^2}}{\sqrt{1+\frac{a^4}{x^4}}} = \frac{\frac{a^2}{x^2}}{\sqrt{\frac{x^4+a^4}{x^4}}} = \frac{a^2}{\sqrt{x^4+a^4}}$

$\cos\varphi = \frac{1}{\sqrt{1+tg^2\varphi}} = \frac{1}{\sqrt{1+\frac{a^4}{x^4}}} = \frac{1}{\sqrt{\frac{x^4+a^4}{x^4}}} = \frac{x^2}{\sqrt{x^4+a^4}}$

n: $0 = mg\cos\varphi + F_p^{in}\sin\varphi - N_n + 0$

$N_n = mg\cos\varphi + F_p^{in}\sin\varphi = mg\frac{x^2}{\sqrt{x^4+a^4}} + m\omega^2 x \frac{a^2}{\sqrt{x^4+a^4}} = \ldots$ (kompliciranje)

y: $0 = mg + 0 - N_n\cos\varphi + 0$

$N_n = \frac{mg}{\cos\varphi} = mg\frac{\sqrt{x^4+a^4}}{x^2} = mg \cdot \frac{\sqrt{\sqrt[3]{\frac{a^8g^4}{\omega^8}} + a^4}}{\sqrt[3]{\frac{a^4g^2}{\omega^4}}} = mg\sqrt{\frac{\left(\frac{a^2g}{\omega^2}\right)^{4/3} + a^4}{\left(\frac{a^2g}{\omega^2}\right)^{4/3}}}$ $\Rightarrow$ $\boxed{N_n = mg\sqrt{1+\left(\frac{\omega^2 a}{g}\right)^{4/3}}}$

z: $0 = 0 + 0 + N_6$ $\Rightarrow$ $\boxed{N_6 = 0}$

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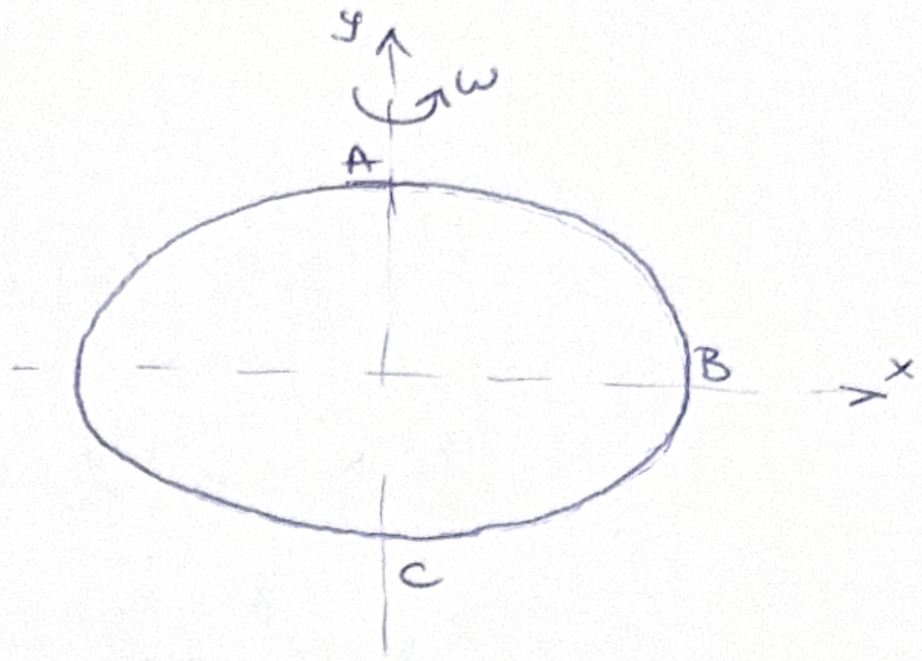
(6.4)



preden: ω

to: $\vec{v}_0 = v_0 \cdot \vec{i}$

položaj $A(0, b)$

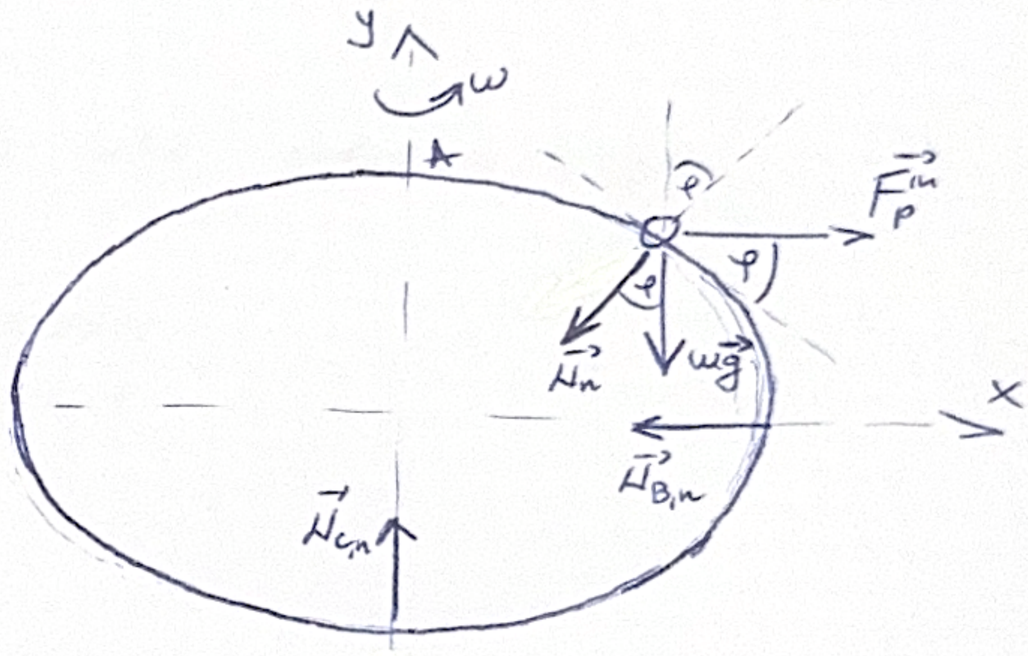
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\omega = \text{konst}$ - obratuje vlice

$N_B = ?$, $N_C = ?$

$B(a, 0)$, $C(0, -b)$

$m\vec{a_r} = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_b$



$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{or} + \vec{\varepsilon} \times \vec{s} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_{rel} = 2m\omega \cdot v_{rel} \cdot \cos\varphi \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (+\vec{n})$

$\vec{N}_b = N_b \cdot (-\vec{k})$

n: $m \cdot a_{rn} = mg\cos\varphi - F_p^{in}\sin\varphi + N_n$

$m \cdot \frac{v_{rel}^2}{R_k} = mg\cos\varphi - m\omega^2 x + N_n$

$N_n = -mg\cos\varphi + m\omega^2 x + m \cdot \frac{v_{rel}^2}{R_k}$, $v_{rel} = ?$, $R_k = ?$

b: $0 = F_{cor}^{in} - N_b \quad \rightarrow \quad N_b = F_{cor}^{in} = 2m\omega \cdot v_{rel} \cdot \cos\varphi$

$T - T_0 = A$ (rel)

$T_0 = \frac{1}{2} m v_0^2$

$T = \frac{1}{2} m v_{rel}^2$

$A = \int_{y_0=b}^{y} -mg\,dy + \int_{x_0=0}^{x} F_p^{in}\,dx = mg(b-y) + \int_0^x m\omega^2 x\,dx = mg(b-y) + \frac{1}{2} m\omega^2 x^2$

$\frac{1}{2} m (v_{rel}^2 - v_0^2) = mg(b-y) + \frac{1}{2} m\omega^2 x^2$

$v_{rel}^2 = v_0^2 + 2g(b-y) + \omega^2 x^2$

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$$R_k = \frac{(1+y'^2)^{3/2}}{|y''|}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} \quad \Rightarrow \quad y = \left(b^2 - x^2\frac{b^2}{a^2}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(b^2 - x^2\frac{b^2}{a^2}\right)^{-\frac{1}{2}} \cdot \left(-2x\frac{b^2}{a^2}\right) = \frac{1}{2}y^{-1} \cdot \left(-2x\frac{b^2}{a^2}\right) = -\frac{x}{y}\frac{b^2}{a^2}$$

$$y'' = -\frac{1}{4}\left(b^2 - x^2\frac{b^2}{a^2}\right)^{-\frac{3}{2}} \cdot \left(-2x\frac{b^2}{a^2}\right)^2 + \frac{1}{2}\left(b^2 - x^2\frac{b^2}{a^2}\right)^{-\frac{1}{2}} \cdot \left(-2\frac{b^2}{a^2}\right) =$$

$$y'' = \frac{1}{2}y^{-3} \cdot \left(x\frac{b^2}{a^2}\right)^2 - y^{-1}\frac{b^2}{a^2} = \frac{1}{2y^3}\left(x\frac{b^2}{a^2}\right)^2 - \frac{1}{y}\frac{b^2}{a^2}$$

<u>položaj B</u> $(x = a, \quad y = 0, \quad \varphi = \frac{\pi}{2})$

$R_k(y=0)$: $y'(y=0) = \infty$

$y''(y=0) = \infty - \infty$ => ne moče ovako, probajmo drugače

$x = a\sin t$, $\dot{x} = a\cos t$, $\ddot{x} = -a\sin t$

$y = b\cos t$, $\dot{y} = -b\sin t$, $\ddot{y} = -b\cos t$

$$R_k = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}$$

<u>položaj B</u>

$\dot{x}_B = 0$, $\dot{y}_B = -b$, $\ddot{x}_B = -a$, $\ddot{y}_B = 0$

$$R_{k,B} = \frac{(0+b^2)^{3/2}}{|0+ab|} = \frac{b^3}{ab} = \frac{b^2}{a}$$

$$v_{rel,B} = v_0^2 + 2gb + \omega^2 a^2$$

$$H_{n,B} = 0 + m\omega^2 a + m\frac{(v_0^2 + 2gb + \omega^2 a)a}{b^2}$$

$$\boxed{H_{n,B} = m\frac{a}{b^2}\left(v_0^2 + 2gb + (a^2+b^2)\omega^2\right)} \quad , \quad \boxed{H_{t,B} = 0} \quad , \quad \boxed{H_B = H_{n,B}}$$

<u>položaj C</u> $(x = 0, \quad y = -b, \quad \varphi = \pi)$

$\dot{x}_C = -a$, $\dot{y}_C = 0$, $\ddot{x}_C = 0$, $\ddot{y}_C = b$

$$R_{k,C} = \frac{(a^2+0)^{3/2}}{|-ab-0|} = \frac{a^3}{ab} = \frac{a^2}{b}$$

$$v_{rel,C}^2 = v_0^2 + 2g(b-(-b)) + \cancel{\omega^2 \cdot 0^{\to 0}} = v_0^2 + 4gb$$

$$H_{n,C} = mg + \cancel{m\omega^2 \cdot 0^{\to 0}} + m\cdot\frac{(v_0^2 + 4gb)b}{a^2} = m\left(g + \frac{b}{a^2}(v_0^2 + 4gb)\right)$$

$$\boxed{H_{n,C} = m\left(g + \frac{b}{a^2}(v_0^2 + 4gb)\right)} \quad , \quad \boxed{H_{t,C} = -2m\omega\sqrt{v_0^2 + 4gb}} \quad , \quad \boxed{H_C = \sqrt{H_{n,C}^2 + H_{t,C}^2}}$$

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(6.5)

ploča: ℓ, $\vec{a}$ $(a = const)$

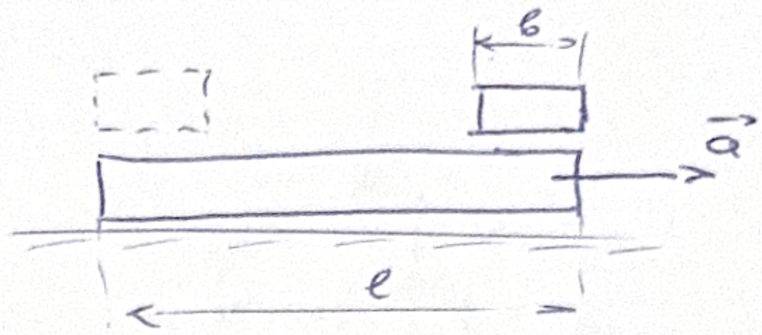
telo A: b

t_0: $v_{A,rel} = 0$

A je na zadnjoj ivici ploče

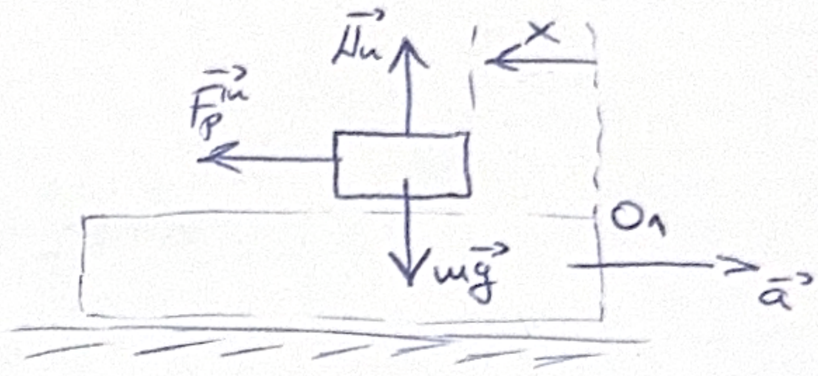
t_1: A je na drugom kraju ploče

$t_1 = ?$



Zadatak: naći $x(t)$, a onda reći da je $x(t_1) = \ell - b$

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \cancel{\vec{F}_{cor}^{in}}^{\,0} + \vec{N}_n + \cancel{\vec{N}_b}^{\,0}$$



$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \cancel{\dot{\vec{\varepsilon}} \times \vec{\rho}}^{\,0} + \vec{\omega} \times \cancel{(\vec{\omega} \times \vec{\rho})}^{\,0}) = ma \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 0$$

$$\vec{N}_n = N_n \cdot (\vec{j})$$

x: $\quad m\ddot{x} = 0 + F_p^{in} + 0$

$$m\ddot{x} = ma \qquad /\int dt$$

$$\dot{x} = \int a\,dt = at + \cancel{c_1}^{\,0} \qquad /\int dt$$

$$x = \frac{1}{2}at^2 + \cancel{c_2}^{\,0}$$

$$x = \frac{1}{2}at^2$$

$$x(t_1) = \frac{1}{2}at_1^2 = \ell - b$$

$$\boxed{t_1 = \sqrt{\frac{2(\ell - b)}{a}}}$$

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(6.6.) nastavak na prethodni zadatak, samo što između tela i ploče postoji trenje koeficijenta μ.

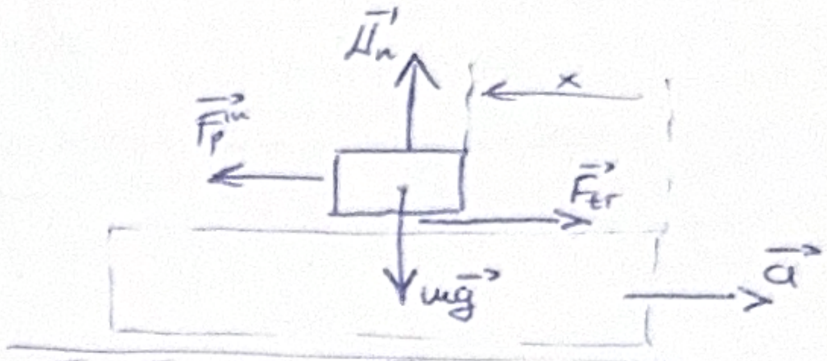
---

x: $m\ddot{x} = F_p^{in} - F_{tr} = ma - \mu \cdot N = ma - \mu \cdot mg$

$\ddot{x} = a - \mu g \quad / \int dt$

$\dot{x} = (a - \mu g)t + c_1^{\,0} \quad / \int dt$

$x = \frac{1}{2}(a - \mu g)t^2 + c_2^{\,0}$



$x(t_1) = \frac{1}{2}(a - \mu g)t_1^2 = l - b$

$\Rightarrow$ $\boxed{t_1 = \sqrt{\frac{2(l-b)}{a - \mu g}}}$, uslov: $a - \mu g > 0$

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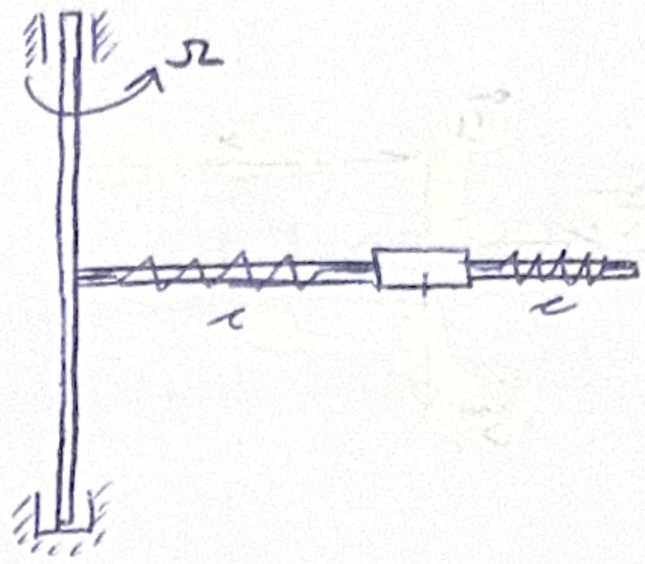
(6.7.) OA: l, $\Omega = \text{const.}$

klizač: m

opruge: c, $l_0 = \frac{l}{2}$

i-na rel. kretanja klizača po štapu?



$l_e = l + l = 2l$

$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$

$m\vec{g} = mg \cdot (-\vec{j})$

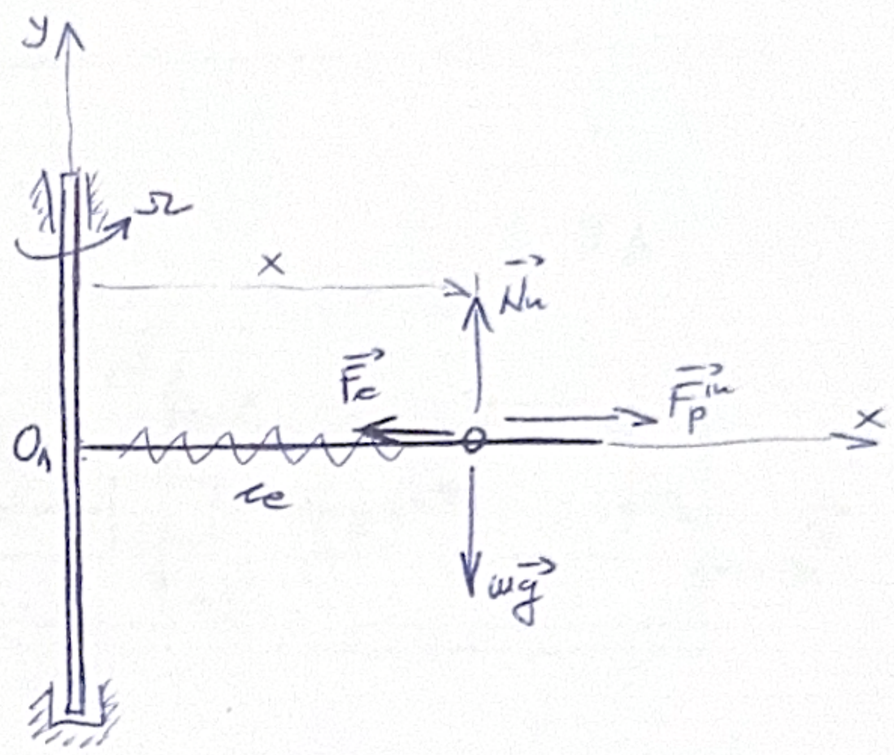
$\vec{F}_c = l_e \cdot \Delta l \cdot (-\vec{i}) = 2c \cdot (x - \frac{1}{2}l) \cdot (-\vec{i}) = c \cdot (2x - l) \cdot (-\vec{i})$

$\vec{F}_p^{in} = -m(\cancel{\vec{a}_{O_1}}^{\,0} + \cancel{\vec{\varepsilon} \times \vec{\rho}}^{\,0} + \vec{\Omega} \times (\vec{\Omega} \times \vec{\rho})) = m\Omega^2 \cdot x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\vec{\Omega} \times \vec{v}_r = 2m \cdot \Omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{j})$

$\vec{N}_6 = N_6 \cdot (-\vec{k})$



x: $m\ddot{x} = -c(2x - l) + m\Omega^2 x \quad / : m$

$\ddot{x} + \frac{c}{m} \cdot 2x - \Omega^2 x = \frac{cl}{m}$

$\ddot{x} + \frac{2c - \Omega^2 m}{m} x = \frac{cl}{m}$

---

1) $2c > \Omega^2 m \quad \Rightarrow \quad \omega^2 = \frac{2c - \Omega^2 m}{m}, \quad h = \frac{cl}{m}$

H: $\lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = 0 \pm i\omega \quad \rightarrow \quad x_h = A_1 \cos\left(\sqrt{\frac{2c - \Omega^2 m}{m}}\, t\right) + A_2 \sin\left(\sqrt{\frac{2c - \Omega^2 m}{m}}\, t\right)$

P: $b(t) = h = e^{0t}(h\cos 0t + 0\sin 0t)$

$x_p = t^0 e^{0t}(A\cos 0t + B\sin 0t) = A, \quad \ddot{x}_p = 0$

$0 + \omega^2 A = h \quad \rightarrow \quad A = \frac{h}{\omega^2} = \frac{\frac{cl}{m}}{\frac{2c - \Omega^2 m}{m}} = \frac{cl}{2c - \Omega^2 m}$

$$x = A_1 \cos\left(\sqrt{\frac{2c - \Omega^2 m}{m}}\, t\right) + A_2 \sin\left(\sqrt{\frac{2c - \Omega^2 m}{m}}\, t\right) + \frac{cl}{2c - \Omega^2 m}$$

2) $2c = \Omega^2 m \quad \Rightarrow \quad \ddot{x} = \frac{cl}{m} \quad \Rightarrow \quad \dot{x} = \frac{cl}{m} t + B_1 \quad \rightarrow \quad \boxed{x = \frac{cl}{2m} t^2 + B_1 t + B_2}$

3) $2c < \Omega^2 m \quad \Rightarrow \quad \omega^2 = \frac{\Omega^2 m - 2c}{m}, \quad h = \frac{cl}{m}$

H: $\lambda^2 - \omega^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = \pm\omega \pm 0i \quad \Rightarrow \quad x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$

P: $x_p = A, \quad \ddot{x}_p = 0 \quad \Rightarrow \quad 0 - \omega^2 A = h \quad \Rightarrow \quad A = -\frac{h}{\omega^2} = -\frac{cl}{2c - \Omega^2 m}$

$$x = C_1 e^{\sqrt{\frac{\Omega^2 m - 2c}{m}}\, t} + C_2 e^{-\sqrt{\frac{\Omega^2 m - 2c}{m}}\, t} + \frac{cl}{2c - \Omega^2 m}$$

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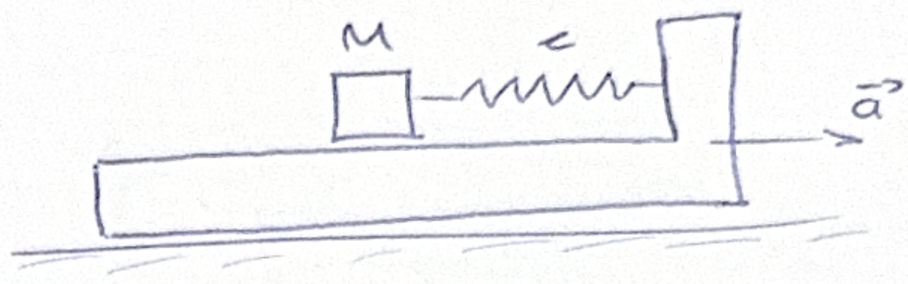
(68.) M: $m = 10\ kg$

opruga: $c = \frac{4\,kN}{m} = 4000\ \frac{N}{m}$

$\Delta l_{max} = ?$

$t_0 = 0$: A miruje, M miruje

$a_A = a = 3\ \frac{m}{s}$



$$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^{in} + \cancel{\vec{F}_{cor}^{in}}^{\,0} + \vec{N}_n + \cancel{\vec{M}_c}^{\,0}$$

$m\vec{g} = mg \cdot (-\vec{j})$

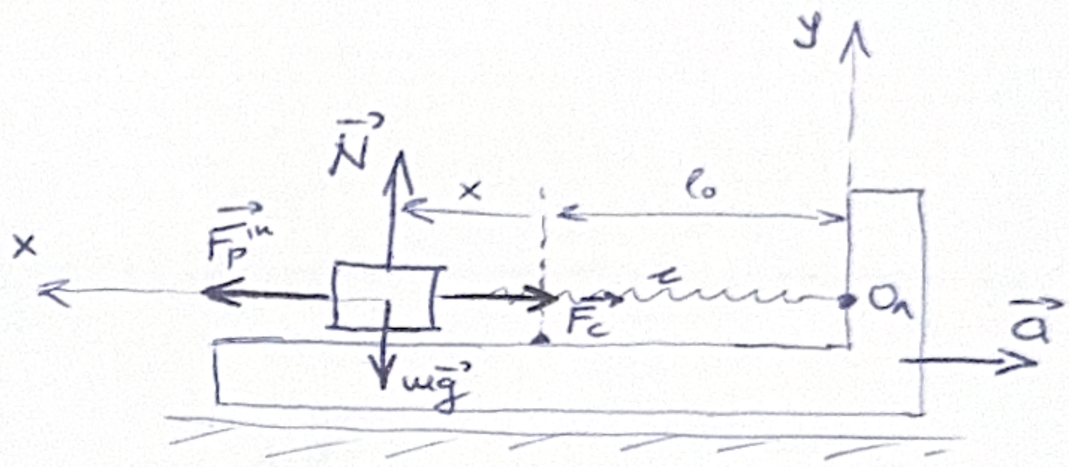
$\vec{F}_c = c \cdot \Delta l \cdot (-\vec{i}) = c \cdot x \cdot (-\vec{i})$

$\vec{F}_p^{in} = -m\left(\vec{a}_{OA} + \cancel{\vec{\varepsilon} \times \vec{\rho}}^{\,0} + \vec{\omega} \times \cancel{(\vec{\omega} \times \vec{\rho})}^{\,0}\right) = m \cdot a \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 0$

$\vec{N}_n = N \cdot (\vec{j}) = \vec{N}$

$\vec{M}_c = 0$



x: $m\ddot{x} = -cx + ma \quad / : m$

$\ddot{x} + \frac{c}{m} x = a \quad , \quad \omega^2 = \frac{c}{m} \quad , \quad h = a$

$\ddot{x} + \omega^2 x = h$

H: $\lambda^2 + \omega^2 \cdot 1 = 0 \rightarrow \lambda_{1/2} = 0 \pm i \cdot \omega \rightarrow x_h = C_1 \cos\left(\sqrt{\frac{c}{m}}\,t\right) + C_2 \sin\left(\sqrt{\frac{c}{m}}\,t\right)$

P: $b(t) = h = e^{0t}(h \cos 0t + 0 \sin 0t)$

$x_p = t^0 e^{0t}(A \cos 0t + B \sin 0t) = A \quad , \quad \ddot{x}_p = 0$

$0 + \omega^2 A = h \rightarrow A = \frac{h}{\omega^2} = \frac{a}{\frac{c}{m}} = \frac{ma}{c}$

$x = C_1 \cos\left(\sqrt{\frac{c}{m}}\,t\right) + C_2 \sin\left(\sqrt{\frac{c}{m}}\,t\right) + \frac{ma}{c}$

$\dot{x} = -\sqrt{\frac{c}{m}}\,C_1 \sin\left(\sqrt{\frac{c}{m}}\,t\right) + \sqrt{\frac{c}{m}}\,C_2 \cos\left(\sqrt{\frac{c}{m}}\,t\right)$

$x(t_0) = C_1 \cdot 1 + 0 + \frac{ma}{c} = 0 \rightarrow C_1 = -\frac{ma}{c}$

$\dot{x}(t_0) = 0 + \sqrt{\frac{c}{m}}\,C_2 \cdot 1 = 0 \rightarrow C_2 = 0$

$x = -\frac{ma}{c} \cos\left(\sqrt{\frac{c}{m}}\,t\right) + \frac{ma}{c}$

$x_{max} = -\frac{ma}{c} \cdot (-1) + \frac{ma}{c} = \frac{2ma}{c} = \frac{2 \cdot 10 \cdot 3}{4000} = 0{,}015\ m = 1{,}5\ cm$

$\boxed{\Delta l_{max} = 1{,}5\ cm}$

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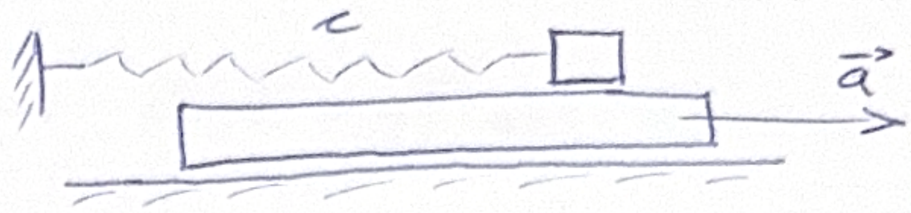
(6.9)



ploča: $\vec{a}$

kliznač: m

opruga: $c = k^2 m$

koef tr: μ

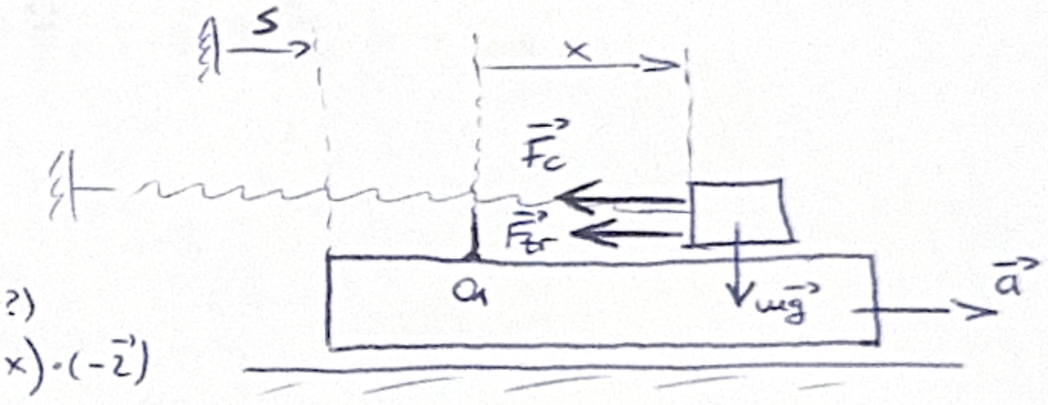
$t_0 = 0$: ploča miruje
opruga sabijena $\Delta l_0 = \frac{\mu g}{k^2}$
kliznač v_0

$t_1 = \frac{\pi}{3k}$: kliznač se prvi put zaustavlja na ploči

$v_0 = ?$

pod kojim uslovima će to početi ravnoteže u kratkom intervalu vremena?

