(4.1.) $Q = 2\,N$

opruga: $F = 0{,}5\,N \rightarrow \Delta l = 1\,cm = 0{,}01\,m$

to: $v_0 = ?$

položaj ramotere

$a = 2\,cm = 0{,}02\,m$

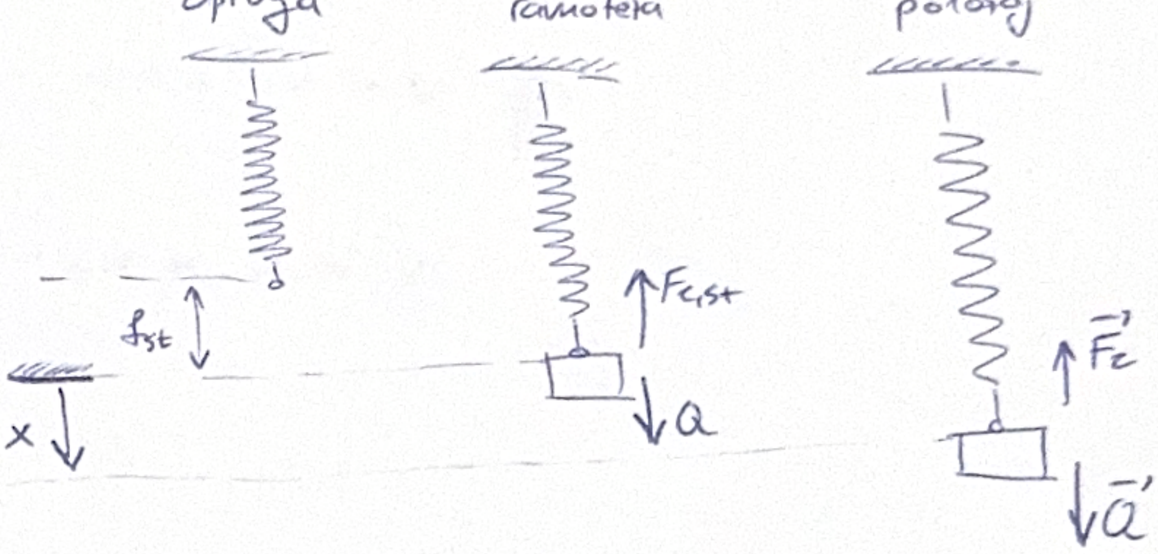
$c = \frac{F}{\Delta l} = \frac{0{,}5\,N}{0{,}01\,m} = 50\,\frac{N}{m}$

položaj statičke ramotere:

$\overleftarrow{m\vec{a} = \vec{Q} + \vec{F}_{c,st}}\rightarrow$

x: $0 = Q - F_{c,st}$

$0 = Q - c \cdot f_{st}$



proizvoljan položaj

$\overleftarrow{m\vec{a} = \vec{Q} + \vec{F}_c}\rightarrow$

x: $m\ddot{x} = Q - F_c$

$\frac{Q}{g}\ddot{x} = Q - c(f_{st} + x)$

$\frac{Q}{g}\ddot{x} = \cancel{Q} - \cancel{c f_{st}} - cx \quad / \cdot \frac{g}{Q}$

$\ddot{x} + \frac{cg}{Q}x = 0 \quad , \quad \omega^2 = \frac{cg}{Q}$

$\ddot{x} + \omega^2 x = 0 \rightarrow \lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i = 0 \pm \sqrt{\frac{cg}{Q}}\,i$

$x = C_1 \cos\left(\sqrt{\frac{cg}{Q}}\,t\right) + C_2 \sin\left(\sqrt{\frac{cg}{Q}}\,t\right)$

$\dot{x} = -C_1 \cdot \sqrt{\frac{cg}{Q}} \sin\left(\sqrt{\frac{cg}{Q}}\,t\right) + C_2 \cdot \sqrt{\frac{cg}{Q}} \cdot \cos\left(\sqrt{\frac{cg}{Q}}\,t\right)$

$x(t_0) = 0 \rightarrow C_1 \cdot 1 + 0 = 0 \rightarrow C_1 = 0$

$\dot{x}(t_0) = 0 + C_2 \cdot \sqrt{\frac{cg}{Q}} \cdot 1 = v_0 \quad , \quad C_2 = ?$

$a^2 = C_1^2 + C_2^2 \rightarrow C_2^2 = a^2 - C_1^2 = 0{,}0004 - 0 \rightarrow C_2 = 0{,}02$

$v_0 = 0{,}02 \cdot \sqrt{\frac{50 \cdot 9{,}81}{2}} = \sqrt{0{,}0004 \cdot \frac{50 \cdot 9{,}81}{2}} = \sqrt{0{,}0981}\ \frac{m}{s}$

$\boxed{v_0 = \sqrt{981}\ \frac{cm}{s}}$

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(4.2.) $Q = 4{,}9\ N$

$T_\omega = \frac{\pi}{4}\ s$

$F = ?$ - da bi $\Delta l = 1\,cm = 0{,}01\,m$

$F = c \cdot \Delta l$, $c = ?$

$T_\omega = \frac{2\pi}{\omega}$ → $\omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{\frac{\pi}{4}} = 8\ s^{-1}$

$\omega = \sqrt{\frac{c}{m}}$ → $\omega^2 = \frac{c}{m}$ → $c = \omega^2 \cdot m = \omega^2 \frac{Q}{g}$

$F = c \cdot \Delta l = \omega^2 \frac{Q}{g} \cdot \Delta l = 64 \cdot \frac{4{,}9}{9{,}81} \cdot 0{,}01 = 0{,}32\ N$

$\boxed{F = 0{,}32\ N}$

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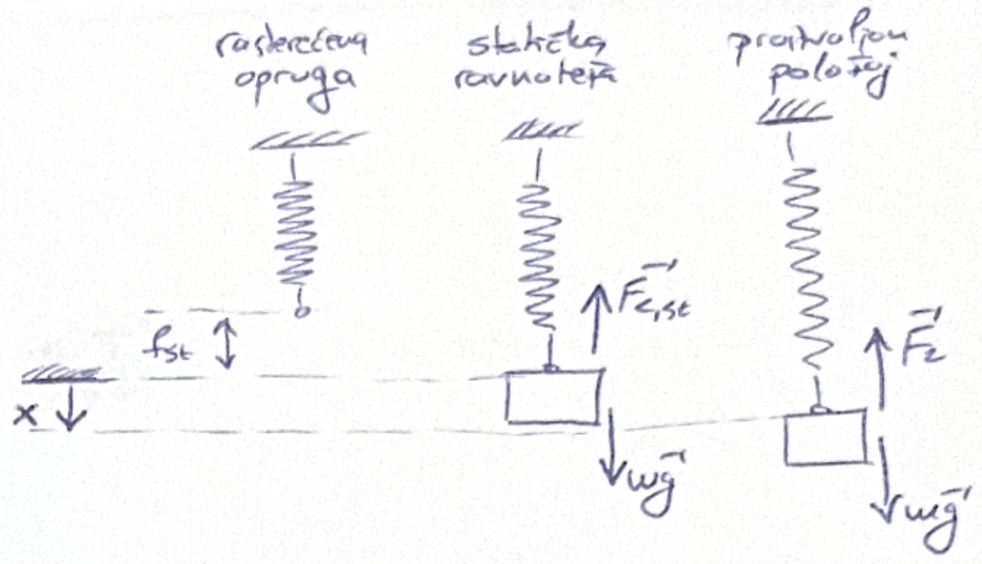
(4.3) $x = 10\sin(7t)$

$\Delta l = ?$ – za $x=0$

$(f_{st} = ?)$



$\omega = 7 \Rightarrow \omega^2 = \frac{c}{m} = 49$

statična ravnotežja

$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$

x: $0 = mg - F_{c,st}$

$0 = mg - cf_{st}$

$f_{st} = \frac{mg}{c} = g \cdot \frac{1}{\frac{c}{m}} = g \cdot \frac{1}{\omega^2} = \frac{9,81}{49} = 0,2\ m$

$f_{st} = 20\ cm$

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(4.4.)

opruga: c

$P = ?$ – tegina

period $T_\omega = T$

$$T_\omega = \frac{2\pi}{\omega} \quad \rightarrow \quad \omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{T}$$

$$\omega^2 = \frac{c}{m} \quad \rightarrow \quad m = \frac{c}{\omega^2} = \frac{c}{\frac{4\pi^2}{T^2}} = \frac{c\,T^2}{4\pi^2}$$

$$P = mg \quad \rightarrow \quad \boxed{P = \frac{cgT^2}{4\pi^2}}$$

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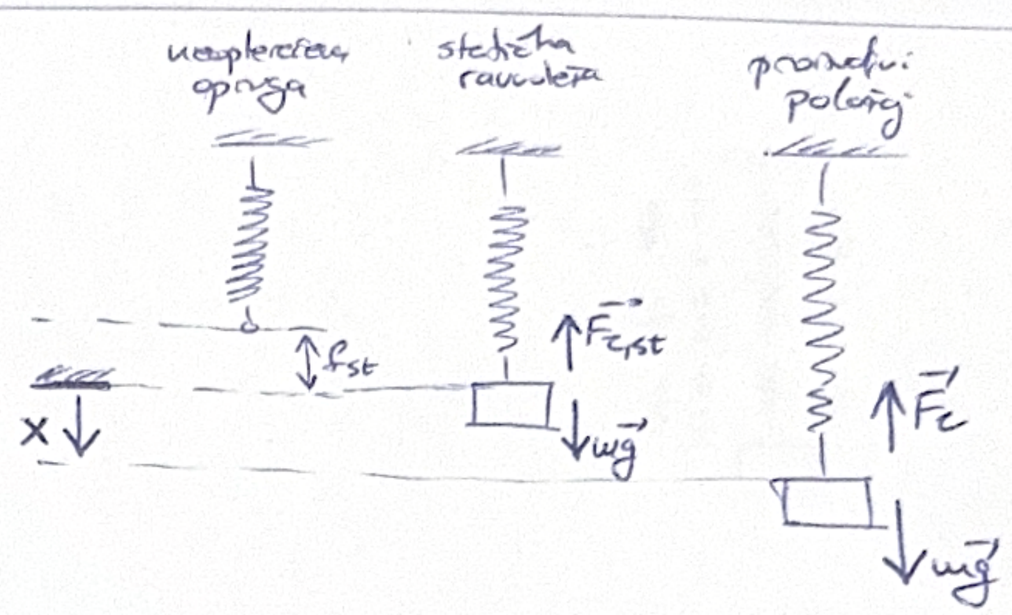
(4.5.) $f_{st} = 10\ cm = 0{,}1\ m$

to: $x_0 = 15\ cm = 0{,}15\ m$

$v_0 = 99\ \frac{cm}{s} = 99 \cdot \frac{0{,}01\ m}{s} = 0{,}99\ \frac{m}{s}$

$x(t) = ?$



<u>statička ravnoteža:</u>

$\underline{m\vec{a} = m\vec{g} + \vec{F}_{c,st}}$

x: $0 = mg - F_{c,st}$

$0 = mg - cf_{st}$

$mg = cf_{st}$

$\frac{c}{m} = \frac{g}{f_{st}} = \frac{9{,}81}{0{,}1} = 98{,}1\ s^{-2}$

$\omega^2 = 98{,}1\ s^{-2} \quad \rightarrow \quad \omega = 9{,}9\ s^{-1}$

<u>proizvoljni položaj</u>

$\underline{m\vec{a} = m\vec{g} + \vec{F}_c}$

x: $m\ddot{x} = mg - F_c$

$m\ddot{x} = mg - c(f_{st} + x)$

$m\ddot{x} = \cancel{mg} - \cancel{cf_{st}} - cx \qquad /:m$

$\ddot{x} + \frac{c}{m}x = 0$

$\ddot{x} + \omega^2 x = 0 \quad \rightarrow \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm \omega i$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = 0{,}15 \quad \rightarrow \quad C_1 = 0{,}15$

$\dot{x}(t_0) = 0 + C_2 \cdot 9{,}9 \cdot 1 = 0{,}99 \quad \rightarrow \quad C_2 = \frac{0{,}99}{9{,}9} = 0{,}1$

$\boxed{x = 0{,}15 \cos(9{,}9\,t) + 0{,}1 \sin(9{,}9\,t)}$

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(4.6.) $T_{\omega_1} = T$

oprugo izdelimo na 3 dela, jedan odbacimo, a druga dva vežemo paralelno

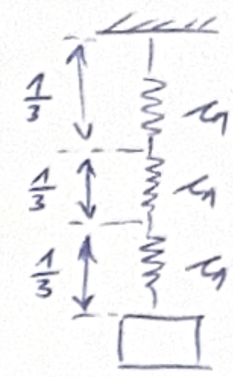
$T_{\omega_2} = ?$

---

c_1, c_2, c_3 – redna veza

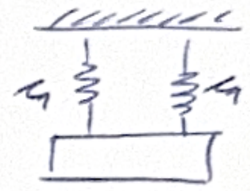
$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_1} + \frac{1}{c_1} = \frac{3}{c_1}$$

$$c_1 = \frac{3c}{1}$$



$c_2 = c_1 + c_1 = 2c_1 = 6c$

$c_2 = 6c$



$$T_{\omega_1} = T = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\frac{c}{m}}}$$

$$T_{\omega_2} = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{\frac{c_2}{m}}} = \frac{2\pi}{\sqrt{\frac{6c}{m}}} = \frac{2\pi}{\sqrt{6}\cdot\sqrt{\frac{c}{m}}} = \frac{T}{\sqrt{6}}$$

$$\boxed{T_{\omega_2} = \frac{T}{\sqrt{6}}}$$

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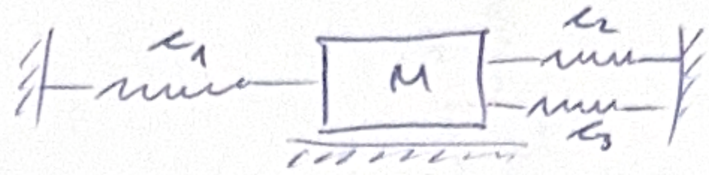
(4.7.)

M: m

c_1, c_2, c_3

$T_\omega = ?$



$c_e = c_1 + c_2 + c_3$

$$T_\omega = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{c_e}{m}}} = 2\pi \cdot \sqrt{\frac{m}{c_e}}$$

$$\boxed{T_\omega = 2\pi \cdot \sqrt{\frac{m}{c_1+c_2+c_3}}}$$

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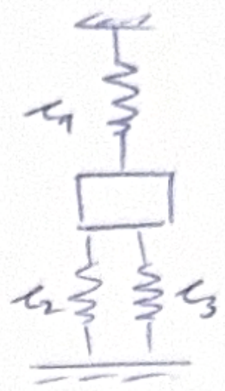
(4.8.) $m = 1\ kg$



$c_1 = 2\frac{N}{cm} = 2\frac{N}{0{,}01\,m} = 200\frac{N}{m}$

$c_2 = c_3 = 7\frac{N}{cm} = 700\frac{N}{m}$

$\omega = ?$

$c_e = c_1 + c_2 + c_3 = 1600\frac{N}{m}$

$\omega = \sqrt{\frac{c}{m}} = \sqrt{\frac{1600}{1}}$

$\boxed{\omega = 40\ s^{-1}}$

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(4.11.) $m = 16\ kg$

$c = 4\frac{N}{cm} = 4\frac{N}{0{,}01\,m} = 400\frac{N}{m}$

t_0: $x_0 = 8\ cm = 0{,}08\ m$

$v_0 = ?$

$a = 10\ cm = 0{,}1\ m$

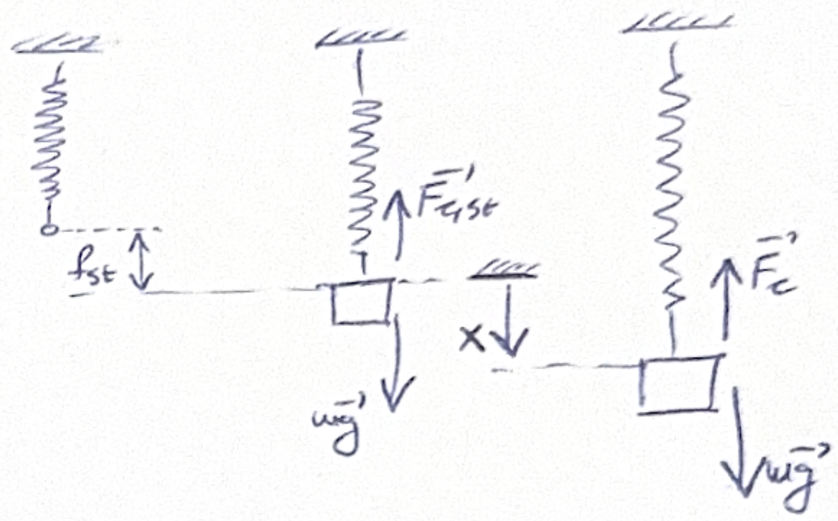
---

statična ravnotežja

$\leftarrow m\vec{a} = m\vec{g} + \vec{F}_{c,st} \rightarrow$

x: $0 = mg - F_{c,st}$

$0 = mg - cf_{st}$



prosto nihanje telesa

$\leftarrow m\vec{a} = m\vec{g} + \vec{F}_c \rightarrow$

x: $m\ddot{x} = mg - F_c$

$m\ddot{x} = mg - c(f_{st} + x)$

$m\ddot{x} = \cancel{mg} - \cancel{cf_{st}} - cx \quad /:m$

$\ddot{x} + \frac{c}{m}x = 0 \quad , \quad \omega^2 = \frac{c}{m} = \frac{400}{16} = 25 \quad , \quad \omega = 5$

$\ddot{x} + \omega^2 x = 0 \quad \rightarrow \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1/2} = 0 \pm \omega i$

