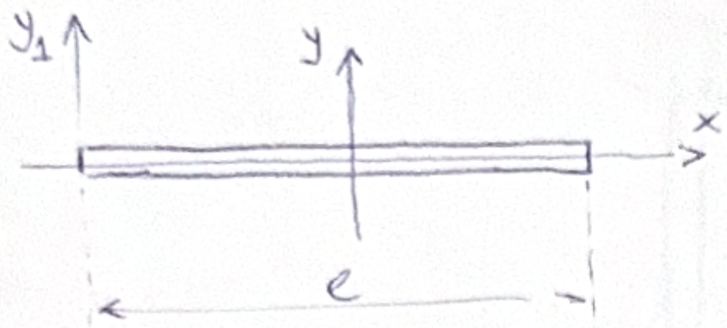
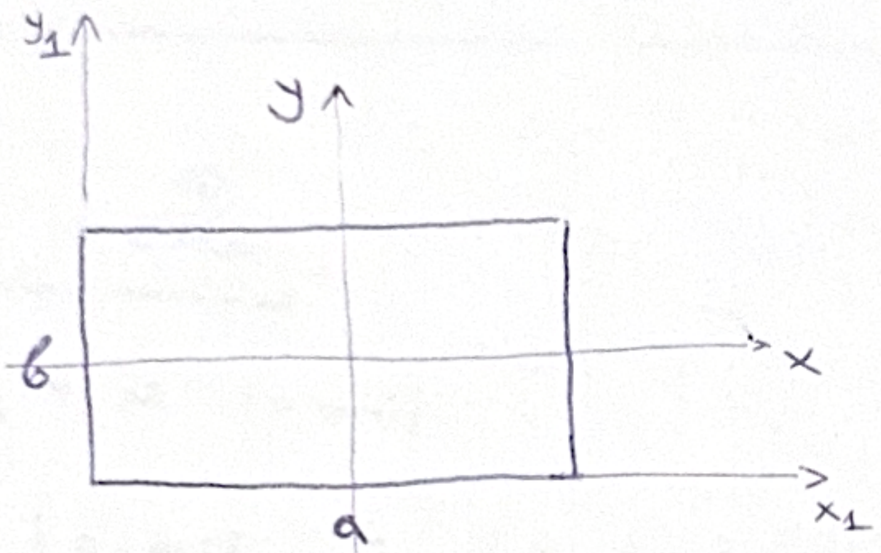