$m\vec{a}_r = m\vec{g} + \vec{F}_{tr} + \vec{F}_c + \vec{F}_p^{in} + \cancel{\vec{F}_{cor}^{in}} + \vec{N}$



$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_{tr} = \mu \cdot N \cdot (-\vec{i}) = \mu \cdot mg \cdot (-\vec{i})$

$\vec{F}_c = c \cdot \Delta l \cdot (-\vec{i}) = c \cdot (-\Delta l_0 + s + x) \cdot (\vec{i}) = k^2 m \left(-\frac{\mu g}{k^2} + s + x\right) \cdot (-\vec{i})$ $\quad (s = ?)$

$\vec{F}_p^{in} = -m(\vec{a}_{on} + \cancel{\vec{\varepsilon} \times \vec{\rho}}^{\,0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m \cdot a \cdot (-\vec{i})$

$\vec{N} = N \cdot (\vec{j})$

$\ddot{s} = a \quad / \int dt$

$\dot{s} = at + \cancel{C_1}^{\,0} \quad / \int dt$

$s = \frac{1}{2} a t^2 + \cancel{C_2}^{\,0}$

x: $m\ddot{x} = -\mu \cdot mg - k^2 \cdot m \left(-\frac{\mu g}{k^2} + \frac{1}{2} a t^2 + x\right) - ma$

$m\ddot{x} + k^2 m x = -\mu mg + m\mu g - \frac{1}{2} k^2 m a t^2 - ma$

$\ddot{x} + k^2 x = -\frac{1}{2} k^2 a t^2 - a$

H: $\lambda^2 + k^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = 0 \pm i \cdot k \quad \rightarrow \quad x_h = C_1 \cos kt + C_2 \sin kt$

P: $f(t) = -\frac{1}{2} k^2 a t^2 - a = e^{\alpha t} (P_1(t) \cos \omega t + P_2(t) \sin \omega t)$

$x_p = t^0 e^{0t} \left((At^2 + Bt + C) \cos 0t + (Dt^2 + Et + F) \sin 0t\right) = At^2 + Bt + C, \quad \ddot{x}_p = 2A$

$2A + k^2 A t^2 + k^2 B t + k^2 C = -\frac{1}{2} k^2 a t^2 - a$

$\rightarrow \quad B = 0, \quad A = -\frac{1}{2} a, \quad C = \frac{-a - 2 \cdot (-\frac{1}{2} a)}{k^2} = 0 \quad \Rightarrow \quad x_p = -\frac{1}{2} a t^2$

$x = C_1 \cos kt + C_2 \sin kt - \frac{1}{2} a t^2$

$\dot{x} = -C_1 k \sin kt + C_2 k \cos kt - at$

$x(t_0) = 0 \quad \rightarrow \quad C_1 \cdot 1 + 0 - 0 = 0 \quad \rightarrow \quad C_1 = 0$

$\dot{x}(t_0) = v_0 \quad \rightarrow \quad -0 + C_2 \cdot k - 0 = v_0 \quad \rightarrow \quad C_2 = \frac{v_0}{k}$

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$$x = \frac{v_0}{k}\sin(kt) - \frac{1}{2}at^2$$

$$\dot{x}\left(t_1 = \frac{\pi}{3k}\right) = 0 \quad \rightarrow \quad \frac{v_0}{k}\cdot k\cos\left(k\cdot\frac{\pi}{3k}\right) - a\cdot\frac{\pi}{3k} = 0 \quad \rightarrow \quad v_0\cdot\frac{1}{2} - \frac{a\pi}{3k} = 0$$

$$\hookrightarrow \boxed{v_0 = \frac{2}{3}\frac{a\pi}{k}}$$

usla relativno ramotefe u trenutku t_1:

$$\underline{m\vec{a}_r\left(t_1 = \frac{\pi}{3k}\right) = 0}$$, stanje ravnoteže: $F_{tr} \leq \mu\cdot mg$, ; to suprotnog smera!

x:

$$0 = +F_{tr} - k^2 m\left(-\frac{\mu g}{k^2} + \frac{1}{2}a t_1^2 + x(t_1)\right) - ma$$

$$0 = F_{tr} - k^2 m\left(-\frac{\mu g}{k^2} + \frac{1}{2}a\frac{\pi^2}{9k^2} + \frac{v_0}{k}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}a\frac{\pi^2}{9k^2}\right) - ma$$

$$0 = F_{tr} + m\mu g - k\cdot m\, v_0\frac{\sqrt{3}}{2} - ma$$

$$0 = F_{tr} + m\mu g - k\cdot m\cdot\frac{2}{3}\frac{a\pi}{k}\frac{\sqrt{3}}{2} - ma$$

$$0 = F_{tr} + m\mu g - \frac{\sqrt{3}}{3}ma\pi - ma$$

$$F_{tr} = ma + \frac{\sqrt{3}}{3}a\pi\cdot m - m\mu g$$

graniči slučaj: $F_{tr} = F_{tr,max} = m\mu g$

$$\cancel{m}\mu g = \cancel{m}a + \frac{\sqrt{3}}{3}a\pi\cancel{m} - \cancel{m}\mu g$$

$$2\mu g = \frac{a}{3}(3 + \pi\sqrt{3})$$

$$\boxed{\mu \geq \frac{a}{6g}(3 + \pi\sqrt{3})}$$

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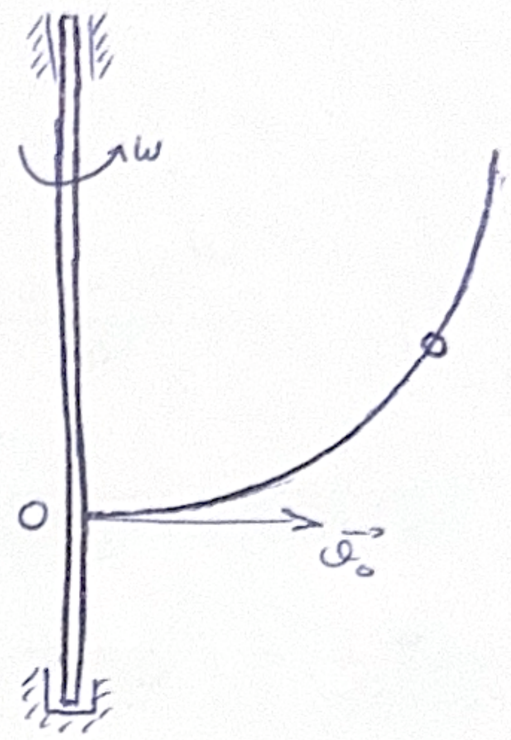
(6.10.) $y = \frac{x^2}{2a}$

$\omega = \sqrt{\frac{g}{a}} = \text{const.}$



vertikalna osa Oy usmerena navise

prsten mase m

$t_0 = 0$: prsten je u temenu parabole

$v_0 = \sqrt{ag}$

N u proizvoljnom položaju ?

$m\vec{a} = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_b$

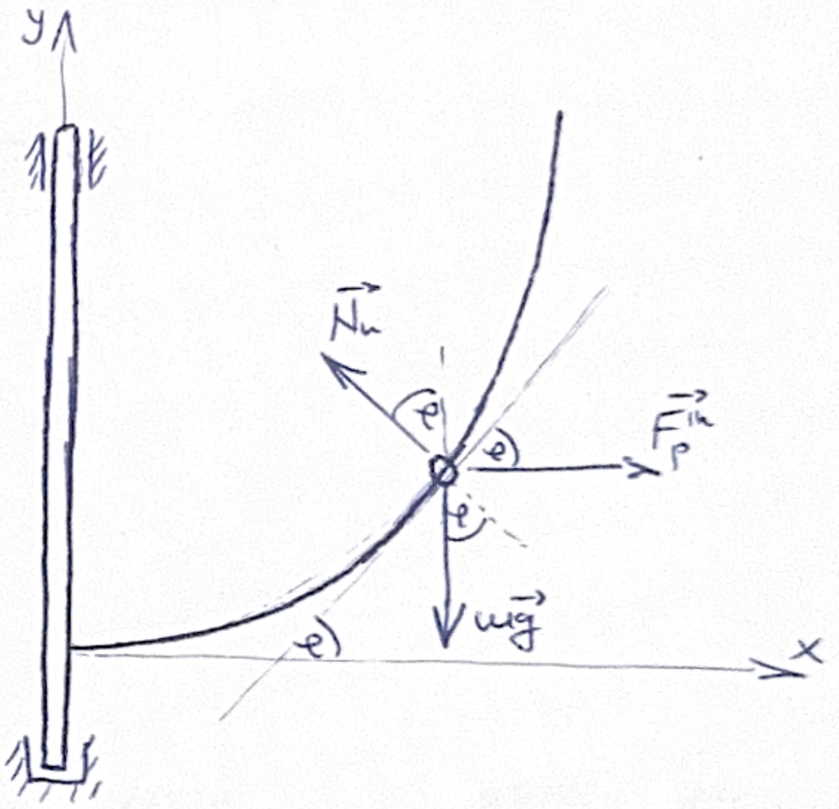
$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{0n}^{\,0} + \vec{\varepsilon} \times \vec{\rho}^{\,0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (+\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{n})$

$\vec{N}_b = N_b \cdot (\vec{k})$



I način: rastavimo savo na x ili savo na y ; dobijemo N_n (dosta komplikacija)

II način: rastavimo na n (lakši)

n: $ma_n = -mg\cos\varphi - F_p^{in}\sin\varphi + N_n$, $a_n = \frac{v_{rel}^2}{R_k}$, $v_{rel} = ?$, $R_k = ?$, $\sin\varphi = ?$, $\cos\varphi = ?$

$T - T_0 = A$ (rel)

$T = \frac{1}{2} m v_{rel}^2$

$T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} mag$

$A = \int_0^{y=\frac{x^2}{2a}} -mg\,dy + \int_0^x F_p^{in}\,dx = -\frac{1}{2} mg \frac{x^2}{a} + \int_0^x m\omega^2 x\,dx = -\frac{1}{2}\frac{mgx^2}{a} + \frac{1}{2} m \frac{g}{a} \cdot x^2 = 0$

$\frac{1}{2} m v_{rel}^2 - \frac{1}{2} mag = 0 \quad \Rightarrow \quad v_{rel} = \sqrt{ag}$

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$$R_k = \frac{(1+y'^2)^{3/2}}{|y''|}, \qquad y' = \frac{x}{a}, \qquad y'' = \frac{1}{a}$$

$$R_k = \frac{\left(1+\frac{x^2}{a^2}\right)^{3/2}}{\frac{1}{a}} = \frac{\frac{(a^2+x^2)^{3/2}}{(a^2)^{3/2}}}{\frac{1}{a}} = \frac{(a^2+x^2)^{3/2}\cdot a}{a^3} = \frac{(a^2+x^2)^{3/2}}{a^2}$$

$$tg\varphi = y' = \frac{x}{a} \rightarrow \sin\varphi = \frac{tg\varphi}{\sqrt{1+tg^2\varphi}} = \frac{\frac{x}{a}}{\sqrt{1+\frac{x^2}{a^2}}} = \frac{\frac{x}{a}}{\sqrt{\frac{a^2+x^2}{a^2}}} = \frac{x}{\sqrt{a^2+x^2}}$$

$$\rightarrow \cos\varphi = \frac{1}{\sqrt{1+tg^2\varphi}} = \frac{1}{\sqrt{1+\frac{x^2}{a^2}}} = \frac{1}{\sqrt{\frac{a^2+x^2}{a^2}}} = \frac{a}{\sqrt{a^2+x^2}}$$

$$H_n = m\cdot a_{n,rel} + mg\cos\varphi + F_p^{in}\sin\varphi$$

$$H_n = m\cdot\frac{ga}{\frac{(a^2+x^2)^{3/2}}{a^2}} + mg\frac{a}{\sqrt{a^2+x^2}} + m\omega^2 x\cdot\frac{x}{\sqrt{a^2+x^2}}$$

$$H_n = m\cdot\frac{ga^3}{(a^2+x^2)^{3/2}} + mg\frac{a}{\sqrt{a^2+x^2}} + m\frac{g}{a}\frac{x^2}{\sqrt{a^2+x^2}}$$

$$H_n = \frac{mg}{(a^2+x^2)^{3/2}}\left(a^3 + a(a^2+x^2) + \frac{x^2}{a}(a^2+x^2)\right)$$

$$\boxed{H_n = \frac{mg}{(a^2+x^2)^{3/2}}\left(2a^3 + 2ax^2 + \frac{x^4}{a}\right)}$$ – isto je rešenje i u zbirci, samo u drugačijem zapisu (malo sređeno)

Iz: $0 = F_{cor}^{in} - H_e$

$$H_e = F_{cor}^{in} = 2m\omega\cdot\dot{x} = 2m\cdot\sqrt{\frac{g}{a}}\cdot\dot{\vartheta}_{rel}\cdot\cos\varphi = 2m\sqrt{\frac{g}{a}}\cdot\sqrt{ag}\cdot\frac{a}{\sqrt{a^2+x^2}}$$

$$\boxed{H_e = \frac{2mga}{\sqrt{a^2+x^2}}}$$

$$\boxed{H = \sqrt{H_n^2 + H_e^2}}$$

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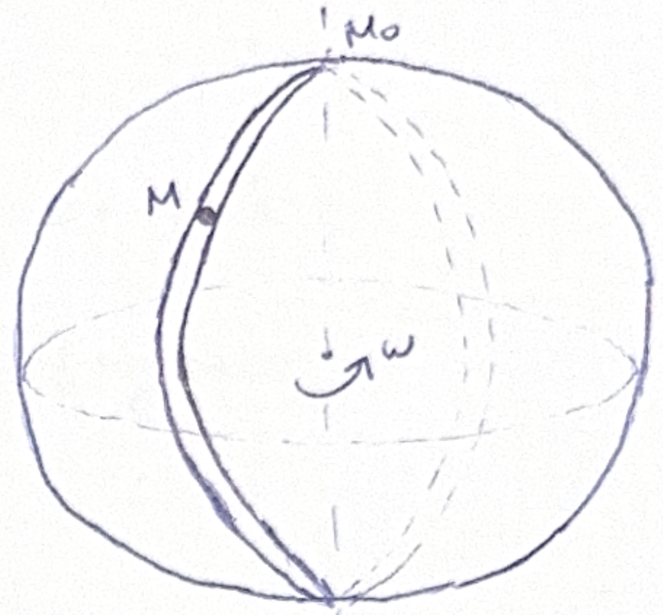
6.11.



kopka R

kuglica: m

$\omega = \sqrt{\frac{g}{2R}} = \text{const}$

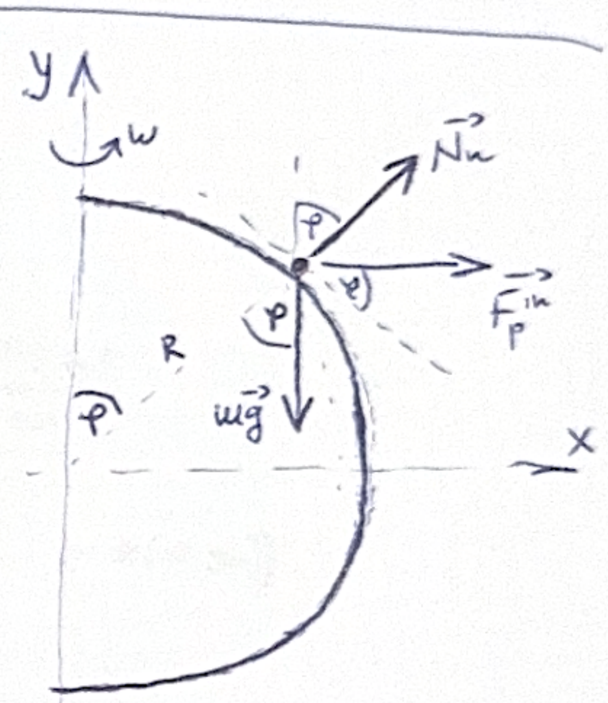
vertikalna ravan

to: kuglica u M_0

$v_0 = \frac{\sqrt{gR}}{5}$

položaj u kojem kuglica napušta žleb?

horizontalna reakcija žleba u tau trenutku?



$m\vec{a} = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_t$

$m\vec{g} = mg\cdot(-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_0^{\,\circ} + \vec{\varepsilon}\times\vec{s}^{\,\circ} + \vec{\omega}\times(\vec{\omega}\times\vec{s})) = m\omega^2 x\cdot(\vec{i})$

$\vec{F}_{cor}^{in} = -2m\vec{\omega}\times\vec{v}_r = +2m\omega\cdot\dot{x}\cdot(-\vec{k})$

$\vec{N}_n = N_n\cdot(-\vec{n})$

$\vec{N}_t = N_t\cdot(\vec{t})$

cilj: naći $N(\varphi)$; reći da je $N(\varphi_1)=0$ => odatle dobijamo φ_1

n: $m\cdot a_n = mg\cos\varphi - F_p^{in}\sin\varphi - N_n$, $a_n = \frac{v_{rel}^2}{R_k}$, $v_{rel}=?$, $R_k = R$

$T - T_0 = A$ (rel)

$T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{gR}{25} = \frac{1}{50}mgR$

$T = \frac{1}{2}mv_{rel}^2$

$A = \int_R^{R\cos\varphi} -mg\,dy + \int_0^{R\sin\varphi} F_p^{in}\,dx = mgR(1-\cos\varphi) + \int_0^{R\sin\varphi} m\omega^2 x\,dx = mgR(1-\cos\varphi) + \frac{1}{2}m\omega^2R^2\sin^2\varphi =$

$A = mgR(1-\cos\varphi) + \frac{1}{2}m\frac{g}{2R}R^2\sin^2\varphi = mgR(1-\cos\varphi+\frac{1}{4}\sin^2\varphi)$

$\frac{1}{2}mv_{rel}^2 - \frac{1}{50}mgR = mgR(1-\cos\varphi+\frac{1}{4}\sin^2\varphi)$

$v_{rel}^2 = \frac{1}{25}gR + 2gR(1-\cos\varphi+\frac{1}{4}\sin^2\varphi)$

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$$N_n = mg\cos\varphi - F_p^{in}\sin\varphi - ma_n$$

$$N_n = mg\cos\varphi - m\cdot\frac{g}{2R}\,k\sin\varphi\cdot\sin\varphi - m\cdot\frac{1}{25}g - m\cdot 2g\left(1-\cos\varphi+\frac{1}{4}\sin^2\varphi\right)$$

$$N_n = mg\cos\varphi - \frac{1}{2}mg\sin^2\varphi - \frac{1}{25}mg - 2mg + 2mg\cos\varphi - \frac{1}{2}mg\sin^2\varphi$$

$$N_n = -\frac{51}{25}mg + 3mg\cos\varphi - mg\sin^2\varphi$$

$$N_n(\varphi_1) = 0 \quad \rightarrow \quad -\frac{51}{25} + 3\cos\varphi - \sin^2\varphi = 0$$

$$-\frac{51}{25} + 3\cos\varphi - (1-\cos^2\varphi) = 0$$

$$-\frac{76}{25} + 3\cos\varphi + \cos^2\varphi = 0$$

$$(\cos\varphi_1)_{1/2} = \frac{-3 \mp \sqrt{9+\frac{4\cdot 76}{25}}}{2} = \frac{-3 \mp \sqrt{\frac{529}{25}}}{2} = -\frac{\frac{15}{5} \mp \frac{23}{5}}{2} = \frac{-15 \mp 23}{10}$$

$$(\cos\varphi_1)_1 = \frac{-15-23}{10} < 0 \quad - \text{otpada resenje} \quad (\text{jer je } \varphi \in (0,90))$$

$$(\cos\varphi_1)_2 = \frac{-15+23}{10} = \frac{8}{10} = \frac{4}{5} \quad - \text{resenje}$$

$$\boxed{\varphi_1 = \arccos\left(\frac{4}{5}\right)}$$

Z: $\quad 0 = -F_{cor}^{in} + N_6 \quad \Rightarrow \quad N_6 = F_{cor}^{in} = 2m\omega\cdot\dot{x}$, $\quad \dot{x}(\varphi_1) = ?$

$$\dot{x}(\varphi_1) = v_{rel}\cdot\cos\varphi_1 = \sqrt{\frac{1}{25}gR + 2gR\left(1-\cos\varphi_1+\frac{1}{4}\sin^2\varphi_1\right)}\cdot\cos\varphi_1 \quad , \quad \cos\varphi_1 = \frac{4}{5} \quad , \quad \sin^2\varphi_1 = 1-\cos^2\varphi_1 = \frac{9}{25}$$

$$\dot{x}(\varphi_1) = \sqrt{\frac{1}{25}gR + 2gR\left(1-\frac{4}{5}+\frac{1}{4}\cdot\frac{9}{25}\right)}\cdot\frac{4}{5} = \sqrt{\frac{1}{25}gR + 2gR\cdot\frac{29}{100}}\cdot\frac{4}{5} = \sqrt{\frac{31}{50}gR}\cdot\frac{4}{5} = \frac{4\sqrt{31}}{25\sqrt{2}}\sqrt{gR}$$

$$N_6 = 2m\cdot\sqrt{\frac{g}{2R}}\cdot\frac{4\sqrt{31}}{25\sqrt{2}}\sqrt{gR} = \frac{4\sqrt{31}}{25}mg$$

$$\boxed{N_6 = \frac{4\sqrt{31}}{25}mg}$$

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(6.12.) vertikalna ravan

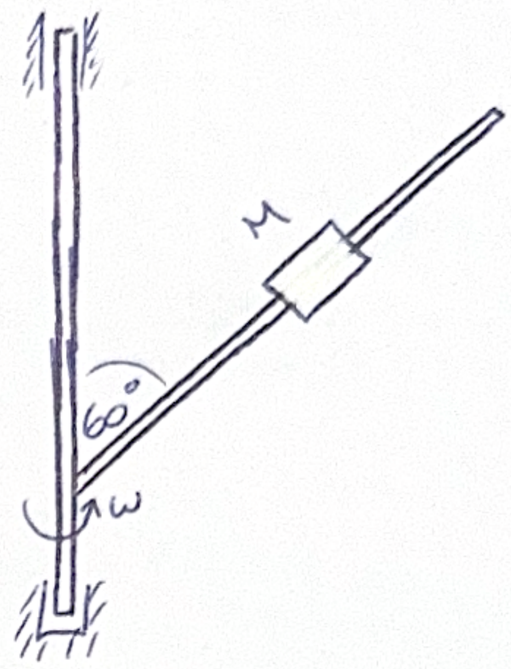
ugaona brzina ω

klizač: m

to: klizač je u položaju $\overline{OM_1} = a$

$\omega(t) = ?$ - da bi se klizač kretao $v_{rel} = v_0 = const.$

$U = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{U}_n + \vec{U}_6$$

$$m\vec{g} = mg\cdot(-\vec{j})$$

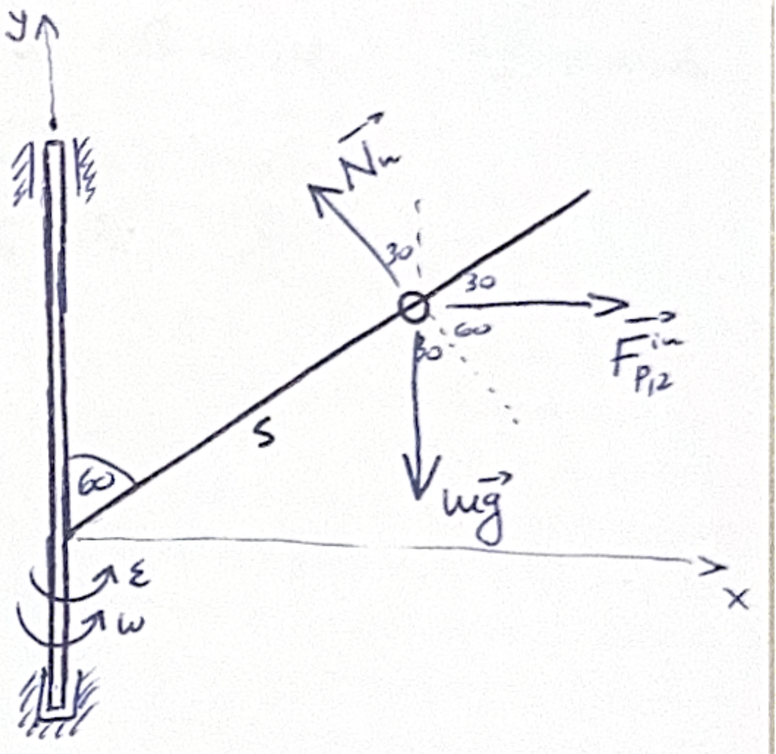
$$\vec{F}_p^{in} = -m(\vec{a}_{o_1}^{\,0} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) =$$

$$= m\cdot\varepsilon\cdot x\cdot(\vec{k}) + m\omega^2 x\cdot(\vec{i}) = \vec{F}_{P,1}^{in} + \vec{F}_{P,2}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = 2m\omega\cdot\dot{x}\cdot(\vec{k})$$

$$\vec{U}_n = U_n\cdot(\vec{n})$$

$$\vec{U}_6 = U_6\cdot(-\vec{k})$$



t: $m\cdot\cancel{a_{r,t}}^{\,0} = -mg\sin 30 + F_{P,2}^{in}\cos 30$

$0 = -\frac{1}{2}mg + m\omega^2 x\frac{\sqrt{3}}{2}$ /:m

$\omega^2 = \frac{1}{2}\frac{g}{x}\frac{2}{\sqrt{3}} = \frac{g}{x\sqrt{3}}$, $x = ?$

$\ddot{s} = 0 \rightarrow \dot{s} = const. = v_0 \rightarrow s = v_0 t + const. = v_0 t + a$

$x = s\cdot\sin 60 = (v_0 t + a)\frac{\sqrt{3}}{2}$ $\Rightarrow$ $\omega^2 = \frac{g}{(v_0 t + a)\frac{\sqrt{3}}{2}\sqrt{3}} = \frac{2g}{3(v_0 t + a)}$ $\Rightarrow$ $\boxed{\omega = \sqrt{\frac{2g}{3(v_0 t + a)}}}$

y: $0 = -mg + U_n\cos 30$

$U_n = \frac{mg}{\cos 30} = \frac{mg}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}mg$

z: $0 = F_{P,1}^{in} + F_{cor}^{in} - U_6$

$U_6 = F_{P,1}^{in} + F_{cor}^{in} = m\cdot\varepsilon\cdot x + 2m\cdot\omega\cdot\dot{x}$, $\varepsilon = ?$, $\dot{x} = v_0\cos 30 = v_0\frac{\sqrt{3}}{2}$

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$$\omega^2 = \frac{2}{3}\,\frac{g}{v_0 t + a} \qquad \Big| \frac{d}{dt}$$

$$2 \cdot \omega \cdot \varepsilon = \frac{2}{3}\,\frac{0 - g \cdot v_0}{(v_0 t + a)^2} = \frac{2}{3}\,\frac{g}{(v_0 t + a)} \cdot \left(-\frac{v_0}{(v_0 t + a)}\right) = \omega^2 \cdot \left(-\frac{v_0}{v_0 t + a}\right) \qquad / : 2\omega$$

$$\varepsilon = -\frac{1}{2}\,\omega \cdot \frac{v_0}{v_0 t + a}$$

$$M_6 = m \cdot \left(-\frac{1}{2}\,\omega\,\frac{v_0}{v_0 t + a}\right) \cdot (v_0 t + a)\,\frac{\sqrt{3}}{2} + 2m\omega \cdot v_0 \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}\,m\omega v_0 + \sqrt{3}\,m\omega v_0 = \frac{3\sqrt{3}}{4}\,m\omega v_0$$

$$M = \sqrt{M_n^2 + M_6^2} = \sqrt{\frac{4}{3}\,m^2 g^2 + m\omega^2 v_0^2 \cdot \frac{27}{16}} = m \cdot \sqrt{\frac{4}{3}\,g^2 + \frac{2}{3}\,\frac{g}{v_0 t + a} \cdot v_0^2 \cdot \frac{27}{16}}$$

$$\boxed{M = m\sqrt{\frac{4}{3}\,g^2 + \frac{9}{8}\,\frac{g v_0^2}{v_0 t + a}}}$$

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(6.13.)

O_1A, AB, BO_2: $\emptyset$, e

vertikalna ravan

O_1A, BO_2: $\omega = const.$

klizač: m

opruge: $c = \frac{m\omega^2}{2}$

$t_0 = 0$:

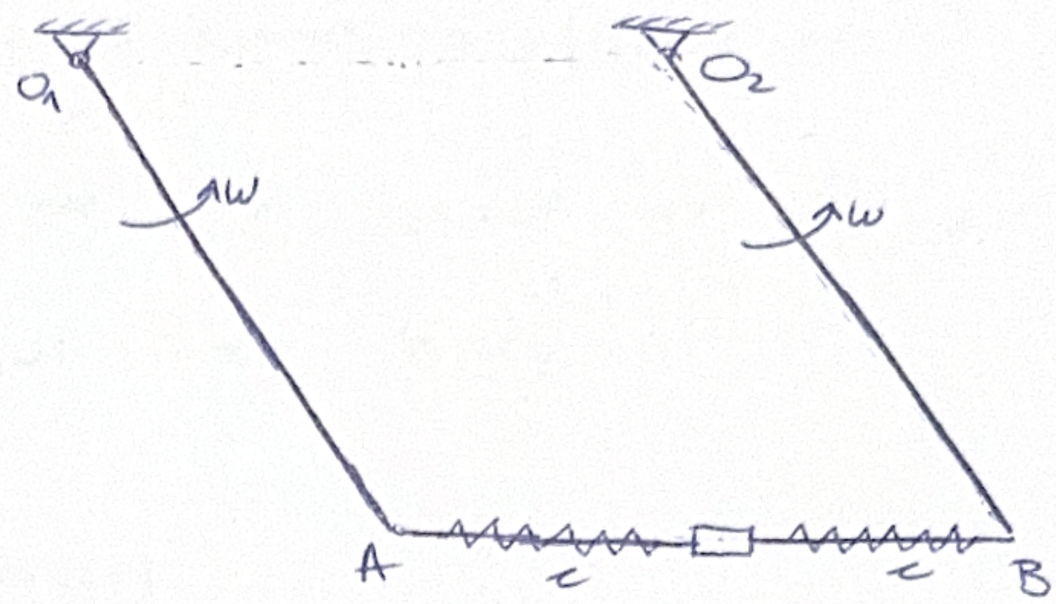
O_1A, O_2B: vertikalni

klizač: na sredini AB relativno mirovao

opruge: nenapregnute

$X(t) = ?$ - j-na relativnog kretanja klizača po štapu AB

$N(t) = ?$ - reakcija između štapa i klizača



$k_e = k_1 + k_2 = 2c = m\omega^2$

$m\vec{a_r} = m\vec{g} + \vec{F_c} + \vec{F_p^{in}} + \vec{F_{cor}^{in}} + \vec{M_u} + \vec{M_G}$ (poništi se)

$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F_c} = k_e \cdot x \cdot (-\vec{i}) = m\omega^2 x \cdot (\vec{i})$

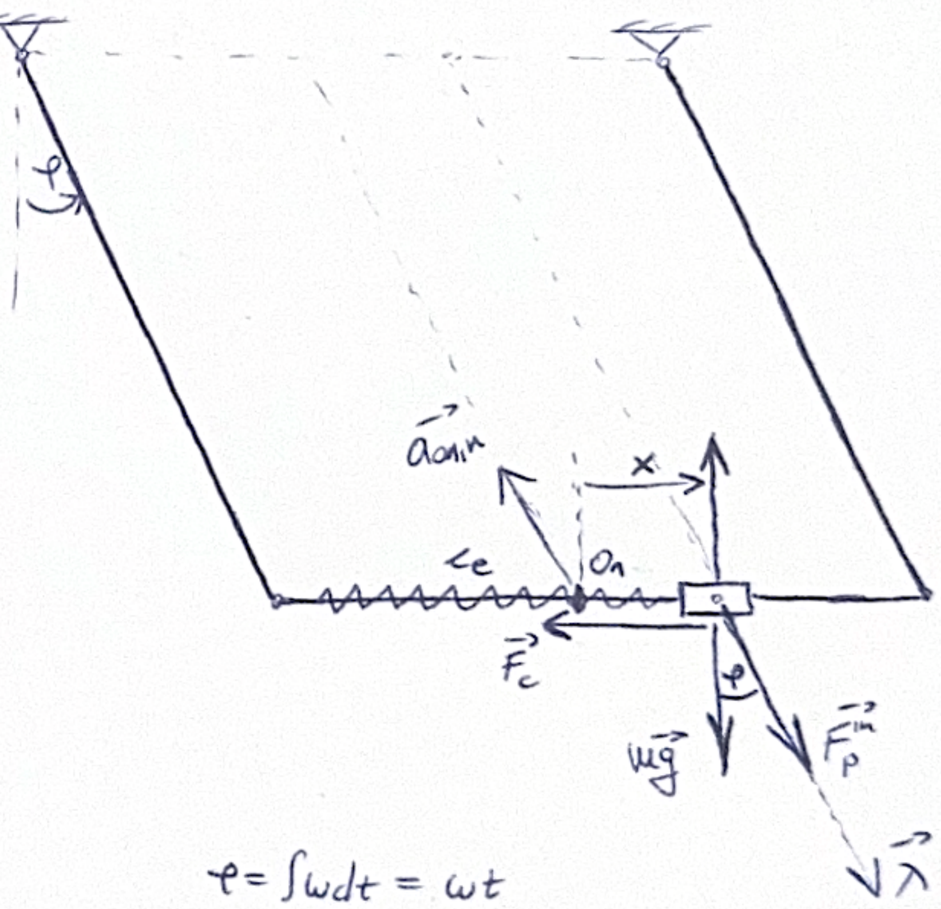
$\vec{F_p^{in}} = -m(\vec{a_{on}} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$ (poništi se)

$= -m(\vec{a_{on,n}} + \vec{a_{on,t}}) = m \cdot \omega^2 \cdot e \cdot (\vec{\lambda})$

$\vec{F_{cor}^{in}} = -2m\vec{\omega} \times \vec{v_r} = 0$

$\vec{M_u} = M_u \cdot (\vec{j})$

$\vec{M_G} = 0$



$\varphi = \int \omega dt = \omega t$

x:

$m\ddot{x} = -m\omega^2 x + m\omega^2 e \sin(\omega t)$

$\ddot{x} + \omega^2 x = \omega^2 e \sin(\omega t)$

H:

$\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega \cdot i$, $(\alpha = 0, \beta = \omega)$

$x_h = C_1 \cos \omega t + C_2 \sin \omega t$

P:

$\ddot{G}(t) = e^{0t}(0 \cos(\omega t) + \omega^2 e \sin(\omega t))$, $(a = 0, \beta = \omega)$

$x_p = t^1 e^{0t}(A \cos \omega t + B \sin \omega t)$

$\dot{x}_p = A \cos \omega t + B \sin \omega t + \omega t(-A \sin \omega t + B \cos \omega t)$

$\ddot{x}_p = (-A \sin(\omega t) + B \cos(\omega t)) \cdot \omega \cdot 2 + \omega^2 t(-A \cos(\omega t) - B \sin(\omega t))$

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$$2w\left(-A\sin\omega t + B\cos\omega t\right) + \omega^2 t\left(-A\cos\omega t - B\sin\omega t\right) + \omega^2 t\left(A\cos\omega t + B\sin\omega t\right) = \omega^2 e\sin\omega t$$

$$\sin\omega t\left(-2A\omega - \cancel{\omega^2 tB} + \cancel{\omega^2 tB}\right) = \omega^2 e\sin\omega t \quad \Rightarrow \quad -2A = \omega e \quad \Rightarrow \quad A = -\tfrac{1}{2}\omega e$$

$$\cos\omega t\left(2B\omega - \cancel{A\omega^2 t} + \cancel{\omega^2 tA}\right) = 0\cos\omega t \quad \Rightarrow \quad B = 0$$

$$x = C_1\cos\omega t + C_2\sin\omega t - \tfrac{1}{2}\omega e\, t\cos(\omega t)$$

$$\dot{x} = -C_1\omega\sin\omega t + C_2\omega\cos\omega t - \tfrac{1}{2}\omega e\cos\omega t + \tfrac{1}{2}\omega^2 e\, t\sin\omega t$$

$$x(t_0) = C_1 + 0 - 0 = 0 \quad \rightarrow \quad C_1 = 0$$

$$\dot{x}(t_0) = 0 + C_2\omega - \tfrac{1}{2}\omega e + 0 = 0 \quad \rightarrow \quad C_2 = \tfrac{1}{2}e$$

$$\boxed{x = \tfrac{1}{2}e\left(\sin\omega t - \omega t\cos\omega t\right)}$$

y: $\quad 0 = -mg - m\omega^2 e\cos\omega t + U_n$

$$\boxed{U_n = mg + m\omega^2 e\cos(\omega t)}$$

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(6.14.) obrtanje: $\omega = const.$