$x = C_1 \cos\omega t + C_2 \sin\omega t$

$\dot{x} = -C_1\omega \sin\omega t + C_2\omega\cos\omega t$

$x(t_0) = C_1 \cdot 1 + 0 = 0{,}08\ m \quad \rightarrow \quad C_1 = 0{,}08$

$\dot{x}(t_0) = 0 + C_2\omega \cdot 1 = v_0 \quad \rightarrow \quad C_2 = \frac{v_0}{\omega} = \frac{v_0}{5}$

$a^2 = C_1^2 + C_2^2 \quad \rightarrow \quad 0{,}01 = 0{,}064 + \frac{v_0^2}{25}$

$v_0^2 = (0{,}01 - 0{,}064) \cdot 25 = 0{,}09$

$v_0 = 0{,}3\ \frac{m}{s}$

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(4.12)

$m = 1\,kg$

vertikalne osalacje

$z = 2\sin(kt+1)\ \ [cm] = 0{,}02\sin(kt+1)\ \ [m]$

$v_{max} = 4\,\frac{cm}{s} = 4 \cdot \frac{0{,}01\,m}{s} = 0{,}04\,\frac{m}{s}$

$c = ?$

---

možemo ovo da ratgijemo:

$z = 0{,}02\sin(kt+1) = 0{,}02\sin(kt)\cos(1) + 0{,}02\cos(kt)\sin(1)$

u svakom slučaju, $\underline{\underline{\omega = k}}$

$\omega^2 = \frac{c}{m} \quad \Rightarrow \quad c = \omega^2 \cdot m = k^2 \cdot m \quad , \quad k = ?$

$\dot{z} = 0{,}02\,k\cos(kt+1)$

$\dot{z}_{max} = 0{,}02\,k \cdot 1 = 0{,}04\,\frac{m}{s} \quad \rightarrow \quad k = \frac{0{,}04}{0{,}02} = 2$

$\boxed{c = 4 \cdot 1 = 4\,\frac{4}{m}}$

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(4.13) $T_\omega = \frac{\pi}{2}\ s$

$\alpha = ?$ - fazna razlika

t_0: $x_0 = 1\,cm = 0{,}01\,m$

$\dot{x}_0 = 4\,\frac{cm}{s} = 0{,}04\,\frac{m}{s}$

$T_\omega = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T_\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4\ s^{-1}$

$x = C_1 \cos(\omega t) + C_2 \sin(\omega t)$

$\dot{x} = -C_1 \omega \sin(\omega t) + C_2 \omega \cos(\omega t)$

$x(t_0) = C_1 \cdot 1 + 0 = 0{,}01 \rightarrow C_1 = 0{,}01$

$\dot{x}(t_0) = 0 + C_2 \cdot \omega \cdot 1 = 0{,}04 \rightarrow C_2 = \frac{0{,}04}{\omega} = \frac{0{,}04}{4} = 0{,}01$

$\alpha = \operatorname{arctg} \frac{C_2}{C_1} = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\boxed{\alpha = \frac{\pi}{4}}$

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(4.14.) $z = A \sin(kt)$

Vertikalno oscilovanje

$f_{st} = ?$

Statična ravnotežna

$$\underbrace{m\vec{a} = m\vec{g} + \vec{F}_{c,st}}$$

z: $\quad 0 = mg - cf_{st}$

$$f_{st} = \frac{mg}{c} = \frac{g}{\frac{c}{m}} = \frac{g}{\omega^2}, \qquad \omega = k$$

$$\boxed{f_{st} = \frac{g}{k^2}}$$

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(4.15.) $P,\ h,\ S$

c

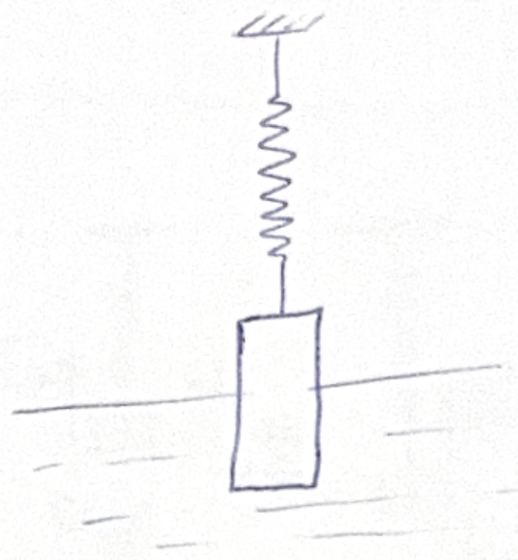
položaj ravnotežje: težišče je na nivoju površine tekočsti

μ – specifična teža tekočsti

t_0: potopljen do $\frac{3}{4}h$

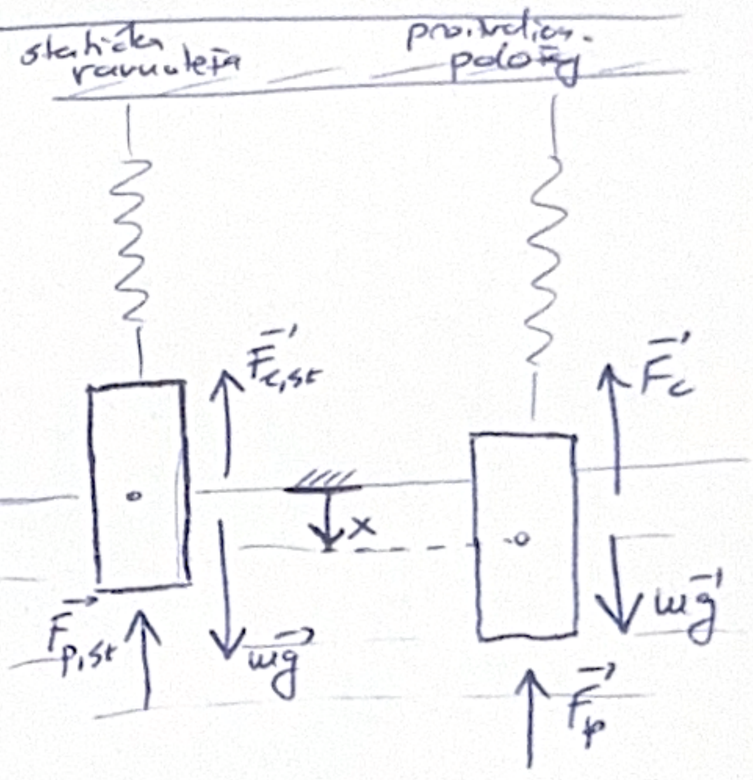
brez poč. hitrosti

$x(t) = ?$



sila potiska je gonilna težilni izstisnjene tekočsti:

$F_p = Q_t = \rho \cdot V$, V – prostornina dela telesa ki je potopljen



statična ravnotežja:

$$m\vec{a}' = m\vec{g}' + \vec{F}'_{c,st} + \vec{F}'_{p,st}$$

x: $0 = mg - cf_{st} - \rho \cdot S \cdot \left(\frac{h}{2}\right)$

prostoljna položaj

$$m\vec{a}' = m\vec{g}' + \vec{F}'_c + \vec{F}'_p$$

x: $m\ddot{x} = mg - c(f_{st} + x) - \rho \cdot S \cdot \left(\frac{h}{2} + x\right) \quad / : m$

$$\ddot{x} + \frac{c}{m}x + \frac{\rho S}{m}x = 0 \quad , \quad \omega^2 = \frac{c + \rho S}{m}$$

$$\ddot{x} + \omega^2 x = 0 \quad \rightarrow \quad \lambda^2 + \omega^2 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm \omega i$$

$$x = C_1 \cos \omega t + C_2 \sin \omega t$$

$$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$$

$$x(t_0) = C_1 + 0 = \frac{1}{4}h \quad \rightarrow \quad C_1 = \frac{1}{4}h$$

$$\dot{x}(t_0) = 0 + C_2 = 0 \quad \rightarrow \quad C_2 = 0$$

$$x = \frac{1}{4}h \cos\left(\sqrt{\frac{c + \rho S}{m}}\, t\right)$$

$$\boxed{x = \frac{1}{4}h \cos\left(\sqrt{\frac{(c + \rho S)g}{P}}\, t\right)}$$

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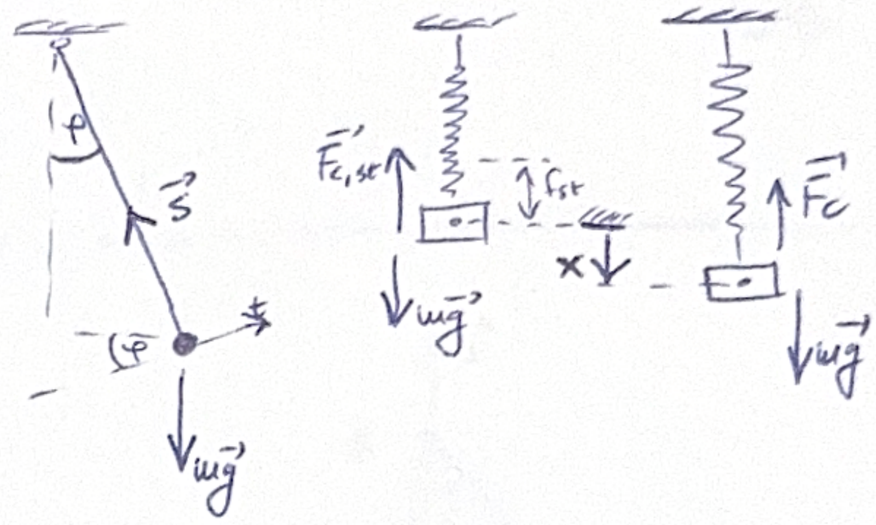
(4.17.) $e = ?$ - da bi osaloga je matematičkog klatna imale isti period oscilovanja kao vertikalne osaloge istog tereta obešenog o oprugu c

m, c, male osaloge



Klatno:

$\leftarrow m\vec{a} = m\vec{g} + \vec{S} \rightarrow$

t: $ma_t = -mg\sin\varphi$

$ml\ddot{\varphi} = -mg\varphi$

$\ddot{\varphi} + \frac{g}{l}\varphi = 0$, $\omega_1^2 = \frac{g}{l}$

$T_{\omega_1} = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}}$

Vertikalne oscilacije

1. ravn:

$\leftarrow m\vec{a}' = m\vec{g}' + \vec{F}_{c,st} \rightarrow$

x: $0 = mg - cf_{st}$

proizvoljni položaj:

$\leftarrow m\vec{a}' = m\vec{g}' + \vec{F}_c' \rightarrow$

x: $m\ddot{x} = mg - c(f_{st} + x)$

$\ddot{x} + \frac{c}{m}x = 0$, $\omega^2 = \sqrt{\frac{c}{m}}$

$T_{\omega_2} = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{\frac{c}{m}}} = 2\pi\sqrt{\frac{m}{c}}$

$T_{\omega_1} = T_{\omega_2} \Rightarrow 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{m}{c}} \quad /^2 \Rightarrow \frac{l}{g} = \frac{m}{c}$

$$\boxed{l = \frac{mg}{c}}$$

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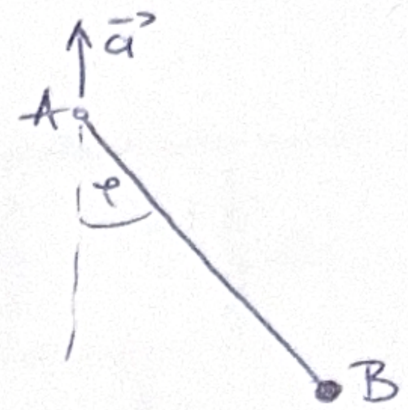
4.18.

e - dužina klatna

a - ubrzanje tačke o koju je obešeno klatno

$T_\omega = ?$

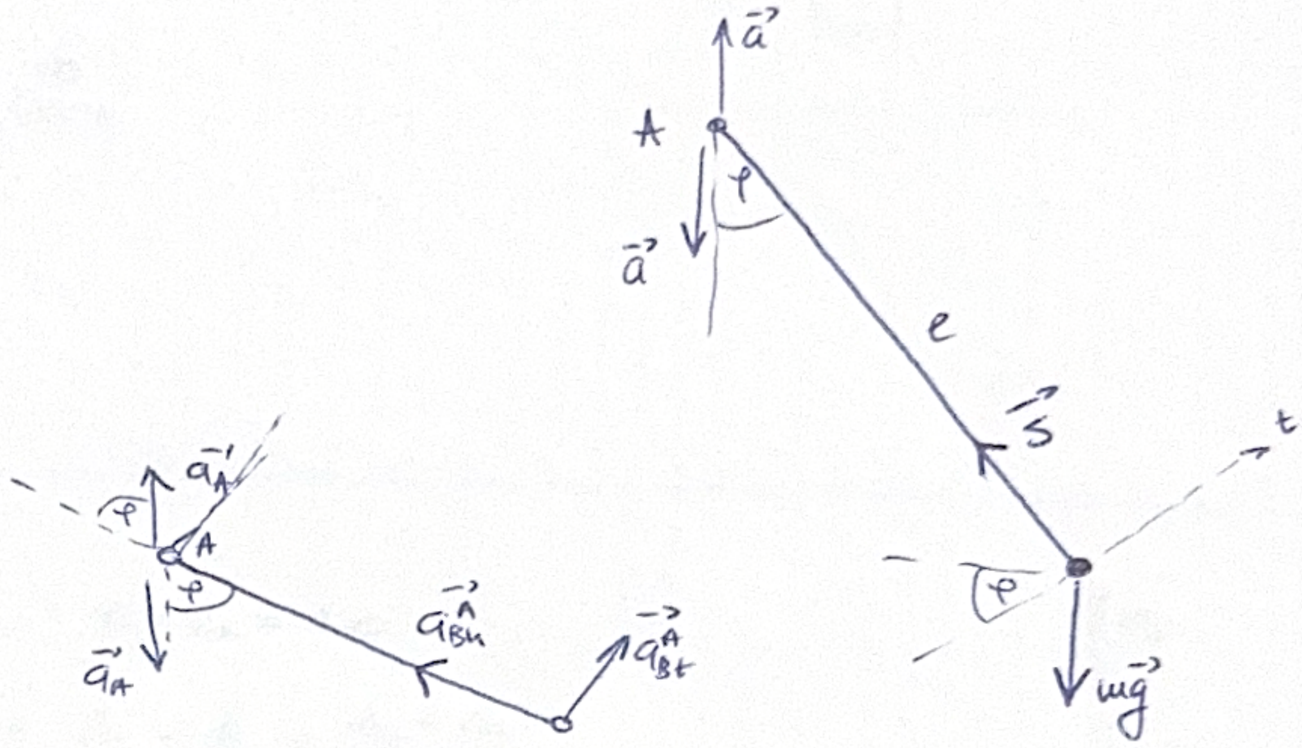
male oscilacije



a) ubrzanje a je naviše

b) ubrzanje a je naniže $(a<g)$

$m\vec{a} = m\vec{g} + \vec{S}$



t: $ma_t = -mg\sin\varphi$

oprez: $a_t \neq e\cdot\ddot{\varphi}$!

$\vec{a}_B = \vec{a}_A + \vec{a}_{Bn}^A + \vec{a}_{Bt}^A$

a) t: $a_{Bt} = a_A\sin\varphi + 0 + a_{Bt}^A$

$a_t = a\cdot\varphi + e\ddot{\varphi}$

$ma\varphi + me\ddot{\varphi} = -mg\varphi \quad /:e$

$\ddot{\varphi} + \left(\frac{a}{e} + \frac{g}{e}\right)\varphi = 0 \quad , \quad \omega^2 = \frac{a+g}{e}$

$T_{\omega_1} = \frac{2\pi}{\omega} \rightarrow \quad \boxed{T_{\omega_1} = 2\pi\sqrt{\frac{e}{a+g}}}$

b) t: $a_{Bt} = -a_A\sin\varphi + a_{Bt}^A$

$a_t = -a\varphi + e\ddot{\varphi}$

$-ma\varphi + me\ddot{\varphi} = -mg\varphi \quad /:e$

$\ddot{\varphi} + \left(\frac{g}{e} - \frac{a}{e}\right)\varphi = 0 \quad , \quad \omega^2 = \frac{g-a}{e}$

$T_{\omega_2} = \frac{2\pi}{\omega} \rightarrow \quad \boxed{T_{\omega_2} = 2\pi\sqrt{\frac{e}{g-a}}}$

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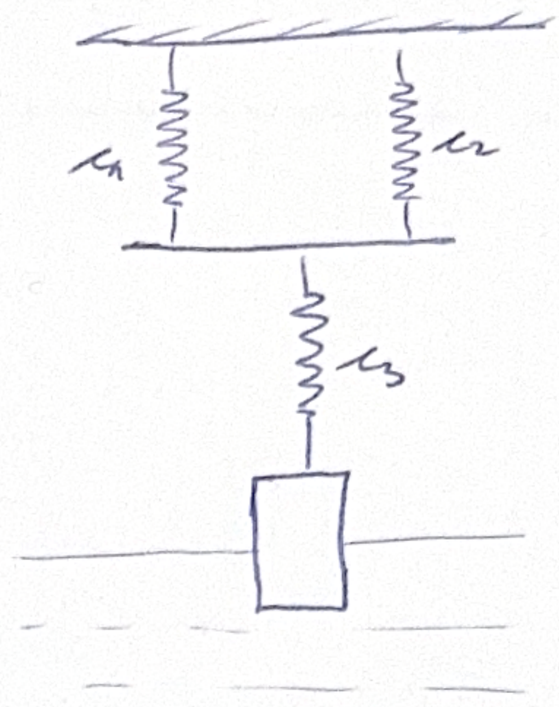
(4.19.)

prav krožni valjak

G_1 – teža

r – polmer osnove

h – višina



$c_1 = c_2 = c$, $c_3 = 2c$

težnost specifične teže μ

ravnotežni položaj: valjak potopljen do polovice svoje teže

to: potopljen $\frac{2}{3}h$, $v_0 = 0$

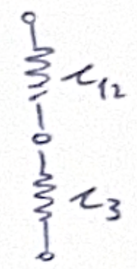
$x = ?$

---

1 ; 2: paralelna veza → $c_{e12} = c_1 + c_2 = 2c$

12 ; 3: redna veza → $\frac{1}{c_e} = \frac{1}{c_{12}} + \frac{1}{c_3} = \frac{c_3 + c_{12}}{c_{12} \cdot c_3}$