$$J_x = 0$$

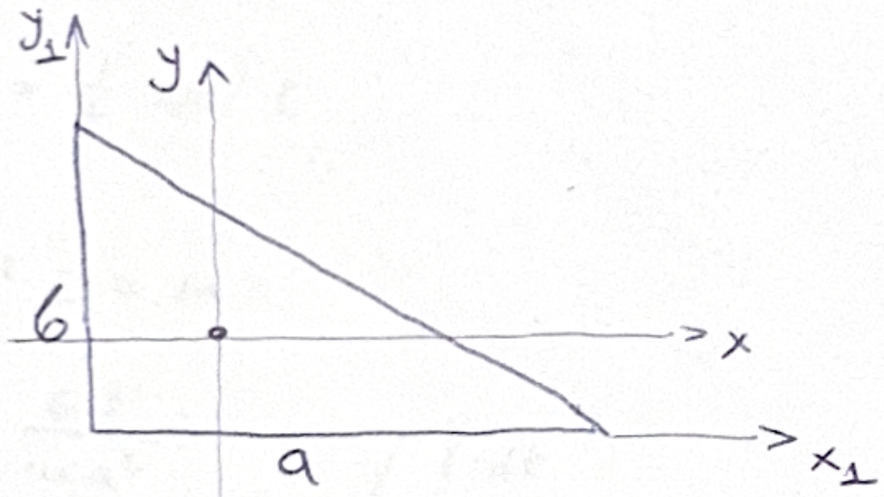
$$J_y = \frac{1}{12} m l^2$$

$$J_{y_1} = \frac{1}{3} m l^2$$



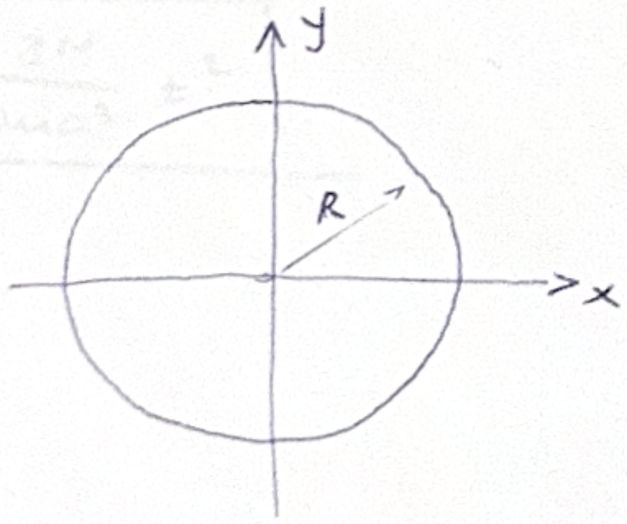
$$J_x = \frac{1}{12} m b^2 \quad , \quad J_y = \frac{1}{12} m a^2 \quad , \quad J_z = \frac{1}{12} m (a^2 + b^2)$$

$$J_{x_1} = \frac{1}{3} m b^2 \quad , \quad J_{y_1} = \frac{1}{3} m a^2$$

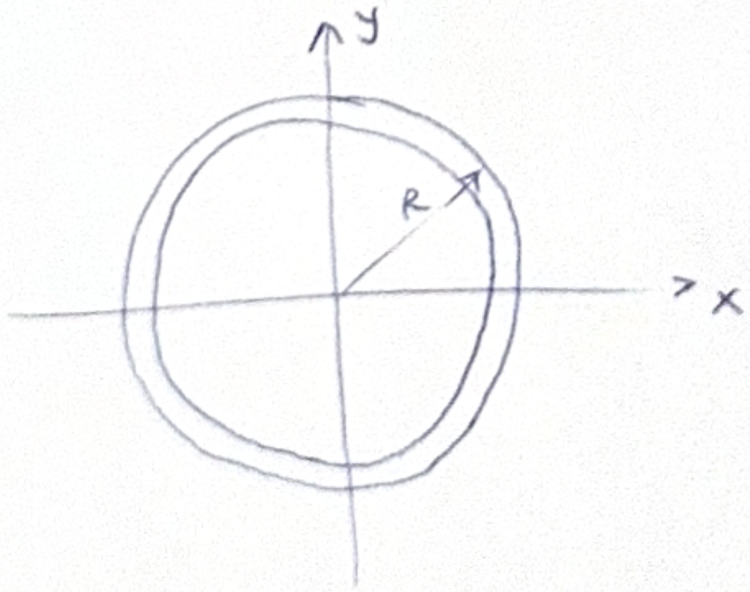


$$J_{x_0} = \frac{1}{18} m b^2 \quad , \quad J_{y_0} = \frac{1}{18} m a^2 \quad , \quad J_z = \frac{1}{18} m (a^2 + b^2)$$

$$J_{x_1} = \frac{1}{6} m b^2 \quad , \quad J_{y_1} = \frac{1}{6} m a^2$$



$$J_x = \frac{1}{4} m R^2 \quad , \quad J_y = \frac{1}{4} m R^2 \quad , \quad J_z = \frac{1}{2} m R^2$$



$$J_x = \frac{1}{2} m R^2 \quad , \quad J_y = \frac{1}{2} m R^2 \quad , \quad J_z = m R^2$$

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(10.1)

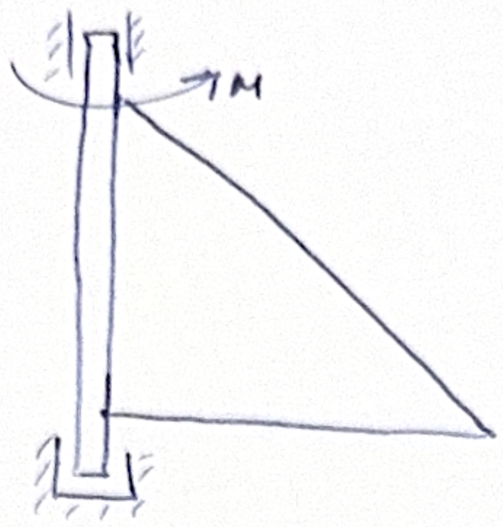
M

a x a

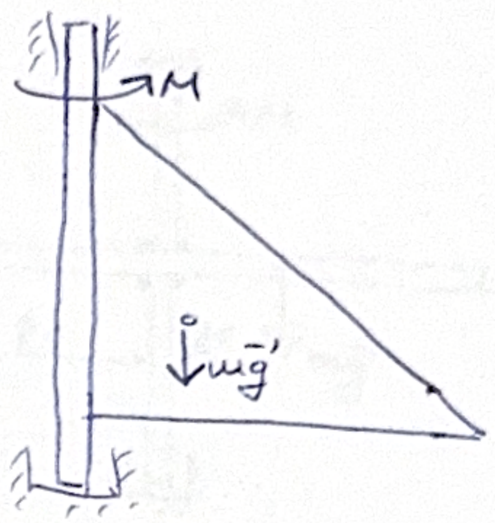
M = const.