AB : OC čep u horizontalnoj ravni

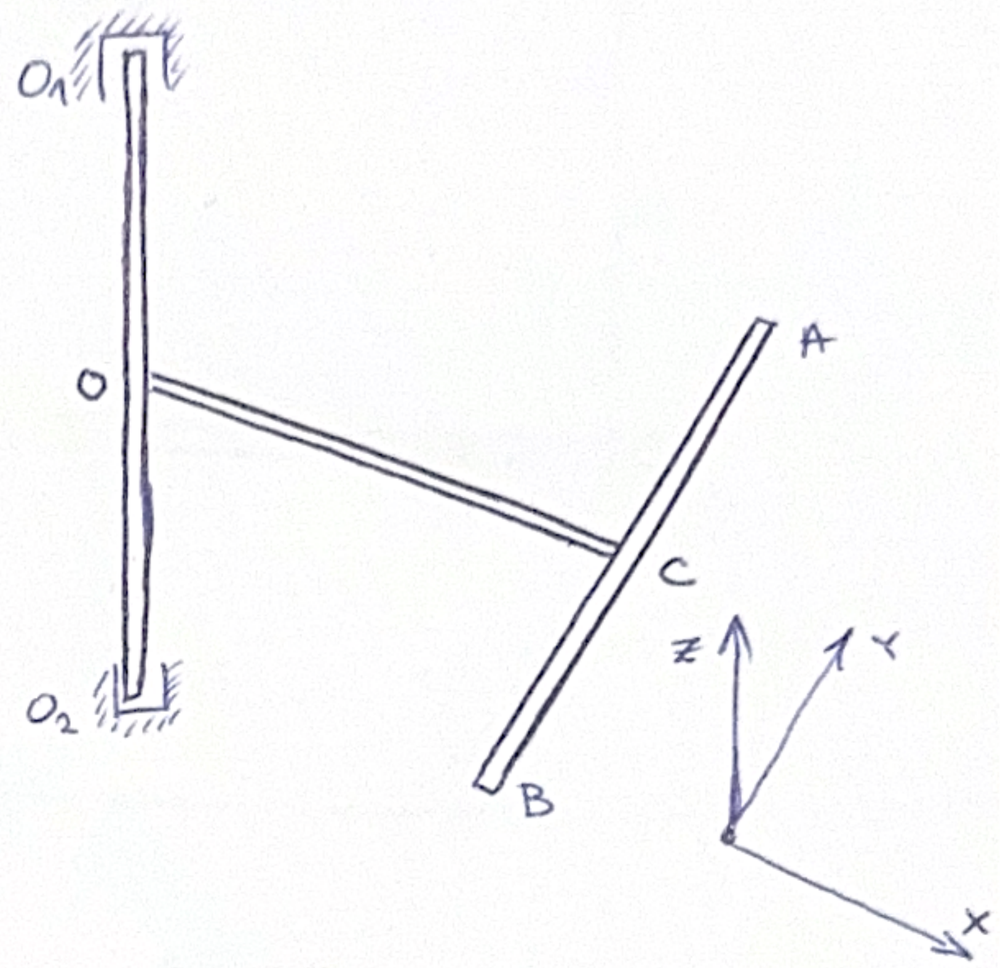
$\overline{AC} = \overline{CB} = e$, $\overline{OC} = e$

tačka mase m

t_0: tačka je u A
relativna brzina v_0

$v_0 = ?$ – da bi tačka dospela u položaj B

$N_C = ?$



$\underline{m\vec{a}_r = m\vec{g} + \vec{F}^{in}_p + \vec{F}^{in}_{cor} + \vec{N}_n + \vec{N}_6}$

$m\vec{g} = mg \cdot (-\vec{k}')$

$\vec{F}^{in}_p = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$

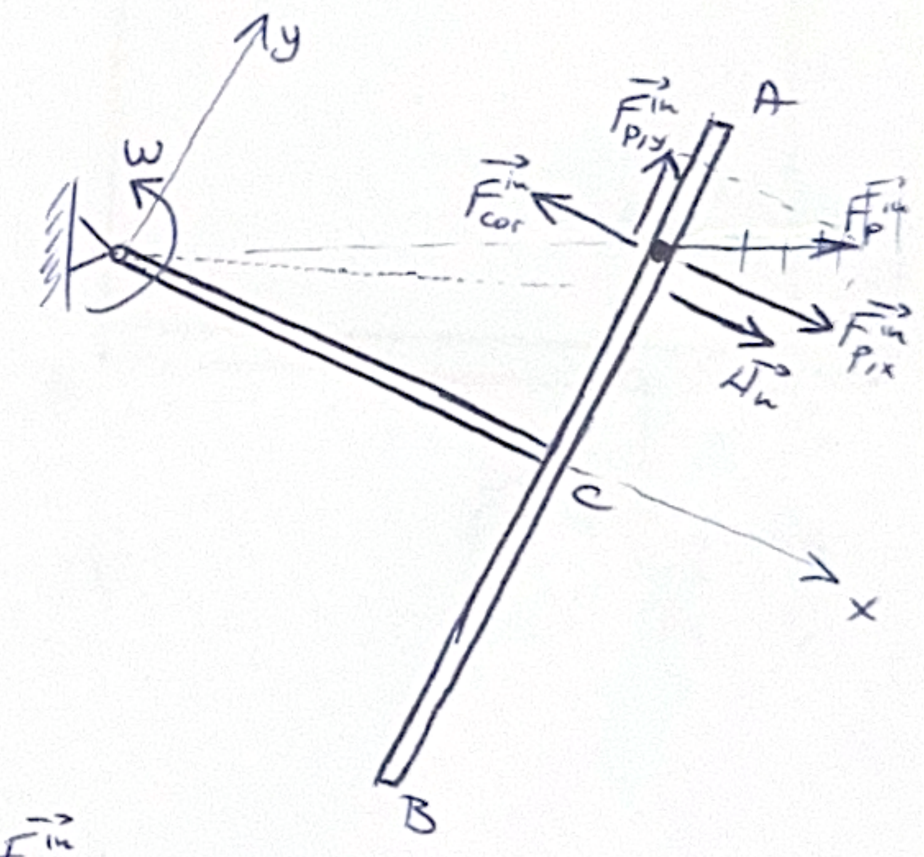
$= m\omega^2 x \cdot (\vec{i}') + m\omega^2 y \cdot (\vec{j}')$

$\vec{F}^{in}_{cor} = -2m\,\vec{\omega} \times \vec{v}_r = +2m\omega \cdot v_r \cdot (-\vec{i}')$

$\vec{N}_n = N_n \cdot (\vec{i}')$

$\vec{N}_6 = N_6 \cdot (\vec{k}')$

pogled odozgo
(horizontalna ravan)



Da bi tačka došla do položaja B, treba proveriti da li će doći do položaja C, jer do C sila $\vec{F}^{in}_{P,y}$ deluje neposredno, a posle toga deluje u pravcu kretanja ka B. Dokle, ako stigne do C, sigurno će i do B.

$\underline{T_C - T_A = A_{AC}}$

$\frac{1}{2}mv_C^2 - \frac{1}{2}mv_0^2 = \int_e^0 F^{in}_{P,y}\,dy = \int_e^0 m\omega^2 y\,dy = \frac{1}{2}m\omega^2(0 - e^2) = -\frac{1}{2}m\omega^2 e^2$

$v_0^2 - v_C^2 = \omega^2 e^2$, u graničnom slučaju $v_C \approx 0$ → $\boxed{v_0 > \omega e}$

z: $0 = -mg + N_C \Rightarrow N_C = mg$

x: $0 = m\omega^2 x - 2m\omega \cdot v_{r,C} + N_n$, $v_C = \sqrt{v_0^2 - \omega^2 e^2}$

$N_n = 2m\omega v_C - m\omega^2 x = 2m\omega\sqrt{v_0^2 - \omega^2 e^2} - m\omega^2 e$

$N = \sqrt{N_n^2 + N_C^2} \Rightarrow \boxed{N = m\sqrt{g^2 + (2\omega\sqrt{v_0^2 - \omega^2 e^2} - \omega^2 e)^2}}$

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(6.15.) horizontalna ravan

kvadratna ploča ABCD

$\omega = const.$

stranica ploče a

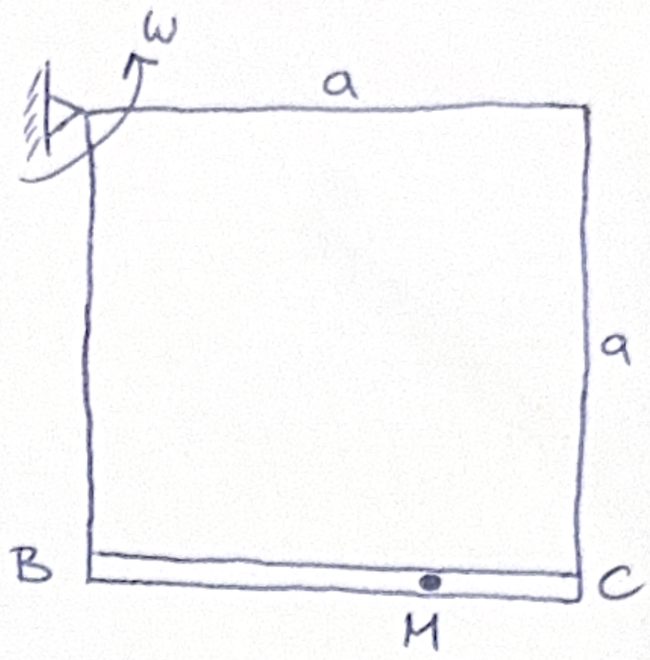
$\omega = ?$

t_0: tačka M je u B

$v_{r0} = v_0$

t_1: tačka M najniža tačka (položaj C)

$v_{r1} = v_1 = 2v_0$



$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_1 + \vec{N}_2$

$m\vec{g} = mg \cdot (-\vec{k})$

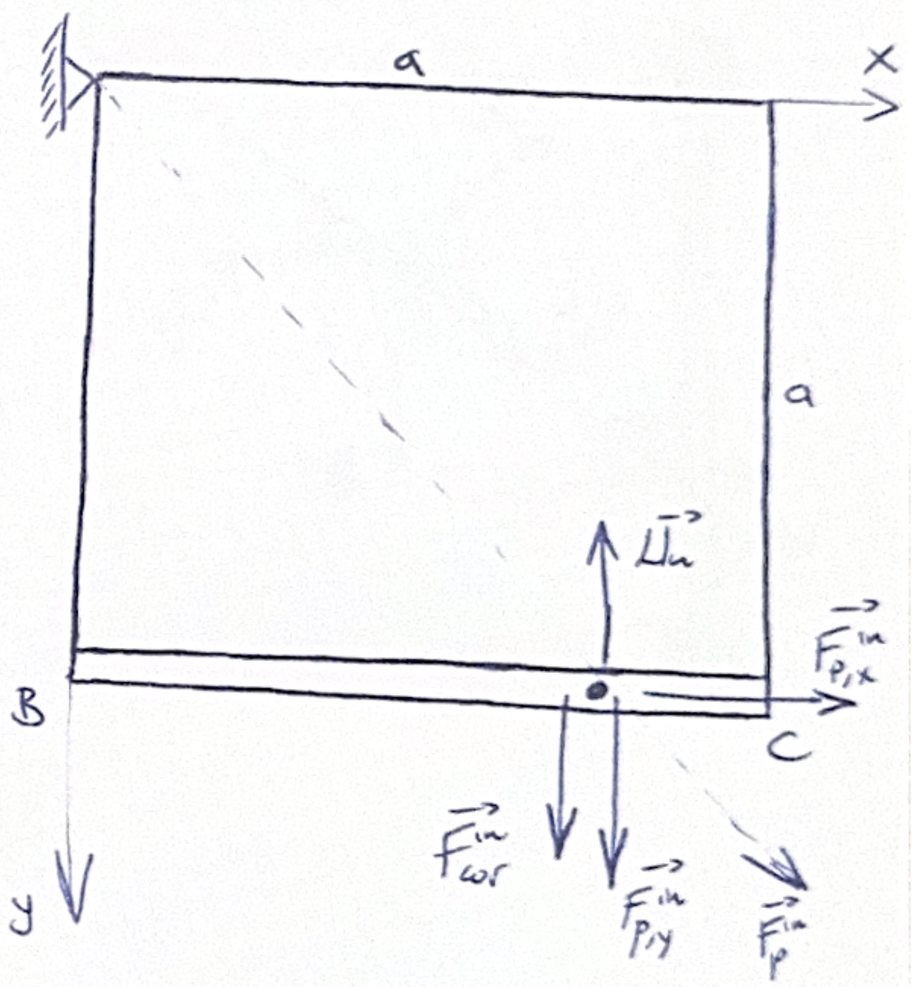
$\vec{F}_p^{in} = -m(\cancel{\vec{a}_0}^{0} + \cancel{\vec{\varepsilon} \times \vec{\rho}}^{0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$

$= m\omega^2 \cdot x \cdot (\vec{i}) + m\omega^2 \cdot y \cdot (\vec{j}) = \vec{F}_{p,x}^{in} + \vec{F}_{p,y}^{in}$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot v_r \cdot (\vec{j})$

$\vec{N}_1 = N_1 \cdot (-\vec{j})$

$\vec{N}_2 = N_2 \cdot (\vec{k})$



$T_C - T_B = A_{BC}$

$\frac{1}{2} m v_C^2 - \frac{1}{2} m v_B^2 = \int_0^a F_{p,x}^{in} \, dx$

$\frac{1}{2} m \cdot 4v_0^2 - \frac{1}{2} m v_0^2 = \int_0^a m\omega^2 x \, dx$

$\frac{3}{2} m v_0^2 = \frac{1}{2} m \omega^2 a^2$

$$\boxed{\omega = \frac{v_0\sqrt{3}}{a}}$$

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(6.16.) vertikalna ravan

tacka mase m

$\omega = const.$

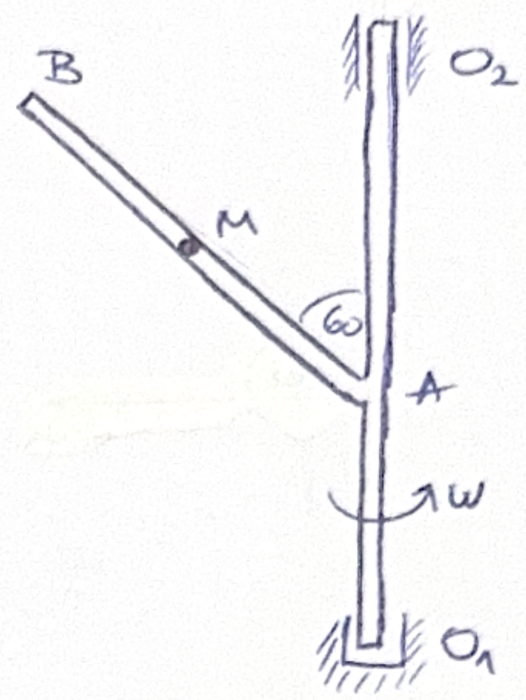
sila otpora sredine proporcionalna prvom stepenu brzine

koef. proporcionalnosti je $m\omega$

$t_0 = 0$: $\quad v = \frac{g}{\omega}$, polozaj A

t_1: $\quad v_r = \frac{1}{2} v$

$N(t_1) = ?$



$\vec{ma_r} = m\vec{g} + \vec{F_v} + \vec{F_p^{in}} + \vec{F_{cor}^{in}} + \vec{N_n} + \vec{N_G}$

$m\vec{g} = mg \cdot (-\vec{j})$

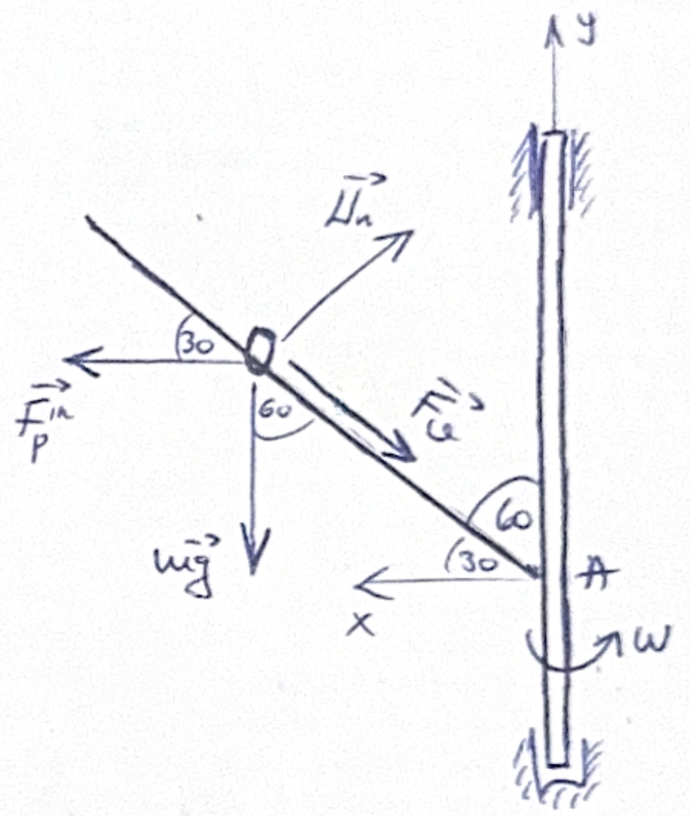
$\vec{F_v} = m \cdot \omega \cdot v_r \cdot (-\vec{t})$

$\vec{F_p^{in}} = -m(\vec{a_{Oz}} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 \cdot x \cdot (\vec{i})$

$\vec{F_{cor}^{in}} = -2m\vec{\omega} \times \vec{v_r} = 2m \cdot \omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N_n} = N_n \cdot (\vec{n})$

$\vec{N_e} = N_G \cdot (-\vec{k})$



n: $\quad 0 = -mg\sin 60 - F_p^{in}\sin 30 + N_n = 0$

$N_n = \frac{\sqrt{3}}{2} mg + \frac{1}{2} m\omega^2 x$, $\quad x(t_1) = ?$

z: $\quad 0 = 2m\omega \cdot \dot{x} - N_G$

$N_G = 2m\omega \cdot \dot{x} = 2m\omega \cdot v_{O130} = 2m\omega \cdot \frac{1}{2} \frac{g}{\omega} \frac{\sqrt{3}}{2} = mg\frac{\sqrt{3}}{2}$

$T_1 - T_0 = A_{01}$ $\quad$ (rel)

$T_1 = \frac{1}{2} m v_{r1}^2 = \frac{1}{2} m (\frac{1}{2} v)^2 = \frac{1}{8} mv^2 = \frac{1}{8} \frac{mg^2}{\omega^2}$

$T_0 = \frac{1}{2} m v_{r0}^2 = \frac{1}{2} m v^2 = \frac{1}{2} \frac{mg^2}{\omega^2}$

$A_{01} = \int_0^x F_p^{in} dx + \int_0^{s_1} -F_v ds =$ ne moze da se resi!

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$t:\quad m\ddot{s} = -F_{tr} + F_p^{in}\cos 30 - mg\cos 60$

$m\ddot{s} = -m\omega\cdot v_r + m\omega^2 x\cdot\frac{\sqrt{3}}{2} - mg\cdot\frac{1}{2}$

$\ddot{s} = -\omega\cdot\dot{s} + \omega^2 s\cos 30\,\frac{\sqrt{3}}{2} - \frac{1}{2}g$

$\ddot{s} + \omega\dot{s} - \omega^2\cdot\frac{3}{4}s = -\frac{1}{2}g$

$\#:\quad \lambda^2 + \omega\lambda - \frac{3}{4}\omega^2 = 0 \quad\rightarrow\quad \lambda_{1/2} = \frac{-\omega\pm\sqrt{\omega^2+3\omega^2}}{2} = \frac{-\omega\pm 2\omega}{2}$

$s_h = C_1 e^{\frac{1}{2}\omega t} + C_2 e^{-\frac{1}{2}\omega t}$

$p:\quad 6(t) = e^{0t}\left(-\frac{1}{2}g\cos 0t + 0\sin 0t\right)$

$s_p = t^0 e^{0t}\left(A\cos 0t + B\sin 0t\right) = A \quad,\quad \dot{s}_p = 0 \quad,\quad \ddot{s}_p = 0$

$-\frac{3}{4}\omega^2 A = -\frac{1}{2}g \quad\rightarrow\quad A = \frac{2}{3}\frac{g}{\omega^2}$

$s = C_1 e^{\frac{1}{2}\omega t} + C_2 e^{-\frac{3}{2}\omega t} + \frac{2}{3}\frac{g}{\omega^2}$

$\dot{s} = C_1\cdot\frac{1}{2}\omega e^{\frac{1}{2}\omega t} - C_2\cdot\frac{3}{2}\omega e^{-\frac{3}{2}\omega t}$

$\dot{s}(t_0) = \frac{g}{\omega} \quad\rightarrow\quad \frac{1}{2}\omega C_1 - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$

$s(t_0) = 0 \quad\rightarrow\quad C_1 + C_2 + \frac{2}{3}\frac{g}{\omega^2} = 0 \quad\rightarrow\quad C_1 = -\frac{2}{3}\frac{g}{\omega^2} - C_2$

$\frac{1}{2}\omega\left(-\frac{2}{3}\frac{g}{\omega^2} - C_2\right) - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$

$-\frac{1}{3}\frac{g}{\omega} - \frac{1}{2}\omega C_2 - \frac{3}{2}\omega C_2 = \frac{g}{\omega}$

$-2\omega C_2 = \frac{4}{3}\frac{g}{\omega} \quad\rightarrow\quad C_2 = -\frac{2}{3}\frac{g}{\omega^2} \quad\rightarrow\quad C_1 = 0$

$$\boxed{s = \frac{2}{3}\frac{g}{\omega^2}\left(1 - e^{-\frac{3}{2}\omega t}\right)}$$

$\dot{s} = \frac{2}{3}\frac{g}{\omega^2}\left(\frac{3}{2}\omega e^{-\frac{3}{2}\omega t}\right) = \frac{g}{\omega}e^{-\frac{3}{2}\omega t}$

$\dot{s}(t_1) = \frac{1}{2}\frac{g}{\omega} \quad\rightarrow\quad \frac{g}{\omega}e^{-\frac{3}{2}\omega t_1} = \frac{1}{2}\frac{g}{\omega} \quad\rightarrow\quad e^{-\frac{3}{2}\omega t_1} = \frac{1}{2}$

$s(t_1) = \frac{2}{3}\frac{g}{\omega^2}\left(1 - \frac{1}{2}\right) = \frac{1}{3}\frac{g}{\omega^2}$

$x(t_1) = s(t_1)\cdot\cos 30 = \frac{\sqrt{3}}{2}\frac{1}{3}\frac{g}{\omega^2} = \frac{\sqrt{3}}{6}\frac{g}{\omega^2}$

$H_n = \frac{\sqrt{3}}{2}mg + \frac{1}{2}m\omega^2\frac{\sqrt{3}}{6}\frac{g}{\omega^2} = \frac{7\sqrt{3}}{12}mg$

$H = \sqrt{H_n^2 + H_G^2} = mg\sqrt{\frac{49\cdot 3}{144} + \frac{3}{4}} = mg\sqrt{\frac{255}{144}} = mg\sqrt{\frac{85}{48}} = \frac{1}{4}mg\sqrt{\frac{85}{3}}$

$$\boxed{H(t_1) = \frac{1}{4}mg\sqrt{\frac{85}{3}}}$$

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(6.17.) kuglica mase m

horizontalna ravan

$\omega = const.$

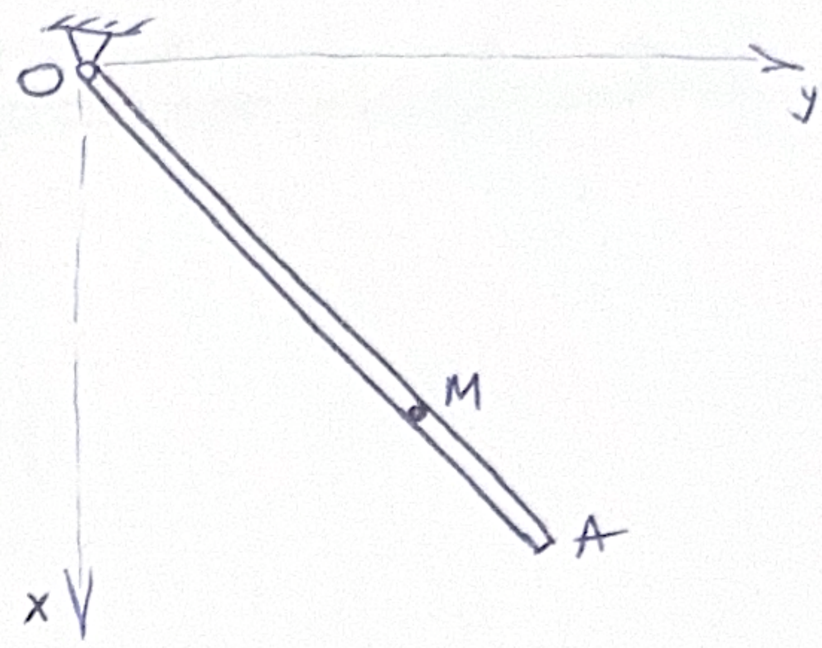
$\overline{OA} = 6$

t_0: M je na sredini cevi

$Q_r = 0$

t_1: tačka napusta cev

$t_A = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$

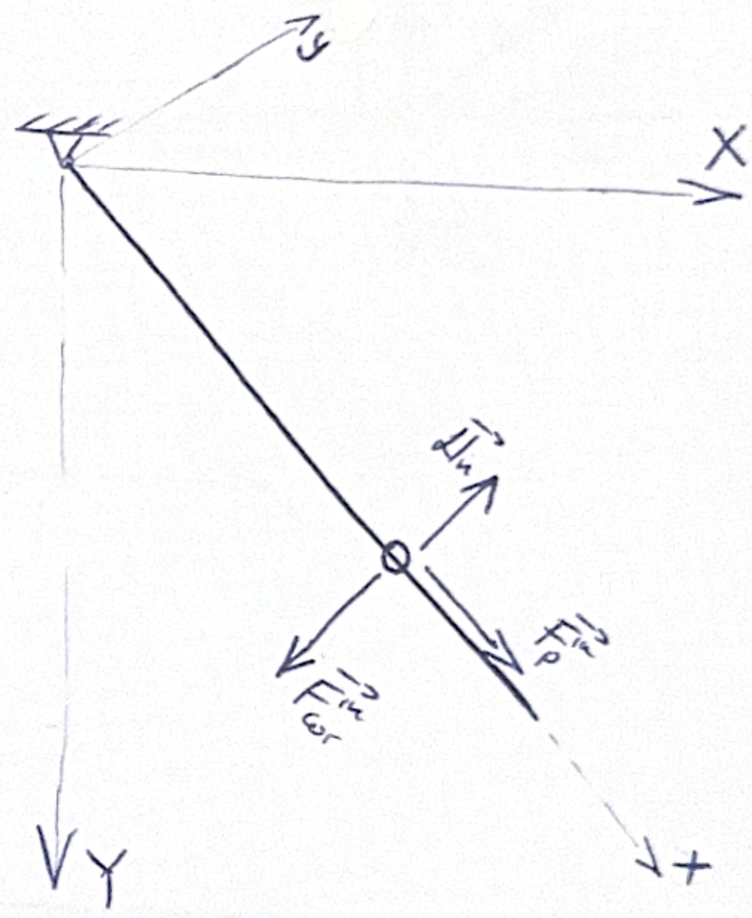
$$m\vec{g} = mg \cdot (-\vec{k})$$

$$\vec{F}_p^{in} = -m(\cancel{\vec{a}_{or}}^{\,0} + \cancel{\vec{\varepsilon} \times \vec{\rho}}^{\,0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 \cdot x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (-\vec{j})$$

$$\vec{N}_n = N_n \cdot (\vec{j})$$

$$\vec{N}_6 = N_6 \cdot (\vec{k})$$



x: $m\ddot{x} = m\omega^2 x$

$\ddot{x} - \omega^2 x = 0$

$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1,2} = \pm\omega \pm 0i$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$

$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$

$x(t_0) = C_1 + C_2 = \frac{6}{2}$

$\dot{x}(t_0) = C_1\omega - C_2\omega = 0 \rightarrow C_1 = C_2$

$\Rightarrow C_1 = C_2 = \frac{1}{4}6$

$x = \frac{1}{4}6(e^{\omega t} + e^{-\omega t}) = \frac{1}{4}6 \cdot 2 \cdot \frac{e^{\omega t} + e^{-\omega t}}{2} = \frac{1}{2}6\cosh(\omega t)$

$x(t_A) = \frac{1}{2}6\cosh(\omega t_A) = 6$

$\cosh(\omega t_A) = 2$

$$\boxed{t_A = \frac{1}{\omega}\operatorname{arccosh}(2)}$$

- U rešenju u zbirci je izračunato preko ln. Rezultat je isti, jer bi jedno rešenje moglo matematičkim operacijama da se transformiše u drugo.

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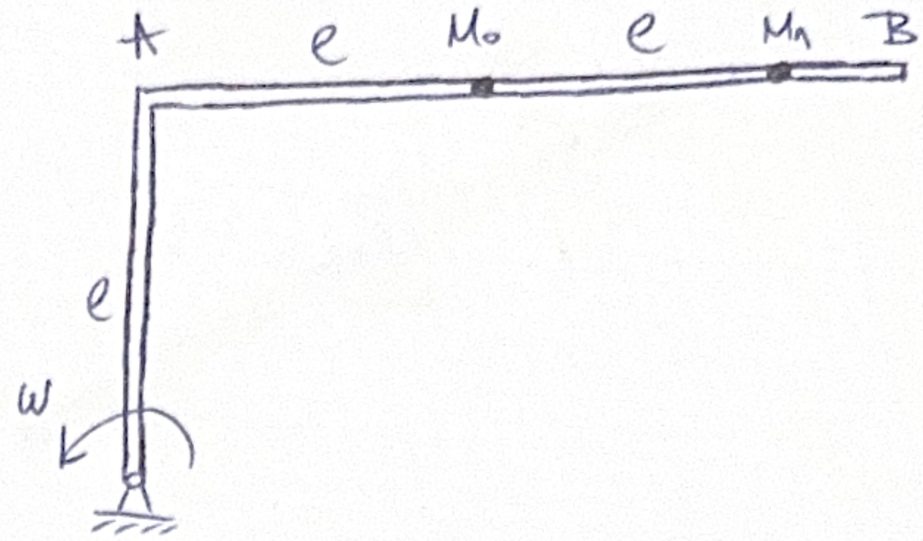
(6.18.) horizontalna ravan

$\omega = \text{const.}$

tačka mase m

to: tačka u M_0

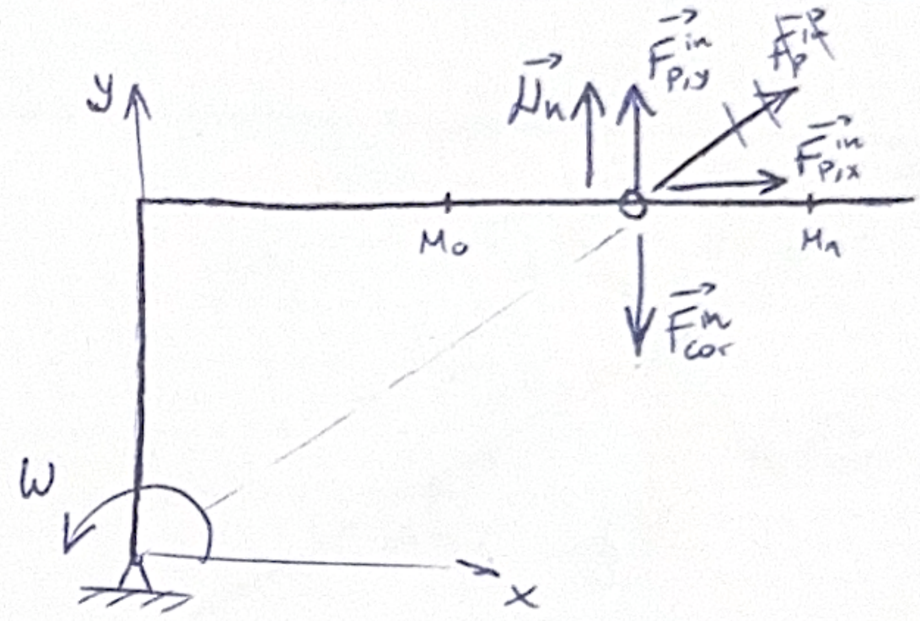
$v_{r0} = e\omega$



1) $t_1 = ?$ - trenutak kada je tačka u M_1

2) $N_n(t_1) = ?$

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$



$$m\vec{g} = mg\cdot(-\vec{k})$$

$$\vec{F}_p^{in} = -m\left(\vec{a}_{on} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})\right) =$$

$$= m\omega^2 x\cdot(\vec{i}) + m\omega^2 y\cdot(\vec{j}) = \vec{F}_{P,x}^{in} + \vec{F}_{P,y}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega}\times\vec{v}_r = 2m\omega\cdot\dot{x}\cdot(-\vec{j})$$

$$\vec{N}_n = N_n\cdot\vec{j}$$

$$\vec{N}_6 = N_6\cdot\vec{k}$$

x: $m\ddot{x} = m\omega^2 x$

$\ddot{x} - \omega^2 x = 0$

$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega \pm 0i$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$

$\dot{x} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t}$

$x(t_0) = C_1 + C_2 = e$

$\dot{x}(t_0) = C_1\cdot\omega - C_2\cdot\omega = e\omega$

$C_1 = e + C_2 \rightarrow e + C_2 + C_2 = e$

$\Rightarrow C_2 = 0, \quad C_1 = e$

$x = e\cdot e^{\omega t}$

$x(t_1) = e\cdot e^{\omega t_1} = 2e$

$e^{\omega t_1} = 2 \quad / \ln$

$\omega t_1 = \ln 2$

$$\boxed{t_1 = \frac{1}{\omega}\ln 2}$$

y: $0 = m\omega^2 y - 2m\omega\dot{x} + N_n$

$\dot{x}(t_1) = e\cdot\omega\cdot e^{\omega\cdot\frac{1}{\omega}\ln 2} = e\cdot\omega\cdot 2$

$N_n = 2m\omega\cdot 2e\omega - m\omega^2\cdot e$

$$\boxed{N_n(t_1) = 3me\omega^2}$$

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(6.19) Vertikalna ravan

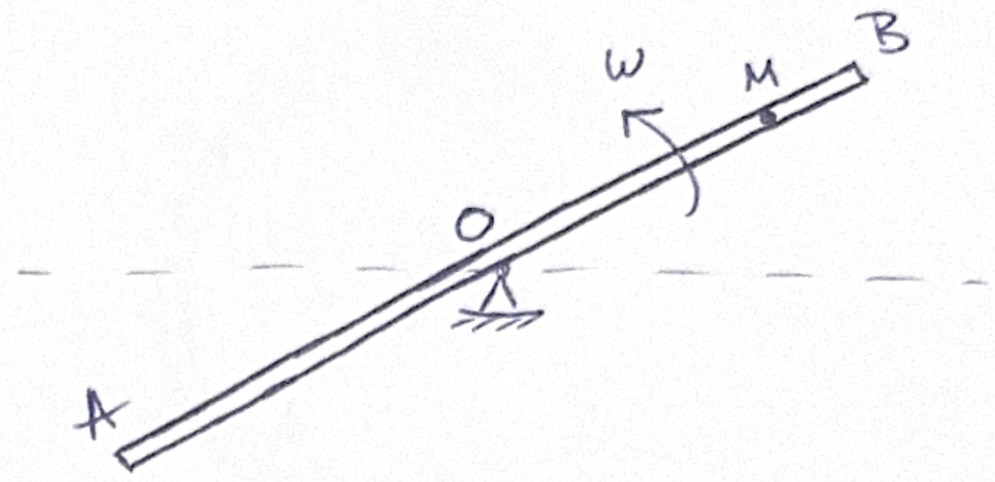
AB: $w = \text{const.}$

M: m

t_0: cev horizontalna

M ie u O

$v_0 = \frac{g}{2w}$



$x(t) = ?$ - j-na rel kretanja tačke M ?