↳ $c_e = \frac{c_{12} \cdot c_3}{c_{12} + c_3} = \frac{2c \cdot 2c}{2c + 2c} = c$



statična ravnotežna

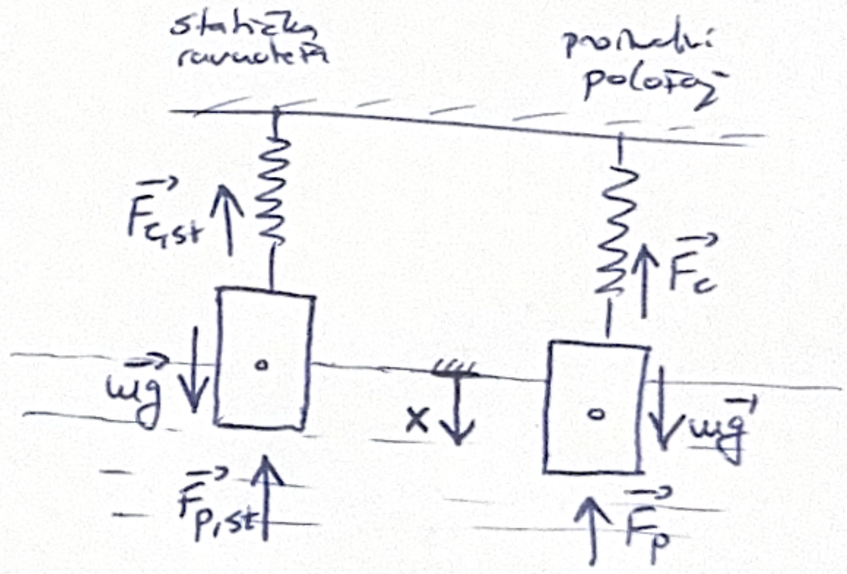
(kapomena: $F_p = Q_t = \mu \cdot V_p$)

$m\vec{a} = m\vec{g} + \vec{F}_{c,st} + \vec{F}_{p,st}$

x: $0 = mg - cf_{st} - \mu(r^2\pi)\frac{h}{2}$

prostorski položaj

$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_p$



x: $m\ddot{x} = mg - c(f_{st} + x) - \mu(r^2\pi)(\frac{h}{2} + x)$ /:m

$\ddot{x} + \frac{c}{m}x + \frac{\mu r^2 \pi}{m}x = 0$

$\ddot{x} + \omega^2 x = 0$ → $\lambda^2 + \omega^2 = 0$ → $\lambda_{1/2} = 0 \pm \omega i$

$\omega^2 = \frac{c + \mu r^2 \pi}{m} = \frac{(c + \mu r^2 \pi) g}{G}$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = (\frac{2}{3}h - \frac{1}{2}h) = \frac{1}{6}h$ → $C_1 = \frac{1}{6}h$

$\dot{x}(t_0) = 0 + C_2 \cdot 1 = 0$ → $C_1 = 0$

$$x = \frac{1}{6}h \cos\left(\sqrt{\frac{(c + \mu r^2 \pi) g}{G}}\, t\right)$$

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(4.20.) nastavak na prethodni zadatak

$v_0 = ?$ - da ć: potpuno izroniо iz vode

$x(t_0) = 0$

$\dot{x}(t_0) = v_0 = ?$

$a = \frac{h}{2}$

$x = C_1 \cos \omega t + C_2 \sin \omega t$

$\dot{x} = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$

$x(t_0) = C_1 \cdot 1 + 0 = 0 \quad \rightarrow \quad C_1 = 0$

$\dot{x}(t_0) = 0 + C_2 \omega \cdot 1 = v_0 \quad \rightarrow \quad C_2 = \frac{v_0}{\omega}$

$a^2 = C_1^2 + C_2^2$

$\frac{h^2}{4} = \frac{v_0^2}{\omega^2}$

$v_0 = \frac{h \cdot \omega}{2}$

$$v_0 \geq \frac{h}{2} \sqrt{\frac{(c + \mu r^2 \pi) g}{G}}$$

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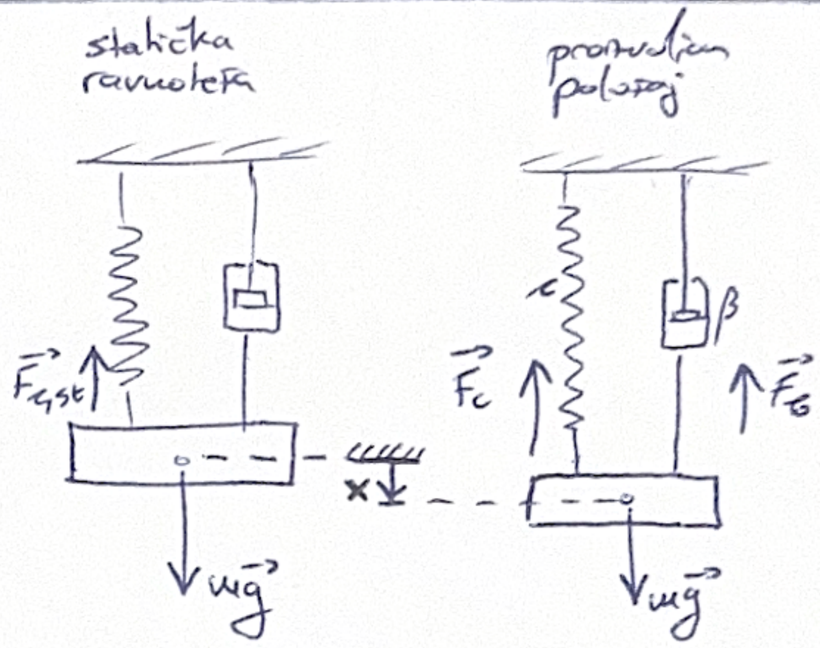
(4.21) telo mase m
opruga kratkosti c
vertikalne oscilacije
otpor β
to: x_0 - vertikalno navpre
v_0 - (verovatno navpre?)

$x(t) = ?$



statična ravnotežja

$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$

x: $0 = mg - cl_{st}$

poljubni položaj

$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_\beta$

x: $m\ddot{x} = mg - c(l_{st} + x) - \beta\dot{x}$ /:m

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 0$, $\omega^2 = \frac{c}{m}$, $2\delta = \frac{\beta}{m}$

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = 0$

$\lambda^2 + 2\delta\lambda + \omega^2 = 0$ ->

$\lambda_{1/2} = \frac{-2\delta \pm \sqrt{4\delta^2 - 4\omega^2}}{2} = -\delta \pm \sqrt{\delta^2 - \omega^2}$

1) $\delta < \omega$ -> $-p^2 = \delta^2 - \omega^2$

$\lambda_{1/2} = -\delta \pm pi$

$x = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$

$\dot{x} = -\delta e^{-\delta t}(C_1 \cos pt + C_2 \sin pt) + e^{-\delta t} \cdot p \cdot (-C_1 \sin pt + C_2 \cos pt)$

$x(t_0) = 1 \cdot (C_1 + 0) = x_0$ -> $C_1 = x_0$

$\dot{x}(t_0) = -\delta(x_0 + 0) + p(0 + C_2)$ -> $C_2 = \frac{v_0 + \delta x_0}{p}$

$$x = e^{-\delta t}\left(x_0 \cos pt + \frac{v_0 + \delta x_0}{p} \sin pt\right)$$

2) $\delta = \omega$ -> $\lambda_{1/2} = -\delta$ (dvostruko rešenje)

$x = e^{-\delta t}(C_1 t + C_2)$

$\dot{x} = -\delta e^{-\delta t}(C_1 t + C_2) + C_1 e^{-\delta t}$

$x(t_0) = 1(0 + C_2) = x_0$ -> $C_2 = x_0$

$\dot{x}(t_0) = -\delta(0 + x_0) + C_1 = v_0$ -> $C_1 = v_0 + x_0\delta$

$$x = e^{-\delta t}((v_0 + x_0\delta)t + x_0)$$

3) $\delta > \omega$ -> $\nu^2 = \delta^2 - \omega^2$

$x = e^{-\delta t}(C_1 \cosh \nu t + C_2 \sinh \nu t)$

$\dot{x} = -\delta e^{-\delta t}(C_1 \cosh \nu t + C_2 \sinh \nu t) + e^{-\delta t} \cdot \nu (C_1 \sinh \nu t + C_2 \cosh \nu t)$

$x(t_0) = 1(C_1 + 0) = x_0$ -> $C_1 = x_0$

$\dot{x}(t_0) = -\delta(x_0 + 0) + \nu(0 + C_2) = v_0$ -> $C_2 = \frac{v_0 + x_0\delta}{\nu}$

$$x = e^{-\delta t}\left(x_0 \cosh \nu t + \frac{v_0 + x_0\delta}{\nu} \sinh \nu t\right)$$

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(4.22) prigušene oscilacije imaju period oscilovanja 25 % veći od odgovarajućih slobodnih

nađi $\frac{\delta}{\omega}$?

$$T_\omega = \frac{2\pi}{\omega}$$

$$T_p = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\omega^2-\delta^2}}$$

$$\frac{T_p}{T_\omega} = 1{,}25 = \frac{5}{4}$$

$$\frac{\frac{2\pi}{\sqrt{\omega^2-\delta^2}}}{\frac{2\pi}{\omega}} = \frac{5}{4} \qquad /^2$$

$$\frac{\omega^2}{\omega^2-\delta^2} = \frac{25}{16}$$

$$25\omega^2 - 25\delta^2 = 16\omega^2$$

$$9\omega^2 = 25\delta^2$$

$$\frac{\omega^2}{\delta^2} = \frac{9}{25} \qquad /\sqrt{\ }$$

$$\boxed{\frac{\omega}{\delta} = \frac{3}{5}}$$

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(4.23.)

prigušene oscilacije

$D = e^{0,75\pi}$ - dekrement

naći $\frac{T_p}{T_\omega}$ odgovarajućih slobodnih oscilacija bez prigušenja

---

$$D = e^{\delta \frac{T_p}{2}} = e^{0,75\pi} \quad \Rightarrow \quad \delta \cdot \frac{T_p}{2} = 0,75\,\pi \quad \Rightarrow \quad \delta \cdot \frac{2\pi}{2p} = 0,75\,\pi$$

$$\delta \cdot \frac{1}{\sqrt{\omega^2 - \delta^2}} = \frac{3}{4} \quad \Rightarrow \quad \frac{\delta^2}{\omega^2 - \delta^2} = \frac{9}{16} \quad \Rightarrow \quad 16\delta^2 = 9\omega^2 - 9\delta^2$$

$$25\delta^2 = 9\omega^2 \quad \Rightarrow \quad \frac{\omega^2}{\delta^2} = \frac{25}{9} \quad \Rightarrow \quad \frac{\omega}{\delta} = \frac{5}{3}$$

$$\frac{T_p}{T_\omega} = \frac{\frac{2\pi}{\sqrt{\omega^2 - \delta^2}}}{\frac{2\pi}{\omega}} \quad /^2$$

$$\frac{T_p^2}{T_\omega^2} = \frac{\omega^2}{\omega^2 - \delta^2} = \frac{1}{\frac{\omega^2}{\omega^2} - \frac{\delta^2}{\omega^2}} = \frac{1}{1 - \left(\frac{3}{5}\right)^2} = \frac{1}{1 - \frac{9}{25}} = \frac{1}{\frac{16}{25}} = \frac{25}{16}$$

$$\boxed{\frac{T_p}{T_\omega} = \frac{5}{4}}$$

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(4.24.) $Q = 1\,N$

Prigušenje osalaje: $T_p = 0,102\ s$

$$\beta = \frac{0,02\,N}{1\,\frac{cm}{s}} = \frac{0,02\,N}{0,01\,\frac{m}{s}} = 2\,\frac{Ns}{m}$$

$$\frac{R_1}{R_5} = ?$$

$$D = \frac{R_n}{R_{n+1}} = e^{\delta \frac{T_p}{2}}$$

$$2\delta = f_m \Rightarrow \delta = \frac{\beta}{2m} = \frac{\beta g}{2Q}$$

$$D = e^{\frac{\beta g}{2Q} \cdot \frac{T_p}{2}} = e^{0,5}$$

$$R_2 = \frac{R_1}{D}$$

$$R_3 = \frac{R_2}{D} = \frac{R_1}{D^2}$$

$$R_4 = \frac{R_3}{D} = \frac{R_1}{D^3}$$

$$R_5 = \frac{R_4}{D} = \frac{R_1}{D^4}$$

$$\frac{R_1}{R_5} = \frac{R_1}{\frac{R_1}{D^4}} = D^4 = \left(e^{0,5}\right)^4 = e^2$$

$$\boxed{\frac{R_1}{R_5} = e^2}$$

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4.25. $\frac{R_1}{R_7} = 10$

$\mathbb{D} = ?$

$$R_7 = \frac{R_1}{D^6}$$

$$\frac{R_1}{R_7} = \frac{R_1}{\frac{R_1}{D^6}} = 10 \quad \rightarrow \quad D^6 = 10 \quad \rightarrow \quad D = (10)^{\frac{1}{6}}$$

$$\mathbb{D} = \ln D = \ln (10)^{1/6} = \frac{1}{6} \ln 10$$

$$\boxed{\mathbb{D} = \frac{1}{6} \ln 10}$$

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4.26. $m = 1\ kg$

$T_p = 0{,}25\ s$

$\mathbb{D} = 0{,}5$

$F_p = ?$ - za $v = 1\frac{cm}{s} = 0{,}01\frac{m}{s}$

$F_p = \beta \cdot \dot{x} = \beta \cdot 0{,}01$, $\beta = ?$

$2\delta = \frac{\beta}{m}$ $\rightarrow$ $\beta = 2\delta \cdot m$, $\delta = ?$

$\mathbb{D} = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \cdot \frac{T_p}{2} = 0{,}5$ $\rightarrow$ $\delta = \frac{2 \cdot 0{,}5}{T_p} = \frac{1}{0{,}25} = 4$

$\beta = 2 \cdot 4 \cdot 1 = 8$

$F_p = 8 \cdot 0{,}01 = 0{,}08\ N$

$F_p = 0{,}08\ N$

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(4.27.) $T_p = \frac{\pi}{2}$ s

$D = \frac{\pi}{10}$

$x(t) = ?$

to: $x = 3$ cm $= 0{,}03$ m

$v_0 = 0$

$x = e^{-\delta t}(C_1 \cos(pt) + C_2 \sin(pt))$, $\delta = ?$, $p = ?$

$\dot{x} = -\delta e^{-\delta t}(C_1 \cos pt + C_2 \sin pt) + p e^{-\delta t}(-C_1 \sin pt + C_2 \cos pt)$

$D = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \frac{T_p}{2}$ $\rightarrow$ $\delta = \frac{2 \cdot D}{T_p} = \frac{2 \cdot \frac{\pi}{10}}{\frac{\pi}{2}} = \frac{4}{10} = \frac{2}{5}$

$T_p = \frac{2\pi}{p}$ $\rightarrow$ $p = \frac{2\pi}{T_p} = \frac{2\pi}{\frac{\pi}{2}} = 4$

$x(t_0) = 1 \cdot (C_1 \cdot 1 + 0) = 0{,}03$ $\rightarrow$ $C_1 = 0{,}03$

$\dot{x}(t_0) = -\frac{2}{5} \cdot 1 (0{,}03 \cdot 1 + 0) + 4 \cdot 1 (0 + C_2 \cdot 1) = 0$

$\rightarrow -\frac{2}{5} \cdot \frac{3}{100} + 4 C_2 = 0$ $\rightarrow$ $4 C_2 = \frac{3}{250}$ $\rightarrow$ $C_2 = \frac{3}{1000} = 0{,}003$

$$x(t) = e^{-\frac{2}{5}t}(0{,}03 \cos 4t + 0{,}003 \sin 4t)$$

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(4.28.) $\frac{T_p}{T_\omega} = ?$

$\mathbb{D} = \frac{5\pi}{12}$

$$\frac{T_p}{T_\omega} = \frac{\frac{2\pi}{p}}{\frac{2\pi}{\omega}} = \frac{\omega}{p} = \frac{\omega}{\sqrt{\omega^2-\delta^2}} \quad /^2$$

$$\frac{T_p^2}{T_\omega^2} = \frac{\omega^2}{\omega^2-\delta^2} = \frac{1}{1-\frac{\delta^2}{\omega^2}}, \qquad \frac{\delta}{\omega} = ?$$

$$\mathbb{D} = \ln D = \ln e^{\delta \frac{T_p}{2}} = \delta \frac{T_p}{2}$$

$$T_p = \frac{\mathbb{D}\cdot 2}{\delta} = \frac{5}{6}\frac{\pi}{\delta}$$

$$T_p = \frac{2\pi}{p} = \frac{2\pi}{\sqrt{\omega^2-\delta^2}}$$

$$\frac{2\pi}{\sqrt{\omega^2-\delta^2}} = \frac{5}{6}\frac{\pi}{\delta} \quad /^2$$

$$\frac{4}{\omega^2-\delta^2} = \frac{25}{36}\frac{1}{\delta^2}$$

$$144\delta^2 = 25\omega^2 - 25\delta^2$$

$$169\delta^2 = 25\omega^2$$

$$\frac{\delta^2}{\omega^2} = \frac{25}{169}$$

$$\frac{T_p^2}{T_\omega^2} = \frac{1}{1-\frac{\delta^2}{\omega^2}} = \frac{1}{1-\frac{25}{169}} = \frac{1}{\frac{144}{169}} = \frac{169}{144}$$

$$\boxed{\frac{T_p}{T_\omega} = \frac{13}{12}}$$

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(4.29.) $m = 0{,}3\ kg$

$x = 24 \cos 13t$ - slobodno neprigušeno

$x(t) = ?$ - slobodno prigušeno pri istim početnim uslovima

$R = 3v \left[N \frac{m}{s}\right]$ - sila otpora

$\omega = 13$

$R = \beta \cdot v \quad \rightarrow \quad \underline{\beta = 3}$

<u>Početni uslovi:</u>

$x(t_0) = 24 \cdot 1 = 24$

$\dot{x} = -24 \cdot 13 \sin(13t)$

$\dot{x}(t_0) = 0$

$x(t) = e^{-\delta t}(C_1 \cos pt + C_2 \sin pt)$

$2\delta = \frac{\beta}{m} = \frac{3}{93} = 10 \quad \rightarrow \quad \underline{\delta = 5}$