φ = ?

t_0: stanje mirovanja



$\frac{dL_{O\uparrow}}{dt} = \overset{\curvearrowleft}{\Sigma M_{O\uparrow}}$



$\Sigma M_{O\uparrow} = M \quad (= \text{const.})$

$L_{Oz} = \frac{1}{3} a \cdot m v_c + J\omega = \frac{1}{3} a \cdot m \cdot \frac{1}{3} a\omega + \frac{1}{18} m a^2 \omega$

$L_{O\uparrow} = \frac{1}{6} m a^2 \omega$

$\frac{dL_{O\uparrow}}{dt} = \frac{1}{6} m a^2 \varepsilon$

$\frac{1}{6} m a^2 \varepsilon = M$

$\varepsilon = \frac{6M}{m a^2} \quad / \int dt$

$\omega = \frac{6M}{m a^2} t \quad / \int dt$

$$\boxed{\varphi = \frac{3M}{m a^2} t^2}$$

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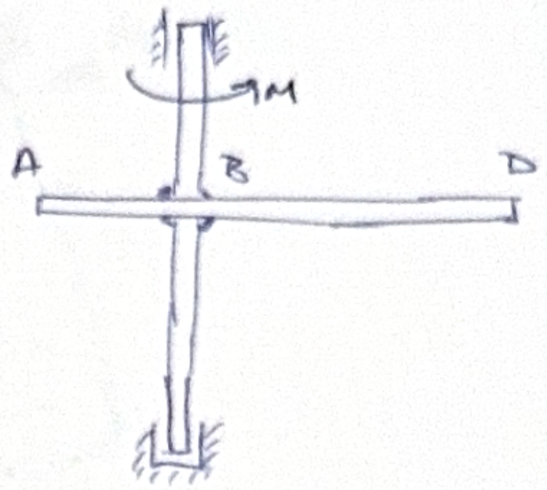
(10.2)

AD: $m = 1\ kg$, $\ell = 2\ m$

$\overline{AB} = \frac{1}{3}\ell$

$M = 4\ kNm$

$\varepsilon = ?$



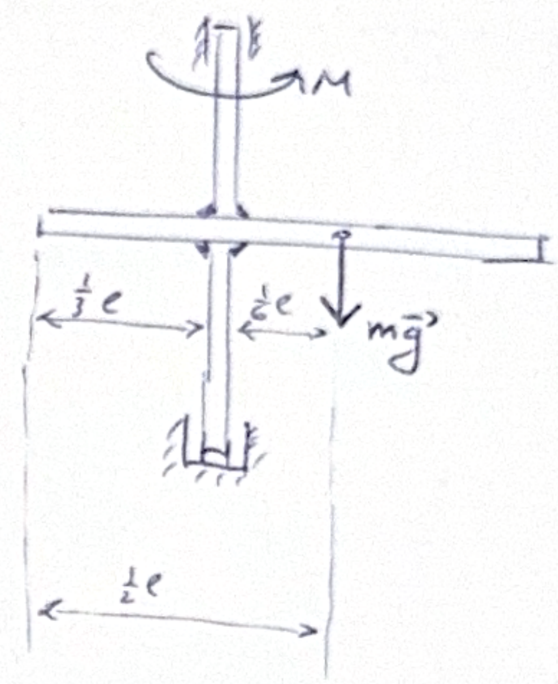
$\frac{dL_{Oz}}{dt} = \Sigma M_{Oz}$

$\Sigma M_{Oz} = M \quad (= const)$

$L_{Oz} = \frac{1}{6}\ell \cdot m v_c + J_c \omega = \frac{1}{6}\ell m \cdot \frac{1}{6}\ell\omega + \frac{1}{12} m\ell^2\omega =$

$L_{Oz} = \frac{1}{9} m\ell^2\omega$

$\frac{dL_{Oz}}{dt} = \frac{1}{9} m\ell^2\varepsilon$



$\frac{1}{9} m\ell^2\varepsilon = M$

$\varepsilon = \frac{9 \cdot M}{m\ell^2} = \frac{9 \cdot 4}{1 \cdot 2^2} = 9$

$\boxed{\varepsilon = 9\ s^{-2}}$

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(10. 3)