$N_{n,max} = ?$

$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{\mu}_6$ ~~$\vec{\mu}_6$~~

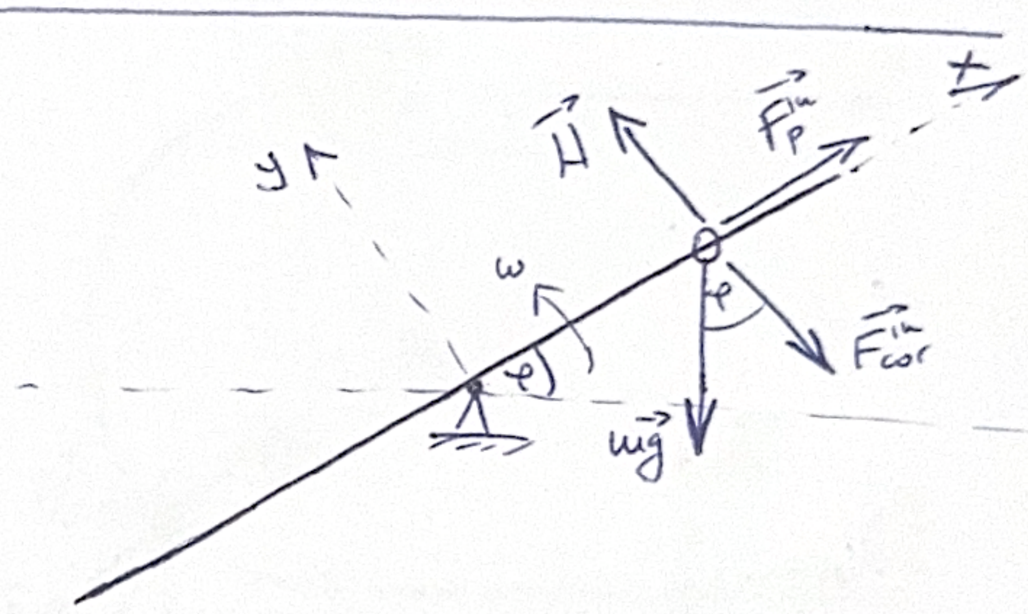
$m\vec{g} = mg \cdot (-\vec{i})$

$\vec{F}_p^{in} = -m\left(\vec{a}_{on} + \vec{\varepsilon}' \times \vec{s}' + \vec{\omega}' \times (\vec{\omega}' \times \vec{s}')\right) = m\omega^2 x \cdot (\vec{i}')$

$\vec{F}_{cor}^{in} = -2m\vec{\omega}' \times \vec{v}_r = +2m\omega \cdot \dot{x} \cdot (-\vec{j}')$

$\vec{N}_n = N \cdot (\vec{j}') = \vec{N}$

$\vec{\mu}_6 = 0$



$\varphi = \int \omega dt = \omega t$

x: $m\ddot{x} = -mg \sin\varphi + m\omega^2 x$

$\ddot{x} - \omega^2 x = -g \sin \omega t$

H: $\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega \pm 0i$

$x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$

P: $6(t) = e^{0t}(0\cos(\omega t) - g\sin(\omega t))$

$x_p = t^0 e^{0t}(A\cos\omega t + B\sin\omega t)$

$\dot{x}_p = -A\omega \sin\omega t + B\omega\cos\omega t$

$\ddot{x}_p = -A\omega^2\cos\omega t - B\omega^2 \sin\omega t$

$\cos(\omega t)\left(-A\omega^2 - \omega^2 A\right) + \sin(\omega t)\left(-B\omega^2 - \omega^2 B\right) =$

$= 0 \cdot \cos(\omega t) + (-g) \cdot \sin(\omega t)$

$\Rightarrow \quad A = 0, \quad 2B\omega^2 = g \rightarrow B = \frac{1}{2}\frac{g}{\omega^2}$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t} + \frac{1}{2}\frac{g}{\omega^2}\sin\omega t$

$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t} + \frac{1}{2}\frac{g}{\omega}\cos\omega t$

$x(t_0) = C_1 + C_2 + 0 = 0 \rightarrow C_1 = -C_2$

$\dot{x}(t_0) = C_1\omega - C_2\omega + \frac{1}{2}\frac{g}{\omega} = \frac{1}{2}\frac{g}{\omega} \rightarrow C_1 = -C_2 = 0$

$$\boxed{x = \frac{1}{2}\frac{g}{\omega^2}\sin(\omega t)}$$

y: $0 = -mg\cos\varphi - 2m\omega\dot{x} + N$

$N = mg\cos\omega t + 2m\omega \cdot \frac{1}{2}\frac{g}{\omega}\cos\omega t$

$N = 2mg\cos\omega t$

$$\boxed{N_{1,max} = 2mg}$$

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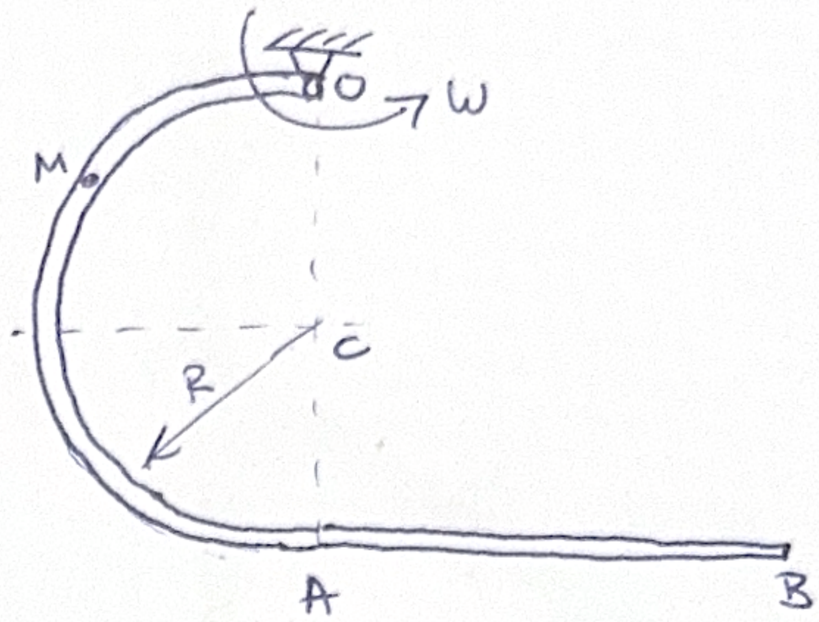
6.20. horizontalna ravan

$\omega = const.$

tačka mase m

to: M je u O, $v_0 = 0$

kružna j-na rel. kretanja na delu AB?



<u>deonica OA</u>

$$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$$

$$m\vec{g} = mg\cdot(\vec{k})$$

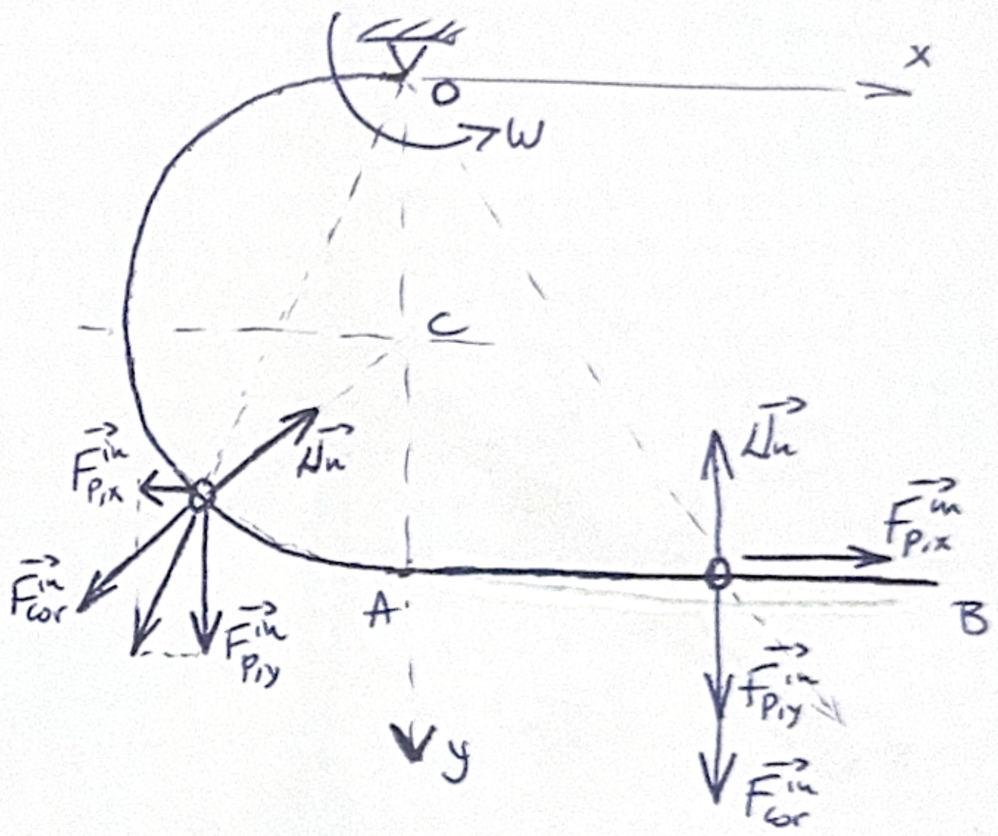
$$\vec{F}_p^{in} = -m\left(\cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\varepsilon}\times\vec{\rho}}^{\,0} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})\right) =$$

$$= m\omega^2\cdot|x|\cdot(-\vec{i}) + m\omega^2 y\cdot(\vec{j}) = \vec{F}_{p,x}^{in} + \vec{F}_{p,y}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = +2m\cdot\omega\cdot v_r\cdot(-\vec{n})$$

$$\vec{N}_n = N_n\cdot(\vec{n})$$

$$\vec{N}_6 = N_6\cdot(-\vec{k})$$



$$\underline{T_A - T_0 = A_{on}} \quad (rel)$$

$$\frac{1}{2}m v_{Ar}^2 - 0 = \int_0^0 \cancel{F_{p,x}^{in}}^{\,0}\,dx + \int_0^{2R} F_{p,y}\,dy = \int_0^{2R} m\omega^2 y\,dy = \frac{1}{2}m\omega^2(4R^2) = 2m\omega^2R^2$$

$$v_{Ar}^2 = 4\omega^2R^2 \;\rightarrow\; v_{A,r} = 2\omega R$$

<u>deonica AB</u>

$$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$$

$$m\vec{g} = mg\cdot(\vec{k})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) =$$

$$= m\omega^2 x\cdot\vec{i} + m\omega^2 y\cdot\vec{j} = \vec{F}_{p,x}^{in} + \vec{F}_{p,y}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = 2m\omega\cdot\dot{x}\cdot(\vec{j})$$

$$\vec{N}_n = N_n\cdot(-\vec{j})$$

$$\vec{N}_6 = N_6\cdot(-\vec{k})$$

x: $m\ddot{x} = m\omega^2 x$

$\ddot{x} - \omega^2 x = 0$

#: $\lambda^2 - \omega^2 = 0 \;\rightarrow\; \lambda_{1/2} = \pm\omega \pm 0i$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$

$\dot{x} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t}$

$x(t_0) = C_1 + C_2 = 0 \;\Rightarrow\; C_1 = -C_2$

$\dot{x}(t_0) = C_1\omega - C_2\omega = 2\omega R \;\Rightarrow\; 2C_1 = 2R$

$\Rightarrow\; C_1 = R = -C_2$

$$\boxed{x = R\left(e^{\omega t} - e^{-\omega t}\right)}$$

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6.21.

α - ugao

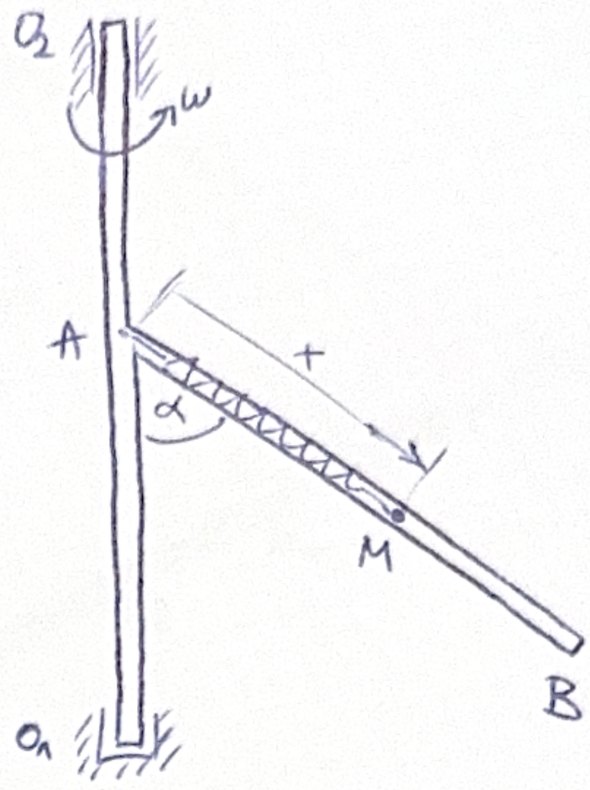
$\omega = const.$

kuglica: m

opruga: c, l_0

1) d.j. j-na rel. kretanja kuglice?
uslov za α da li kretanje bilo oscilatorno?

2) u slučaju da je ispunjen uslov za oscilatorno kretanje, j-na rel. kretanja?
t₀: $x_0 = l_0$, $v_{r0} = 0$



---

1)

$$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_t$$

$m\vec{g} = mg \cdot (-\vec{j})$

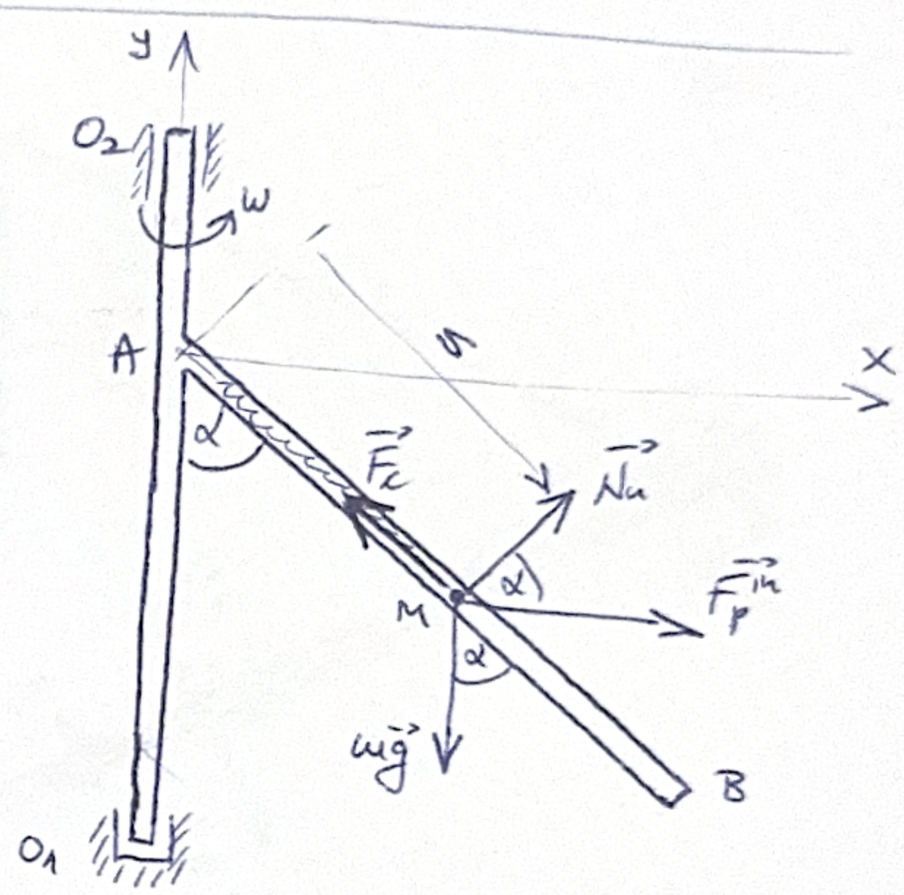
$\vec{F}_c = c \cdot (s - l_0) \cdot (-\vec{t})$

$\vec{F}_p^{in} = -m(\vec{a}_{on}^{\,0} + \dot{\vec{\varepsilon}} \times \vec{\rho}^{\,0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{t})$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{n})$

$\vec{N}_t = N_t \cdot (-\vec{k})$



t: $m\ddot{s} = mg\cos\alpha - c(s - l_0) + m\omega^2 x \sin\alpha$

$\ddot{s} = g\cos\alpha - \frac{c}{m}(s - l_0) + \omega^2 \cdot s \cdot \sin^2\alpha$

$$\boxed{\ddot{s} + \left(\frac{c}{m} - \omega^2\sin^2\alpha\right) \cdot s = g\cos\alpha + \frac{c}{m} l_0}$$

$\ddot{s} + \omega_1^2 s = h_1$, $\omega_1^2 = \frac{c}{m} - \omega^2\sin^2\alpha$, $h_1 = g\cos\alpha + \frac{c}{m} l_0$

uslov za osc. kretanje: $\omega_1 > 0 \rightarrow \frac{c}{m} - \omega^2\sin^2\alpha > 0 \rightarrow$

$$\boxed{\alpha < \arcsin\left(\sqrt{\frac{c}{m\omega^2}}\right)}$$

2) H: $\lambda^2 + \omega_1^2 = 0 \rightarrow x_h = C_1\cos(\omega_1 t) + C_2\sin(\omega_1 t)$

P: $b(t) = e^{0t}(h_1\cos(0t) + 0\sin(0t)) \rightarrow x_p = t^0 e^{0t}(A\omega\cos 0t + B\sin 0t) = A$, $\ddot{x}_p = 0$

$\omega_1^2 A = h_1 \rightarrow A = \frac{h_1}{\omega_1^2} \rightarrow x_p = \frac{h_1}{\omega_1^2}$

$x = C_1\cos(\omega_1 t) + C_2\sin(\omega_1 t) + \frac{h_1}{\omega_1^2}$

$\dot{x} = -C_1\omega_1\sin(\omega_1 t) + C_2\omega_1\cos(\omega_1 t)$

$x(t_0) = C_1 + 0 + \frac{h_1}{\omega_1^2} = l_0 \rightarrow C_1 = l_0 - \frac{h_1}{\omega_1^2}$

$\dot{x}(t_0) = 0 + C_2\omega_1 = 0 \rightarrow C_2 = 0$

$x = \left(l_0 - \frac{h_1}{\omega_1^2}\right)\cos(\omega_1 t) + \frac{h_1}{\omega_1^2}$

$$\boxed{x = \left(l_0 - \frac{g\cos\alpha + \frac{c}{m} l_0}{\frac{c}{m} - \omega^2\sin^2\alpha}\right)\cos\left(\sqrt{\frac{c}{m} - \omega^2\sin^2\alpha} \cdot t\right) + \frac{g\cos\alpha + \frac{c}{m} l_0}{\frac{c}{m} - \omega^2\sin^2\alpha}}$$

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(6.22.) horizontalna ravan

jednakostranični trougao stranice a

$\omega = const.$

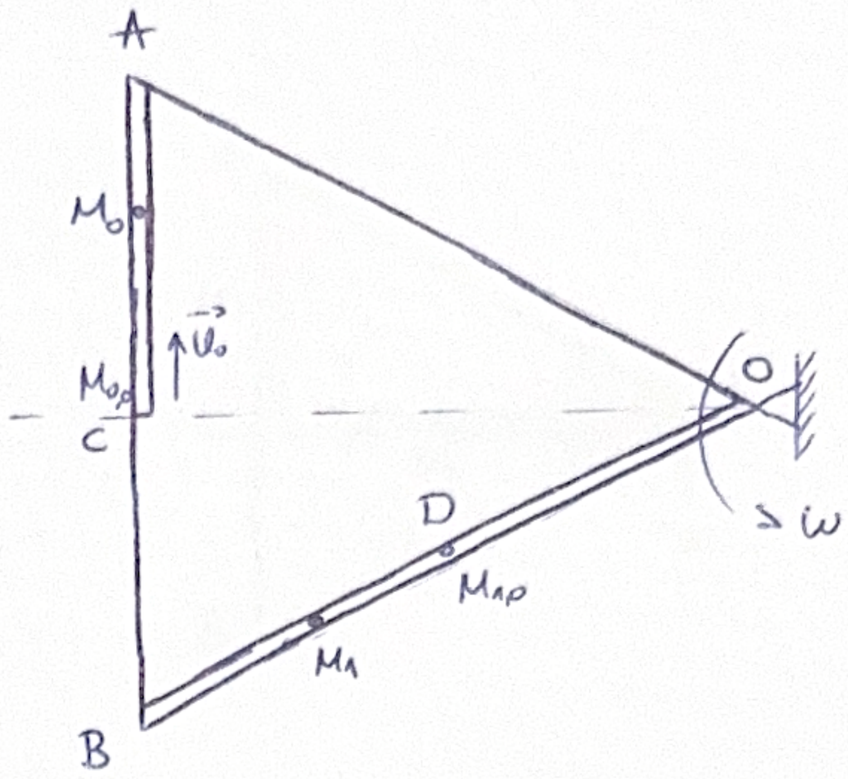
to: M_0 u C (C na sredini)

$v_{r,0} = v_0$

M_1 u D (D na sredini)

$v_{r,1} = 0$

$v_0 = ?$ - da bi obe tačke istovremeno napustile kanal



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tačka M_1:

$m\vec{a}_{1r} = m\vec{g} + \vec{F}^{in}_{p1} + \vec{F}^{in}_{cor1} + \vec{M}_{1n} + \vec{M}_{1G}$

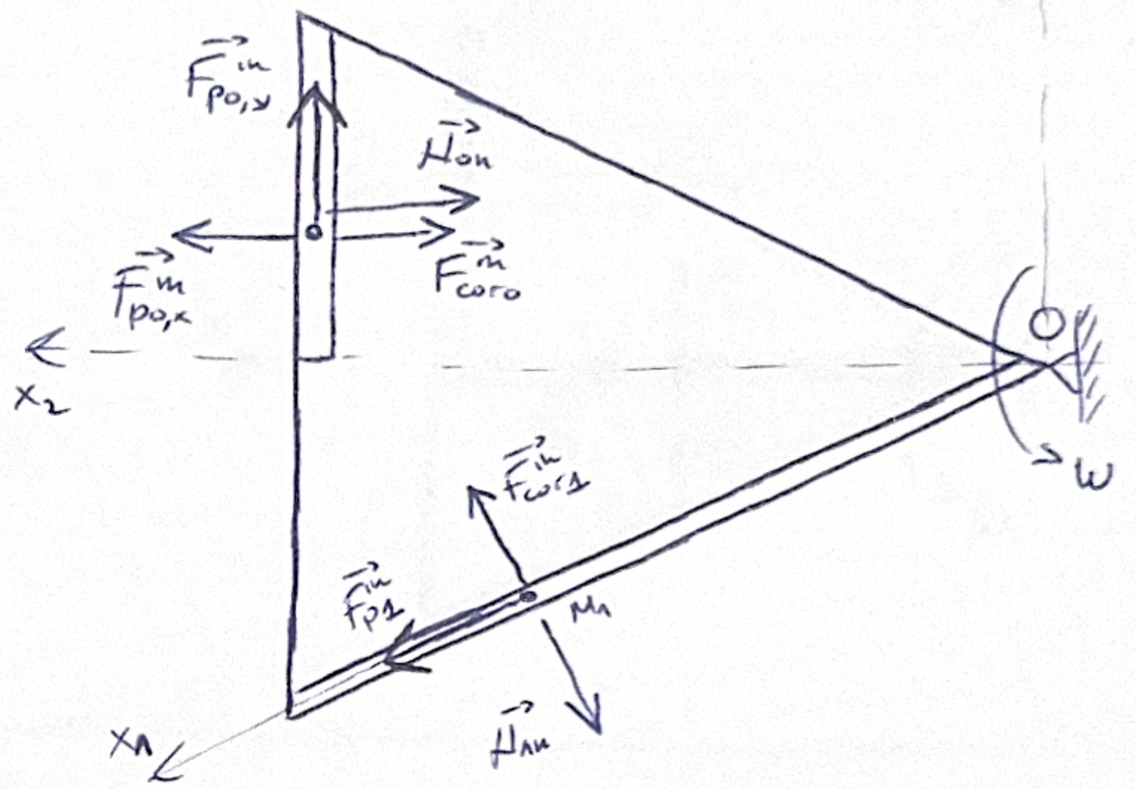
$m\vec{g} = mg\cdot(-\vec{k}_1)$

$\vec{F}^{in}_{p1} = -m(\vec{a}_{0n}^{\,0} + \vec{\varepsilon}\times\vec{\rho}^{\,0} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) = m\omega^2 x_1\cdot(\vec{i}_1)$

$\vec{F}^{in}_{cor1} = -2m\,\vec{\omega}\times\vec{v}_r = 2m\cdot\omega\cdot\dot{x}\cdot(\vec{j}_1)$

$\vec{M}_{1n} = M_{1n}\cdot(-\vec{j}_1)$

$\vec{M}_{1G} = M_G\cdot(\vec{k}_1)$



x_1: $m\ddot{x}_1 = m\omega^2 x_1$

$\ddot{x}_1 - \omega^2 x_1 = 0$

$\lambda_{1/2}^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega \pm 0i$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$

$\dot{x} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t}$

$\dot{x}(t_0) = C_1\omega - C_2\omega = 0 \rightarrow C_1 = C_2$

$x(t_0) = C_1 + C_2 = \frac{a}{2} \rightarrow C_1 = C_2 = \frac{a}{4}$

$x = \frac{1}{4}a(e^{\omega t} + e^{-\omega t}) = \frac{1}{4}a\cdot 2\,\frac{e^{\omega t} + e^{-\omega t}}{2}$

$x = \frac{1}{2}a\cosh(\omega t)$

$x(t_1) = \frac{1}{2}a\cosh(\omega t_1) = a$

$\cosh(\omega t_1) = 2$

---

tačka M_0

$m\vec{a}_{0r} = m\vec{g} + \vec{F}^{in}_{p0} + \vec{F}^{in}_{cor0} + \vec{M}_{0n} + \vec{M}_{0G}$

$m\vec{g} = mg\cdot(-\vec{k}_2)$

$\vec{F}^{in}_{p0} = -m(\vec{a}_{0n}^{\,0} + \vec{\varepsilon}\times\vec{\rho}^{\,0} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) = m\omega^2\cdot x_2\cdot(\vec{i}_2) + m\omega^2 y_2\cdot(\vec{j}_2)$

$\vec{F}^{in}_{cor0} = -2m\cdot\vec{\omega}\times\vec{v}_r = 2m\omega\dot{y}_2(-\vec{i}_2)$

$\vec{M}_{0n} = M_{0n}\cdot(-\vec{i}_2)$

$\vec{M}_{0G} = M_{0G}\cdot(\vec{k}_2)$

y_2: $m\ddot{y}_2 = m\omega^2 y$

$\ddot{y}_2 - \omega^2 y = 0$

$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega \pm 0i$

$y = C_1 e^{\omega t} + C_2 e^{-\omega t}$, $\dot{y} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t}$

$y(t_0) = C_1 + C_2 = 0$, $\dot{y}(t_0) = C_1\omega - C_2\omega = v_0$

$\Rightarrow C_1 = \frac{v_0}{2\omega}$, $C_2 = -\frac{v_0}{2\omega}$

$y = \frac{v_0}{2\omega}(e^{\omega t} - e^{-\omega t}) = \frac{v_0}{\omega}\sinh(\omega t)$

$y(t_1) = \frac{v_0}{\omega}\sinh(\omega t_1) = \frac{a}{2}$, $\cosh^2(\omega t_1) - \sinh^2(\omega t_1) = 1$

$4 - \sinh^2(\omega t_1) = 1 \rightarrow \sinh(\omega t_1) = \sqrt{3}$

$y(t_1) = \frac{v_0}{\omega}\sqrt{3} = \frac{a}{2} \Rightarrow v_0 = \frac{a\omega}{2\sqrt{3}} \rightarrow \boxed{v_0 = a\omega\frac{\sqrt{3}}{6}}$

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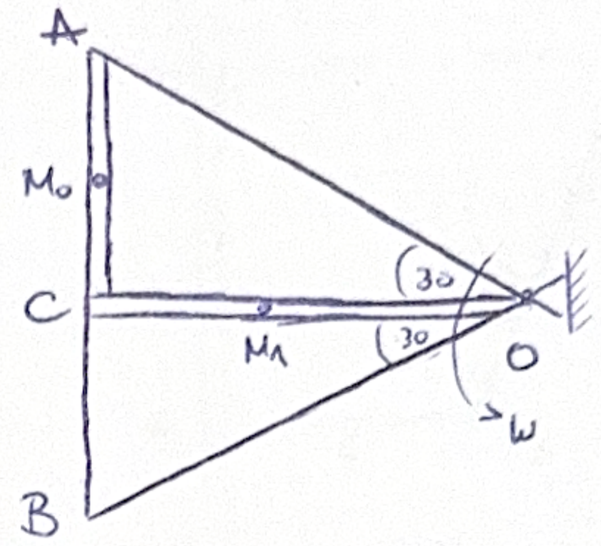
(6.23) horizontalna ravan
jednakostranični trougao
$\omega = const.$



to: $M_0 \cup C$

$\vartheta_{r0,0} = \vartheta_0$

$M_1 \cup O$

$\vartheta_{r0,1} = \vartheta_1$

$\frac{\vartheta_0}{\vartheta_1} = ?$ – da bi istovremeno napustile ploču

---

tačka M_1:

$m\vec{a}_{r1} = m\vec{g} + \vec{F}^{in}_{P1} + \vec{F}^{in}_{cor1} + \vec{M}_{n1} + \vec{M}_{G1}$

$m\vec{g} = mg \cdot (-\vec{k})$

$\vec{F}^{in}_{P1} = -m(\vec{a}_{o1} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}^{in}_{cor1} = -2m\vec{\omega} \times \vec{\vartheta}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$

$\vec{M}_{n1} = M_{n1} \cdot (-\vec{j})$

$\vec{M}_{G1} = M_{G1} \cdot (\vec{k})$

x: $m\ddot{x} = mw^2 x$

$\ddot{x} - \omega^2 x = 0$

$\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega \pm 0i$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t}$

$\dot{x} = C_1 \omega e^{\omega t} - C_2 \omega e^{-\omega t}$

$x(t_0) = 0 \rightarrow C_1 + C_2 = 0 \rightarrow C_1 = -C_2$

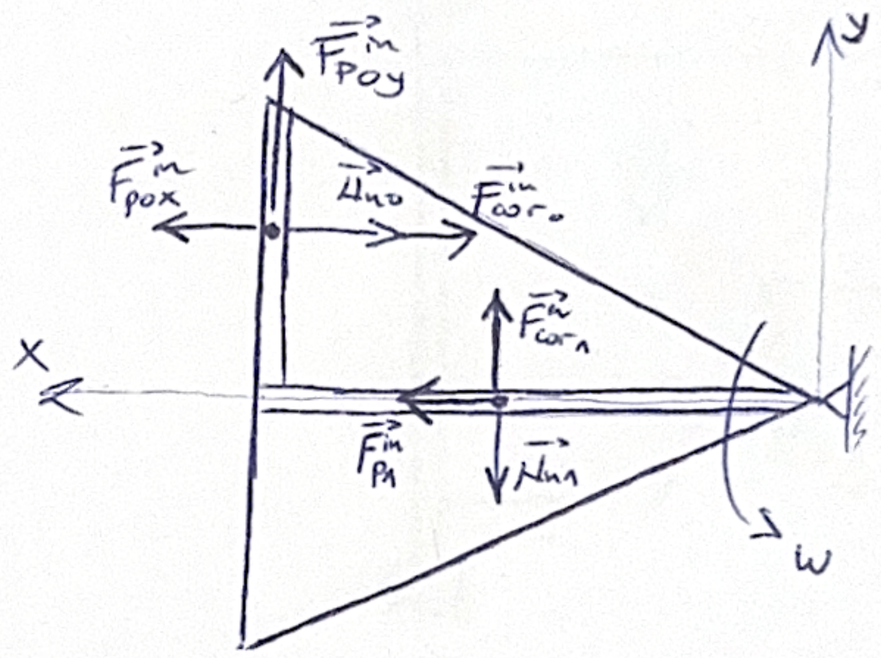
$\dot{x}(t_0) = C_1\omega - C_2\omega = \vartheta_1$