$p^2 = \omega^2 - \delta^2 = 169 - 25 = 144 \quad \rightarrow \quad \underline{p = 12}$

$x(t) = e^{-5t}(C_1 \cos 12t + C_2 \sin 12t)$

$\dot{x}(t) = -5e^{-5t}(C_1 \cos 12t + C_2 \sin 12t) + 12e^{-5t}(-C_1 \sin 12t + C_2 \cos 12t)$

$x(t_0) = 1 \cdot (C_1 + 0) = 24 \quad \rightarrow \quad C_1 = 24$

$\dot{x}(t_0) = -5 \cdot 1(24 \cdot 1 + 0) + 12 \cdot 1(0 + C_2 \cdot 1) = 0 \quad \rightarrow \quad 12C_2 = 120 \quad \rightarrow \quad C_2 = 10$

$$\boxed{x(t) = e^{-5t}(24 \cos(12t) + 10 \sin(12t))}$$

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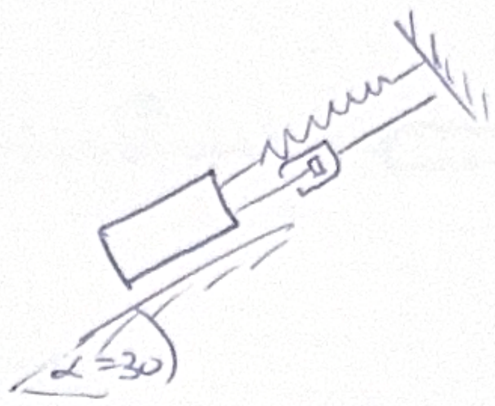
(4.30.) $m, c, \alpha = 30°, \beta$

$\beta = ?$ - da bi: $T_p = 2T_\omega$



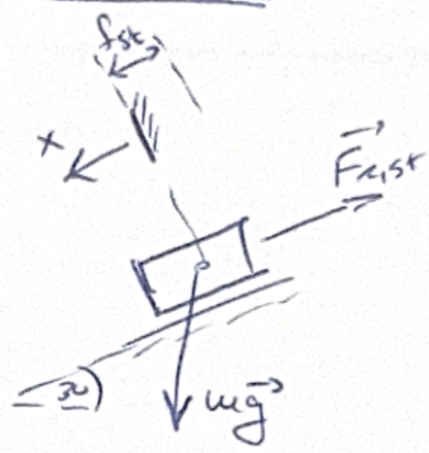
$T_\omega = \frac{2\pi}{\omega}$, $T_p = \frac{2\pi}{p}$, $p = \sqrt{\omega^2 - \delta^2}$, ali ...

da bi su su $\omega^2 = \frac{c}{m}$? ; da bi je $2\delta = \frac{\beta}{m}$? ; ... ker je pod oglom ...

Stat. ravn.:

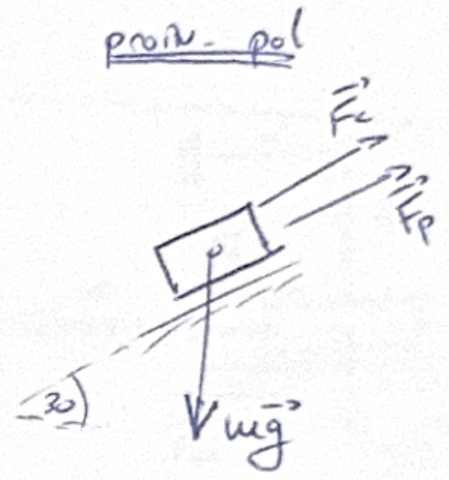
$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$

x: $0 = mg\sin 30 - cf_{st}$



Pron. pobitaj:

$m\vec{a} = m\vec{g} + \vec{F}_c + \vec{F}_{p_1}$



x: $m\ddot{x} = mg\sin 30 - c(x + f_{st}) - \beta\dot{x}$ $\quad / : m$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 0$ $\Rightarrow$ ok, tj. $2\delta = \frac{\beta}{m}$; $\omega^2 = \frac{c}{m}$

$\frac{T_p}{T_\omega} = 2 \rightarrow \frac{\frac{2\pi}{p}}{\frac{2\pi}{\omega}} = 2 \rightarrow \frac{\omega}{p} = 2$

$\frac{\omega}{\sqrt{\omega^2 - \delta^2}} = 2 \rightarrow \frac{\omega^2}{\omega^2 - \delta^2} = 4 \rightarrow 4\omega^2 - 4\delta^2 = \omega^2$

$4\delta^2 = 3\omega^2 \rightarrow 4 \cdot \left(\frac{\beta}{2m}\right)^2 = 3\frac{c}{m} \rightarrow \frac{\beta^2}{m^2} = 3\frac{c}{m}$

$\boxed{\beta = \sqrt{3cm}}$

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Á.31. $Q = 20\,N$

$c_1 = 1{,}5\frac{N}{cm} = 1{,}5\frac{N}{0{,}01\,m} = 150\frac{N}{m}$

$c_2 = 2{,}5\frac{N}{cm} = 250\frac{N}{m}$

položaj 0: neraspregnute opruge

t_0: $x_0 = 15\,cm = 0{,}15\,m$, $v_0 = 0$

t_1: telo se zaustavlo

$t_1 = ?$, $x(t_1) = ?$

$\mu_1 = 0{,}2$ – kor. trenja mirovanja

$\mu_2 = 0{,}1$ – kor. trenja pri kretanju

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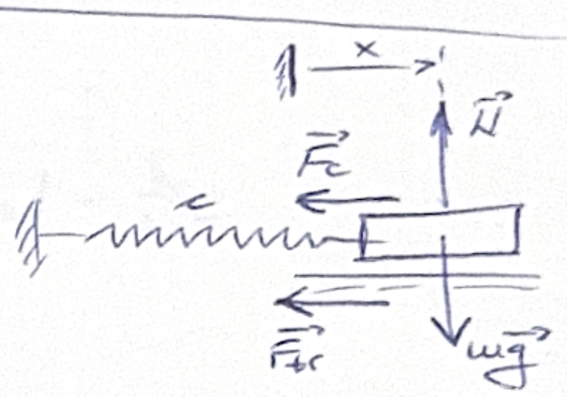
paralelne opruge: $c_e = c_1 + c_2 = 400 = c$

$f_{st} = 0$ (horizontalne oscilacije)



uslov da se telo zaustavi:
↳ da u amplitudnom položaju bude $\Sigma F_{xi} = 0$

$\Sigma F_{xi} = 0 \Rightarrow F_c - F_{tr} = c \cdot x_k - \mu_1 \cdot N = 0$, $N = mg$ (jer je $m\ddot{y} = mg - N = 0$)

$\Rightarrow x_k = \frac{\mu_1 \cdot mg}{c} = 0{,}01\,m$

zadatak: naći $x_1, x_2, \dots, x_k$! (to su amplitude, tj. koordinate pri svakom zaustavljanju)

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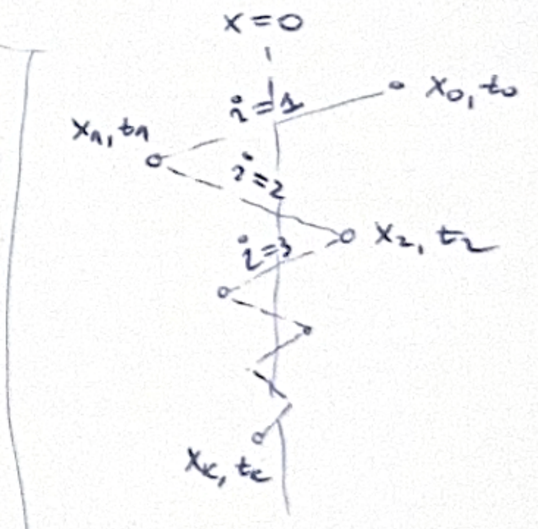
svaki prolaz od jednog do drugog amplitudnog položaja je druga dif. j-na, jer sila trenja deluje u suprotnom smeru



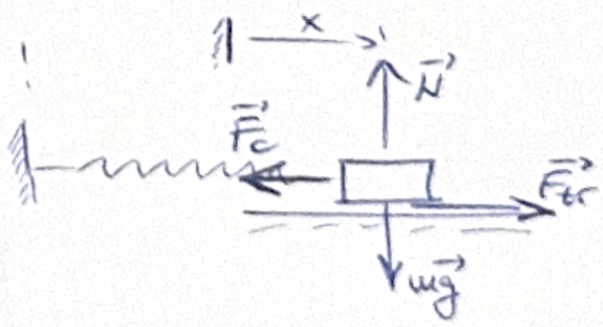
posmatrajmo i-ti prolaz, uzećemo da je $x > 0$, $\dot{x} < 0$:

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F_c} + \vec{F_{tr}}$ ← j-na koje važi za i-ti prolaz, od t_{i-1} do t_i

x: $m\ddot{x} = -cx + \mu_2 mg$ /:m

$\ddot{x} + \frac{c}{m}x = \mu_2 g$

$\ddot{x} + \omega^2 x = h$



#: $\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1,2} = 0 \pm \omega i$

$x_h = A\cos(\omega t) + B\sin(\omega t)$

za t_{i-1} → desni amplitudni položaj
$\cos(\omega t_{i-1}) = +1$
$\sin(\omega t_{i-1}) = 0$

za t_i → levi amplitudni položaj
$\cos(\omega t_i) = -1$
$\sin(\omega t_i) = 0$

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P: $\quad e(t) = \mu_2 g = e^{\sigma t}\left(\mu_2 g \cos\omega t + 0 \sin\omega t\right)$

$$x_p = s^0 e^{\sigma t}\left(C\cos\omega t + D\sin\omega t\right) = C \quad , \quad \ddot{x}_p = 0$$

$$0 + \omega^2 C = h \quad \rightarrow \quad C = \frac{h}{\omega^2}$$

$$x = A_i \cos(\omega t) + B_i \sin(\omega t) + \frac{h}{\omega^2}$$

$$\dot{x} = -\omega A_i \sin(\omega t) + \omega B_i \cos(\omega t)$$

na osnovu početnih uslova i-tog prolaza:

$$\dot{x}(t_{i-1}) = -0 + \omega_i B_i \cdot 1 = 0 \quad \rightarrow \quad \underline{B_i = 0}$$

$$x(t_{i-1}) = A_i \cdot 1 + 0 + \frac{h}{\omega^2} = |x_{i-1}| \quad \rightarrow \quad A_i = |x_{i-1}| - \frac{h}{\omega^2}$$

na osnovu krajnjih uslova i-tog prolaza:

$$x(t_i) = A_i \cdot (-1) + 0 + \frac{h}{\omega^2} = -|x_i| \quad \rightarrow \quad A_i = |x_i| + \frac{h}{\omega^2}$$

$$|x_i| + \frac{h}{\omega^2} = |x_{i-1}| - \frac{h}{\omega^2}$$

$$|x_{i-1}| = |x_i| + \frac{2h}{\omega^2}$$

$$|x_{i-2}| = |x_i| + \frac{4h}{\omega^2}$$

...

$$|x_0| = |x_i| + 2i \cdot \frac{h}{\omega^2}$$

$$|x_i| = x_0 - 2i\frac{h}{\omega^2} = 0{,}15 - (2i)\cdot\frac{\mu_2 g}{\omega^2} = 0{,}15 - 2i \cdot 0{,}005$$

$$|x_k| = 0{,}15 - 2k \cdot 0{,}005$$

$$0{,}01 \leq 0{,}15 - (2k)\cdot 0{,}005$$

$$k \leq \frac{0{,}15 - 0{,}01}{0{,}005 \cdot 2} = 14 \quad \Rightarrow \quad \underline{k = 14}$$

$$x_k = 0{,}15 - 2 \cdot 14 \cdot 0{,}005 = 0{,}01 \text{ m} \quad \Rightarrow \quad \underline{x_k = 1 \text{ cm}}$$

$$t_k = k \cdot \frac{T_\omega}{2} = 14 \cdot \frac{2\pi}{2\omega} = 14 \cdot \frac{\pi}{14} \quad \Rightarrow \quad \boxed{t_k = \pi}$$

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(4.32.)

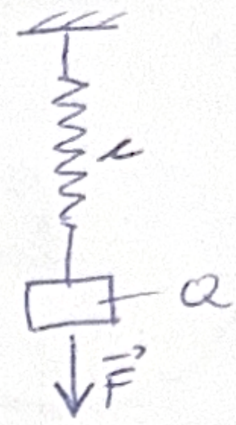
$Q = 2{,}45\ N$

$c = 1\frac{N}{cm} = 100\frac{N}{m}$

$F = 1{,}8\ sin(16t)$

j-na prinudnih osc?

vertikalne oscilacije



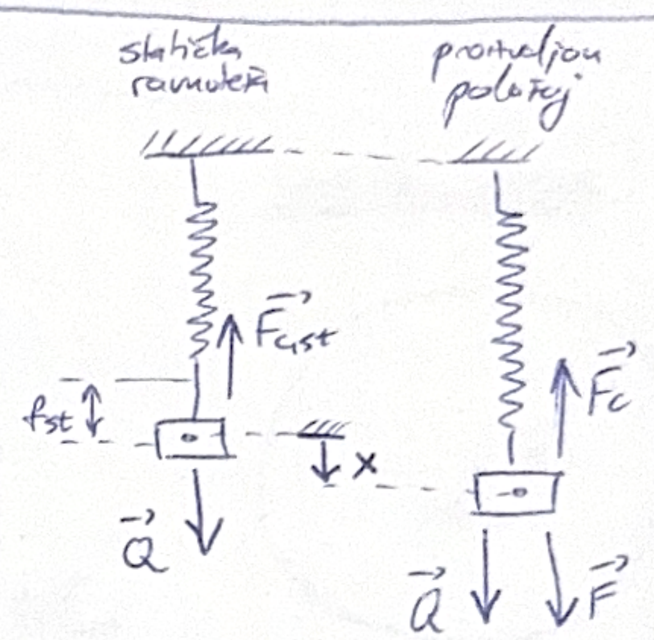
<u>statička ravnoteža:</u>

$$m\vec{a} = \vec{Q} + \vec{F}_{c,st}$$

x: $0 = Q - cf_{st}$

<u>pomaknuti položaj:</u>

$$m\vec{a} = \vec{Q} + \vec{F}_c + \vec{F}$$



x: $m\ddot{x} = Q - c(f_{st} + x) + 1{,}8\ sin(16t)$ $\quad /:m$

$\ddot{x} + \frac{c}{m}x = \frac{1{,}8}{m}sin(16t)$, $\omega^2 = \frac{c}{m} = \frac{cg}{Q} = 400$, $\omega = 20$, $h = \frac{1{,}8}{m} = \frac{1{,}8g}{Q} = 7{,}20$

$\ddot{x} + \omega^2 x = h\ sin\ 16t$

sopstvene oscilacije → homogeno rešenje (nema potrebe da ga tražimo, jer se traže prinudne oscilacije. Bitno je samo da znamo da nema dođi do rezonance (neće jer $\lambda^2 + \omega^2 = 0 \rightarrow \lambda = 0 \pm \omega i$) $(\omega = 20)$

prinudne oscilacije → partikularno rešenje

P: $f(t) = h\ sin(16t) = e^{0t}(0\ cos(16t) + h\ sin(16t))$

$x_p = t^0 e^{0t}(A\ cos(16t) + B\ sin(16t))$, $\ddot{x}_p = -256(A\ cos\ 16t + B\ sin\ 16t)$

$cos(16t)(-256A + 400A) + sin(16t)(-256B + 400B) = 0\ cos\ 16t + h\ sin\ 16t$

$A = 0$, $144B = h$ → $B = 0{,}05$

$$\boxed{x_p = 0{,}05 \cdot sin(16t)}$$

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(4.33) disk: r

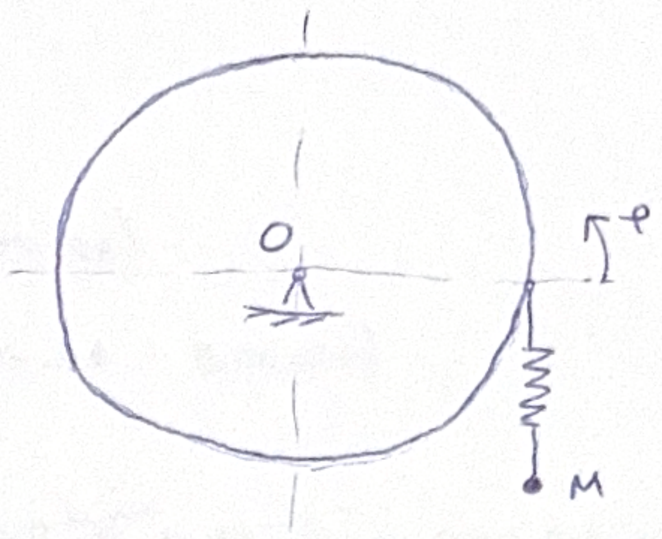
M: m

$\varphi = \varphi_0 \sin \Omega t$ – oscilacije diska

f-ua prinudnih osc?