$m = 10$ kg

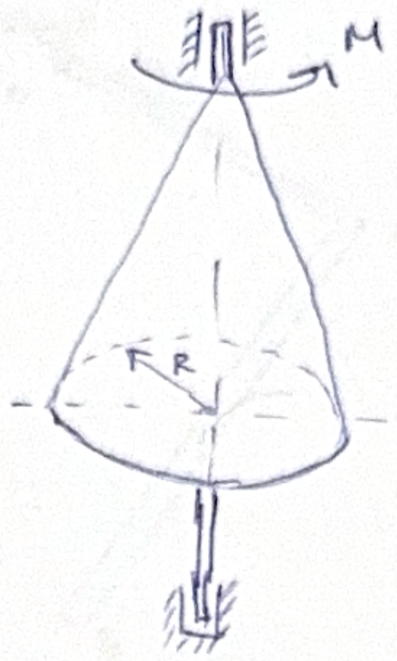
$R = 0,2$ m

$M = 6$ Nm

$t_0 = 0$: mirovanje

$t_1 = ?$

$\omega(t_1) = 20\ s^{-1}$



$$\frac{d\vec{L}_{Ot}}{dt} = \sum \vec{M}_{Ot}$$

$$\sum M_{Ot} = M \quad (= \text{const.})$$

$$L_{Ot} = 0 + J_{Ct}\,\omega = \frac{3}{10} m R^2 \omega$$

$$\frac{dL_{Ot}}{dt} = \frac{3}{10} m R^2 \varepsilon$$

$$\frac{3}{10} m R^2 \varepsilon = M$$

$$\varepsilon = \frac{10\,M}{3\,mR^2} = \frac{10 \cdot 6}{3 \cdot 10 \cdot 0,2^2} = 50 \qquad / \int dt$$

$$\omega = 50\,t$$

$$\omega(t_1) = 50 \cdot t_1 = 20 \quad \Rightarrow \quad t_1 = \frac{2}{5} \quad \Rightarrow \quad \boxed{t_1 = 0,4\ s}$$

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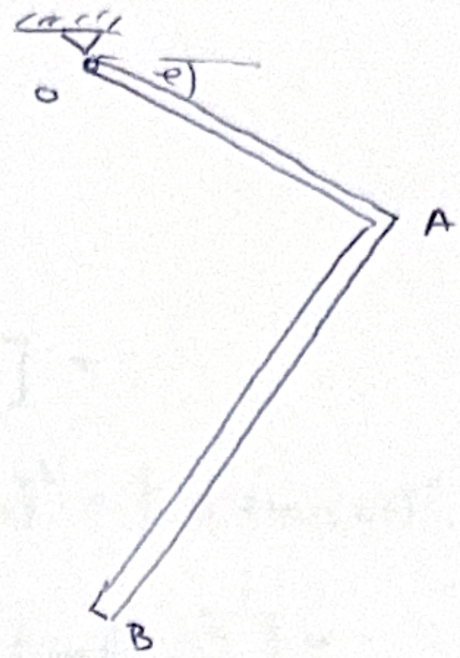
(10. 5.)

OA: m, ℓ

AB: $2m$, 2ℓ

$t_0 = 0$: OA je horiz. mirovanje

$\omega(\varphi)$? $\varepsilon(\varphi)$? $\vec{N}(\varphi)$?



Zadatak je moguće uraditi na 3 načina:

1) $\frac{dL_{Ot}}{dt} = \Sigma M_{Ot}$ → odavde dobijemo ε, zatim integralimo da dobijemo ω

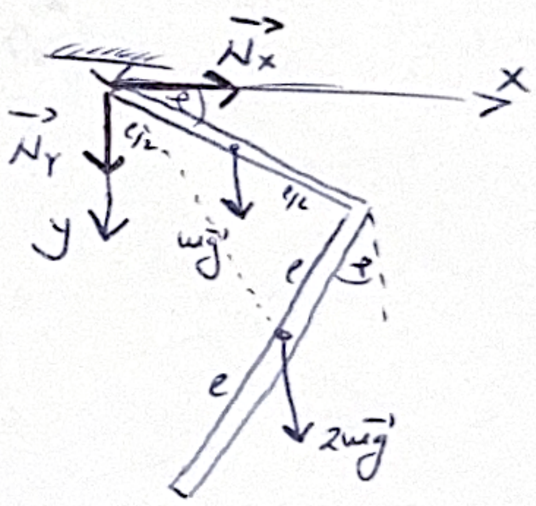
2) $T - T_0 = A$ → odavde dobijemo ω, zatim radimo izvod da dobijemo ε

3) $\frac{dT}{dt} = \frac{d'A}{dt}$ → odavde dobijemo ε, zatim integralimo da dobijemo ω

Uvek je lakše raditi izvod nego integraliti. Ali svakako ću ua oba načina uraditi:

I način

$\frac{dL_{Ot}}{dt} = \Sigma M_{Ot}$



$\Sigma M_{Ot} = mg \cdot \frac{\ell}{2}\cos\varphi + 2mg(\ell\cos\varphi - \ell\sin\varphi) = \frac{5}{2}mg\ell\cos\varphi - 2mg\ell\sin\varphi$

$L_{Ot} = \left[\frac{\ell}{2} \cdot m\frac{\ell}{2} + J_1 \cdot \omega\right] + \left[\ell\sqrt{2} \cdot 2m\ell_2 + J_2\omega\right] =$

$L_{Ot} = \left[\frac{\ell}{2} m \frac{\ell}{2}\omega + \frac{1}{12}m\ell^2\omega\right] + \left[\ell\sqrt{2} \cdot 2m \cdot \ell\sqrt{2} \cdot \omega + \frac{1}{12} 2m(2\ell)^2\omega\right] =$

$L_{Ot} = \frac{1}{3}m\ell^2\omega + \frac{14}{3}m\ell^2\omega = 5m\ell^2\omega$

$\frac{dL_{Ot}}{dt} = 5m\ell^2\varepsilon$

$5m\ell^2\varepsilon = \frac{5}{2}mg\ell\cos\varphi - 2mg\ell\sin\varphi \quad \Rightarrow \quad \boxed{\varepsilon = \frac{1}{10}\frac{g}{\ell}(5\cos\varphi - 4\sin\varphi)}$

$\varepsilon = \frac{d\omega}{dt} \cdot \frac{d\varphi}{d\varphi} = \frac{d\omega \cdot \omega}{d\varphi} = \frac{1}{10}\frac{g}{\ell}(5\cos\varphi - 4\sin\varphi)$

$\omega\, d\omega = \frac{1}{10}\frac{g}{\ell}(5\cos\varphi - 4\sin\varphi)\, d\varphi \qquad / \int$

$\frac{1}{2}\omega^2 = \frac{1}{10}\frac{g}{\ell}(5\sin\varphi + 4\cos\varphi) + C_1$

$\omega = \sqrt{\frac{1}{5}\frac{g}{\ell}(5\sin\varphi + 4\cos\varphi) + C_1}$

$\omega(t_0 = 0) = \sqrt{\frac{1}{5}\frac{g}{\ell}(0 + 4) + C_1} = 0 \quad \Rightarrow \quad C_1 = -\frac{4}{5}\frac{g}{\ell} \quad \Rightarrow \quad \boxed{\omega = \sqrt{\frac{1}{5}\frac{g}{\ell}(5\sin\varphi + 4\cos\varphi - 4)}}$

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II način

$T - T_0 = A$

$T_0 = 0$

$T = \left[\frac{1}{2} m v_1^2 + \frac{1}{2} J_1 \omega^2\right] + \left[\frac{1}{2} 2m v_2^2 + \frac{1}{2} J_2 \omega^2\right] =$

$T = \left[\frac{1}{2} m \left(\frac{\ell}{2}\omega\right)^2 + \frac{1}{2}\frac{1}{12} m \ell^2 \omega^2\right] + \left[\frac{1}{2} 2m \left(\ell\sqrt{2}\,\omega\right)^2 + \frac{1}{2}\frac{1}{12} 2m (2\ell)^2 \omega^2\right] =$

$T = \left[\frac{1}{8} m \ell^2 \omega^2 + \frac{1}{24} m \ell^2 \omega^2\right] + \left[2m\ell^2\omega^2 + \frac{1}{3} m \ell^2 \omega^2\right] =$

$T = \frac{1}{6} m \ell^2 \omega^2 + \frac{7}{3} m \ell^2 \omega^2 = \frac{5}{2} m \ell^2 \omega^2$

$A = \int mg\, dy_1 + \int 2mg\, dy_2 =$

$y_1 = \frac{1}{2}\ell \sin\varphi \quad \Rightarrow \quad \frac{dy_1}{dt} = \frac{1}{2}\ell \cos\varphi \cdot \frac{d\varphi}{dt}$

$y_2 = \ell \sin\varphi + \ell\cos\varphi \quad \Rightarrow \quad \frac{dy_2}{dt} = \ell\cos\varphi \frac{d\varphi}{dt} - \ell \sin\varphi \frac{d\varphi}{dt}$

$A = \int_{\varphi_0=0}^{\varphi} mg \cdot \frac{1}{2}\ell \cos\varphi\, d\varphi + \int_{\varphi_0=0}^{\varphi} 2mg \cdot (\ell\cos\varphi - \ell\sin\varphi)\, d\varphi =$