$2C_1\omega = \vartheta_1$

$C_1 = \frac{\vartheta_1}{2\omega} = -C_2$

$x = \frac{\vartheta_1}{2\omega}\left(e^{\omega t} - e^{-\omega t}\right) = \frac{\vartheta_1}{\omega}\sinh(\omega t)$

$x(t_1) = \frac{\vartheta_1}{\omega}\sinh(\omega t_1) = a\cos 30 = \frac{a\sqrt{3}}{2}$



tačka M_0

$m\vec{a}_{r0} = m\vec{g} + \vec{F}^{in}_{P0} + \vec{F}^{in}_{cor} + \vec{M}_{no} + \vec{M}_{G0}$

$m\vec{g} = mg \cdot (-\vec{k})$

$\vec{F}^{in}_{P0} = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$

$= m\omega^2 x \cdot (\vec{i}) + m\omega^2 y \cdot (\vec{j}) = \vec{F}^{in}_{Pox} + \vec{F}^{in}_{Poy}$

$\vec{F}^{in}_{cor0} = -2m\vec{\omega} \times \vec{\vartheta}_r = 2m\omega \cdot \dot{y} \cdot (-\vec{i})$

$\vec{M}_{no} = M_{no} \cdot (-\vec{i})$

$\vec{M}_{G0} = M_{G0} \cdot (\vec{k})$

y: $m\ddot{y} = mw^2 y$

$\ddot{y} - \omega^2 y = 0$

$y = \frac{\vartheta_0}{\omega}\sinh(\omega t_1)$

$y(t_1) = \frac{\vartheta_0}{\omega}\sinh(\omega t_1) = \frac{a}{2}$

$x(t_1) = y(t_1)\sqrt{3}$

$\frac{\vartheta_1}{\omega}\sinh(\omega t_1) = \frac{\vartheta_0}{\omega}\sinh(\omega t_1)\sqrt{3}$

$\boxed{\frac{\vartheta_0}{\vartheta_1} = \frac{\sqrt{3}}{3}}$

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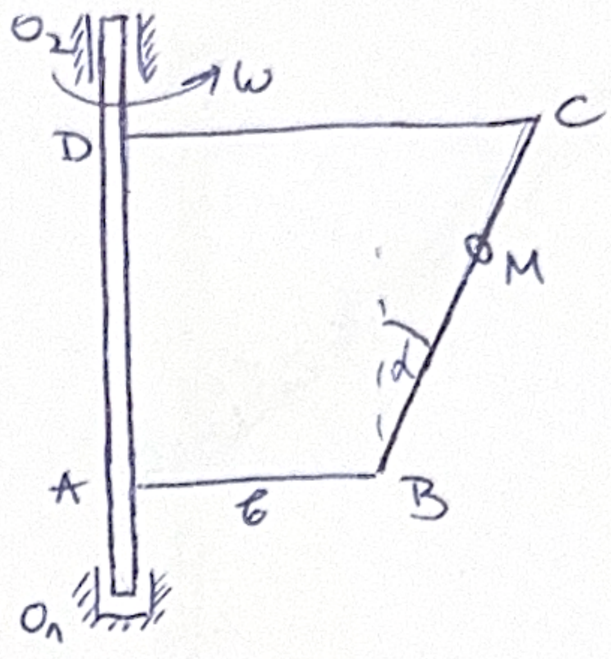
(6.24.) vertikalna ravan

ugao α

prsten: m



1) $w = ?$ - da bi $a_r = a = const.$

tj: prsten u B, $v_{r0} = 0$

2) $w_{max} = ?$ - da bi prsten ostao u B

---

1) $m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_A + \vec{N}_B$

$m\vec{g} = mg \cdot (-\vec{j})$

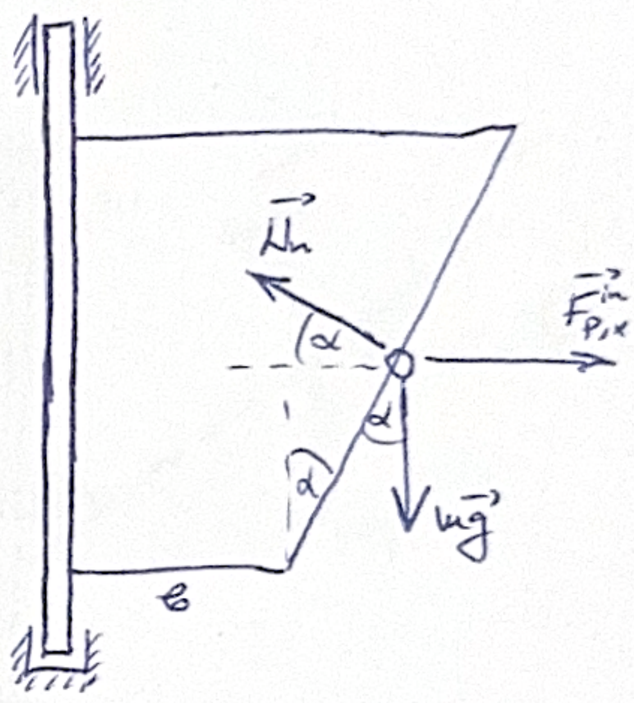
$\vec{F}_p^{in} = -m(\vec{a}_{O_1}^{\,0} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$

$= m \cdot \varepsilon \cdot x \cdot (\vec{k}) + m\omega^2 x \cdot (\vec{i}) = \vec{F}_{p,z}^{in} + \vec{F}_{p,x}^{in}$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_A = N_A \cdot (\vec{n})$

$\vec{N}_B = N_B \cdot (-\vec{k})$



tj: $m\ddot{s} = -mg\cos\alpha + F_{p,x}^{in}\sin\alpha$

$m\ddot{s} = -mg\cos\alpha + m\omega^2 x \sin\alpha$

$\omega = \sqrt{\dfrac{a + g\cos\alpha}{x\sin\alpha}}$, $x = ?$

$\ddot{s} = a \rightarrow \dot{s} = at + C_1^{\,0} \rightarrow s = \frac{1}{2}at^2 + C_2^{\,0} \rightarrow x = 6 + s\cdot\sin\alpha = 6 + \frac{1}{2}at^2\sin\alpha$

$$\boxed{\omega = \sqrt{\frac{a + g\cos\alpha}{(6 + \frac{1}{2}at^2\sin\alpha)\sin\alpha}}}$$

2) tj: $0 = -mg\cos\alpha + F_{p,x}^{in}\sin\alpha$

$0 = -mg\cos\alpha + m\omega^2 6\sin\alpha$

$$\boxed{\omega_{max} = \sqrt{\frac{g\cos\alpha}{6\sin\alpha}}}$$

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(6.25)

OA: 6

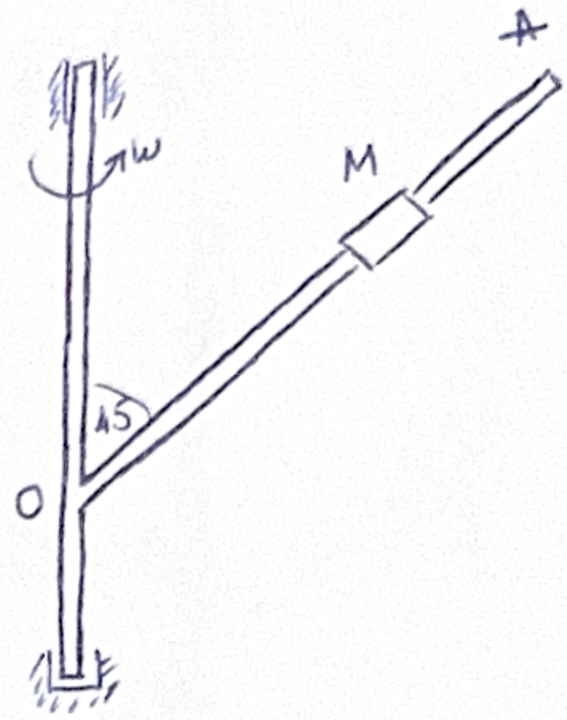
vertikalna ravan

ω = const.

klizač: m

to: klizač relativno miruje na sredini OA

$t_1 = ?$ – trenutak kada klizač napusti štap



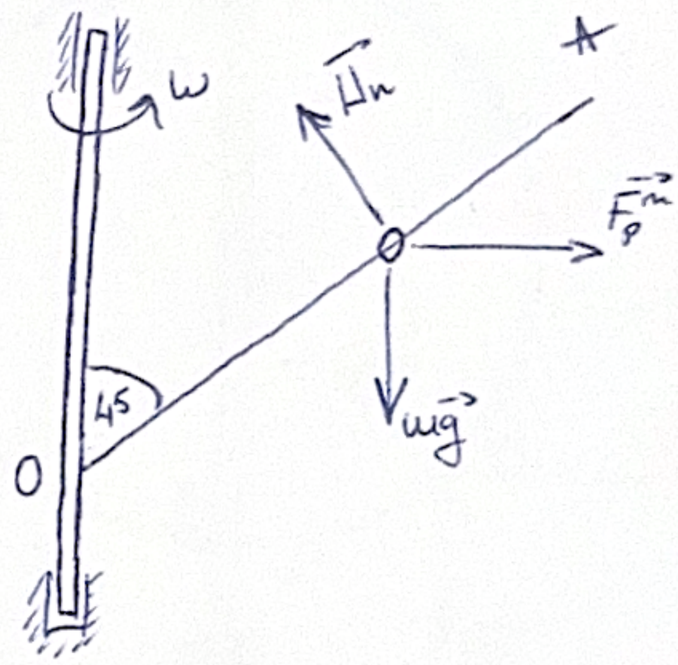
$\vec{a}_r = a_r \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{or} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{n})$

$\vec{N}_6 = N_6 \cdot (-\vec{k})$



t: $m\ddot{s} = -mg\frac{\sqrt{2}}{2} + m\omega^2 x \frac{\sqrt{2}}{2}$

$\ddot{s} = -g\frac{\sqrt{2}}{2} + \omega^2 \cdot s\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}$

$\ddot{s} - \frac{1}{2}\omega^2 s = -g\frac{\sqrt{2}}{2}$

H: $\lambda^2 - \frac{1}{2}\omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\frac{\sqrt{2}}{2}\omega \pm 0i$

$x_h = C_1 e^{\frac{\sqrt{2}}{2}\omega t} + C_2 e^{-\frac{\sqrt{2}}{2}\omega t}$

P: $6(t) = e^{0t}(-g\frac{\sqrt{2}}{2}\cos 0t + 0\sin 0t)$

$x_p = t^0 e^{0t}(A\cos 0t + B\sin 0t), \quad \ddot{x}_p = 0$

$-\frac{1}{2}\omega^2 \cdot A = -g\frac{\sqrt{2}}{2} \rightarrow A = \frac{g\sqrt{2}}{\omega^2}$

$x = C_1 e^{\frac{\sqrt{2}}{2}\omega t} + C_2 e^{-\frac{\sqrt{2}}{2}\omega t} + \frac{g\sqrt{2}}{\omega^2}$

$\dot{x} = \frac{\sqrt{2}}{2}\omega C_1 e^{\frac{\sqrt{2}}{2}\omega t} - \frac{\sqrt{2}}{2}\omega C_2 e^{-\frac{\sqrt{2}}{2}\omega t}$

$\dot{x}(t_0) = \frac{\sqrt{2}}{2}\omega C_1 - \frac{\sqrt{2}}{2}\omega C_2 = 0 \rightarrow C_1 = C_2$

$x(t_0) = C_1 \cdot 1 + C_2 \cdot 1 + \frac{g\sqrt{2}}{\omega^2} = \frac{6}{2}$

$2C_1 = \frac{6}{2} - \frac{g\sqrt{2}}{\omega^2} \rightarrow$

$C_1 = C_2 = \frac{6}{4} - \frac{g}{\omega^2}\frac{\sqrt{2}}{2}$

$$x = \left(\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}\right)\cosh\left(\frac{\sqrt{2}}{2}\omega t\right) + \frac{g\sqrt{2}}{\omega^2}$$

$$x(t_1) = \left(\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}\right)\cosh\left(\frac{\sqrt{2}}{2}\omega t_1\right) + \frac{g\sqrt{2}}{\omega^2} = 6$$

$$\cosh\left(\frac{\sqrt{2}}{2}\omega t_1\right) = \frac{6 - \frac{g\sqrt{2}}{\omega^2}}{\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}}$$

$$t_1 = \frac{\sqrt{2}}{\omega}\operatorname{arccosh}\left(\frac{6 - \frac{g\sqrt{2}}{\omega^2}}{\frac{6}{2} - \frac{g\sqrt{2}}{\omega^2}}\right)$$

napomena: Rezultat u zbirci je izražen preko logaritamske f-je, a ne preko hiperboličke. Ta dva rezultata su potpuno jednaka, jedno rezultat bi se matematičkim operacijama mogao transformisati u drugo.

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(6.26.)

krivina cev: r

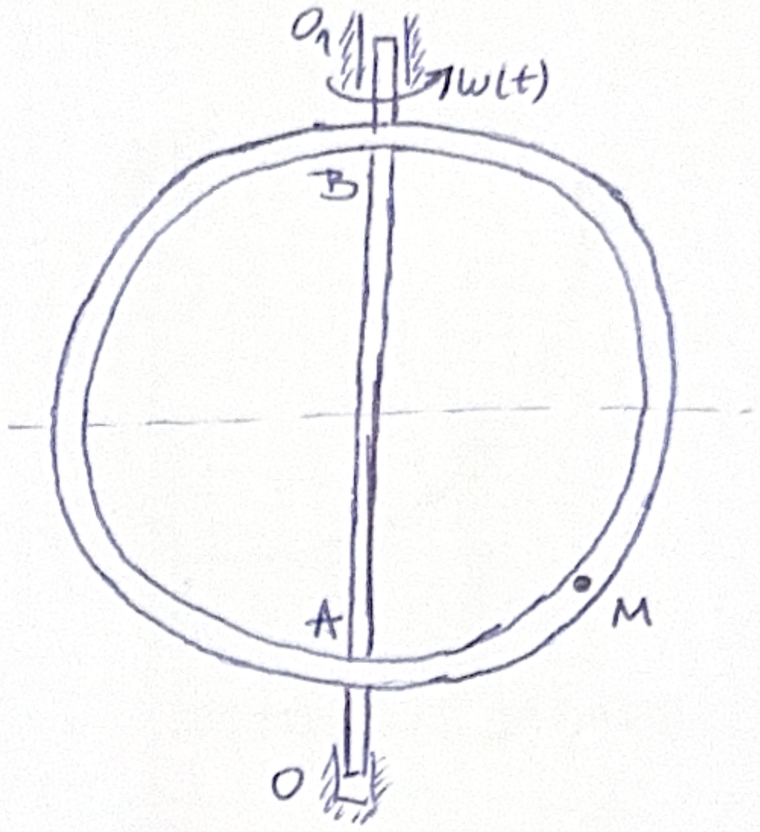
vertikalna ravan

kuglica: m

to: kuglica v A, v_0

$\omega(t) = ?$

$a_{rt} = a = const.$ – relativno tang. ubrzanje kuglice



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_t$$

$$m\vec{g} = mg\cdot(-\vec{j})$$

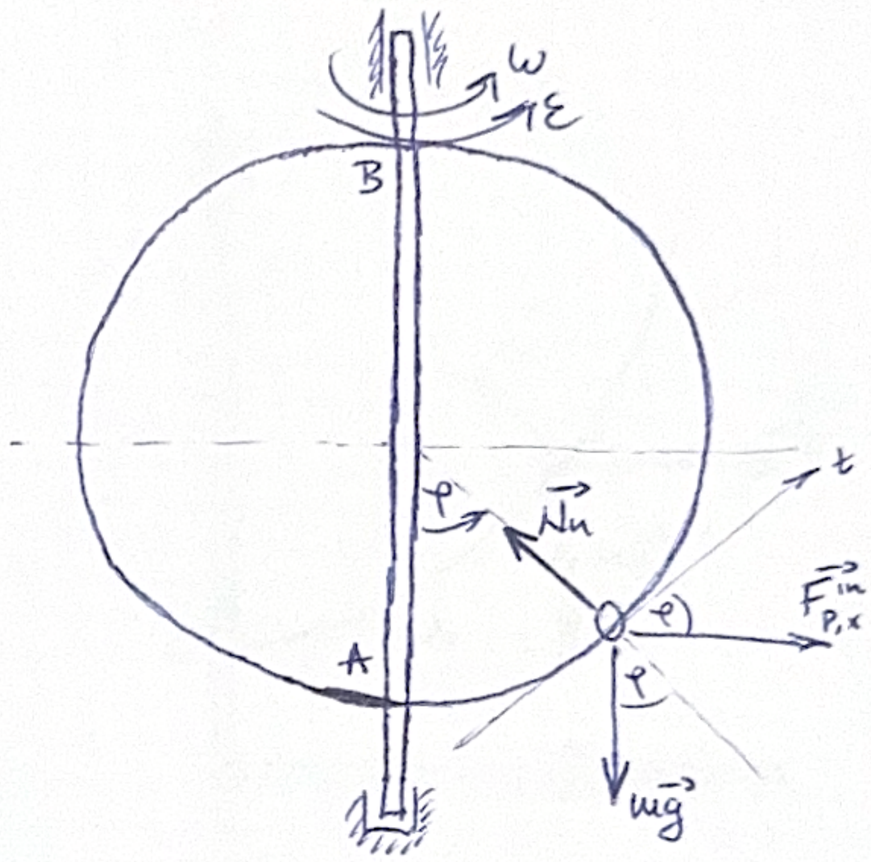
$$\vec{F}_p^{in} = -m\left(\vec{a}_{on} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})\right) =$$

$$= m\cdot\varepsilon\cdot x\cdot(\vec{k}) + m\cdot\omega^2\cdot x\cdot(\vec{i}) = \vec{F}_{p,z}^{in} + \vec{F}_{p,x}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = 2m\,\omega\cdot\dot{x}\cdot(\vec{k})$$

$$\vec{N}_n = N_n\cdot(\vec{n})$$

$$\vec{N}_t = N_t\cdot(-\vec{k})$$



t: $ma_{rt} = -mg\sin\varphi + m\omega^2 x\cos\varphi$, $\varphi = ?$

$$\varepsilon = \frac{a_{rt}}{r} = \frac{a}{r} \quad \Rightarrow \quad \omega = \frac{a}{r}t + c_1 = \frac{a}{r}t + \frac{v_0}{r} \quad \Rightarrow \quad \varphi = \frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t$$

$$a = -g\sin\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right) + \omega^2\cdot r\cdot\sin\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)\cos\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)$$

$$\omega^2 = \frac{a + g\sin\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)}{r\sin\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)\cos\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)}$$

$$\boxed{\omega^2 = \frac{a + g\sin\left(\frac{1}{2}\frac{a}{r}t^2 + \frac{v_0}{r}t\right)}{\frac{1}{2}r\sin\left(\frac{a}{r}t^2 + \frac{2v_0}{r}t\right)}}$$

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(6.27.) Klizač: m

opruga: c

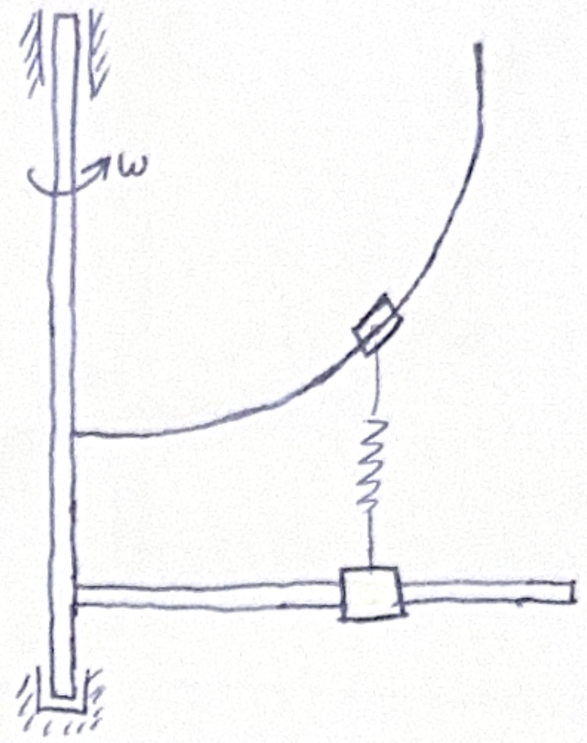
vertikalna ravan

$\omega = const.$

to: klizač u koord. početku, v_0

opruga nenapregnuta

$y(x) = ?$ - tako da $v_r = const.$



$$m\vec{a}_r = m\vec{g} + \vec{F}_c + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_G$$

$$m\vec{g} = mg \cdot (-\vec{j})$$

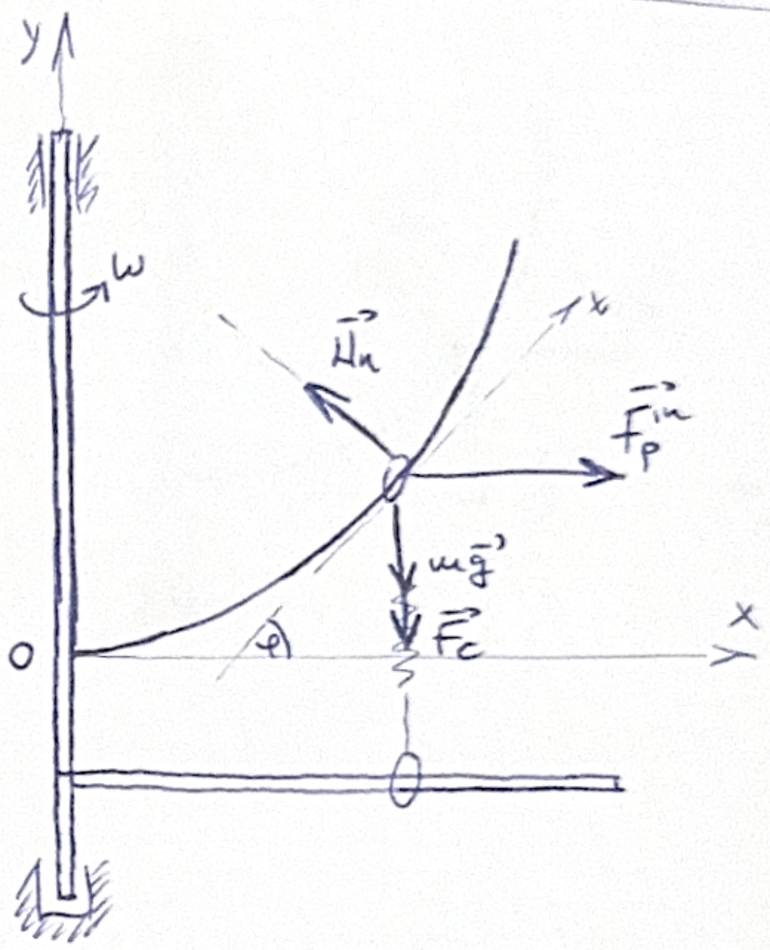
$$\vec{F}_c = c \cdot y \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{OA} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_G = N_G \cdot (-\vec{k})$$



t: $0 = -mg\sin\varphi - F_c\sin\varphi + F_p^{in}\cos\varphi$

$0 = -mg\sin\varphi - c \cdot y \cdot \sin\varphi + m\omega^2 x\cos\varphi$, $\varphi = ?$ $\rightarrow$ $tg\varphi = y' = \frac{dy}{dx}$

$0 = -mg\,tg\varphi - cy\,tg\varphi + m\omega^2 x$

$0 = -(mg + cy) \cdot y' + m\omega^2 x$

$(mg + cy)\frac{dy}{dx} = m\omega^2 x$

$mg\,dy + cy\,dy = m\omega^2 x\,dx \quad / \int dt$

$mg \cdot y + \frac{1}{2}cy^2 = \frac{1}{2}m\omega^2 x^2 \quad / \cdot 2$

$$\boxed{cy^2 + 2mgy - m\omega^2 x^2 = 0}$$

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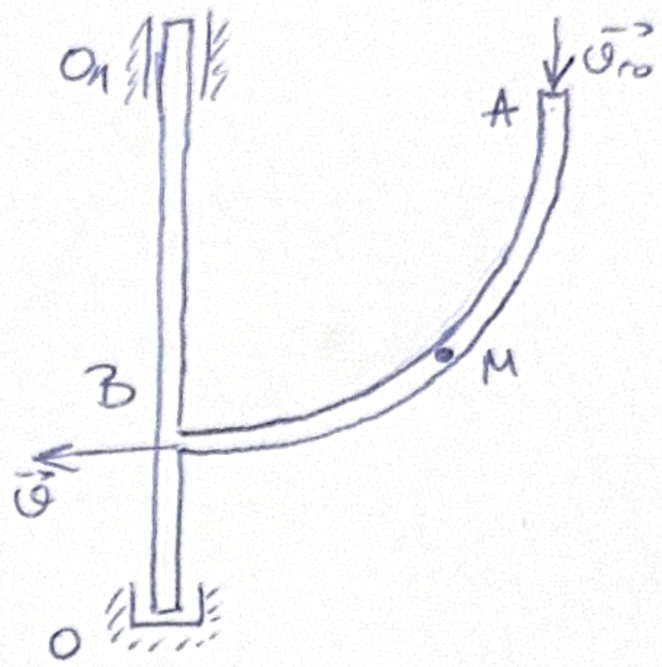
(6.28.) kružni luk: R

vertikalna ravan

$\omega = \text{const.}$

$v_0 = ?$ - da li na otvoru B brzina ima vrednost v?

kuglica: m



$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$

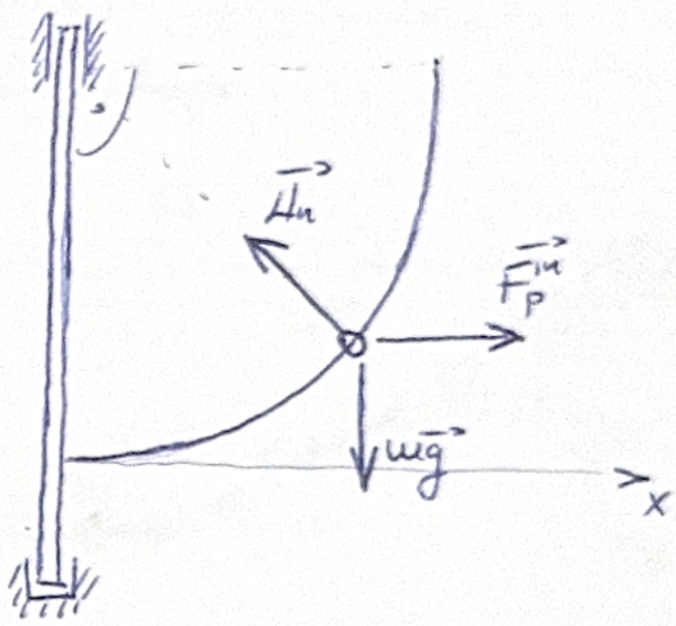
$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{O_1}^{\,0} + \vec{\varepsilon}' \times \vec{\rho}^{\,0} + \vec{\omega}' \times (\vec{\omega}' \times \vec{\rho}')) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}' \times \vec{v}_r' = 2m \cdot \omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{n}')$

$\vec{N}_6 = N_6 \cdot (-\vec{k})$



$\underline{T_B - T_A = A_{AB}}$

$A_{AB} = \int_R^0 -mg\,dy + \int_R^0 F_p^{in}\,dx = mgR + \int_0^R -m\omega^2 x\,dx = mgR - \frac{1}{2}m\omega^2 R^2$

$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgR - \frac{1}{2}m\omega^2 R^2$

$$\boxed{v_0^2 = v^2 - 2gR + \omega^2 R^2}$$

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(6.29.) $\omega = \text{const}, \quad \overline{OA} = a$

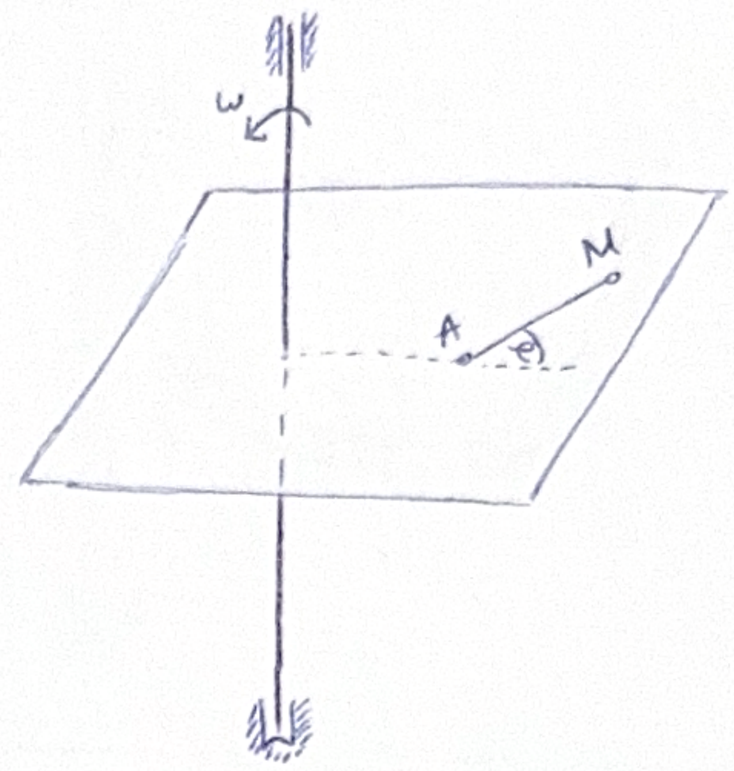
AB: $\varphi, \ 6$

t₀: $\varphi_0 = 0, \ \dot{\varphi}_0 = \omega\sqrt{\frac{a}{6}}$

M: m

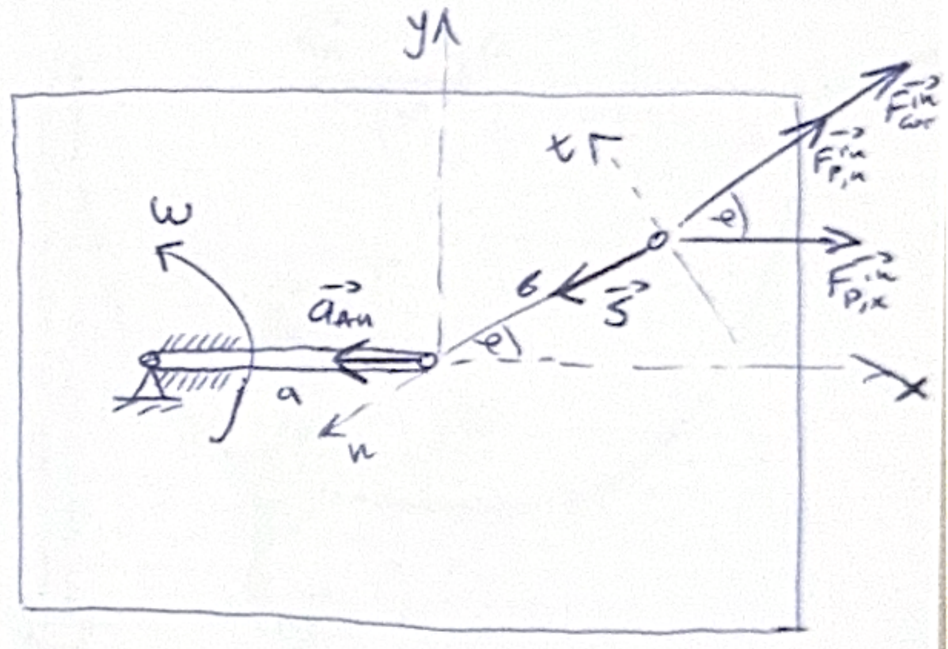
$v_r(\varphi) = ?$

$S_{max} = ?$



$\underline{m\vec{a}_r = m\vec{g} + \vec{S} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}}$

pogled odotgo ↓



$m\vec{g} = mg\cdot(-\vec{k})$

$\vec{S} = S\cdot(\vec{n})$

$\vec{F}_p^{in} = -m(\vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) =$

$= m\cdot a_{An}\cdot(\vec{i}) + m\omega^2\cdot 6\cdot(-\vec{n}) = F_{P,x}^{in} + F_{P,n}^{in}$

$\vec{F}_{cor}^{in} = -2m\vec{\omega}\times\vec{v}_r = 2m\cdot\omega\cdot v_r\cdot(-\vec{n})$

$\vec{N} = N\cdot(\vec{k})$

$a_{An} = \omega^2\cdot a$

$\underline{T - T_0 = A}$ (rel)

$$A = \int_6^{6\cos\varphi} F_{P,x}^{in}\cdot dx = \int_6^{6\cos\varphi} m\cdot\omega^2 a\, dx = m\omega^2 a(6\cos\varphi - 6) = -m\omega^2 a\cdot 6(1-\cos\varphi)$$

$\frac{1}{2}m\cdot v_r^2 - \frac{1}{2}m\cdot(6\cdot\dot{\varphi}_0)^2 = -m\omega^2 a\cdot 6\cdot(1-\cos\varphi) \quad /\cdot\frac{2}{m}$

$v_r^2 - 6^2\cdot\omega^2\cdot\frac{a}{6} = -2\omega^2 a6(1-\cos\varphi)$

$v_r^2 = \omega^2\cdot a6 - 2\omega^2 a6 + 2\omega^2 a6\cos\varphi \quad \Rightarrow \quad \boxed{v_r = \omega\sqrt{a6(2\cos\varphi - 1)}}$

n: $m\cdot a_{rn} = S - m\cdot a_{An}\cos\varphi - m\omega^2 6 - 2m\omega\cdot v_r$

$S = m\cdot\frac{v_r^2}{R_k} + m\omega^2 a\cdot\cos\varphi + m\omega^2 6 + 2m\omega v_r$