1) $\Omega \neq \sqrt{\frac{c}{m}}$

2) $\Omega = \sqrt{\frac{c}{m}}$

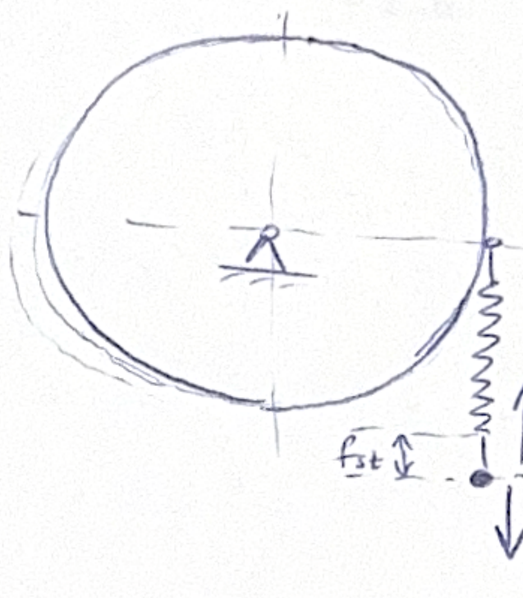


<u>slobodna raztegnjena:</u>

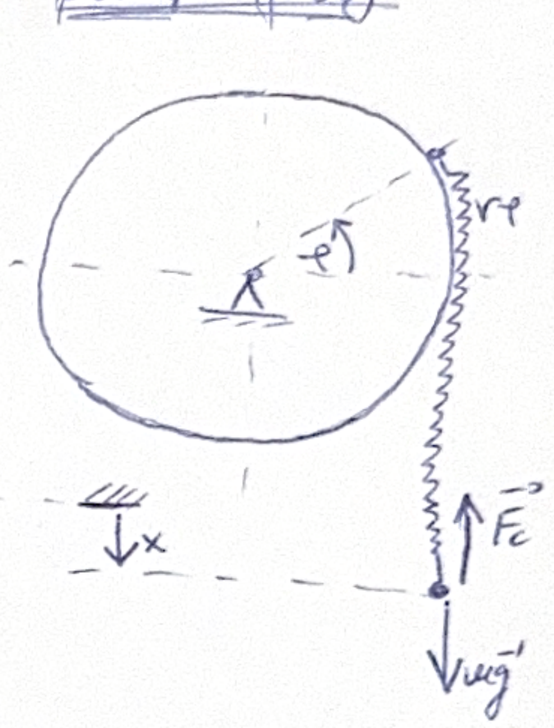
$m\vec{a} = m\vec{g} + \vec{F}_{c,st}$

x: $0 = mg - cf_{st}$

<u>slobodna raztegnjena</u>



<u>prostorski položaj</u>



<u>poljubni položaj:</u>

$m\vec{a} = m\vec{g} + \vec{F}_c$

x: $m\ddot{x} = mg - c(x + f_{st} + r\varphi)$

$m\ddot{x} = -cx - cr\varphi_0 \sin(\Omega t)$ / : m

$\ddot{x} + \frac{c}{m} = -\frac{c}{m} r\varphi_0 \sin(\Omega t)$

$\ddot{x} + \omega^2 x = h \sin(\Omega t)$, $\omega^2 = \frac{c}{m}$, $h = -\frac{c}{m} r\varphi_0$

H: $\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i$ $\left(\begin{matrix}\alpha = 0 \\ \beta = \omega\end{matrix}\right)$

P: $b(t) = h \sin(\Omega t) = e^{0t}(0 \cos \Omega t + h \sin \Omega t)$ $\left(\begin{matrix}a = 0 \\ b = \Omega\end{matrix}\right)$

1) $\omega \neq \Omega$

$x_p = t^0 e^{0t}(A \cos \Omega t + B \sin \Omega t)$, $\ddot{x}_p = -\Omega^2(A \cos \Omega t + B \sin \Omega t)$

$\cos \Omega t(-A\Omega^2 + \omega^2 A) + \sin \Omega t(-B\Omega^2 + B\omega^2) = 0 \cos \Omega t + h \sin \Omega t$

$A = 0$, $B(\omega^2 - \Omega^2) = h \rightarrow B = \frac{h}{\omega^2 - \Omega^2}$

$x_p = B \sin \Omega t = \frac{h}{\omega^2 - \Omega^2} \sin \Omega t = -\frac{cr\varphi_0}{m(\frac{c}{m} - \Omega^2)} \sin \Omega t$

$$\boxed{x_p = \frac{cr\varphi_0}{m\Omega^2 - c} \sin(\Omega t)}$$

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2) $\omega = \Omega$

$$x_p = t^1 e^{\alpha t}(A\cos\Omega t + B\sin\Omega t)$$

$$\dot{x}_p = (A\cos\Omega t + B\sin\Omega t) + t\Omega(-A\sin\Omega t + B\cos\Omega t)$$

$$\ddot{x}_p = \Omega(-A\sin\Omega t + B\cos\Omega t)\cdot 2 + t\Omega^2(-A\sin\Omega t - B\cos\Omega t)$$

$$\cos\Omega t(2\Omega B - \cancel{t\Omega^2 A + tA\omega^2}) + \sin\Omega t(-2\Omega A - \cancel{t\Omega^2 B + tB\omega^2}) = 0\cos\Omega t + h\sin\Omega t$$

$$B = 0 \quad , \quad -2\Omega A = h \quad \rightarrow \quad A = \frac{h}{-2\Omega} = \frac{cr\rho_0}{2m\Omega}$$

$$\boxed{x_p = \frac{cr\rho_0}{2m\Omega} t\cos\Omega t}$$

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(4.34) $v_x = 30 \frac{m}{s} = \text{const}$

$A = ?$ – amplituda prinudnih oscilacija

$G = 2\ kN$

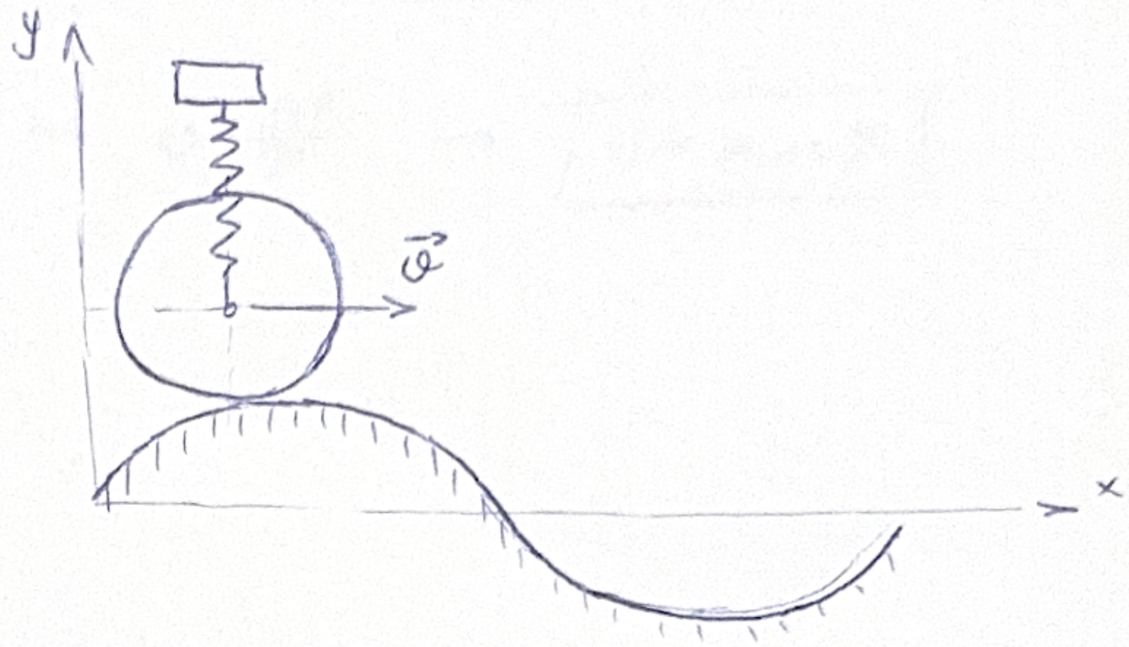
$c = 40 \frac{kN}{cm}$

$y = a \sin\left(\frac{\pi x}{e}\right)$

$a = 1{,}5\ cm = 0{,}015\ m$

$e = 2{,}5\ m$

(zanemarimo dimenzije tačkova)



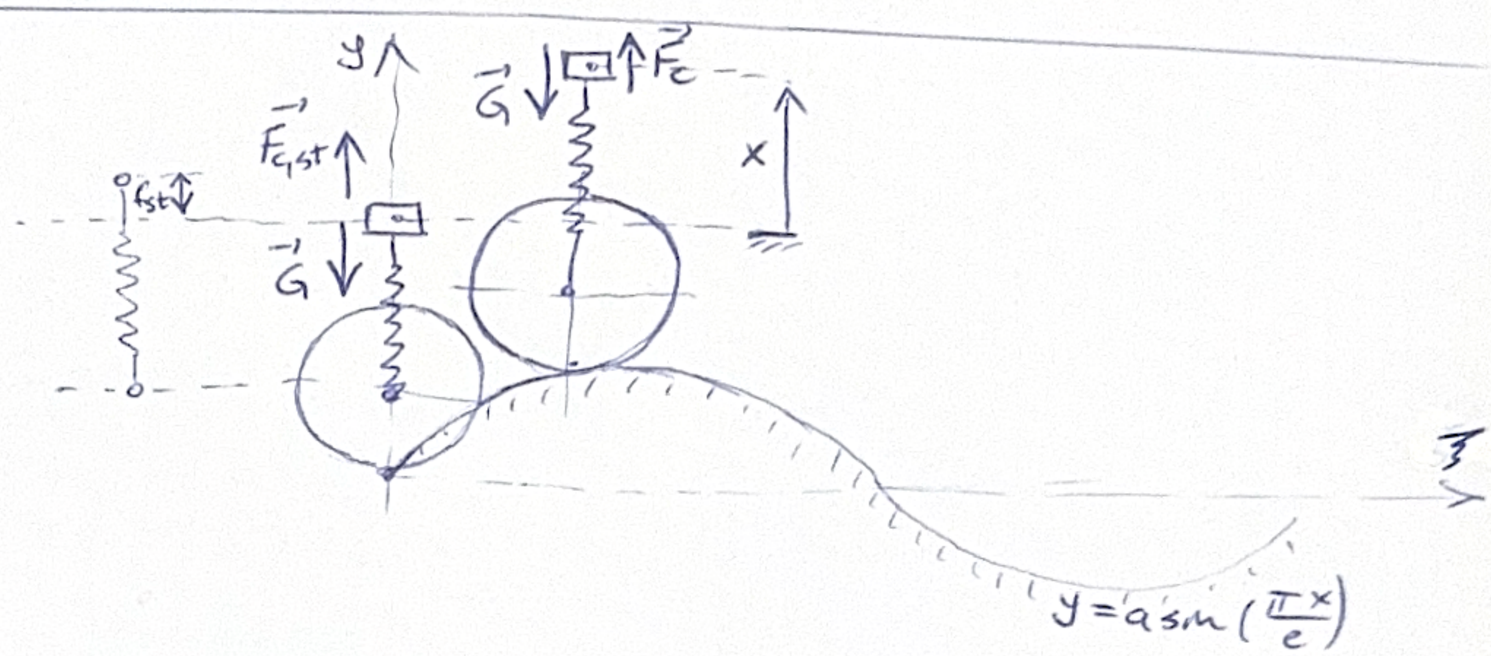
<u>st. ravn.</u>

$m\vec{a} = \vec{G} + \vec{F}_{c,st}$

x: $0 = -G + c f_{st}$

<u>pravilan položaj:</u>

$m\vec{a} = \vec{G} + \vec{F}_c$



x: $m\ddot{x} = -G + c(y + f_{st} - x)$

$m\ddot{x} + cx = cy \quad / :m$

$\ddot{x} + \frac{c}{m} x = \frac{c}{m} a \sin\left(\frac{\pi}{e}\right)$, $\omega^2 = \frac{c}{m} = \frac{cg}{G} = 196{,}2$, $\omega = 14$, $h = \frac{c}{m} \cdot a = 2{,}94$

$\ddot{x} + \omega^2 x = h \sin(\Omega t)$ $\quad z = \int v_x dt = 30\,t$, $\Omega = \frac{\pi \cdot 30}{e} = 12\pi$

#: $\lambda^2 + \omega^2 \cdot 1 = 0 \rightarrow \lambda_{1/2} = 0 \pm i\omega \quad (\alpha = 0,\ \beta = \omega = 14)$

P: $f(t) = e^{0t}(0 \cos(\Omega t) + h \sin(\Omega t)) \quad (a = 0,\ b = \Omega = 12\pi)$

$x_p = t^0 e^{0t}(A \cos \Omega t + B \sin \Omega t)$, $\ddot{x}_p = -\Omega^2(A \cos \Omega t + B \sin \Omega t)$

$\cos \Omega t(-A\Omega^2 + A\omega^2) + \sin \Omega t(-B\Omega^2 + B\omega^2) = 0 \cos \Omega t + h \sin \Omega t$

$A = 0$, $B = \frac{h}{\omega^2 - \Omega^2} \rightarrow x_p = \frac{h}{\omega^2 - \Omega^2} \sin(\Omega t)$

$x_{max} = \frac{h}{|\omega^2 - \Omega^2|} = \frac{2{,}94}{|196{,}2 - 144\pi^2|} = 0{,}0024\ m \Rightarrow \boxed{x_{max} = 0{,}241\ cm}$

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(4.35.) nastavak na prethodni zadatak

$v = ?$ - da bi: $\Omega = \omega$

$$\Omega = \frac{\pi^{\wedge}}{\ell t} = \frac{\pi v}{\ell} \qquad \left.\begin{array}{l} \\ \omega = 14 \end{array}\right\} \Rightarrow \frac{\pi v}{\ell} = 14 \Rightarrow v = \frac{14\ell}{\pi} \Rightarrow \boxed{v = 11{,}14\,\frac{m}{s}}$$

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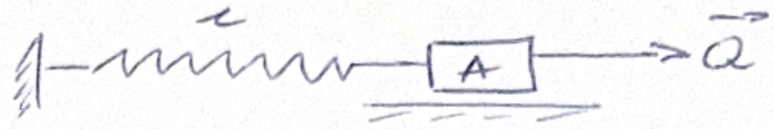
437.

$m = 1\,kg$

$c = 0{,}1\,\frac{kN}{m} = 100\,\frac{N}{m}$

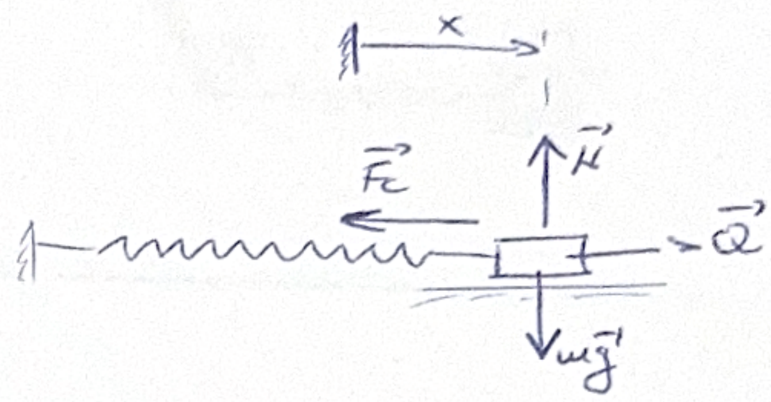
$Q = 10\sin(10t + \frac{1}{3})$ $\quad [N, s]$

$x_p = ?$



$f_{st} = 0$ (hor. oscilacije)

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_c + \vec{Q}$



x: $m\ddot{x} = -cx + Q \quad /:m$

$\ddot{x} + \frac{c}{m}x = 10\sin(10t + \frac{1}{3})$, $\omega^2 = \frac{c}{m} = 100$, $\omega = 10$, $h = 10$, $\Omega = 10$, $\varphi = \frac{1}{3}$

$\ddot{x} + \omega^2 x = h\sin(\Omega t + \varphi)$

#: $\lambda^2 + \omega^2 \cdot 1 = 0 \rightarrow \lambda_{1/2} = 0 \pm \omega i \quad \begin{pmatrix} \alpha = 0 \\ \beta = \omega = 10 \end{pmatrix}$

P: $G(t) = e^{\alpha t}(P_1\cos(\Omega t + \varphi) + P_2\sin(\Omega t + \varphi)) \quad \begin{pmatrix} \alpha = 0 \\ \beta = \Omega = 10 \end{pmatrix}$ => rezonansa!

$x_p = t^1 e^{\alpha t}(A\cos(\Omega t + \varphi) + B\sin(\Omega t + \varphi))$

$\dot{x}_p = (A\cos(\Omega t + \varphi) + B\sin(\Omega t + \varphi)) + \Omega t(-A\sin(\Omega t + \varphi) + B\cos(\Omega t + \varphi))$

$\ddot{x}_p = \Omega(-A\sin(\Omega t + \varphi) + B\cos(\Omega t + \varphi)) \cdot 2 + \Omega^2 t(-A\cos(\Omega t + \varphi) - B\sin(\Omega t + \varphi))$

$\cos(\Omega t + \varphi)(2B\Omega - \cancel{\Omega^2 Bt + B\omega^2 t}) + \sin(\Omega t + \varphi)(-2A\Omega - B\Omega^2 t + B\omega^2 t) = 0\cos(\Omega t + \varphi) + h\sin(\Omega t + \varphi)$

$B = 0$, $\quad A = \frac{h}{-2\Omega} = \frac{10}{-2 \cdot 10} = -\frac{1}{2}$

$$x_p = -0{,}5\,t\sin(10t + \frac{1}{3})$$

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(Ä.38.)