$A = \frac{1}{2} mg\ell(\sin\varphi - \sin 0^{70}) + 2mg\ell(\sin\varphi - \sin 0^{0}) + 2mg\ell(\cos\varphi - \cos 0^{1})$

$A = \frac{5}{2} mg\ell \sin\varphi + 2mg\ell\cos\varphi - 2mg\ell$

$\frac{5}{2} m\ell^2\omega^2 = \frac{5}{2} mg\ell\sin\varphi + 2mg\ell\cos\varphi - 2mg\ell$

$$\omega = \sqrt{\frac{1}{5}\frac{g}{\ell}(5\sin\varphi + 4\cos\varphi - 4)}$$

$\varepsilon = \frac{d\omega}{dt} = \frac{1}{2\cdot\sqrt{\ldots}} \cdot \frac{1}{5}\frac{g}{\ell}(5\cos\varphi - 4\sin\varphi)\cdot\omega$

$$\varepsilon = \frac{1}{10}\frac{g}{\ell}(5\cos\varphi - 4\sin\varphi)$$

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Nastavak (na prvi način ili na drugi način):

$$M\vec{a}_c = m\vec{g} + 2m\vec{g} + \vec{N}_x + \vec{N}_y$$

$$\underline{m\vec{a}_1 + 2m\vec{a}_2 = m\vec{g} + 2m\vec{g} + \vec{N}_x + \vec{N}_y}$$

x: $m\ddot{x}_1 + 2m\ddot{x}_2 = N_x$

$x_1 = \frac{1}{2}\ell\cos\varphi \Rightarrow \dot{x}_1 = -\frac{1}{2}\ell\sin\varphi\cdot\omega \Rightarrow \ddot{x}_1 = -\frac{1}{2}\omega^2\ell\cos\varphi$

$x_2 = \ell\cos\varphi - \ell\sin\varphi \Rightarrow \dot{x}_2 = -\ell\omega\sin\varphi - \ell\omega\cos\varphi \Rightarrow \ddot{x}_2 = \omega^2\ell\sin\varphi - \omega^2\ell\cos\varphi$

$$N_x = -\frac{1}{2}m\ell\omega^2\cos\varphi + 2m\ell\omega^2\sin\varphi - 2m\ell\omega^2\cos\varphi$$

$$\boxed{N_x = 2m\ell\omega^2\sin\varphi - \frac{5}{2}m\ell\omega^2\cos\varphi}$$

y: $m\ddot{y}_1 + 2m\ddot{y}_2 = mg + 2mg + N_y$

$y_1 = \frac{1}{2}\ell\sin\varphi \Rightarrow \dot{y}_1 = \frac{1}{2}\ell\omega\cos\varphi \Rightarrow \ddot{y}_1 = -\frac{1}{2}\ell\omega^2\sin\varphi$

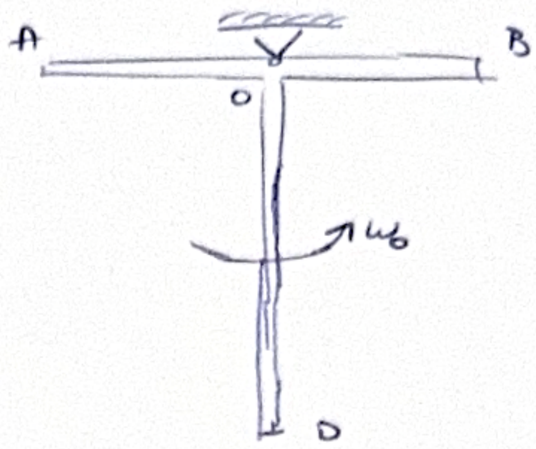
$y_2 = \ell\sin\varphi + \ell\cos\varphi \Rightarrow \dot{y}_2 = \ell\omega\cos\varphi - \ell\omega\sin\varphi \Rightarrow \ddot{y}_2 = -\ell\omega^2\sin\varphi - \ell\omega^2\cos\varphi$

$$-\frac{1}{2}m\ell\omega^2\sin\varphi - 2m\ell\omega^2\sin\varphi - 2m\ell\omega^2\cos\varphi = 3mg + N_y$$

$$\boxed{N_y = -3mg - 2m\ell\omega^2\cos\varphi - \frac{5}{2}m\ell\omega^2\sin\varphi}$$

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(10.6)