$S = m\cdot\frac{\omega^2\cdot a6(2\cos\varphi - 1)}{6} + m\omega^2 a\cos\varphi + m\omega^2 6 + 2m\omega^2\sqrt{a6(2\cos\varphi - 1)}$

$S_{max} = S(\cos\varphi = 1) = m\omega^2 a + m\omega^2 a + m\omega^2 6 + 2m\omega^2\sqrt{a6}$

$\boxed{S_{max} = m\omega^2(2a + 6 + 2\sqrt{a6})}$

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(6.30.)

cev: r

vertikalna ravan

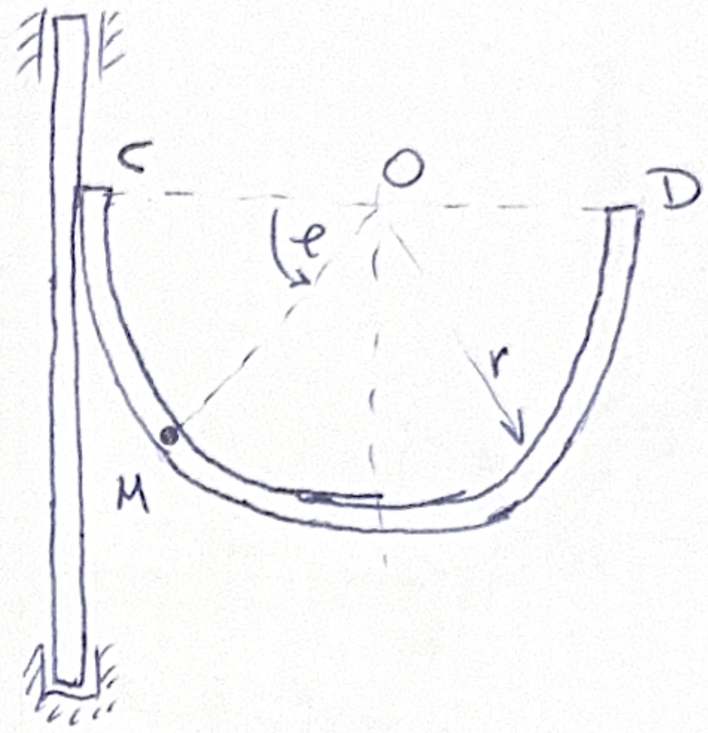
$\omega = \text{konst.}$

$t_0 = 0$: M, C

$\vartheta_{r0} = 0$

$\vartheta_r(\varphi) = ?$

$\omega = ?$ - da bi absolutna hitrost točke M pri izpustanju cevi bila ϑ_D



$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$

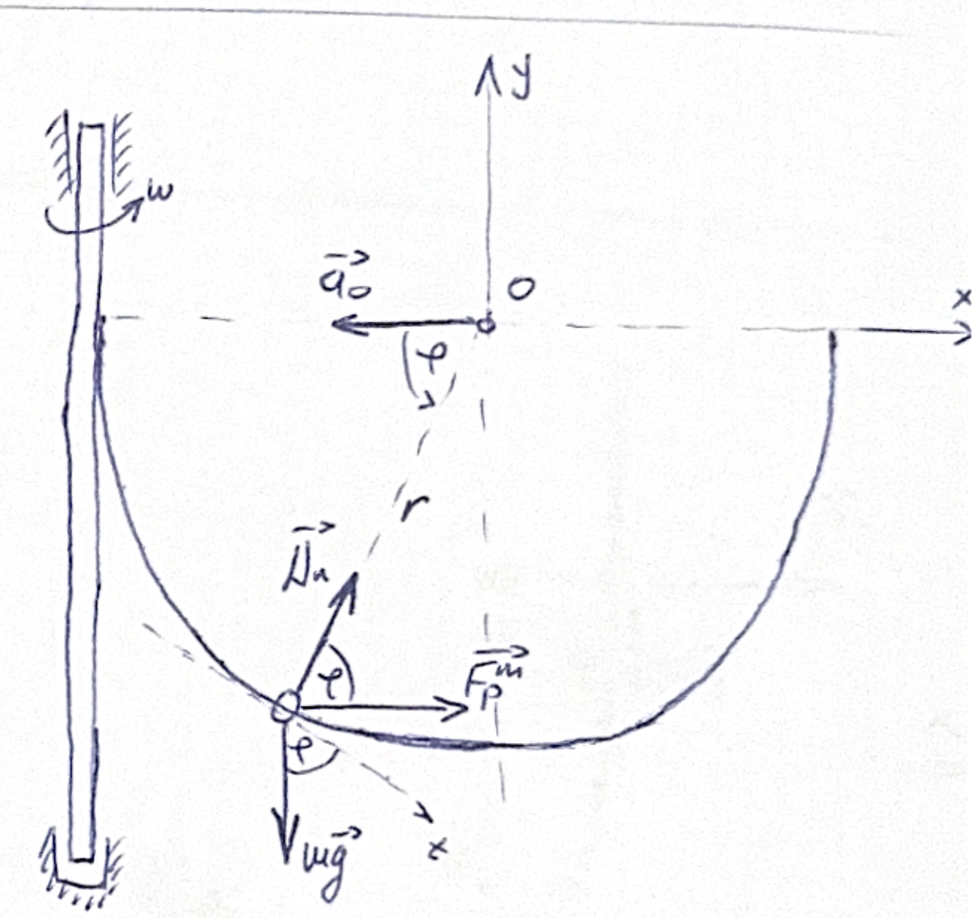
$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m \cdot a_0 \cdot (\vec{i}) + m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{\vartheta}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$$

$$\vec{N}_n = N_n \cdot (\vec{n})$$

$$\vec{N}_6 = N_6 \cdot (-\vec{k})$$



$$T - T_0 = A \quad \text{(rel)}$$

$$a_0 = r\omega^2$$

$$A = \int_0^{r\sin\varphi} -mg\,dy + \int_{-r}^{-r\cos\varphi} F_p^{in}\,dx = mg \cdot r\sin\varphi + \int_{-r}^{-r\cos\varphi} (m \cdot r\omega^2 + m\omega^2 x)\,dx =$$

$$A = mgr\sin\varphi + mr\omega^2(-r\cos\varphi + r) + \frac{1}{2}m\omega^2\left((-r\cos\varphi)^2 - (-r)^2\right) =$$

$$A = mgr\sin\varphi + mr^2\omega^2 - mr^2\omega^2\cos\varphi + \frac{1}{2}m\omega^2 r^2\cos^2\varphi - \frac{1}{2}m\omega^2 r^2$$

$$\frac{1}{2}m\vartheta_r^2 - 0 = mgr\sin\varphi + \frac{1}{2}mr^2\omega^2 - mr^2\omega^2\cos\varphi + \frac{1}{2}m\omega^2 r^2\cos^2\varphi \qquad / \cdot \frac{2}{m}$$

$$\vartheta_r^2 = 2gr\sin\varphi + r^2\omega^2(1 - 2\cos\varphi + \cos^2\varphi)$$

$$\boxed{\vartheta_r = \sqrt{2gr\sin\varphi + r^2\omega^2(1-\cos\varphi)^2}}$$

$$\vartheta_{r,D} = \vartheta_r(\varphi = 180) = \sqrt{2gr \cdot 0 + r\omega^2(1-(-1))^2} = \sqrt{4r^2\omega^2} = 2r\omega$$

$$\vartheta_{P,D} = \omega \cdot 2r$$

$$\vartheta_D^2 = \vartheta_{r,D}^2 + \vartheta_{P,D}^2 = 4r^2\omega^2 + 4\omega^2 r^2 = 8r^2\omega^2$$

$$\omega = \frac{\vartheta_D}{r\sqrt{8}} = \frac{\vartheta_D}{r \cdot 2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\vartheta_D\sqrt{2}}{4r} \quad \Rightarrow \quad \boxed{\omega = \frac{\vartheta_D\sqrt{2}}{4r}}$$

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(6.31.) vertikalna ravan

$\omega = \sqrt{\frac{g}{R}} = \text{const.}$

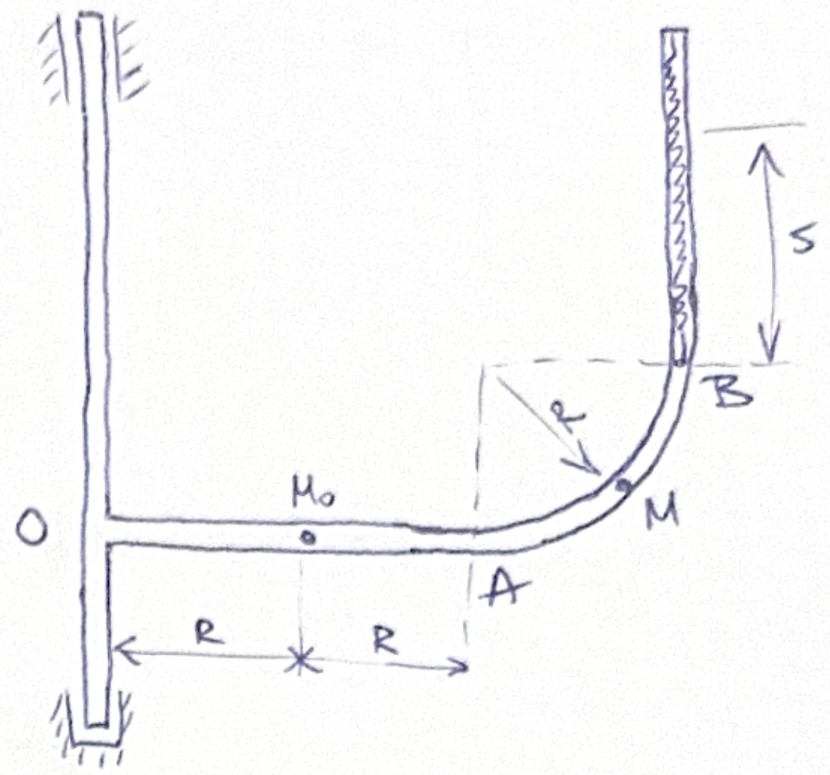
M: m

$t_0 = 0$: $M_{1e} \cup M_0$
relativno mirovanje

t_1: $M_{1e} \cup M_1$

1) $N_A = ?$

2) $\tau = ?$ - da G: $s_{max} = R$



<u>dekartov $M_0 - A$

$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$

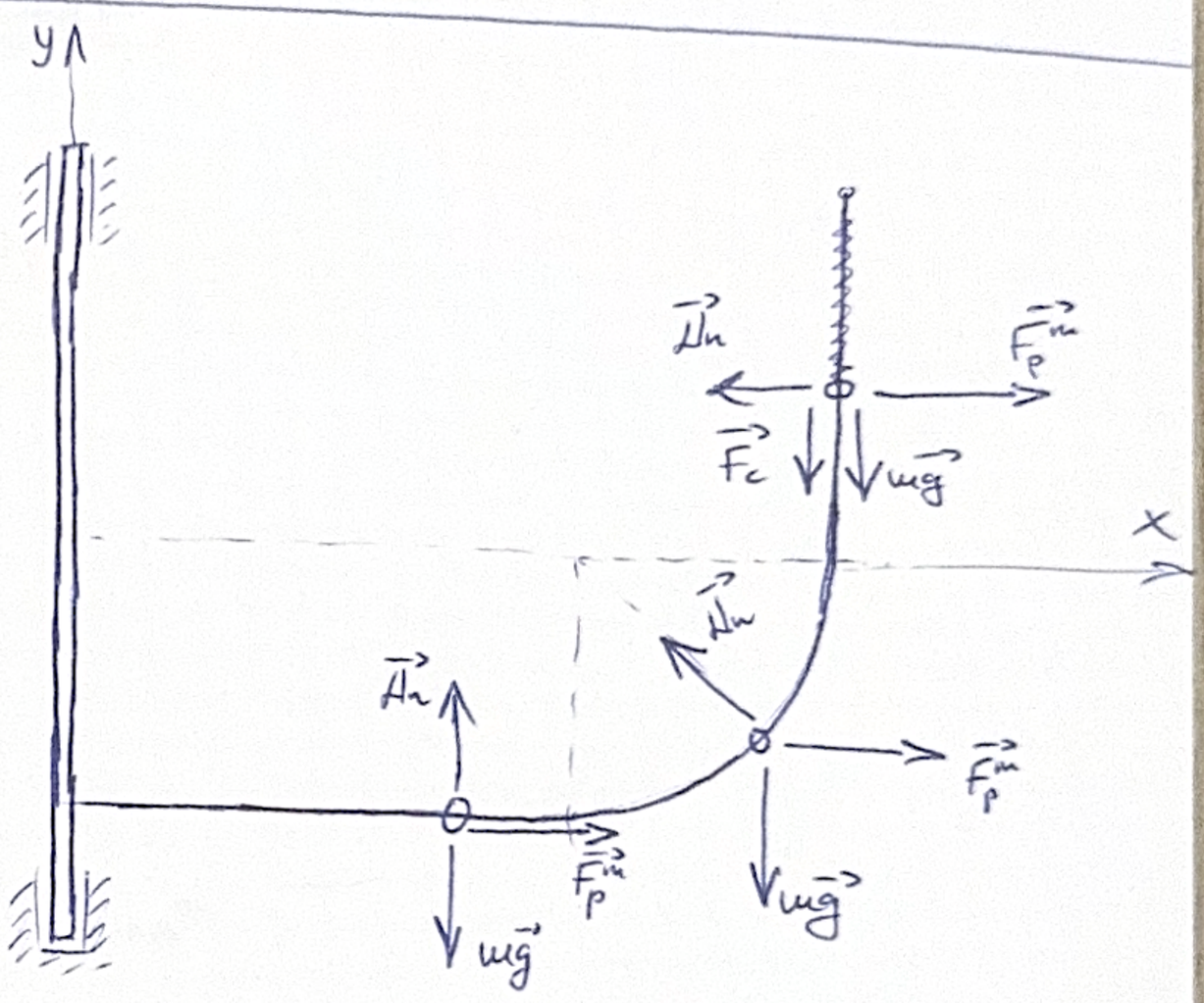
$m\vec{g} = mg \cdot (-\vec{j})$

$\vec{F}_p^{in} = -m(\vec{a}_{0x}^{\,0} + \vec{\varepsilon} \times \vec{\rho}^{\,0} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$

$\vec{F}_{Cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{k})$

$\vec{N}_n = N_n \cdot (\vec{j})$

$\vec{N}_6 = N_6 \cdot (-\vec{k})$



<u>položaj A</u>

↳ $x = x_A - \varepsilon \Rightarrow 0 = -mg + N_{An} \Rightarrow \underline{N_{An} = mg}$

↳ $x = x_A + \varepsilon \Rightarrow m a_{Ar,n} = -mg + N_{An} \Rightarrow N_{An} = mg + m a_{Ar,n}$, $a_{Ar,n} = \frac{v_{Ar}^2}{R_c}$, $R_c = R$ $\quad v_{Ar} = ?$

$\underline{T_A - T_{M_0} = A_{M_0M}}$ (rel)

$A_{M_0M} = \int_R^{2R} F_p^{in}\,dx = \int_R^{2R} m\omega^2 x\,dx = \frac{1}{2} m\omega^2 (4R^2 - R^2) = \frac{3}{2} mR^2\omega^2$

$\frac{1}{2} m v_{Ar}^2 - 0 = \frac{3}{2} m\omega^2 R^2 \rightarrow v_{Ar}^2 = 3\omega^2 R^2$

$N_{An} = mg + m\frac{3\omega^2 R^2}{R} = mg + m \cdot 3\frac{g}{R}R = mg + 3mg = 4mg \Rightarrow \underline{N_{An} = 4mg}$

z: $0 = 2m\omega\dot{x} - N_{A6} \rightarrow N_{A6} = 2m\omega \cdot v_r = 2m\omega \cdot \sqrt{3}\,\omega R = 2m\frac{g}{R}\sqrt{3}R = 2\sqrt{3}\,mg$

$$N_A = \sqrt{N_{An}^2 + N_{A6}^2} = \begin{cases} mg\sqrt{1+12} = mg\sqrt{13}, & x = x_A - \varepsilon \\ mg\sqrt{16+12} = mg \cdot 2\sqrt{7}, & x = x_A + \varepsilon \end{cases}$$

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dionica A-B: ista kao $N_0 - A$, samo što je $\vec{M}_n = M_n \cdot (\vec{i})$

(izrati za ostale sile ru ish)

$T_B - T_{N_0} = A_{N_0B}$ (rel)

$$A_{N_0B} = \int_R^{3R} F_p^m \, dx + \int_0^R -mg\,dy = \int_R^{3R} m\omega^2 x\,dx - mgR = \frac{1}{2} m\omega^2 (9R^2 - R^2) - mgR =$$

$$A_{N_0B} = 4m\omega^2 R^2 - mgR = 4m\frac{g}{R}R^2 - mgR = 3mgR$$

$$\frac{1}{2} m v_B^2 - 0 = 3mgR \quad \Rightarrow \quad \underline{v_B^2 = 6gR}$$

dionica B-M_n: sve sile su iste osim:

$$\vec{M}_n = +M_n \cdot (-\vec{i})$$
$$\vec{F}_e = c \cdot y \cdot (-\vec{j})$$

$T_{M_n} - T_B = A_{BM_n}$ (rel)

$$A_{BM_n} = \int_0^R -mg\,dy + \int_0^R -cy\,dy = -mgR - \frac{1}{2}cR^2$$

$$0 - \frac{1}{2} m v_B^2 = -mgR - \frac{1}{2}cR^2$$

$$-\frac{1}{2} m \cdot 6gR = -mgR - \frac{1}{2}cR^2$$

$$\frac{1}{2}cR^2 = 2mgR$$

$$\boxed{c = \frac{4mg}{R}}$$

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(6.33.) vertikalna ravan

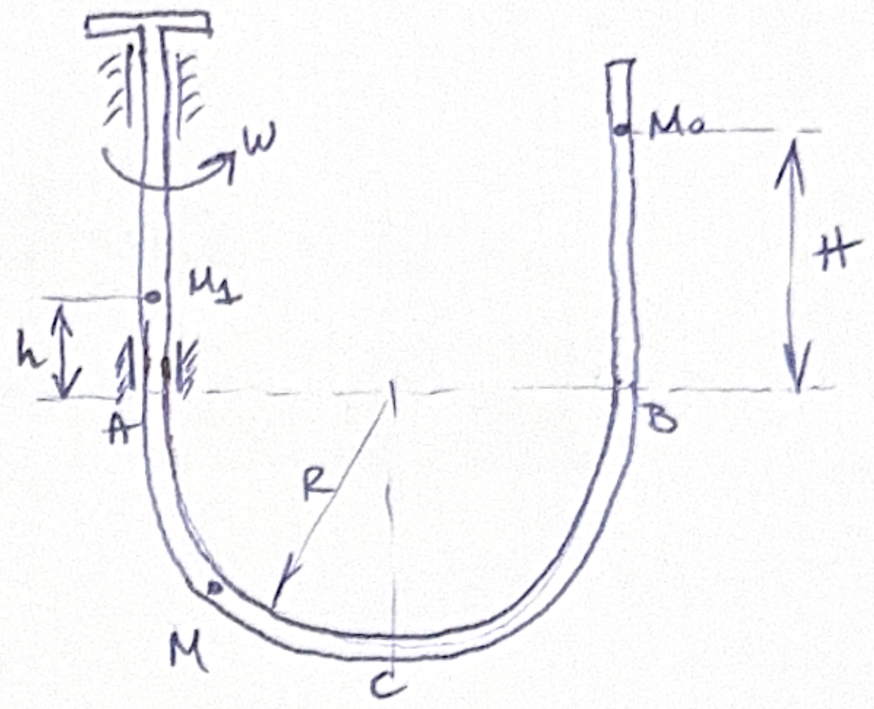
$\omega = \sqrt{\frac{g}{R}}$

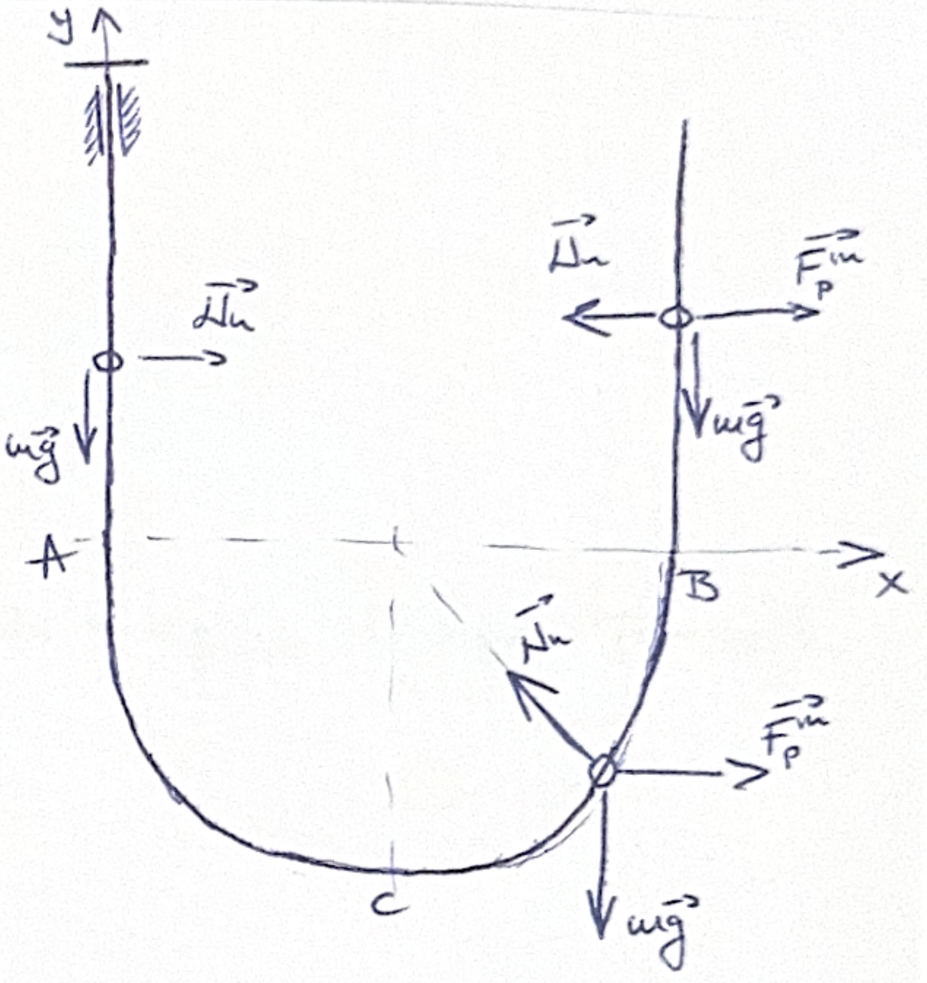
tačka mase m

t_0: $A = 3R$, $v_{r_0} = 0$

1) $h = ?$ – max. položaj

2) $N_C = ?$





dionica $M_0 - B$:

$$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_b}$$

$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 0$$

$$\vec{N}_n = N_n \cdot \vec{n}$$

$$\vec{N}_b = N_b \cdot \vec{b}$$

dionica $B - A$: sve sile su iste, osim:

$$\vec{F}_{cor}^{in} = +2m\omega \cdot \dot{x} \cdot (\vec{k})$$

položaj C:

n: $\quad m \cdot a_{rn,C} = -mg + N_{n,C}$

$$N_{n,C} = m(g + a_{rn,C}), \quad a_{rn,C} = \frac{v_{rC}^2}{R_C}, \quad v_{rC} = ?, \quad R_C = R$$

$$\underline{T_C - T_{M_0} = A_{M_0C}} \quad \text{(rel)}$$

$$A_{M_0C} = \int_{H}^{-R} -mg\,dy + \int_{2R}^{R} F_p^{in}\,dx = -mg(-R-H) + \int_{2R}^{R} m\omega^2 x\,dx = mg(R+H) + \frac{1}{2}m\omega^2(R^2 - 4R^2) =$$

$$A_{M_0C} = 4mgR - \frac{3}{2}m\frac{g}{R}R^2 = \frac{5}{2}mgR$$

$$\frac{1}{2}mv_C^2 - 0 = \frac{5}{2}mgR$$

$$v_C^2 = 5gR$$

$$N_{n,C} = m\left(g + \frac{5gR}{R}\right) = 6mg$$

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$z: \quad 0 = +2m\omega \dot{x}_c + N_{6,c}$

$$N_{6,c} = -2m\omega \cdot \dot{x}_c = -2m\sqrt{\frac{g}{R}} \cdot (-v_{r,c}) = 2m\sqrt{\frac{g}{R}} \cdot \sqrt{5gR} = 2\sqrt{5}\, mg$$

$$N_c = \sqrt{N_{h,c}^2 + N_{6,c}^2} = mg\sqrt{36+20} = \sqrt{56}\, mg \quad \Rightarrow \quad \boxed{N_c = \sqrt{56}\, mg}$$

deonica $A - M_1$: ista kao prethodna, samo gde važi:

$\vec{F}_p^{in} = 0$

$f_{vr}^{in} = 0$

$T_{M_1} - T_{M_0} = A_{M_0M_1}$

$$A_{M_0M_1} = \int_{H}^{h} -mg\,dy + \int_{2R}^{0} f_p^{in}\,dx = mg(H-h) + \int_{2R}^{0} m\omega^2 x\,dx = mg(3R-h) + \frac{1}{2}m\omega^2(0-4R^2)$$

$$A_{M_0M_1} = 3mgR - mgh - 2m\frac{g}{R}R^2 = mgR - mgh$$

$$0 - 0 = mgR - mgh \quad \Rightarrow \quad \boxed{h = R}$$

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(6.34) ABCD: a

ω = const.

horizontalna ravan

M: m

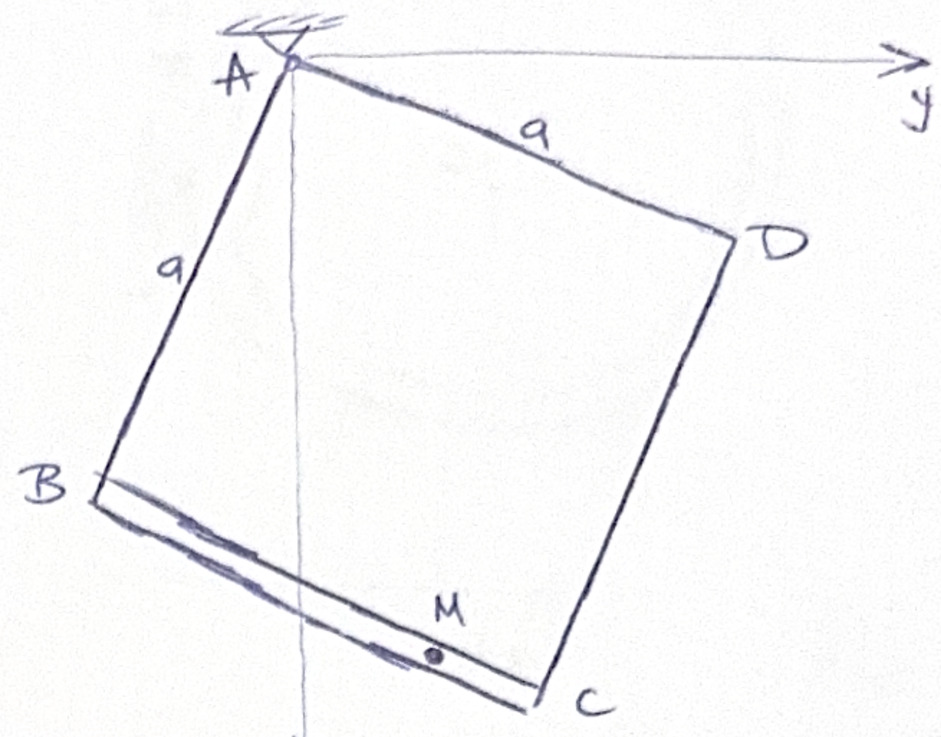
t_0: M je u B

$v_{r0} = v_0$

t_1: M je u C

$v_{r1} = 2v_0$

$AB \parallel A_x$ (A_x usmerena od A ka B)



Kolicine i-ne kretanja tacke M kada ona klizi po ploci u odnosu na dekartov koordinatni sistem A_{xyz} ?

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_G$$

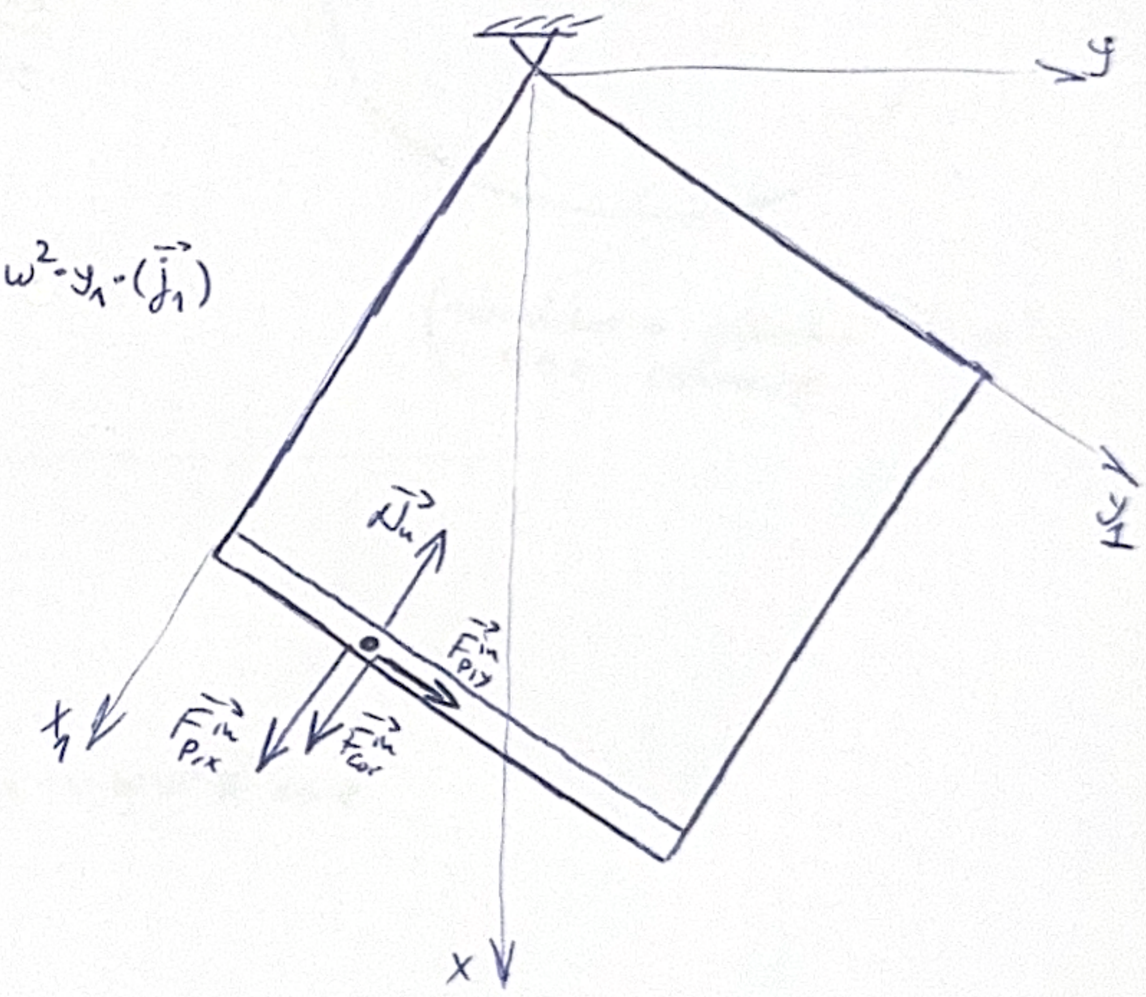
$$m\vec{g} = mg(-\vec{k}_1)$$

$$\vec{F}_p^{in} = -m(\vec{a}_{on} + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) = m\omega^2 x_1\cdot(\vec{i}_1) + m\omega^2\cdot y_1\cdot(\vec{j}_1)$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = 2m\omega\cdot v_r\cdot(\vec{i}_1)$$

$$\vec{N}_n = +N_n\cdot(-\vec{j}_1)$$

$$\vec{N}_G = N_G\cdot(\vec{k}_1)$$

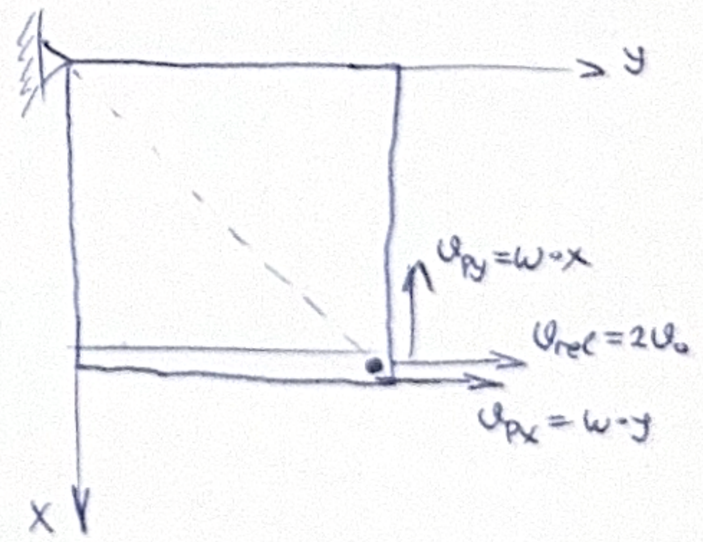


$$T_C - T_B = A_{BC} \quad (rcl)$$

$$A_{BC} = \int_0^a F_{p1y}^{in}\,dy_1 = \int_0^a m\omega^2 y_1\,dy_1 = \frac{1}{2}m\omega^2 a^2$$

$$\frac{1}{2}m(2v_0)^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\omega^2 a^2$$

$$\frac{3}{2}mv_0^2 = \frac{1}{2}m\omega^2 a^2 \Rightarrow \underline{\omega = \frac{v_0}{a}\sqrt{3}}$$



Resavamo vreme u trenutak kada tacka M napusta plocu:

$t_0 = 0$: $x_0 = a$, $y_0 = a$, $\dot{y}_0 = \omega a + 2v_0 = \frac{v_0}{a}a\sqrt{3} + 2v_0$, $\dot{x}_0 = -\omega a = -v_0\sqrt{3}$

$$m\vec{a} = m\vec{g}$$

y: $m\ddot{y} = 0$ → $\dot{y} = \text{const} = v_0(2+\sqrt{3})$ →

x: $m\ddot{x} = 0$ → $\dot{x} = \text{const} = -v_0\sqrt{3}$ →

z: $m\ddot{z} = -mg$ → $\dot{z} = -gt$ →

$$y = v_0(2+\sqrt{3})\,t + a$$
$$x = -v_0\sqrt{3}\,t + a$$
$$z = -\frac{1}{2}gt^2$$

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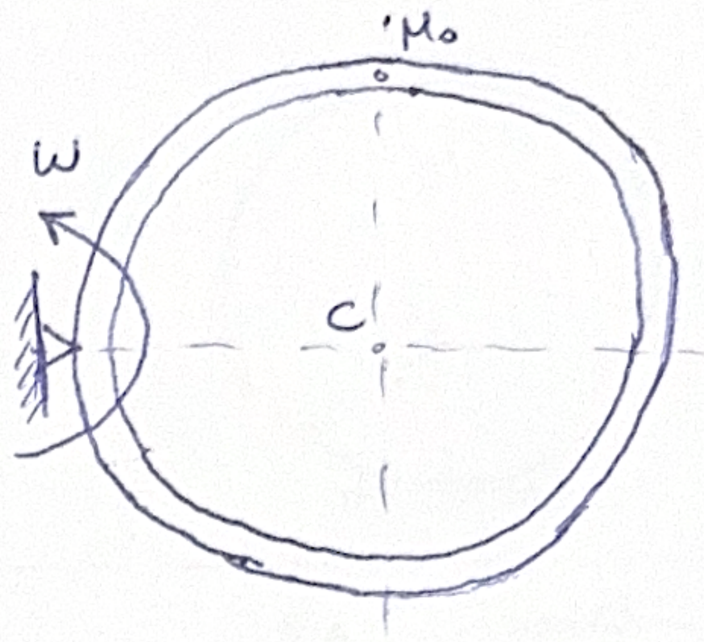
(6.35) horizontalna ravan

$\omega = \text{const.}$

kuglica: m

t_0: M je u M_0, miruje

$v_r(\varphi) = ?$, $N(\varphi) = ?$



$$m\vec{a}_r = m\vec{g} + \vec{F}^{in}_p + \vec{F}^{in}_{cor} + \vec{N}_n + \vec{N}_6$$

$$m\vec{g} = +mg\cdot(-\vec{k})$$

$$\vec{F}^{in}_p = -m(\vec{a}_c + \cancel{\vec{\varepsilon}\times\vec{\rho}}^{\,0} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) =$$

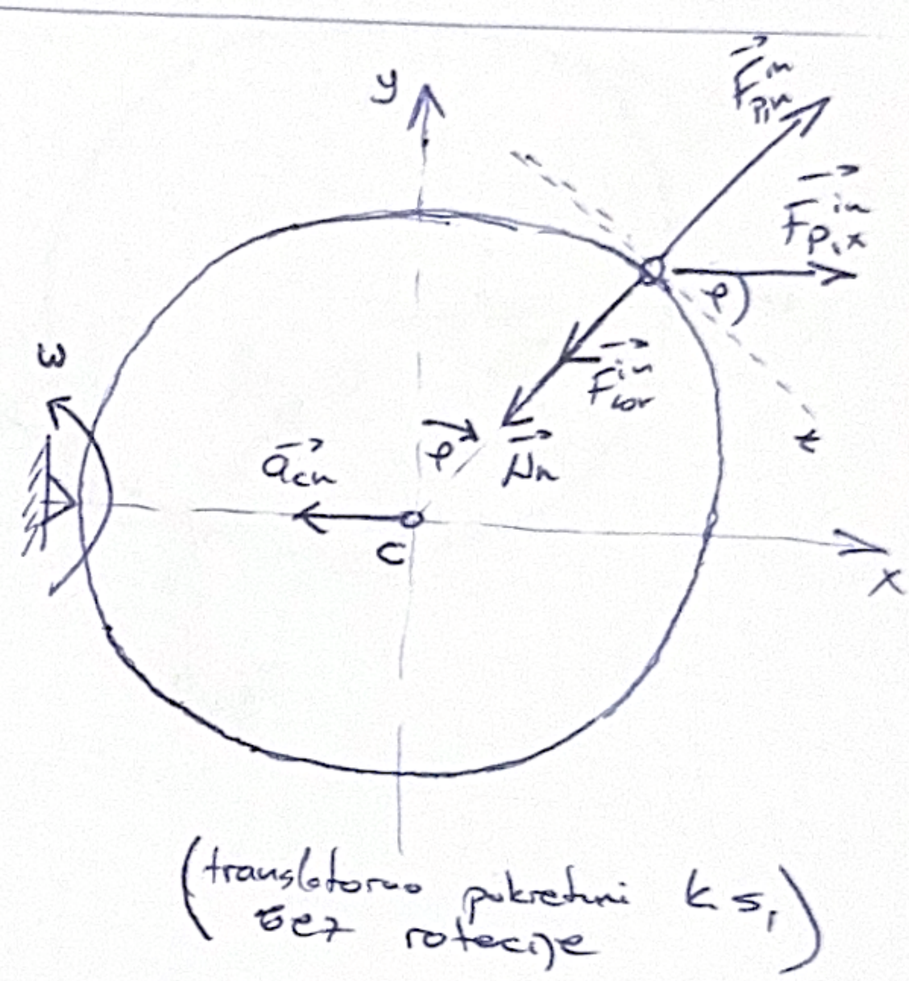
$$= m\,a_{cn}\cdot(\vec{i}) + m\omega^2 R\cdot(-\vec{n}) =$$

$$= m\omega^2 R\cdot(\vec{i}) + m\omega^2 R\cdot(-\vec{n}) = \vec{F}^{in}_{P,x} + \vec{F}^{in}_{P,n}$$

$$\vec{F}^{in}_{cor} = -2m\vec{\omega}\times\vec{v}_r = 2m\omega\cdot v_r\cdot(\vec{n})$$

$$\vec{N}_n = N_n\cdot(\vec{n})$$

$$\vec{N}_6 = N_6\cdot(\vec{k})$$



(translatorno pokretni K_S, brez rotacije)

$$T_M - T_{M0} = A_{M_0M} \qquad (rel)$$

$$A_{M_0M} = \int_{M_0}^{M} \vec{F}^{in}_{P,x}\,d\vec{\rho} = \int_0^{R\sin\varphi} F^{in}_{P,x}\,dx = \int_0^{R\sin\varphi} m\omega^2 R\,dx = m\omega^2 R^2\sin\varphi$$

$$\frac{1}{2}m v_r^2 - 0 = m\omega^2 R^2\sin\varphi$$

$$\boxed{v_r = \omega R\sqrt{2\sin\varphi}}$$

n: $$m\cdot a_{r,n} = -F^{in}_{P,x}\sin\varphi - F^{in}_{P,n} + F^{in}_{cor} + N_n$$

$$N_n = m\,a_{rn} + F^{in}_{P,x}\sin\varphi + F^{in}_{P,n} - F^{in}_{cor}$$

$$N_n = m\frac{v_{rel}^2}{R_K} + m\omega^2 R\sin\varphi + m\omega^2 R - 2m\omega v_{rel}$$

$$N_n = m\frac{\omega^2R^2\cdot 2\sin\varphi}{R} + m\omega^2 R\sin\varphi + m\omega^2 R - 2m\omega^2 R\sqrt{2\sin\varphi}$$

$$\underline{\underline{N_n = m\omega^2 R(1 + 3\sin\varphi - 2\sqrt{2\sin\varphi})}}$$

z: $$0 = -mg + N_6 \quad \Rightarrow \quad \underline{\underline{N_6 = mg}}$$

$$\boxed{N = \sqrt{N_n^2 + N_6^2}}$$

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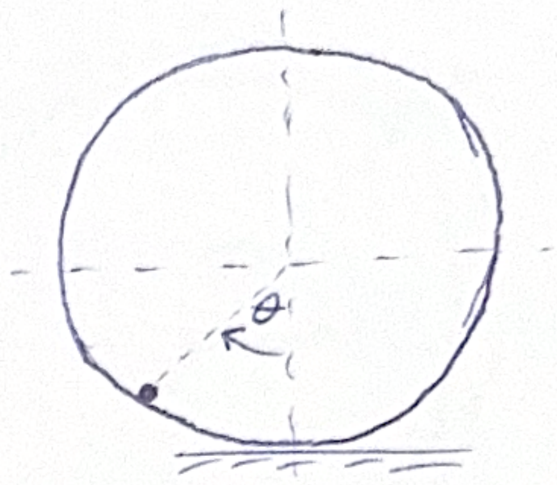
(6.36.) vertikalna ravan

μ - koef. trenja između kanala i cilindra

$\theta = ?$ - konstantan ugao otklona usled trenja

1) $v_c = const. = v$

2) $a_c = const. = 2g$



$m\vec{a}_r = m\vec{g} + \vec{F}_{tr} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_t$ (→0)

y ↑ translatorno pokretni ks (bez rotacije!)

$m\vec{g} = mg\cdot(-\vec{j})$

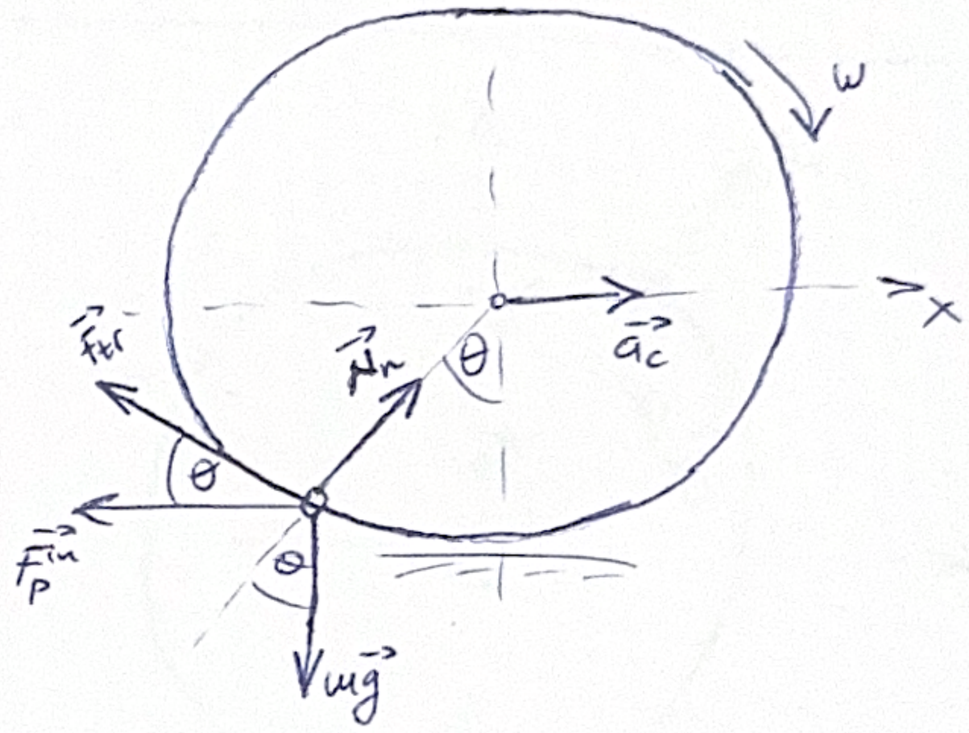
$\vec{F}_{tr} = \mu \cdot N_n \cdot (-\vec{t})$

$\vec{F}_p^{in} = -m(\vec{a}_c + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})) = m\cdot a_c\cdot(-\vec{i})$ (→0, →0)

$\vec{F}_{cor}^{in} = -2m\vec{\omega}\times\vec{v}_r = 0$ (→0)

$\vec{N}_n = N_n\cdot\vec{n}$

$\vec{N}_t = 0$



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t: $0 = mg\sin\theta - F_{tr} - F_p^{in}\cos\theta$

$0 = mg\sin\theta - \mu\cdot N_n - ma_c\cdot\cos\theta$, $N_n = ?$

n: $0 = -mg\cos\theta - F_p^{in}\sin\theta + N_n$

$N_n = mg\cos\theta + ma_c\sin\theta$

t: $0 = mg\sin\theta - \mu\cdot m(g\cos\theta + a_c\sin\theta) - ma_c\cos\theta$

1) $v_c = const.$ → $a_c = 0$

$0 = g\sin\theta - \mu g\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \mu = tg\theta$

$\boxed{\theta = arctg\,\mu}$

2) $a_c = 2g$

$0 = g\sin\theta - \mu g\cos\theta - \mu\cdot 2g\sin\theta - 2g\cos\theta$

$0 = \sin\theta(g - 2\mu g) - \cos\theta(2g + \mu g)$

$\frac{\sin\theta}{\cos\theta} = \frac{2g+\mu g}{g-2\mu g} = \frac{2+\mu}{1-2\mu} = tg\theta$

$\boxed{\theta = arctg\,\frac{2+\mu}{1-\mu}}$

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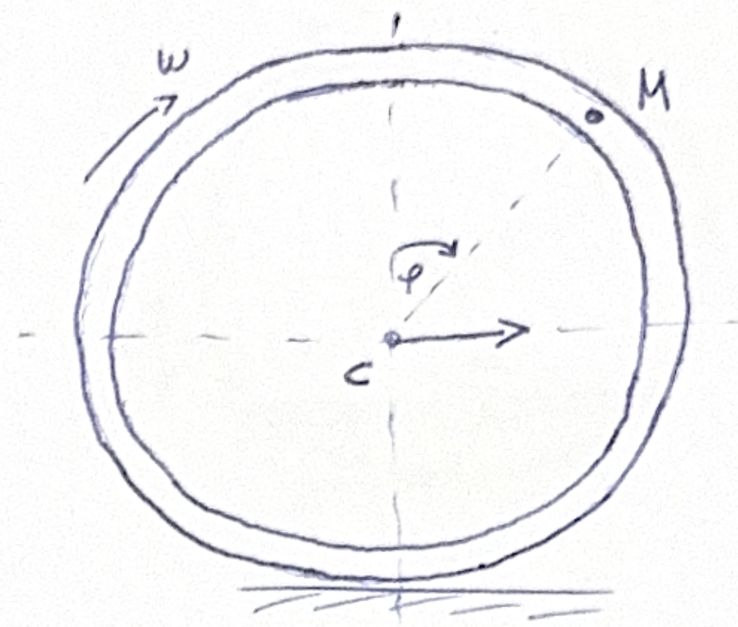
(6.38) vertikalna ravan



M: m

t_0: M je u relativnom mirovanju u najvišem položaju

$N_n(\varphi) = ?$

1) $\omega = \text{const.}$

2) $\varepsilon = \text{const.}, \quad \omega(t_0) = \omega_0$

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$

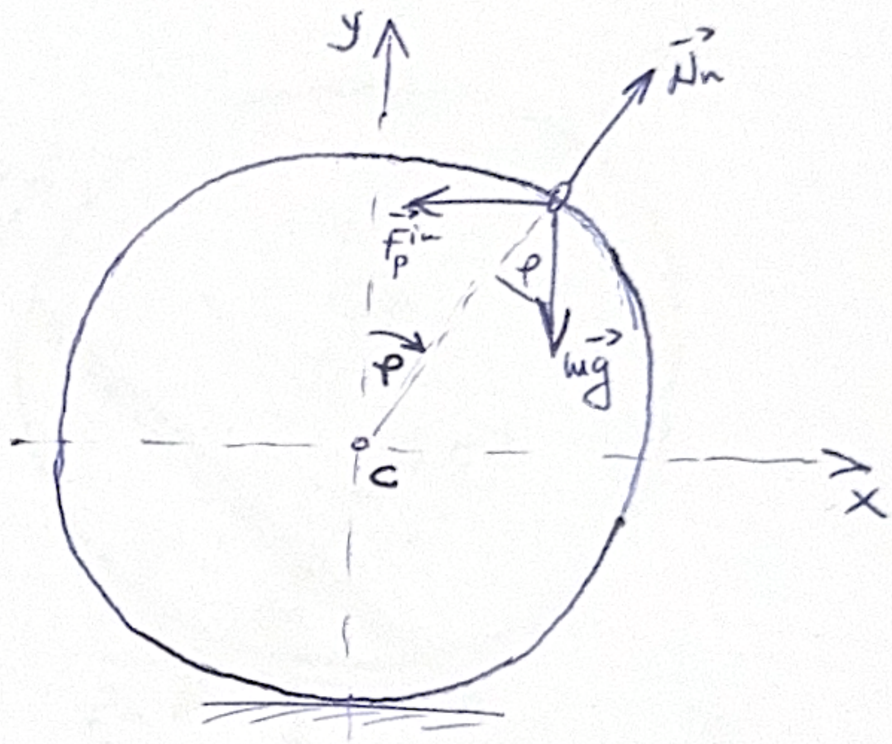
$$m\vec{g} = +mg\cdot(-\vec{j})$$

$$\vec{F}_p^{in} = -m\left(\vec{a}_c + \vec{\varepsilon}\times\vec{\varrho} + \vec{\omega}\times(\vec{\omega}\times\vec{\varrho})\right) = m\cdot a_c\cdot(-\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\cdot\vec{\omega}\times\vec{\varrho}_r = 0$$

$$\vec{N}_n = +N_n\cdot(-\vec{n})$$

$$\vec{N}_6 = 0$$



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n: $\quad m\cdot\frac{\varrho_r^2}{R} = mg\cos\varphi + F_p^{in}\sin\varphi - N_n$

$$N_n = mg\cos\varphi + ma_c\cdot\sin\varphi - m\frac{\varrho_r^2}{R}, \qquad \varrho_r = ?$$

$$T - T_0 = A_{1\ldots} \quad (rel)$$

$$A_{on} = \int_R^{R\cos\varphi} -mg\,dy + \int_0^{R\sin\varphi} -F_p^{in}\,dx = mgR(1-\cos\varphi) - m\cdot a_c\cdot R\sin\varphi$$

$$\frac{1}{2}m\varrho_{rel}^2 - \frac{1}{2}m\varrho_{r0}^2 = mgR(1-\cos\varphi) - m\cdot a_c\cdot R\sin\varphi$$

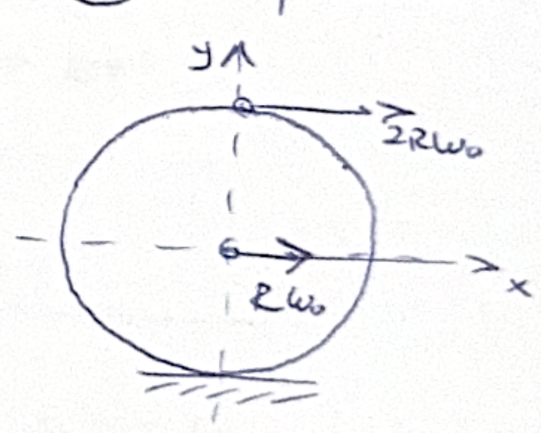
$$\frac{1}{2}m\varrho_{rel}^2 - \frac{1}{2}mR^2\omega_0^2 = mgR(1-\cos\varphi) - ma_c\cdot R\sin\varphi$$

$$\varrho_{rel}^2 = R^2\omega_0^2 + 2gR(1-\cos\varphi) - 2a_c R\sin\varphi$$

$$N_n = mg\cos\varphi + ma_c\sin\varphi - mR\omega_0^2 - 2mg(1-\cos\varphi) + 2ma_c\sin\varphi$$

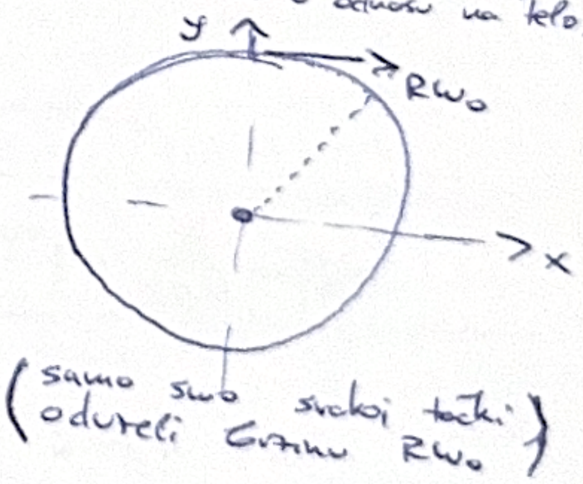
$$N_n = 3mg\cos\varphi + 3ma_c\sin\varphi - mR\omega_0^2 - 2mg$$

t_0 - apsolutno



t_0 - relativno

(u odnosu na Cxy, a ne u odnosu na tlo)



(samo smo svaki tački oduzeli brzinu $R\omega_0$)

1) $\omega = \text{const} \Rightarrow a_c = R\varepsilon = 0 \Rightarrow$ $\boxed{N_n = 3mg\cos\varphi - mR\omega_0^2 - 2mg}$

2) $\varepsilon = \text{const.} \Rightarrow a_c = R\varepsilon \Rightarrow$ $\boxed{N_n = 3mg\cos\varphi + 3mR\varepsilon\cdot R\sin\varphi - mR\omega_0^2 - 2mg}$

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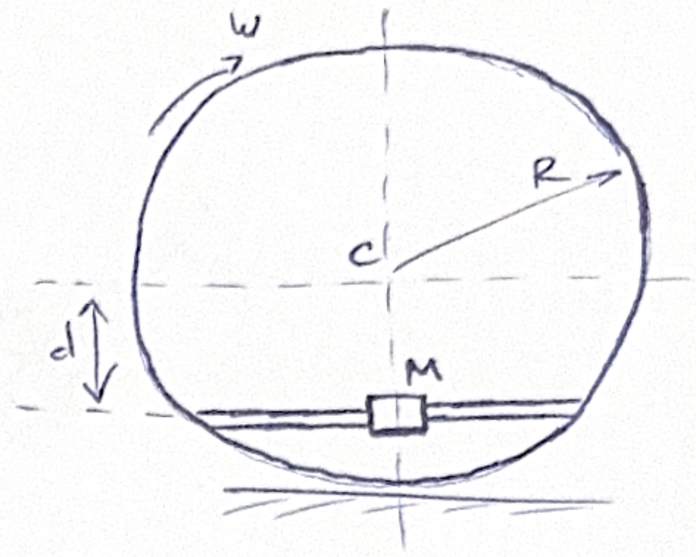
(6.59.)



disk: R

$\omega = const.$

vertikalna ravan

klizač: m

t₀: štap je horizontalan

$v_{r0} = v_0$

f-ja rel kretanja u odnosu na disk?

$N(t) = ?$

$$m\vec{a}_r = m\vec{g}' + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$

$$m\vec{g}' = mg\cdot(-\vec{j})$$

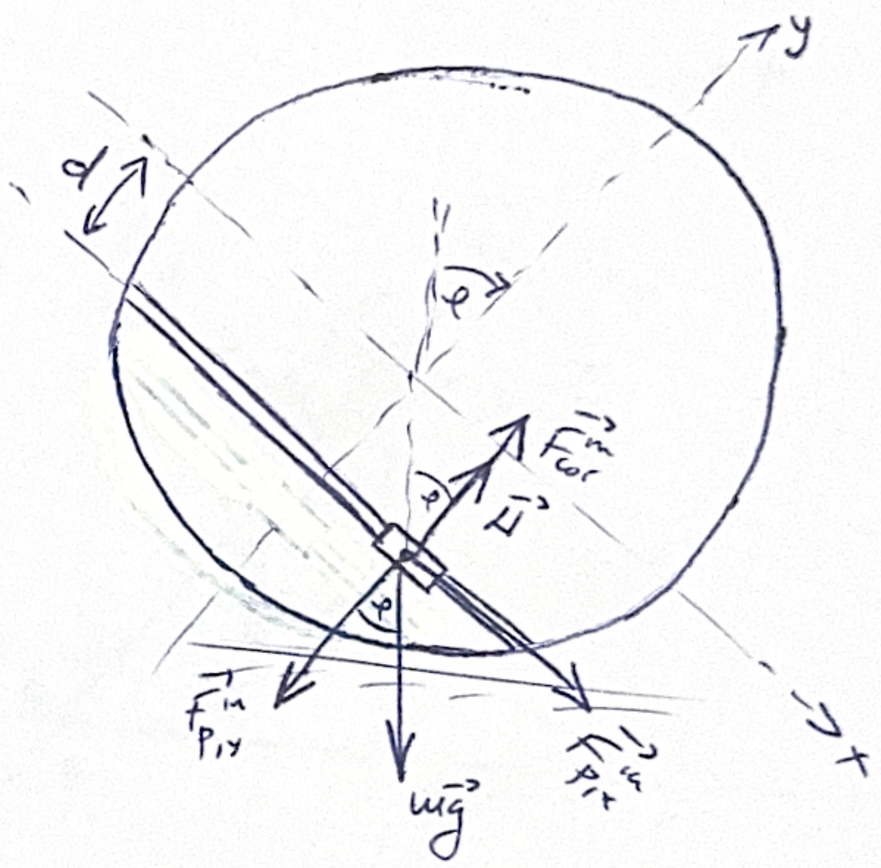
$$\vec{F}_p^{in} = -m\left(\vec{a}_c + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})\right) =$$

$$= m\omega^2 d\cdot(-\vec{j}) + m\omega^2 x\cdot(\vec{i}) = \vec{F}_{p,y}^{in} + \vec{F}_{p,x}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}'\times\vec{v}_r = 2m\omega\cdot\dot{x}\cdot(\vec{j})$$

$$\vec{N}_n = N\cdot\vec{n}' = \vec{N}'$$

$$\vec{N}_6 = 0$$



$$\varphi = \int\omega\,dt = \omega t$$

x: $m\ddot{x} = mg\sin\varphi + m\omega^2 x$

$$\ddot{x} - \omega^2 x = g\sin(\omega t)$$

H: $\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega + 0i$

$$x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$$

P: $G(t) = e^{0t}(0\cos(\omega t) + g\sin(\omega t))$

$$x_p = t^0 e^{0t}(A\cos(\omega t) + B\sin(\omega t))$$

$$\ddot{x}_p = -\omega^2(A\cos(\omega t) + B\sin(\omega t))$$

$$\cos(\omega t)(-A\omega^2 - \omega^2 A) + \sin(\omega t)(-B\omega^2 - \omega^2 B) =$$
$$= \cos(\omega t)\cdot 0 + \sin(\omega t)\cdot g$$

$\Rightarrow A = 0,\ -2B\omega^2 = g \rightarrow B = -\frac{g}{2\omega^2}$

$$x = C_1 e^{\omega t} + C_2 e^{-\omega t} - \frac{g}{2\omega^2}\sin(\omega t)$$

$$\dot{x} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t} - \frac{g}{2\omega}\cos(\omega t)$$

$$x(t_0) = C_1 + C_2 - 0 = 0 \rightarrow C_1 = -C_2$$

$$\dot{x}(t_0) = C_1\omega - C_2\omega - \frac{g}{2\omega} = v_0$$

$$C_1\omega + C_1\omega = v_0 + \frac{g}{2\omega}$$

$$\Rightarrow C_1 = \frac{v_0}{2\omega} + \frac{g}{4\omega^2} = -C_2$$

$$\boxed{x = \left(\frac{v_0}{2\omega} + \frac{g}{4\omega^2}\right)\left(e^{\omega t} - e^{-\omega t}\right) - \frac{g}{2\omega^2}\sin(\omega t)}$$

y: $0 = -mg\cos\varphi - m\omega^2 d + 2m\omega\dot{x} + N$

$$N = mg\cos(\omega t) + m\omega^2 d - 2m\omega\left(\left(\frac{v_0}{2\omega} + \frac{g}{4\omega^2}\right)\cdot\right.$$
$$\left.\cdot(\omega e^{\omega t} + \omega e^{-\omega t}) - \frac{g}{2\omega}\cos(\omega t)\right)$$

$$\boxed{N = M\omega^2 d + 2mg\cos(\omega t) - m\left(v_0\omega + \frac{g}{2}\right)\left(e^{\omega t} + e^{-\omega t}\right)}$$

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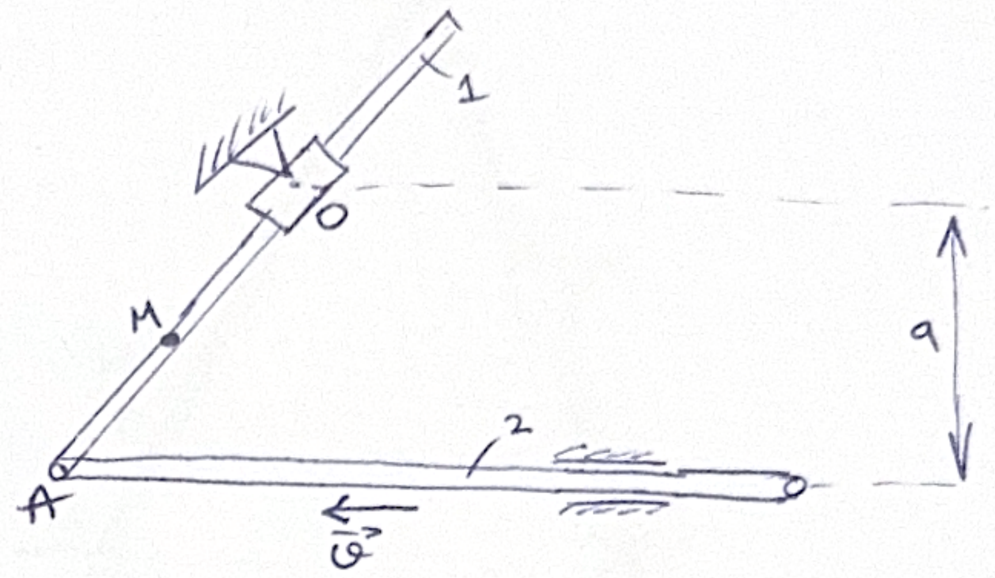
(6.40.)

2: $v_0 = ak = const.$

kuglica: m

dif. j-na rel. kretnje kuglice?

$t_0 = 0$: 1: r vertikalna



$$\omega = \frac{v \omega_1 \varphi}{\overline{OA}} = \frac{v \omega_1 \varphi}{\frac{a}{\omega_1 \varphi}} = \frac{v_0}{a} \omega_1^2 \varphi$$

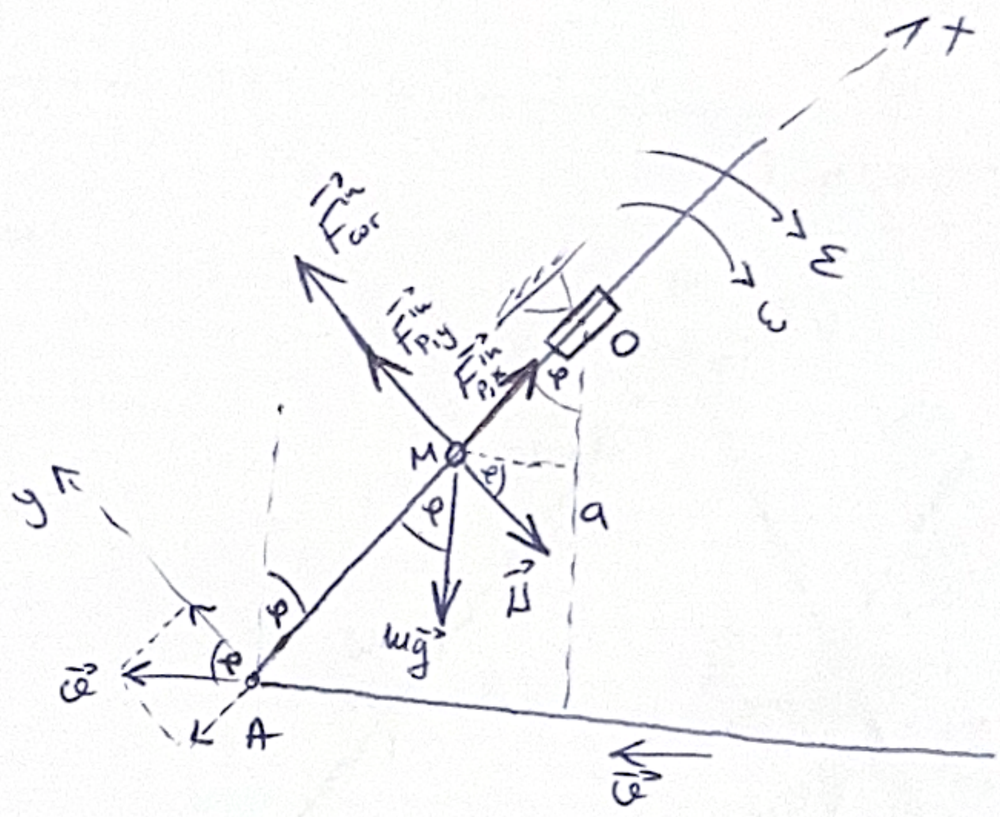
$$\frac{d\varphi}{dt} = \frac{v_0}{a} \omega_1^2 \varphi$$

$$\frac{d\varphi}{\omega_1^2 \varphi} = \frac{v_0}{a} dt \qquad //$$

$$tg\,\varphi = \frac{v_0}{a} t + \cancel{C_1}^{\to 0}$$

$$\varphi = arctg\left(\frac{v_0}{a}\right)$$

$$\omega = \frac{1}{1 + \left(\frac{v_0}{a} t\right)^2} \cdot \frac{v_0}{a}$$



$$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \cancel{\vec{N}_t}^{\to 0}}$$

$$m\vec{g} = mg \cdot (-\vec{j})$$

$$\vec{F}_p^{in} = -m\left(\cancel{\vec{a}_A}^{\to 0} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})\right) = m \cdot \varepsilon \cdot x \cdot (\vec{j}) + m\omega^2 x \cdot (\vec{i})$$

$$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$$

$$\vec{N}_n = N \cdot (-\vec{j}) = \vec{N}$$

$$\vec{N}_t = 0$$

x: $\quad m\ddot{x} = -mg\,\omega_1 \varphi + m\omega^2 x$

$$\omega_1 \varphi = \frac{1}{\sqrt{1 + tg^2 \varphi}} = \frac{1}{\sqrt{1 + \left(\frac{v_0}{a} t\right)^2}}, \qquad v_0 = ak$$

$$\ddot{x} = -g \cdot \frac{1}{\sqrt{1 + k^2 t^2}} + \frac{1}{(1 + (kt)^2)^2} k^2 x$$

$$\boxed{\ddot{x} - \frac{k^2}{(1 + k^2 t^2)^2} x + \frac{g}{\sqrt{1 + k^2 t^2}} = 0}$$

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(6.41.)