OA: $\Omega = \sqrt{\frac{c}{m}} = \text{const.}$

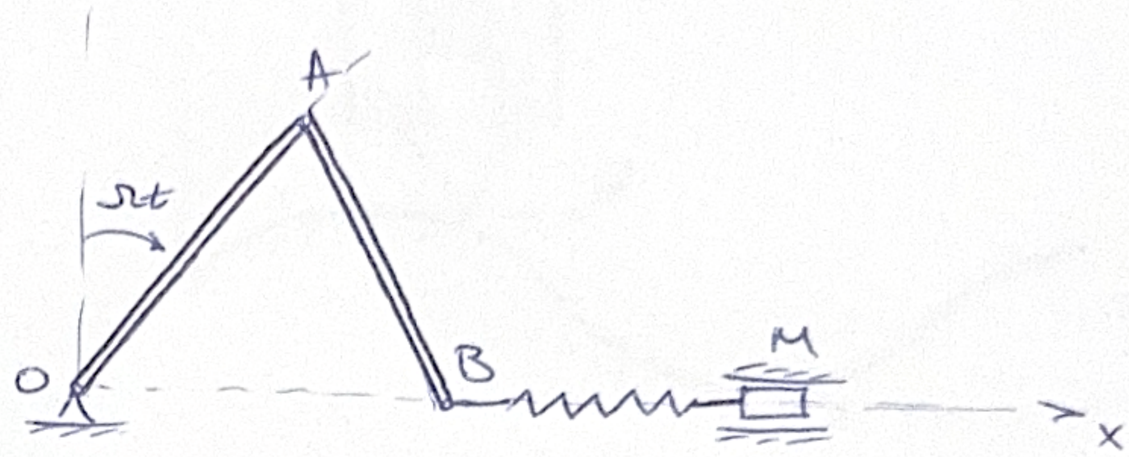
opruga: c

M: m

$t_0 = 0$: opruga neopregnuta
M mirovalo

$x(t) = ?$, koord. početak u C, pozitivni položaj točke M

$\overline{OA} = \overline{AB} = R$



$f_{st} = 0$ (hor. osc.)

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F_c}$

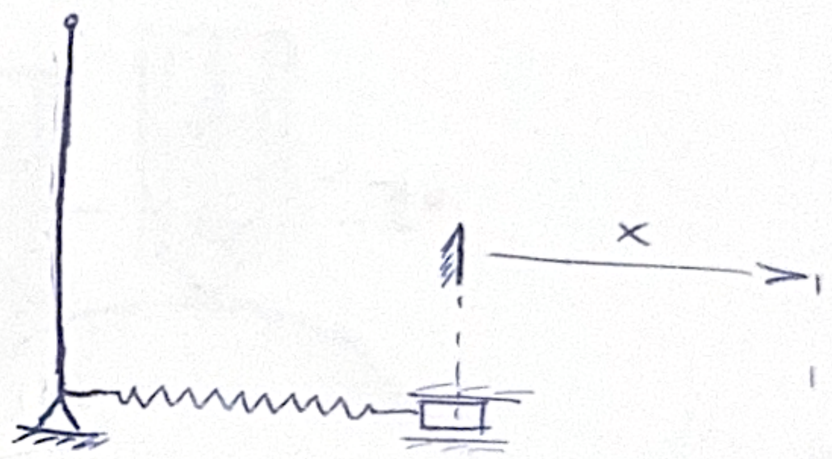
x: $m\ddot{x} = -c(-2R\sin\Omega t + x) \quad / : m$

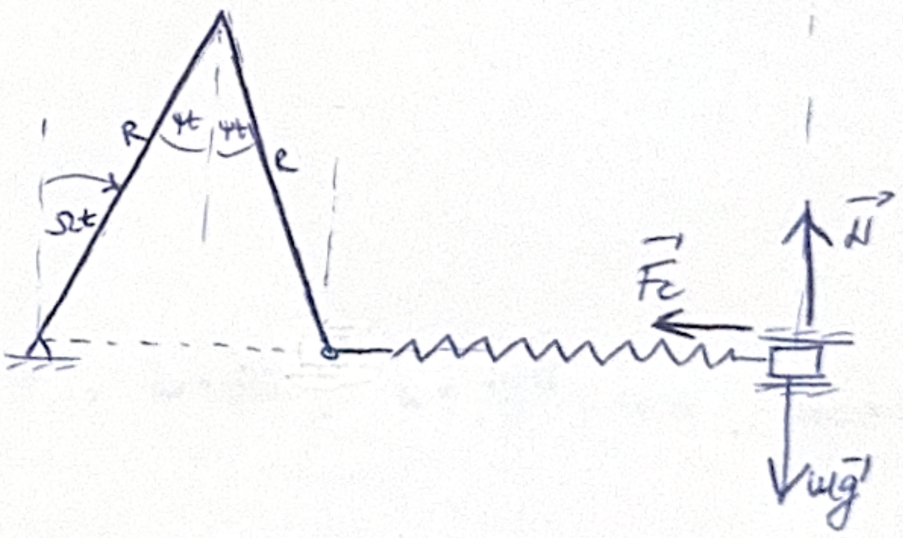
$\ddot{x} + \frac{c}{m}x = \frac{2Rc}{m}\sin\Omega t$, $\omega^2 = \frac{c}{m}$, $h = 2R\omega^2$

$\ddot{x} + \omega^2 x = h\sin\Omega t$

H: $\lambda^2 + \omega^2 \cdot 1 = 0 \rightarrow \lambda_{1,2} = 0 \pm \omega i \quad (\alpha = 0, \beta = \omega = \sqrt{\frac{c}{m}})$

$x_h = C_1\cos\omega t + C_2\sin\omega t$





P: $b(t) = h\sin\Omega t = e^{\alpha t}(0\cos\Omega t + h\sin\Omega t)$, $\alpha = 0$, $\beta = \Omega = \sqrt{\frac{c}{m}}$

$x_p = t^1 e^{\alpha t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{x}_p = A\cos\Omega t + B\sin\Omega t + t\Omega(-A\sin\Omega t + B\cos\Omega t)$

$\ddot{x}_p = -2\Omega A\sin\Omega t + 2B\Omega\cos\Omega t + t\Omega^2(-A\cos\Omega t - B\sin\Omega t)$

$\cos\Omega t(2B\Omega - \cancel{At\Omega^2} + \cancel{At\omega^2}) + \sin\Omega t(-2A\Omega - \cancel{Bt\Omega^2} + \cancel{Bt\omega^2}) = 0\cos\Omega t + h\sin\Omega t$

$B = 0$, $A = -\frac{h}{2\Omega} = -\frac{2R\omega^2}{2\Omega} = -R\omega$

$x = C_1\cos\omega t + C_2\sin\omega t - t \cdot R\omega\cos\Omega t$

$\dot{x} = -C_1\omega\sin\omega t + C_2\omega\cos\omega t - R\omega\cos\Omega t + t \cdot R\omega^2\sin\Omega t$

$x(t_0) = 0 \rightarrow C_1 \cdot 1 + 0 + 0 = 0 \rightarrow C_1 = 0$

$\dot{x}(t_0) = 0 \rightarrow 0 + C_2\omega - R\omega = 0 \rightarrow C_2 = R$

$$x = R\sin\left(\sqrt{\frac{c}{m}}\,t\right) - t \cdot R\sqrt{\frac{c}{m}}\cos\left(\sqrt{\frac{c}{m}}\,t\right)$$

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(4.39.)

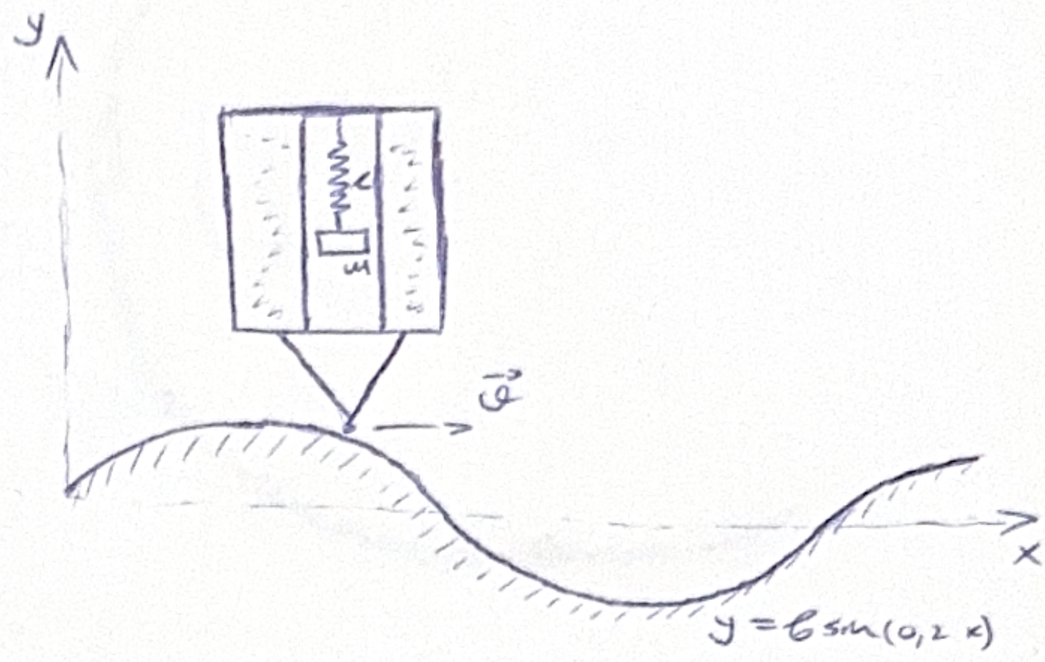
$v_x = 10 \frac{m}{s}$

$y = 6 \sin(0{,}2x)$

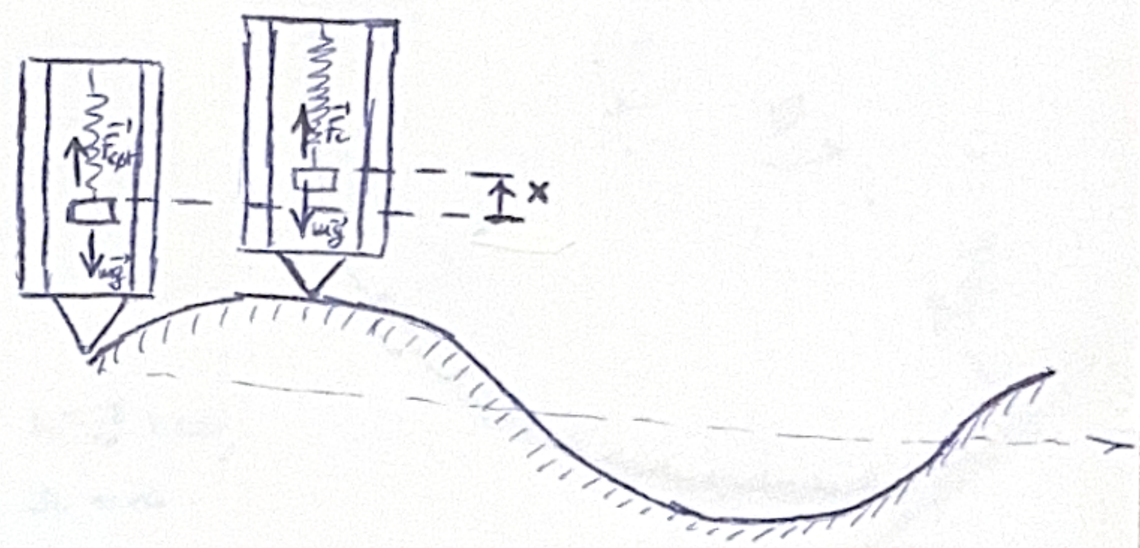
$c = 0{,}1 \frac{kN}{m} = 100 \frac{N}{m}$

$m = 10\ kg$

odredit B, ako je amplituda prisilnih vertikalnih osc $A = 2cm = 0{,}02\ m$



$y = 6 \sin(0{,}2x) = 6 \sin(0{,}2vt) = 6 \sin(2t)$



stat. ravn.

$m\vec{a} = m\vec{g} + \vec{F}_{Gst}$

x: $0 = -mg + cf_{st}$

prostor. položaj

$m\vec{a} = m\vec{g} + \vec{F}_c$

x: $m\ddot{x} = -mg + c(y - x + f_{st})$ $\quad / : m$

$\ddot{x} + \frac{c}{m}x = \frac{c6}{m}\sin(2t)$, $\omega^2 = \frac{c}{m} = \frac{100}{10} = 10$, $\omega = \sqrt{10}$, $h = \frac{c6}{m} = 10 \cdot 6$, $\Omega = 2$

$\ddot{x} + \omega^2 x = h \sin(\Omega t)$

H: $\lambda^2 + \omega^2 \cdot 1 = 0 \quad \rightarrow \quad \lambda_{1,2} = 0 \pm i\omega = 0 \pm i\sqrt{10}$

P: $x_p = t^0 e^{0t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{x}_p = -A\Omega \sin\Omega t + B\Omega\cos\Omega t$

$\ddot{x}_p = -A\Omega^2\cos\Omega t - B\Omega^2 \sin\Omega t$

$\cos\Omega t(-A\Omega^2 + A\omega^2) + \sin\Omega t(-B\Omega^2 + B\omega^2) = 0\cos\Omega t + h\sin\Omega t$

$B(\omega^2 - \Omega^2) = h \quad \rightarrow \quad B = \frac{h}{\omega^2 - \Omega^2}$, $A = 0$

$x_p = \frac{h}{\omega^2 - \Omega^2}\sin\Omega t$, $A = \frac{h}{\omega^2 - \Omega^2} = 0{,}02$

$\frac{10 \cdot 6}{10 - 4} = 0{,}02 \Rightarrow \quad 6 = \frac{0{,}02 \cdot 6}{10} = 0{,}012\ m \Rightarrow$ $\boxed{6 = 1{,}2\ cm}$

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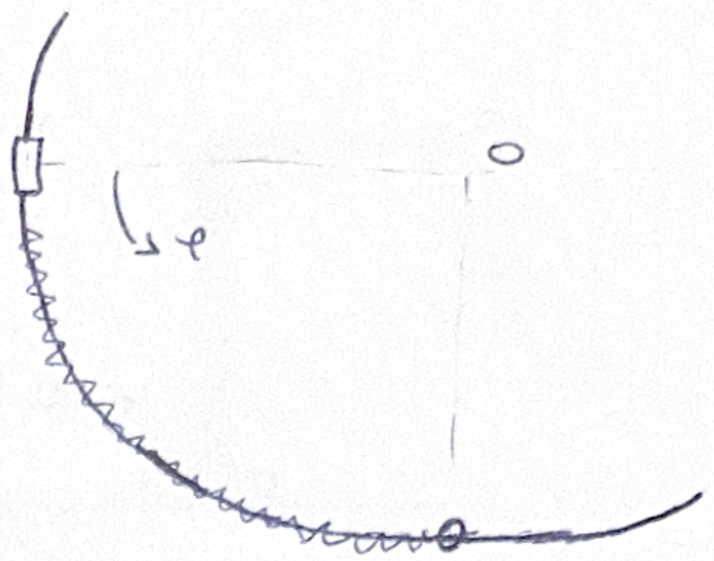
(4.40.) horizontalna ravan

$m = 1\,kg$

$c = 1\,\frac{N}{cm} = 100\,\frac{N}{m}$

$\varphi = \sin(10t)$

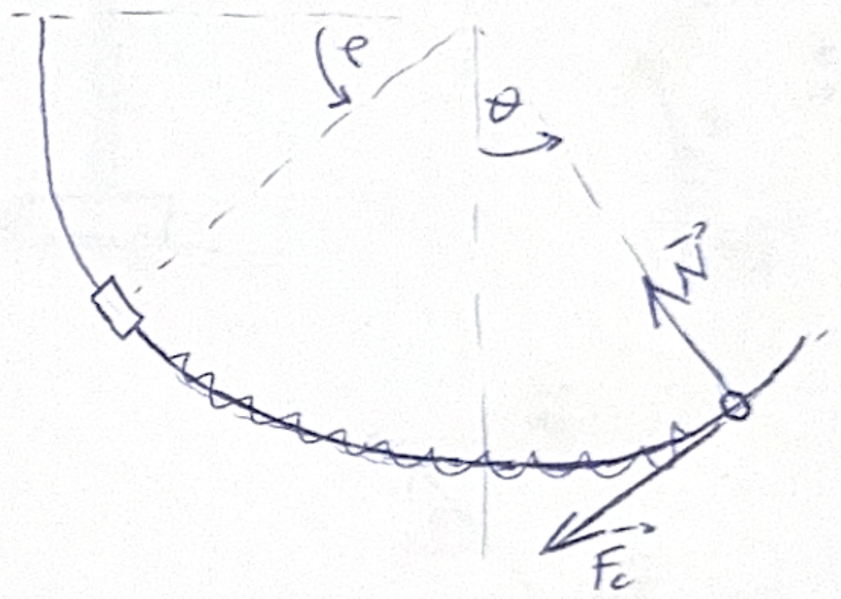


to: opruga neopregnuta
tacka mirovala

j-na kretanja tacke?

$\vartheta_{st} = 0$ (horizontalna ravan)

$\underline{m\vec{a} = \vec{F}_c + \vec{N}}$



t: $ma_t = -c(\vartheta \cdot R - \varphi \cdot R)$

$mR\ddot{\vartheta} = -c\vartheta R + cR\varphi \quad / : mR$

$\ddot{\vartheta} + \frac{c}{m}\vartheta = \frac{c}{m}\sin(10t)$, $\omega^2 = \frac{c}{m} = 100$, $h = \frac{c}{m} = 100$,

$\ddot{\vartheta} + \omega^2\vartheta = h\sin(\Omega t)$ $\omega = 10$, $\Omega = 10$

H: $\lambda^2 + \omega^2 = 0 \rightarrow \lambda_{1/2} = 0 \pm i\omega$, $\alpha = 0$, $\beta = \omega = 10 \Rightarrow \vartheta_h = C_1\cos(\omega t) + C_2\sin(\omega t)$

P: $b(t) = e^{\alpha t}(0\cos(\Omega t) + h\sin(\Omega t))$, $a = 0$, $b = \Omega = 10 \Rightarrow$ rezonansa!