OD: m, ℓ

AB: m, ℓ

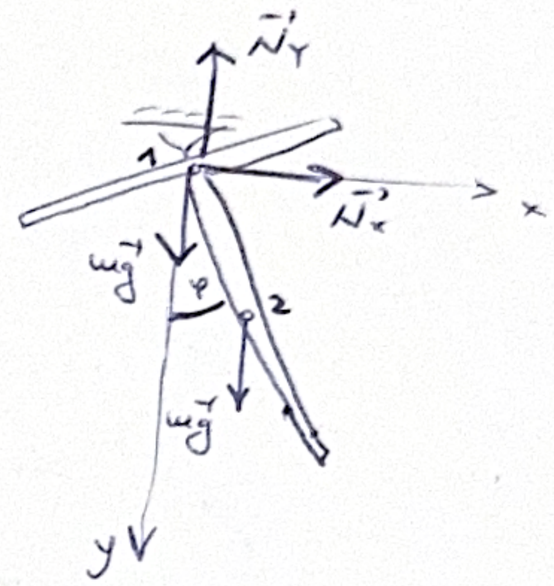
$\varepsilon(\varphi)$? $\omega(\varphi)$? $N(\varphi)$?

t₀: ravnotežni položaj

$\omega_0 = \sqrt{\frac{12}{5}\frac{g}{\ell}}$

Napomena: opet vrste na vrlo nadinu, potro o prethodnom zadatku nismo koristili treći nadin, sada ćemo.

$$\frac{dT}{dt} = \frac{d'A}{dt}$$



$$T = \left[0 + \frac{1}{2} J_1 \omega^2\right] + \left[\frac{1}{2} m v_1^2 + \frac{1}{2} J_2 \omega^2\right] =$$

$$T = \left[\frac{1}{2}\frac{1}{12} m\ell^2\omega^2\right] + \left[\frac{1}{2} m \left(\frac{1}{2}\omega\ell\right)^2 + \frac{1}{2}\frac{1}{12} m\ell^2\omega^2\right] =$$

$$T = \frac{1}{24} m\ell^2\omega^2 + \frac{1}{8} m\ell^2\omega^2 + \frac{1}{24} m\ell^2\omega^2 =$$

$$T = \frac{5}{24} m\ell^2\omega^2$$

$$\frac{dT}{dt} = \frac{5}{12} m\ell^2\omega\varepsilon$$

$$d'A = 0 + mg\,dy_2 = -\frac{1}{2} mg\ell\sin\varphi\,d\varphi$$

$$y_2 = \frac{1}{2}\ell\cos\varphi \quad \Rightarrow \quad \frac{dy_2}{dt} = -\frac{1}{2}\ell\sin\varphi\frac{d\varphi}{dt}$$

$$\frac{d'A}{dt} = -\frac{1}{2} mg\ell\sin\varphi\cdot\omega$$

$$\frac{5}{12} m\ell^2\omega\varepsilon = -\frac{1}{2} mg\ell\sin\varphi$$

$$\boxed{\varepsilon = -\frac{6}{5}\frac{g}{\ell}\sin\varphi}$$

$$\varepsilon = \frac{d\omega}{dt}\frac{d\varphi}{d\varphi} = \frac{d\omega\cdot\omega}{d\varphi} = -\frac{6}{5}\frac{g}{\ell}\sin\varphi$$

$$\omega\,d\omega = -\frac{6}{5}\frac{g}{\ell}\sin\varphi\,d\varphi \qquad / \int dt$$

$$\frac{1}{2}\omega^2 = \frac{6}{5}\frac{g}{\ell}\cos\varphi + C_1$$

$$\omega = \sqrt{\frac{12}{5}\frac{g}{\ell}\cos\varphi + C_1}$$

$$\omega(t_0 = 0) = \sqrt{\frac{12}{5}\frac{g}{\ell}\cdot 1 + C_1} = \sqrt{\frac{12}{5}\frac{g}{\ell}} \quad \Rightarrow \quad \underline{C_1 = 0}$$

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$$\boxed{\omega = \sqrt{\frac{12}{5}\,\frac{g}{\ell}}}$$

$$M\vec{a}_C = m\vec{g} + m\vec{g} + \vec{N}_x + \vec{N}_y$$

$$m\vec{a}_1 + m\vec{a}_2 = m\vec{g} + m\vec{g} + \vec{N}_x + \vec{N}_y$$

x: $\quad m\ddot{x}_1 + m\ddot{x}_2 = N_x$

$$\boxed{\begin{aligned} x_1 &= 0 \quad \Rightarrow \quad \dot{x}_1 = 0 \quad \Rightarrow \quad \ddot{x}_1 = 0 \\ x_2 &= \tfrac{1}{2}\ell\sin\varphi \quad \Rightarrow \quad \dot{x}_2 = +\tfrac{1}{2}\ell\omega\cos\varphi \quad \Rightarrow \quad \ddot{x}_2 = -\tfrac{1}{2}\ell\omega^2\sin\varphi \end{aligned}}$$