OA, AB: R

vertikalna ravan

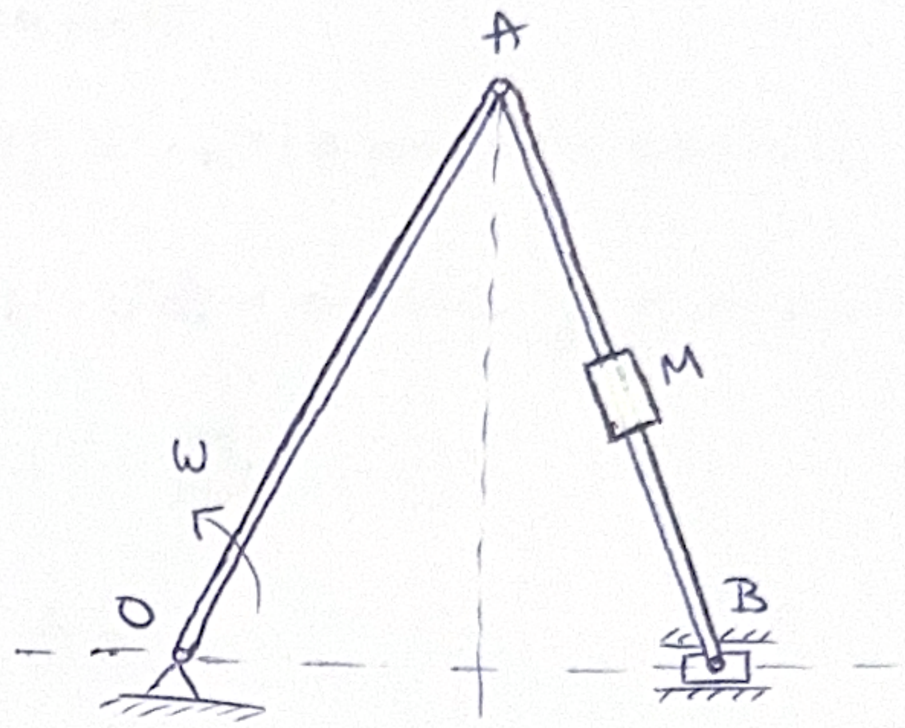
$\omega = \sqrt{\frac{3g}{R}} = const.$

M: m

to: štapovi homogeni

M relativno miruje

položaj A



Konačna j-na rel. kretanja klizača M?

$N(t) = ?$

---

oba štapa zaklapaju ugao φ sa horizontalom

=> iste su im ugaone brzine ω

$$m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6$$

$m\vec{g} = mg \cdot (-\vec{j})$

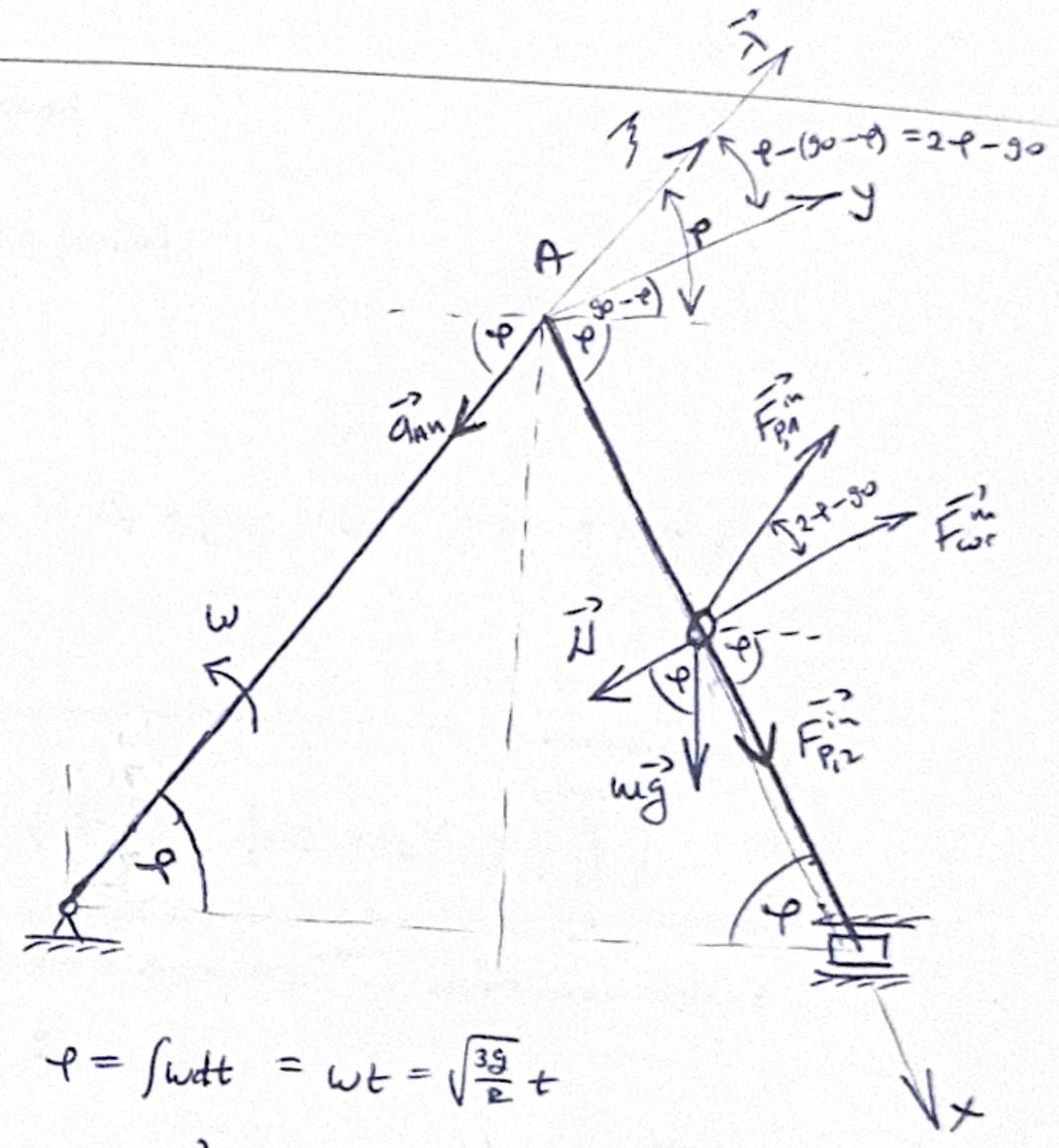
$\vec{F}_p^{in} = -m(\vec{a}_A + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) =$

$= m \cdot a_{An} \cdot (\vec{\lambda}) + m\omega^2 x \cdot (\vec{i}) = \vec{F}_{P,1}^{in} + \vec{F}_{P,2}^{in}$

$\vec{F}_{cor}^{in} = -2m\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{x} \cdot (\vec{j})$

$\vec{N}_n = N \cdot (-\vec{j}) = \vec{N}$

$\vec{N}_6 = 0$



$\varphi = \int \omega dt = \omega t = \sqrt{\frac{3g}{R}}\, t$

$a_{An} = \omega^2 \cdot R = \frac{3g}{R} \cdot R = 3g$

---

x: $m\ddot{x} = mg \sin\varphi - m a_{An} \cdot \sin(2\varphi - 90) + m\omega^2 x$ ($\sin(2\varphi - 90) = \sin 2\varphi \cos 90^\circ - \cos 2\varphi \sin 90 = -\cos 2\varphi$)

$\ddot{x} - \frac{3g}{R} x = g \sin\left(\sqrt{\frac{3g}{R}}\, t\right) + 3g \cos\left(2\sqrt{\frac{3g}{R}}\, t\right)$

$\ddot{x} - \omega^2 x = g \sin(\omega t) + 3g \cos(2\omega t)$

#: $\lambda^2 - \omega^2 = 0 \rightarrow \lambda_{1/2} = \pm\omega + 0i \rightarrow x_h = C_1 e^{\omega t} + C_2 e^{-\omega t}$

P_1: $C_1(t) = g \sin(\omega t) = e^{0t}(0 \cos(\omega t) + g \sin(\omega t))$

$x_{P1} = t^0 e^{0t}(A \cos(\omega t) + B \sin(\omega t)), \quad \ddot{x}_{P1} = -\omega^2(A \cos(\omega t) + B \sin(\omega t))$

$\cos(\omega t)(-\omega^2 A - \omega^2 A) + \sin(\omega t)(-\omega^2 B - \omega^2 B) = 0 \cos(\omega t) + g \sin(\omega t)$

$A = 0, \quad -2\omega^2 B = g \Rightarrow B = -\frac{g}{2\omega^2}$

$x_{P1} = -\frac{g}{2\omega^2} \sin(\omega t) = -\frac{g}{2\frac{3g}{R}} = -\frac{1}{6} R$

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P_2: $\quad f_2(t) = 3g\cos(2\omega t) = e^{0t}\left(3g\cos(2\omega t) + 0\sin(2\omega t)\right)$

$x_{P_2} = t^0 e^{0t}\left(A\cos(2\omega t) + B\sin(2\omega t)\right), \quad \ddot{x}_{P_2} = -4\omega^2\left(A\cos(2\omega t) + B\sin(\omega t)\right)$

$\cos(2\omega t)\left(-4\omega^2 A - \omega^2 \cdot A\right) + \sin(2\omega t)\left(-4\omega^2 B - \omega^2 B\right) = \cos(2\omega t)\cdot 3g + \sin(2\omega t)\cdot 0$

$\Rightarrow \quad B = 0 \quad , \qquad -5\omega^2 = 3g \;\rightarrow\; B = -\frac{3g}{5\omega^2}$

$x_{P_2} = -\frac{3g}{5\omega^2}\cos(2\omega t) = -\frac{3g}{5\cdot\frac{3g}{R}} = -\frac{1}{5}R$

$x = C_1 e^{\omega t} + C_2 e^{-\omega t} - \frac{1}{6}R\sin(\omega t) - \frac{1}{5}R\cos(2\omega t)$

$\dot{x} = C_1\omega e^{\omega t} - C_2\omega e^{-\omega t} - \frac{1}{6}R\omega\cos(\omega t) + \frac{2}{5}R\omega\sin(2\omega t)$

$x(t_0) = C_1 + C_2 - \frac{1}{5}R = 0 \;\rightarrow\; C_1 = \frac{1}{5}R - C_2$

$\dot{x}(t_0) = C_1\omega - C_2\omega - \frac{1}{6}R\omega = 0 \;\rightarrow\; \frac{1}{5}R - C_2 - C_2 - \frac{1}{6}R = 0 \;\rightarrow\; 2C_2 = \frac{1}{30}R$

$\rightarrow \quad C_2 = \frac{1}{60}R \quad , \qquad C_1 = \frac{11}{60}R$

$$\boxed{x = \frac{1}{60}R\left(11e^{t\sqrt{\frac{3g}{R}}} + e^{-t\sqrt{\frac{3g}{R}}}\right) - \frac{1}{6}R\sin\left(t\sqrt{\frac{3g}{R}}\right) - \frac{1}{5}R\cos\left(2t\sqrt{\frac{3g}{R}}\right)}$$

y: $\quad 0 = -mg\cos\varphi + m\cdot a_{An}\cos(2\varphi - 90) + 2m\omega\cdot\dot{x} - N$

$\cos(2\varphi - 90) = \cos 2\varphi \cos 90^{\to 0} + \sin 2\varphi \sin 90 = \sin 2\varphi$

$N = -mg\cos\left(t\sqrt{\frac{3g}{R}}\right) + 3mg\sin\left(2t\sqrt{\frac{3g}{R}}\right) + 2m\sqrt{\frac{3g}{R}}\left(\frac{1}{60}R\cdot\sqrt{\frac{3g}{R}}\left(11e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}}\right) - \right.$

$\left. - \frac{1}{6}R\sqrt{\frac{3g}{R}}\cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{2}{5}R\sqrt{\frac{3g}{R}}\cdot\sin\left(2t\sqrt{\frac{3g}{R}}\right)\right)$

$N = -mg\cos\left(t\sqrt{\frac{3g}{R}}\right) + 3mg\sin\left(2t\sqrt{\frac{3g}{R}}\right) + \frac{1}{10}mg\left(11e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}}\right) -$

$- mg\cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{12}{5}mg\sin\left(2t\sqrt{\frac{3g}{R}}\right)$

$$\boxed{N = \frac{27}{5}mg\sin\left(2t\sqrt{\frac{3g}{R}}\right) - 2mg\cos\left(t\sqrt{\frac{3g}{R}}\right) + \frac{1}{10}mg\left(11e^{t\sqrt{\frac{3g}{R}}} - e^{-t\sqrt{\frac{3g}{R}}}\right)}$$

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6.42.

vertikalna ravan

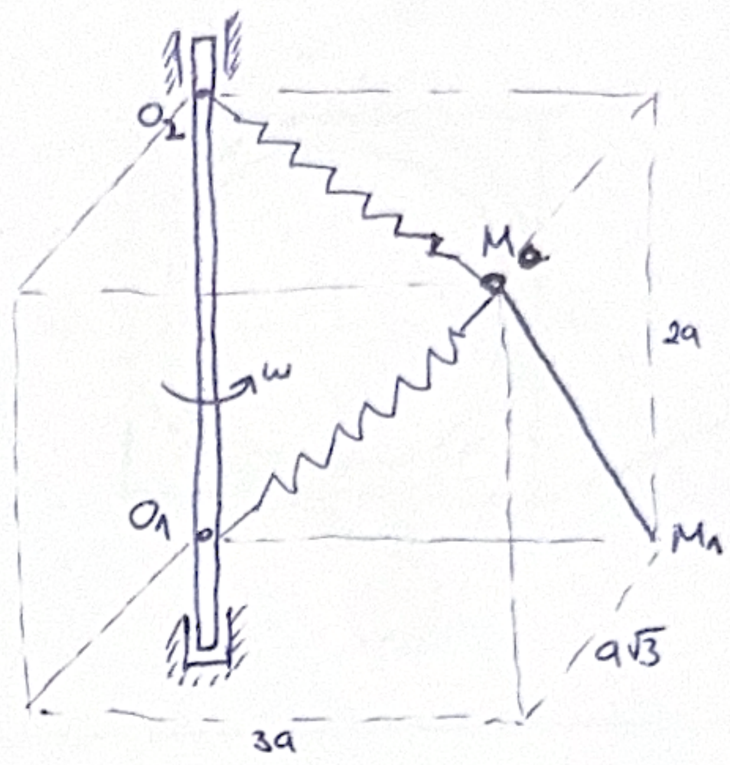
$\omega = \text{const.}$

kliza\v{c} mase m

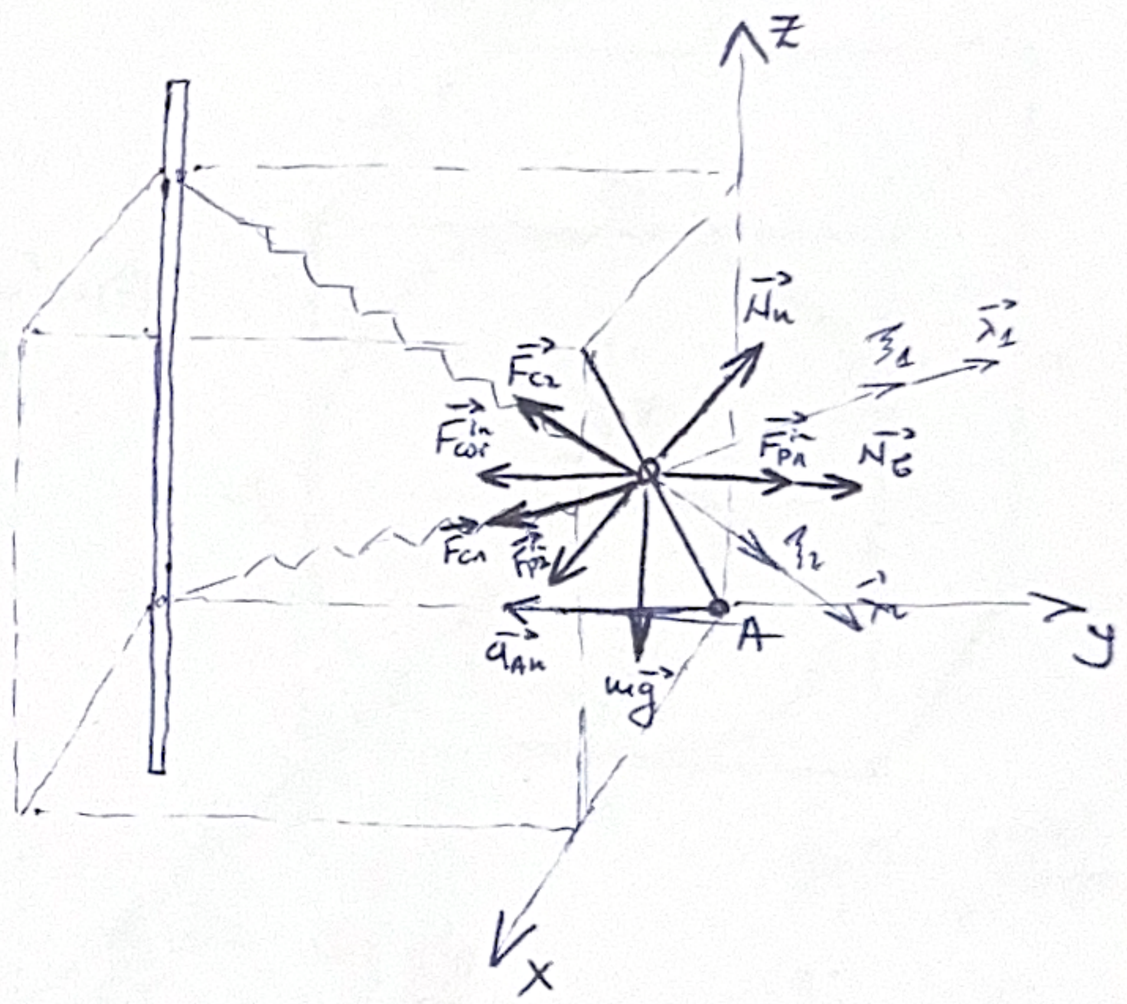
opruge: c, $l_0 = 3a$

$\omega = ?$ - da bi kliza\v{c} stigao u polo\v{z}aj M_1

to: kliza\v{c} u M_0, $v_{r0} = 0$



$$m\vec{a}_r = m\vec{g} + \vec{F}_{c1} + \vec{F}_{c2} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_G$$



$$m\vec{g} = +mg\cdot(-\vec{k})$$

$$\vec{F}_{c1} = c\cdot\Delta l_1\cdot(-\vec{\lambda}_1) = c\vartheta_1\cdot(-\vec{\lambda}_1)$$

$$\vec{F}_{c2} = c\cdot\Delta l_2\cdot(-\vec{\lambda}_2) = c\cdot\vartheta_2\cdot(-\vec{\lambda}_2)$$

$$\vec{F}_p^{in} = -m(\vec{a}_A + \cancel{\vec{\varepsilon}\times\vec{\vartheta}}^{\,0} + \vec{\omega}\times(\vec{\omega}\times\vec{\vartheta})) =$$

$$= m\cdot a_{An}\cdot(-\vec{j}) + m\omega^2 x\cdot(\vec{i}) = \vec{F}_{P1}^{in} + \vec{F}_{P2}^{in}$$

$$\vec{F}_{cor}^{in} = -2m\,\vec{\omega}\times\vec{v}_r = +2m\omega\cdot\dot{x}\cdot(-\vec{j})$$

$$\vec{N}_n = N_n\cdot(-\vec{i})$$

$$\vec{N}_G = N_G\cdot(\vec{j})$$

$$T_{M_1} - T_{M_0} = A_{M_0M_1}$$

$$A_{M_0M_1} = \int_0^{2a} -mg\,dz + \int_{\vartheta_{10}}^{\vartheta_{11}} -c\vartheta_1\,d\vartheta_1 + \int_{\vartheta_{20}}^{\vartheta_{21}} -c\vartheta_2\,d\vartheta_2 + \int_0^{a\sqrt{3}} m\omega^2 x\,dx$$

$\vartheta_{10} = 3a - 3a = 0$, $\vartheta_{11} = \sqrt{(3a)^2 + (a\sqrt{3})^2 + (2a)^2} - 3a = a$,

$\vartheta_{20} = \sqrt{(3a)^2 + (2a)^2} - 3a = a(\sqrt{13} - 3)$, $\vartheta_{21} = \sqrt{(3a)^2 + (a\sqrt{3})^2} - 3a =$

$= a(\sqrt{12} - 3)$

$$A_{M_0M_1} = -2mga + \frac{1}{2}c(\vartheta_{10}^2 - \vartheta_{11}^2) + \frac{1}{2}c(\vartheta_{20}^2 - \vartheta_{21}^2) + \frac{1}{2}m\omega^2\cdot 3a^2 =$$

$$A_{M_0M_1} = -2mga + \frac{1}{2}c(0 - a^2) + \frac{1}{2}c(a^2(13 - 6\sqrt{13} + 9) - a^2(12 - 6\sqrt{12} + 9)) + \frac{3}{2}m\omega^2 a =$$

$$A_{M_0M_1} = -2mga - \cancel{\frac{1}{2}ca^2} + \frac{1}{2}ca^2(\cancel{1} - 6\sqrt{13} + 6\sqrt{12}) + \frac{3}{2}m\omega^2 a = -2mga - 3ca^2(\sqrt{13} - \sqrt{12}) + \frac{3}{2}m\omega^2 a$$

$$0 - 0 = -2mga - 3ca^2(\sqrt{13} - \sqrt{12}) + \frac{3}{2}m\omega^2 a$$

$$\frac{3}{2}m\omega^2 a = 2mg + 3ca(\sqrt{13} - \sqrt{12})$$

$$\boxed{\omega = \sqrt{\frac{\frac{4}{3}mg + 2ca(\sqrt{13} - \sqrt{12})}{ma}}}$$

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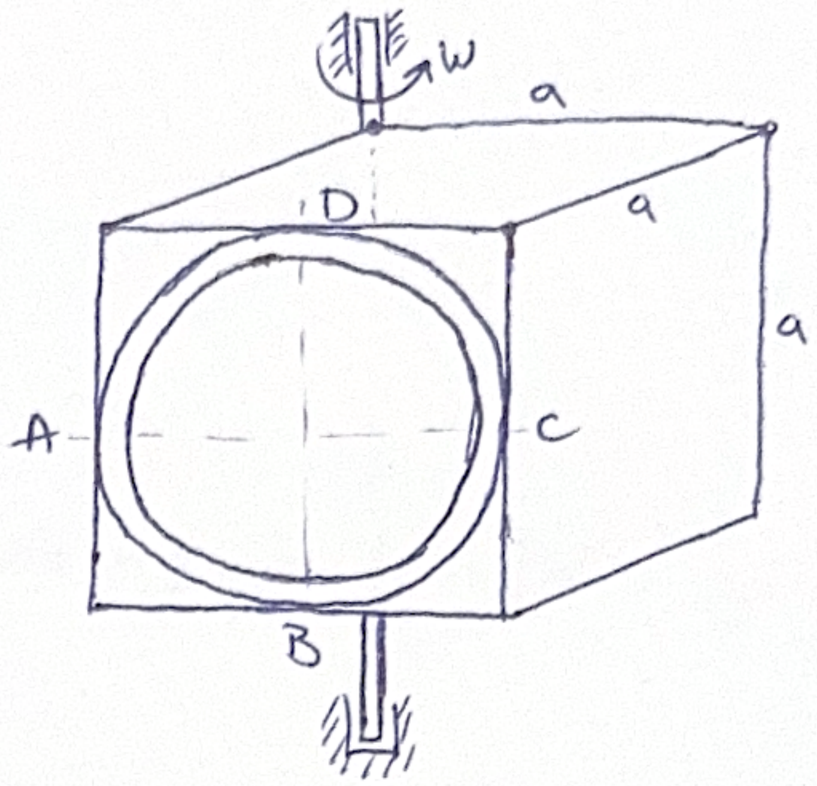
(6.43) kuglica: m

$\omega = \text{const.}$

to: kuglica v A, $v_{r0} = 0$

1) $\omega = ?$ - da bi kuglica stigla v D

2) $N_B = ?$, $N_C = ?$



$\underline{m\vec{a}_r = m\vec{g} + \vec{F}_p^{in} + \vec{F}_{cor}^{in} + \vec{N}_n + \vec{N}_6}$

$m\vec{g} = +mg \cdot (-\vec{k})$

$\vec{F}_p^{in} = -m(\vec{a}_{01} + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})) = \frac{1}{2} m\omega^2 a \cdot (\vec{j}) + m\omega^2 a \cdot (\vec{i}) +$

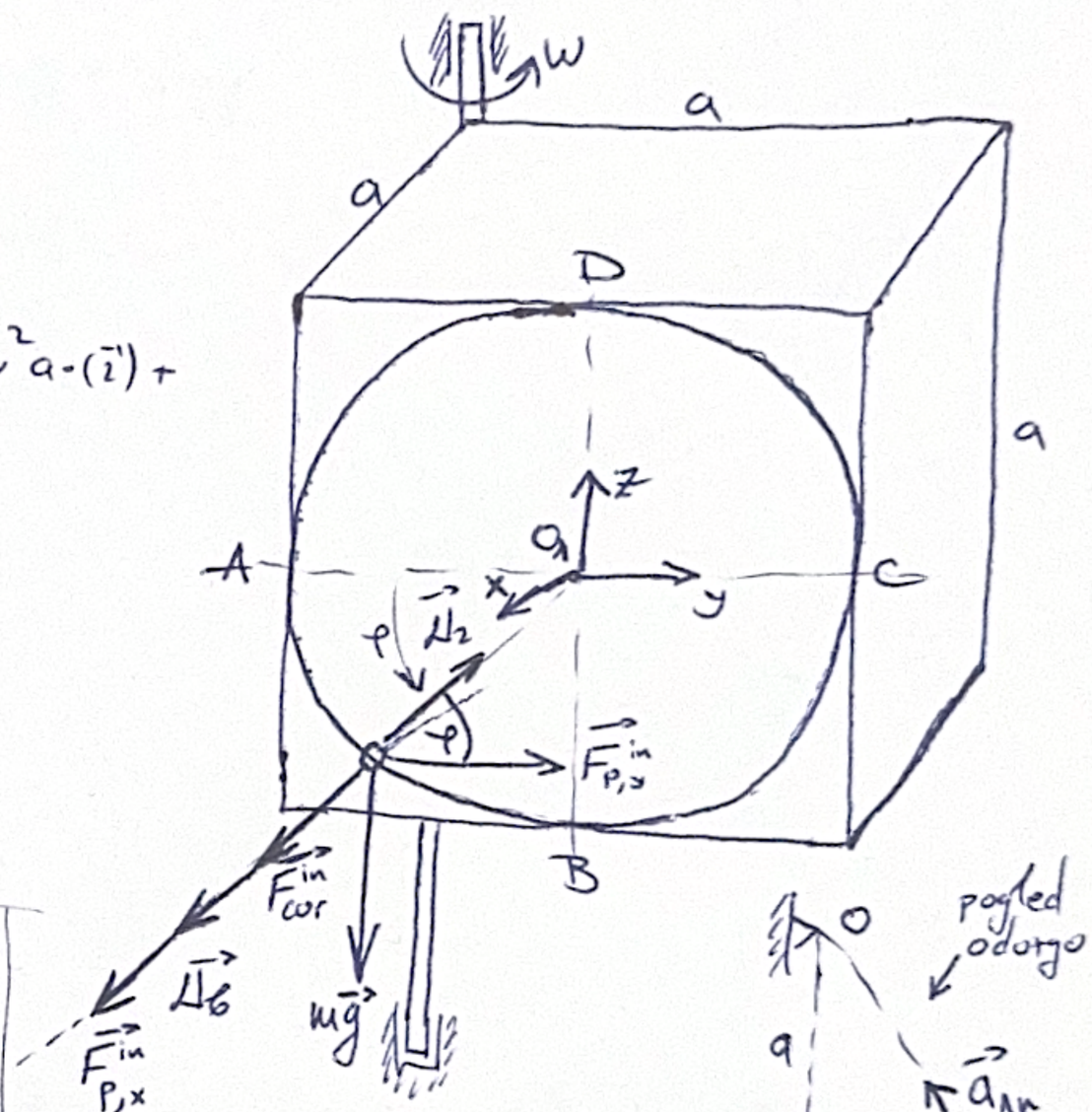
$+ m\omega^2 \cdot y \cdot \vec{j} =$

$= m\omega^2 (y + \frac{1}{2}a)\vec{j} + m\omega^2 a \cdot (\vec{i}) = \vec{F}_{P,y}^{in} + \vec{F}_{P,x}^{in}$

$\vec{F}_{cor}^{in} = -2m\,\vec{\omega} \times \vec{v}_r = 2m\omega \cdot \dot{y} \cdot (\vec{i})$

$\vec{N}_n = N_n \cdot (\vec{n})$

$\vec{N}_6 = N_6 \cdot (\vec{i})$



$\vec{a}_{1n} = -\omega^2 \frac{a}{2} \cdot \vec{j} - \omega^2 \cdot a \cdot (\vec{i})$

$\underline{T_0 - T_A = A_{AD}}$ (rel)

$A_{AD} = \int_0^{\frac{a}{2}} -mg\,dz + \int_{-\frac{a}{2}}^{0} m\omega^2 (y + \frac{1}{2}a)\,dy =$

$= -\frac{1}{2}mga + \frac{1}{2}m\omega^2 \left(0^2 - \left(-\frac{a}{2}\right)^2\right) + \frac{1}{2}m\omega^2 a \left(0 - \left(-\frac{a}{2}\right)\right) =$

$= -\frac{1}{2}mga - \frac{1}{8}m\omega^2 a^2 + \frac{1}{4}m\omega^2 a^2 = -\frac{1}{2}mga + \frac{1}{8}m\omega^2 a^2$

$0 - 0 = -\frac{1}{2}mga + \frac{1}{8}m\omega^2 a^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{4g}{a}} \quad \Rightarrow \quad \underline{\boxed{\omega = 2\sqrt{\frac{g}{a}}}}$

n: $ma_n = -mg\sin\varphi + m\omega^2 (y - \frac{1}{2}a)\cos\varphi + N_n$

$N_n = m \cdot \frac{v_{rel}^2}{R_k} + mg\sin\varphi + m\omega^2 (y - \frac{1}{2}a\cos\varphi)$

x: $0 = m\omega^2 a + 2m\omega\dot{y} + N_6$

$N_6 = -(m\omega^2 a + 2m\omega\dot{y})$

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tačka B

$T_B - T_A = A_{AB}$ (rel)

$$A_{AB} = \int_0^{-\frac{a}{2}} -mg\,dz + \int_{-\frac{a}{2}}^{0} m\omega^2\left(y+\tfrac{1}{2}a\right)dy = \tfrac{1}{2}mga - \tfrac{1}{8}m\omega^2a^2 + \tfrac{1}{4}m\omega^2 a = \tfrac{1}{2}mga + \tfrac{1}{8}m\cdot 4\tfrac{g}{a}a^2 = mga$$

$$\tfrac{1}{2}m v_{Brel}^2 - 0 = mga \quad \Rightarrow \quad v_{Brel}^2 = 2ga$$

$$N_{n,B} = m\cdot\frac{2ga}{\frac{1}{2}a} + mg\cdot 1 + m\omega^2\cancel{(0-0)}^{\to 0} = 4mg + mg = 5mg$$

$$N_{G,B} = -\left(m\cdot 4\tfrac{g}{a}a + 2m\cdot 2\sqrt{\tfrac{g}{a}}\cdot\sqrt{2ga}\right) = -4mg(1+\sqrt{2})$$

$$N_B = \sqrt{N_{n,B}^2 + N_{G,B}^2} = mg\cdot\sqrt{25+16(1+2\sqrt{2}+2)}$$

$$\boxed{N_B = mg\sqrt{73+32\sqrt{2}}}$$

tačka C

$T_C - T_A = A_{AC}$ (rel)

$$A_{AC} = \cancel{\int_0^{0} -mg\,dz}^{\to 0} + \int_{-\frac{a}{2}}^{\frac{a}{2}} m\omega^2\left(y+\tfrac{1}{2}a\right)dy = \cancel{\tfrac{1}{2}m\omega^2\left(\left(\tfrac{a}{2}\right)^2 - \left(-\tfrac{a}{2}\right)^2\right)}^{\to 0} + \tfrac{1}{2}m\omega^2 a\left(\tfrac{a}{2} - \left(-\tfrac{a}{2}\right)\right) = \tfrac{1}{2}m\cdot 4\tfrac{g}{a}\cdot a\cdot a = 2mga$$

$$\tfrac{1}{2}m v_{Crel}^2 - 0 = 2mga \quad \Rightarrow \quad v_{Crel}^2 = 4ga$$

$$N_{n,C} = m\cdot\frac{4ga}{\frac{1}{2}a} + mg\cdot 0 + m\omega^2\left(\tfrac{1}{2}a - \tfrac{1}{2}a\cdot(-1)\right) = 8mg + m\cdot 4\tfrac{g}{a}\cdot a = 12mg$$

$$N_{G,C} = -\left(m\cdot 4\tfrac{g}{a}\cdot a + 0\right) = -4mg$$

$$N_C = \sqrt{N_{n,C}^2 + N_{G,C}^2} = mg\sqrt{144+16} = mg\cdot\sqrt{160}$$

$$\boxed{N_C = 4mg\sqrt{10}}$$

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