$\vartheta_p = t^1 e^{\alpha t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{\vartheta}_p = A\cos\Omega t + B\sin\Omega t + t\Omega(-A\sin\Omega t + B\cos\Omega t)$

$\ddot{\vartheta}_p = -2A\Omega\sin\Omega t + 2B\Omega\cos\Omega t + t\Omega^2(-A\cos\Omega t - B\sin\Omega t)$

$\cos\Omega t\,(2B\Omega - \cancel{At\Omega^2} + \cancel{At\omega^2}) + \sin\Omega t\,(-2A\Omega - \cancel{Bt\Omega^2} + \cancel{Bt\omega^2}) = 0\cos\Omega t + h\sin\Omega t$

$B = 0$, $-2A\Omega = h \Rightarrow A = -\frac{h}{2\Omega} = -\frac{100}{2\cdot 10} = -5$

$\vartheta = C_1\cos 10t + C_2\sin 10t - 5t\cos 10t$

$\dot{\vartheta} = -10C_1\sin 10t + 10C_2\cos 10t - 5\cos 10t + 50t\sin 10t$

$\vartheta(t_0) = C_1 \cdot 1 + 0 - 0 = 0 \rightarrow \underline{C_1 = 0}$

$\dot{\vartheta}(t_0) = 0 + 10C_2 \cdot 1 - 5 + 0 = 0 \rightarrow \underline{C_2 = \frac{1}{2}}$

$\boxed{\vartheta = 0{,}5\sin(10t) - 5t\cos(10t)}$

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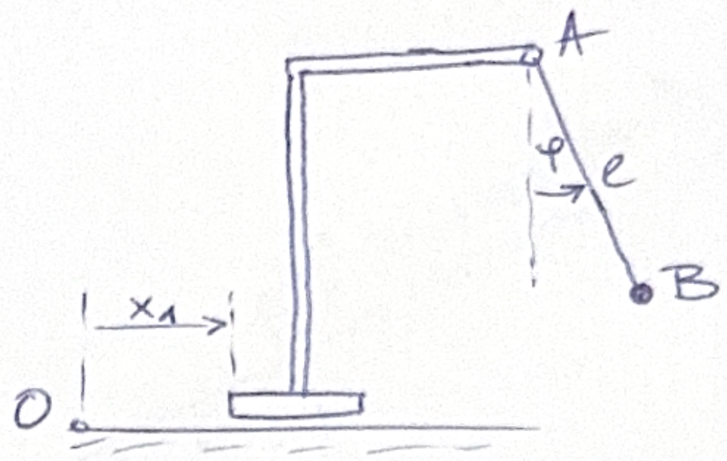
(4.41.) $x_1 = a\sin(\Omega t)$

l – dužina klatna

$\omega = 0{,}2\,\Omega$ – frekvencija oscilovanja matematičkog klatna

$\varphi_p = ?$ – i-na prinudnih osc.

male oscilacije



$\vec{ma} = \vec{mg} + \vec{S}$

t: $ma_t = -mg\sin\varphi \quad /:m$

$a_t = -g\varphi$

$\vec{a_B} = \vec{a_A} + \vec{a_{Bn}^A} + \vec{a_{Bt}^A}$

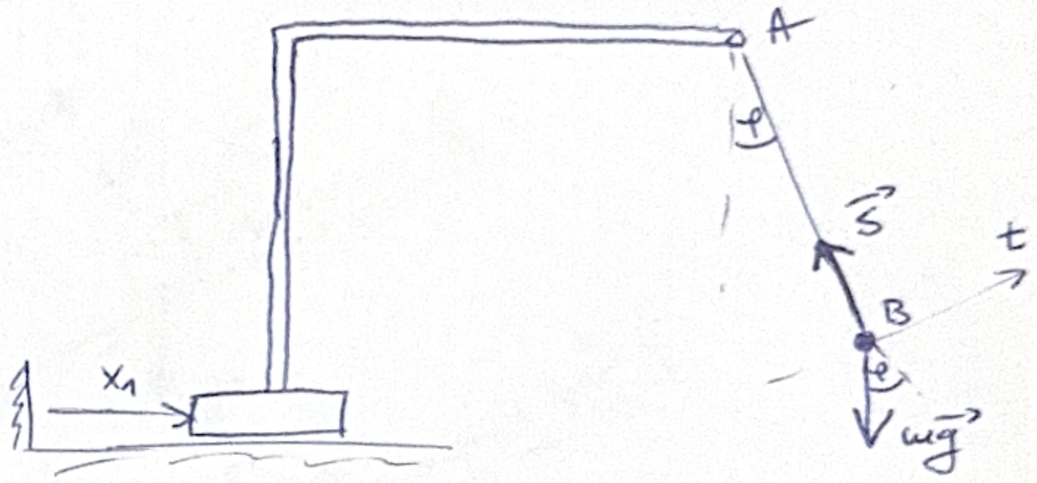
t: $a_{Bt} = \ddot{x}_1 + a_{Bt}^A$

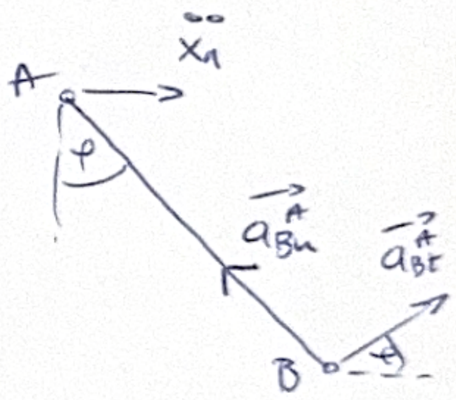
$a_t = \ddot{x}_1 + l\ddot{\varphi}$

$\ddot{x}_1 + l\ddot{\varphi} = -g\varphi \quad /:l$

$\ddot{\varphi} + \frac{g}{l}\varphi = \frac{a}{l}\Omega^2\sin(\Omega t)$, $\omega^2 = \frac{g}{l}$, $\omega = \sqrt{\frac{g}{l}} = 0{,}2\,\Omega$, $h = \frac{a}{l}\Omega^2$

$\ddot{\varphi} + \omega^2\varphi = h\sin(\Omega t)$





H: nema potrebe da ga računamo (ne traži se, a i vidimo da nece doći do rezonanse)

P: $G(t) = h\sin(\Omega t) = e^{0t}(0\cos(\Omega t) + h\sin(\Omega t))$

$\varphi_p = A\cos(\Omega t) + B\sin(\Omega t)$, $\ddot{\varphi}_p = -A\Omega^2\cos(\Omega t) - B\Omega^2\cos(\Omega t)$

$\cos(\Omega t)(-A\Omega^2 + A\omega^2) + \sin(\Omega t)(-B\Omega^2 + B\omega^2) = 0\cos(\Omega t) + h\sin(\Omega t)$

$A = 0$, $B(\omega^2 - \Omega^2) = h$ → $B = \frac{h}{\omega^2 - \Omega^2} = \frac{\frac{a}{l}\Omega^2}{(\frac{1}{5}\Omega)^2 - \Omega^2} = \frac{\frac{a}{l}\Omega^2}{-\frac{24}{25}\Omega^2} = -\frac{25}{24}\frac{a}{l}$

$$\boxed{\varphi_p = -\frac{25}{24}\frac{a}{l}\sin(\Omega t)}$$

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(4.42.) $\alpha = 30°$

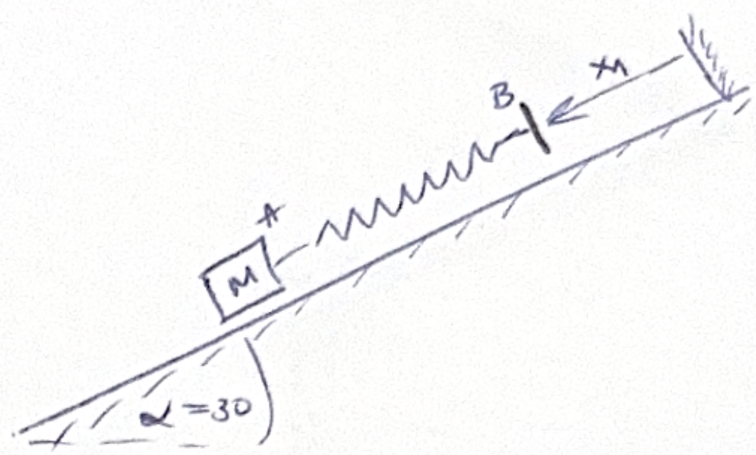
M: $Q = 100\ N$

$c = 50 \frac{N}{cm} = 5000 \frac{N}{m}$

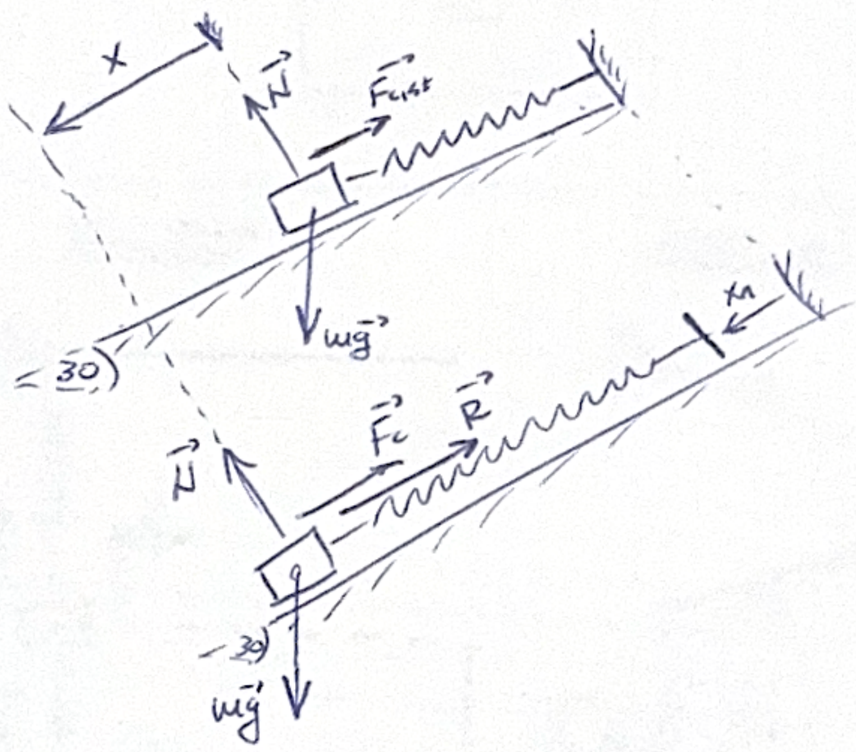
$x_1 = 2\ sin(15t)$

$x_p = ?$

sila otpora: $\vec{R} = -2\vec{v}\ \left[\frac{cm}{s}\right]$



$R = \beta \cdot v \quad \Rightarrow \quad \beta = \frac{R}{v} = 2 \frac{Ns}{cm} = 200 \frac{Ns}{m}$



stat. ravnoteža

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{el\,st}$

x: $0 = mg\sin 30 - c f_{st}$

prost. poloFaj

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_c + \vec{R}$

x: $m\ddot{x} = mg\sin 30 - c(f_{st} + x - x_1) - \beta\dot{x} \quad / : m$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 2\frac{c}{m}\sin(15t)$

$\omega^2 = \frac{c}{m} = \frac{cg}{Q} = 490,5 \quad , \quad \omega = 22,15$

$2\delta = \frac{\beta}{m} = \frac{\beta g}{Q} = 19,62 \quad , \quad h = \frac{2c}{m} = 981$

$\Omega = 15$

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h\sin(\Omega t)$

#: ne treba nam (ne traži se) f_a i očigledno je da nema retonanse)

P: $f(t) = h\sin(\Omega t) = e^{0t}(0\cos(\Omega t) + h\sin(\Omega t))$

$x_p = t^0 e^{0t}(A\cos\Omega t + B\sin\Omega t)$

$\dot{x}_p = -A\Omega\sin\Omega t + B\Omega\cos\Omega t$

$\ddot{x}_p = -A\Omega^2\cos\Omega t - B\Omega^2\sin\Omega t$

$\cos\Omega t(-A\Omega^2 + B\Omega - 2\delta + A\omega^2) + \sin\Omega t(-B\Omega^2 - A\Omega\cdot 2\delta + B\omega^2) = 0\cos\Omega t + h\sin\Omega t$

$A(\omega^2 - \Omega^2) + 2\delta\Omega\cdot B = 0 \quad \rightarrow \quad B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega}$

$B(\omega^2 - \Omega^2) + 2\delta\Omega\cdot A = h$

$-\frac{A(\omega^2 - \Omega^2)^2}{2\delta\Omega} - 2\delta\Omega\cdot A = h \quad \rightarrow \quad A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -1,84 \quad \rightarrow B = 1,66$

$\boxed{x_p = -1,84\cos(15t) + 1,66\sin(15t)}$

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(4.43.)

$c = 0,5 \frac{N}{cm} = 50 \frac{N}{m}$

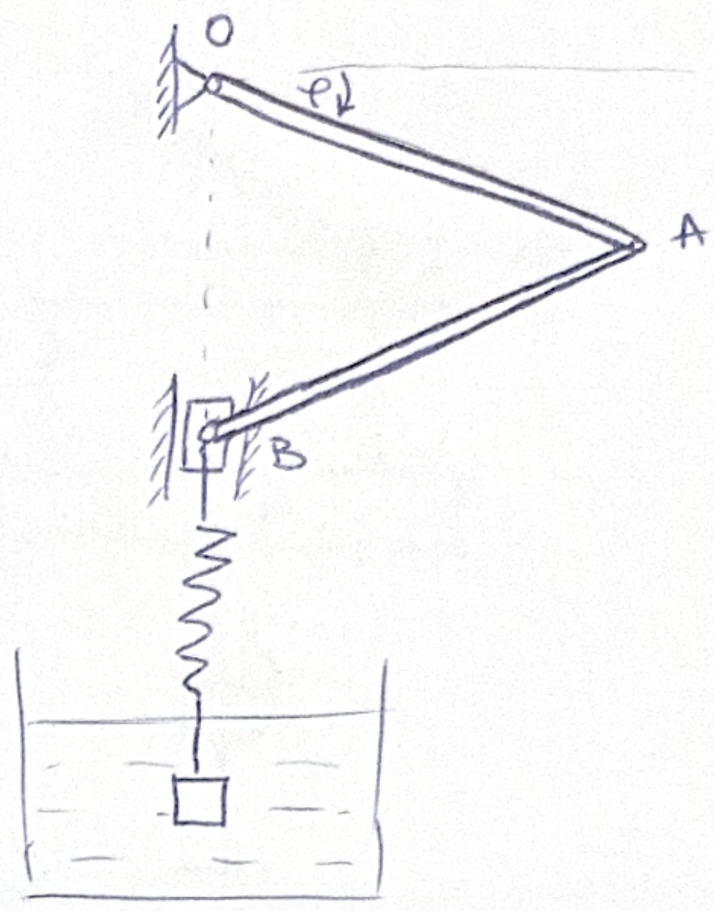
$G = 1\ N$

$\omega = 4\pi$

$\vec{v} = 1 \frac{cm}{s} = 0,01 \frac{m}{s} \Rightarrow R = 0,02\ N$

$x_p = ?$ — oko položaja skalske ravnoteže koji odgovara uglu $\varphi = 0$

$\overline{OA} = \overline{AB} = 3\ cm = 0,03\ m$



$F_{pot} = \rho_t \cdot V \cdot g = ?$ $\quad (V = ?)$

$F_{pot} = const.$ (jer je ovek ista zapremina potopljena)

st. ravn.

$m\vec{a} = \vec{G} + \vec{F}_{c,st} + \vec{F}_{pot}$

xi: $0 = G - cf_{st} - F_{pot}$

pomer. položaj

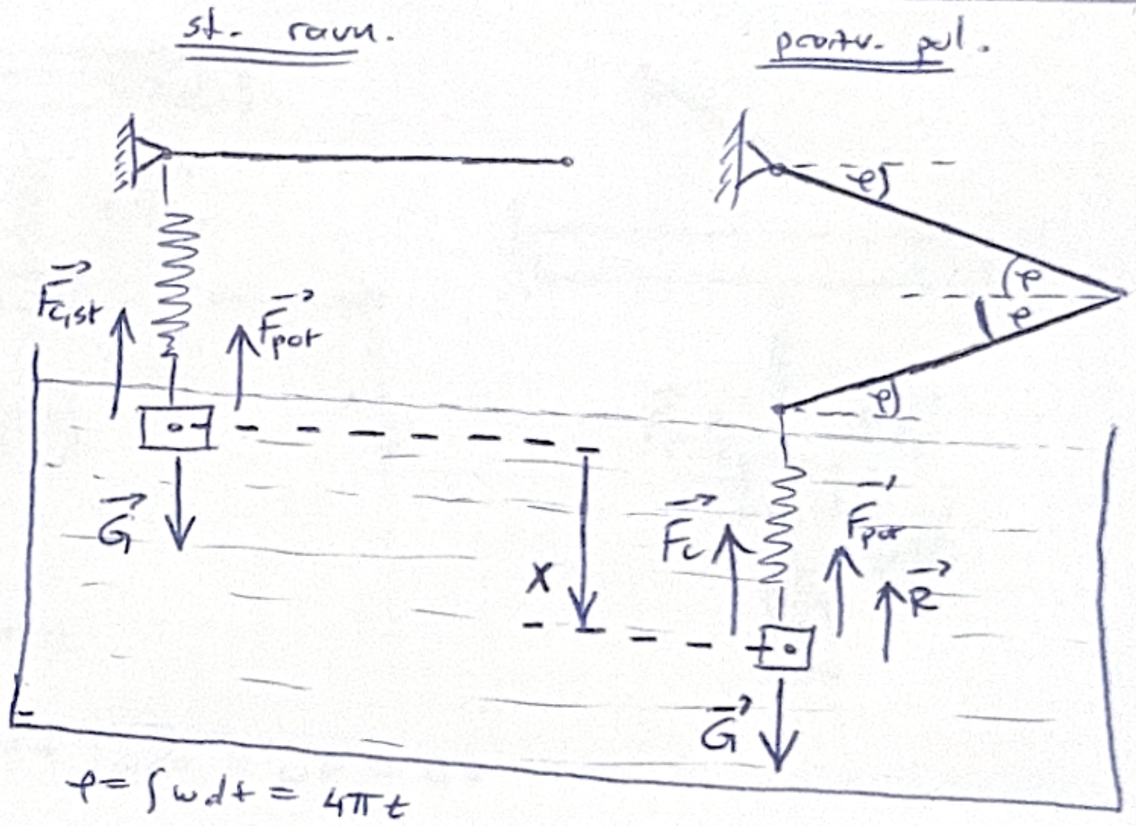
$m\vec{a} = \vec{G} + \vec{F}_c + \vec{F}_{pot} + \vec{R}$

xi: $m\ddot{x} = G - c(f_{st} + x - 2 \cdot 0,03\ sm\varphi) - \beta x$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = \frac{0,06 \cdot c}{m} sm(4\pi t)$

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h\, sm(\Omega t)$

H: necu da ga računam, ne traži se (ocigledno je i da nema retonanse)



$\varphi = \int \omega\, dt = 4\pi t$

$\vartheta = 0,01 \frac{m}{s} \Rightarrow R = 0,02\ N$

$\vartheta = 1 \frac{m}{s} \Rightarrow R = 2\ N$

$R = 2 \cdot \vartheta = \beta \cdot \vartheta \Rightarrow \beta = 2$

$\omega^2 = \frac{c}{m} = \frac{c \cdot g}{G} = 490,5$, $\omega = 22,15$,

$2\delta = \frac{\beta}{m} = \frac{\beta \cdot g}{G} = 19,62$, $\Omega = 4\pi$,

$h = \frac{0,06\,c}{m} = \frac{0,06 \cdot c \cdot g}{G} = 29,43$

P: (identično rešavanje kao u 4.42.)

$A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -0,0423$, $\quad B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega} = 0,0571$

$x_p = -0,0423 \cos(4\pi t) + 0,0571 \sin(4\pi t)$

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4.44.

$Q = 98$

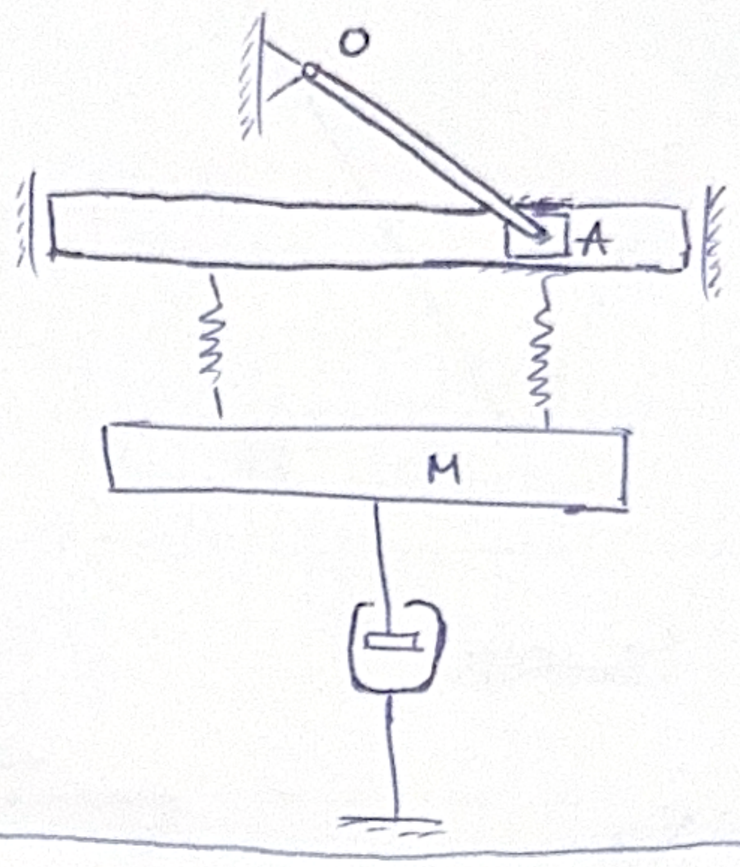
$c = 6{,}8 \frac{N}{cm}$

$\delta = \omega_s$ - prigušenje

$x_p = ?$

$\overline{OA} = 7{,}5\ cm$

$\omega = 6\ s^{-1}$

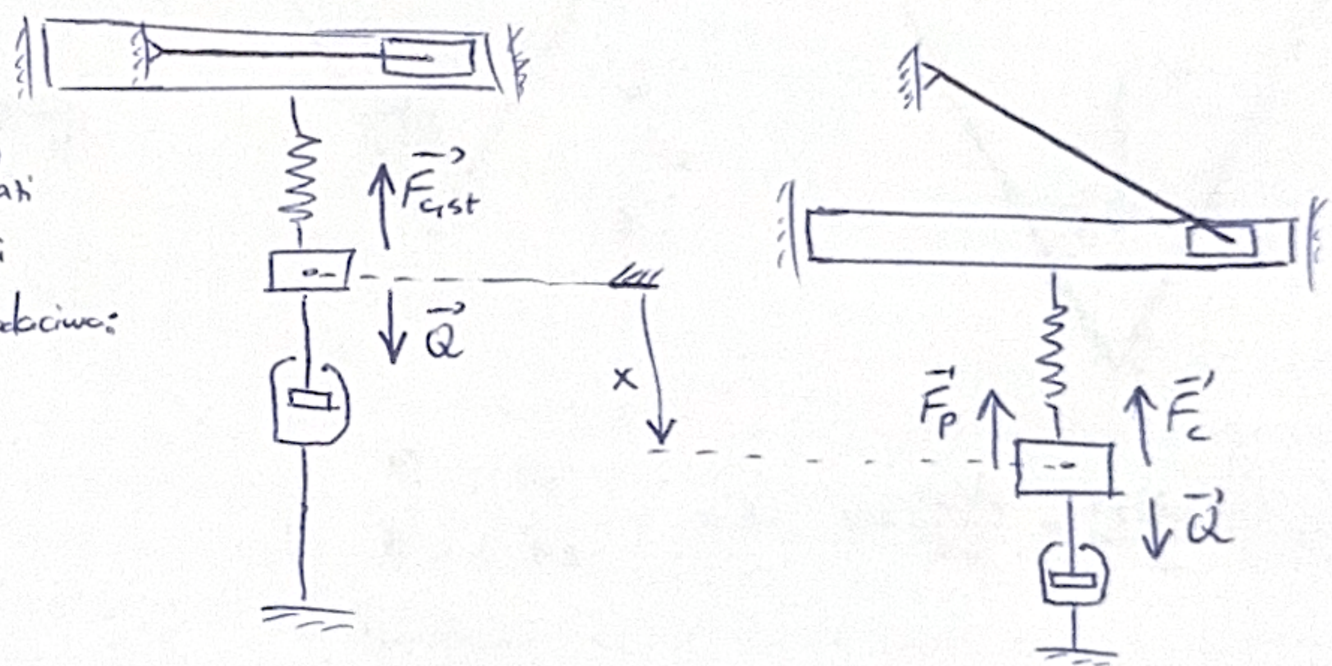


Napomena:

Analizirajući rezultat koji se nalazi u zbirci sam shvatio da su oni zadatak radili sa sledećim podacima:

$c = 6{,}8 \frac{N}{m}$, $\overline{OA} = 7{,}5\ m$

$c_e = c + c = 2c = 13{,}6 \frac{N}{m}$



$\varphi = \int \omega dt = \omega t = 6t$

st. ravnoteža:

$m\vec{a} = \vec{Q} + \vec{F}_{c,st}$

x: $0 = Q - c f_{st}$

proizvoljni položaj:

$m\vec{a} = \vec{Q} + \vec{F}_c' + \vec{F}_p$

x: $m\ddot{x} = Q - c(f_{st} + x - 7{,}5 \sin\varphi) - \beta \cdot \dot{x}$

$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{c}{m}x = 7{,}5 \cdot \frac{c}{m}\sin(6t)$, $\omega^2 = \frac{c}{m} = \frac{cQ}{g} = 135{,}86$, $\omega = 11{,}656$,

$\ddot{x} + 2\delta\dot{x} + \omega^2 x = h\sin(\Omega t)$ | $2\delta = \frac{\beta}{m} = 2\omega = 23{,}312$, $\Omega = 6$

$h = 7{,}5 \frac{c}{m} = 1018{,}96$

H: nema potrebe da radimo, ne traži se (a očigledno je: da nema rezonanse)

P: (već smo rešavali ovo u zadatku 4.42)

$A = \frac{h}{-\left(\frac{(\omega^2 - \Omega^2)^2}{2\delta\Omega} + 2\delta\Omega\right)} = -4{,}83$, $B = -\frac{A(\omega^2 - \Omega^2)}{2\delta\Omega} = 3{,}45$

$\boxed{x_p = -4{,}83\cos(6t) + 3{,}45\sin(6t)}$

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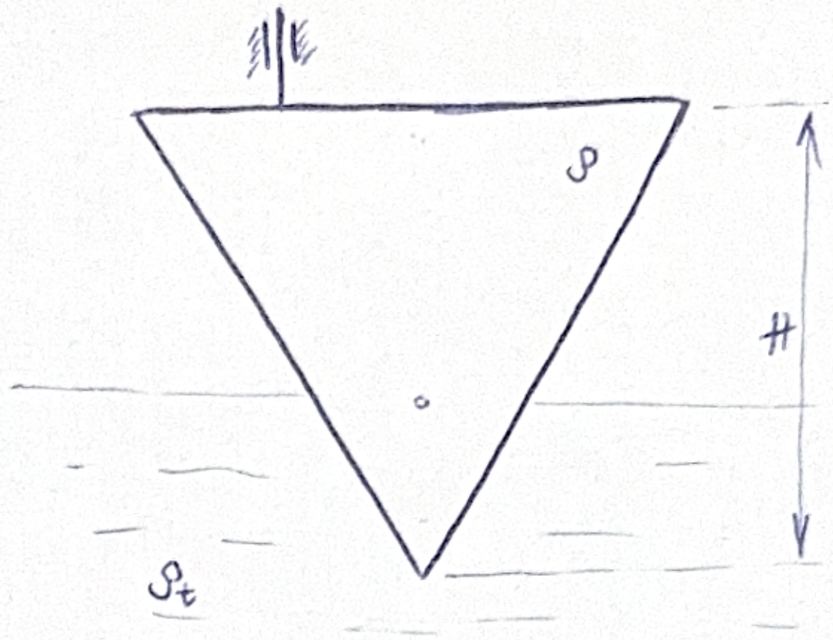
(4.45.)

plovak: $H, \rho,$ kovus

tekočina: ρ_t , $(\rho_t > \rho)$

T_ω ? – perioda majhnih vertikalnih oscilacij



$T_\omega = \frac{2\pi}{\omega}$, $\omega = ?$

majhne oscilacije $\Rightarrow$ koord. x je majhna $\Rightarrow$
$\Rightarrow$ članom višjega reda se odločimo ($x^2, x^3, \ldots$)
+ $\sin x = x$, $\cos x = 1$, ...

$F_p = Q_{pot} = \rho_t \cdot V_{pot} \cdot g = \rho_t \cdot \frac{1}{3} r^2 \pi h g$

st. ravn.

$m\vec{a} = m\vec{g} + \vec{F}_{pist}$

x: $0 = mg - \rho_t \cdot \frac{1}{3} r_s^2 \pi h_s \cdot g$ (1)

protv. pol.

$m\vec{a} = m\vec{g} + \vec{F}_p'$

x: $m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} r^2 \pi h g$

$m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} r_s^2 \left(1 + \frac{x}{h_s}\right)^2 \pi (h_s + x) g$

$m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} \left(r_s^2 + 2r_s^2 \frac{x}{h_s} + r_s^2 \frac{x^2}{h_s^2}\right) \pi (h_s + x) g$

$m\ddot{x} = mg - \rho_t \cdot \frac{1}{3} \left(r_s^2 \pi h_s + r_s^2 \pi x + 2r_s^2 \frac{x}{h_s} \pi h_s + \right.$
$\left. + 2r_s^2 \frac{x}{h_s} \pi x + r_s^2 \frac{x^2}{h_s^2} \pi h_s + r_s^2 \frac{x^2}{h_s^2} \pi x\right) g$

($x^2 \approx 0$, $x^2 \approx 0$, $x^3 \approx 0$)

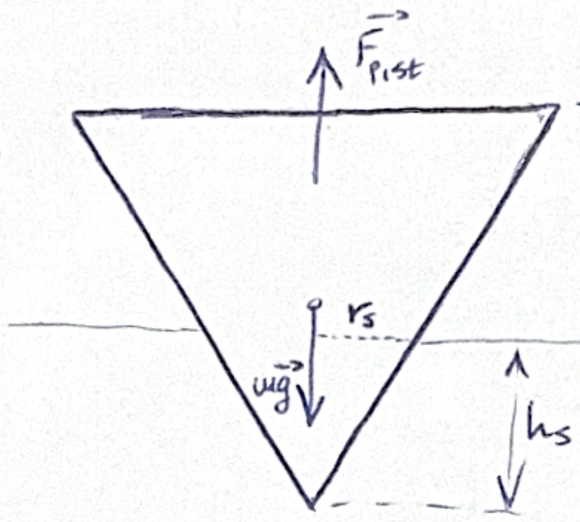
$m\ddot{x} = -\frac{1}{3} \rho_t (3 r_s^2 \pi x) g \quad / : m$

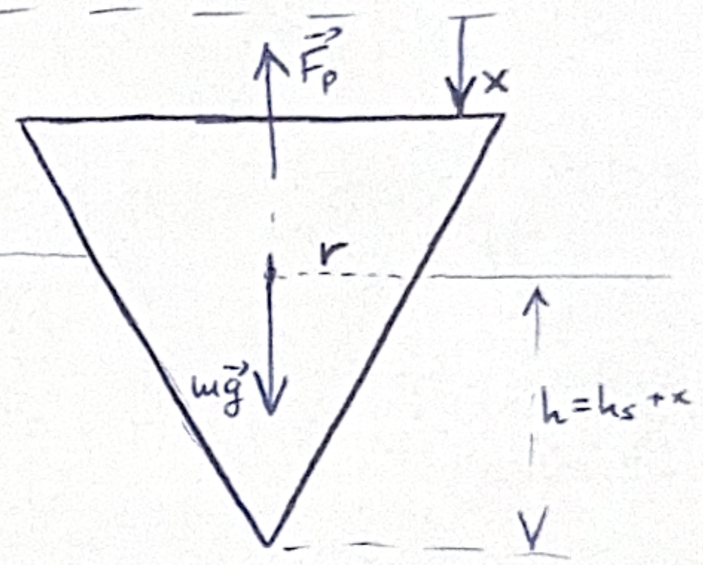
$\ddot{x} + \frac{\rho_t}{m} r_s^2 \pi g x = 0$, $\omega^2 = \frac{\rho_t}{m} r_s^2 \pi g$, $m = ?$, $r_s = ?$

$m = \frac{1}{3} \rho \cdot V = \frac{1}{3} \rho \cdot R^2 \pi \cdot H$

$\omega^2 = \frac{3\rho_t}{\rho R^2 \pi H} r_s^2 \pi g = \frac{3\rho_t r_s^2 g}{\rho R^2 H}$, $r_s = ?$, $R = ?$

$r_s \ldots R$
$h_s \ldots H$ $\Rightarrow$ $r_s = \frac{R h_s}{H}$





$r \ldots r_s$
$h \ldots h_s$ $\Rightarrow$ $r = \frac{r_s \cdot h}{h_s} = \frac{r_s (h_s + x)}{h_s} = r_s \left(1 + \frac{x}{h_s}\right)$

$\omega^2 = \frac{3\rho_t R^2 h_s^2 g}{\rho \cdot R^2 \cdot H^2 \cdot H} = \frac{3\rho_t h_s^2 g}{\rho H^3}$, $h_s = ?$

(1) $\Rightarrow$ $mg - \rho_t \cdot \frac{1}{3} r_s^2 \pi \cdot h_s \cdot g = 0$

$\Rightarrow h_s = \frac{3m}{\rho_t \cdot r_s^2 \cdot \pi} = \frac{3 \cdot \frac{1}{3} \rho R^2 \pi H}{\rho_t r_s^2 \pi}$

$\Rightarrow h_s = \frac{\rho R^2 H}{\rho_t \frac{R^2 h_s^2}{H^2}} = \frac{\rho R^2 H^3}{\rho_t R^2 h_s^2}$

$\Rightarrow h_s^3 = \frac{\rho H^3}{\rho_t}$ $\Rightarrow h_s = \sqrt[3]{\frac{\rho}{\rho_t}} \cdot H$

$\omega^2 = \frac{3\rho_t \sqrt[3]{\frac{\rho}{\rho_t}} H^2 g}{\rho \cdot H^3} = \frac{3g}{H} \frac{\rho^{-1}}{\rho_t^{-\frac{1}{3}}} \cdot \frac{\rho^{\frac{2}{3}}}{\rho_t^{\frac{2}{3}}}$

$\omega^2 = \frac{3g}{H} \frac{\rho^{-\frac{1}{3}}}{\rho_t^{-\frac{1}{3}}} = \frac{3g}{H} \sqrt[3]{\frac{\rho_t}{\rho}}$

$T_\omega = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{H}} \cdot \sqrt[6]{\frac{\rho_t}{\rho}}}$

$$\boxed{T_\omega = 2\pi \cdot \sqrt{\frac{H}{3g}} \cdot \sqrt[6]{\frac{\rho}{\rho_t}}}$$

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