$$\boxed{N_x = -\frac{1}{2}m\ell\omega^2\sin\varphi}$$

y: $\quad m\ddot{y}_1 + m\ddot{y}_2 = mg + mg - N_y$

$$\boxed{\begin{aligned} y_1 &= 0 \quad \Rightarrow \quad \dot{y}_1 = 0 \quad \Rightarrow \quad \ddot{y}_1 = 0 \\ y_2 &= \tfrac{1}{2}\ell\cos\varphi \quad \Rightarrow \quad \dot{y}_2 = -\tfrac{1}{2}\ell\omega\sin\varphi \quad \Rightarrow \quad \ddot{y}_2 = -\tfrac{1}{2}\ell\omega^2\cos\varphi \end{aligned}}$$

$$0 - \frac{1}{2}m\ell\omega^2\cos\varphi = 2mg - N_y$$

$$\boxed{N_y = 2mg + \frac{1}{2}m\ell\omega^2\cos\varphi}$$

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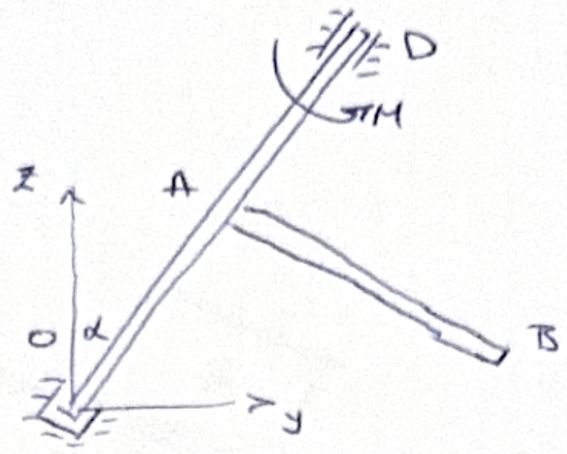
10.7.

AB : m, ℓ

$\alpha = 30^\circ$

$M = \text{const.}$

$\varepsilon(\varphi) = ?$



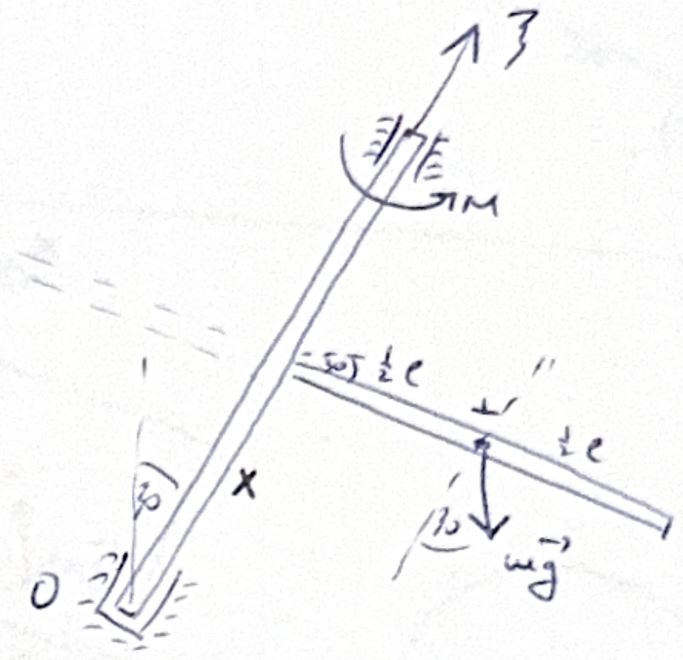
$$\frac{dT}{dt} = \frac{d'A}{dt}$$

$$T = \frac{1}{2} m v_1^2 + \frac{1}{2} J_1 \omega^2 =$$

$$T = \frac{1}{2} m \left(\frac{1}{2}\ell\omega\right)^2 + \frac{1}{2}\frac{1}{12} m\ell^2\omega^2 =$$

$$T = \frac{1}{8} m\ell^2\omega^2 + \frac{1}{24} m\ell^2\omega^2 =$$

$$T = \frac{1}{6} m\ell^2\omega^2$$



$$\frac{dT}{dt} = \frac{1}{3} m\ell^2\omega\varepsilon$$

$$d'A = -mg\,dz_\perp + M\,d\varphi = +\frac{1}{2} mg\ell\cos\varphi\,d\varphi + M\,d\varphi$$

$$z_1 = x - \frac{1}{2}\ell\sin 30\sin\varphi \Rightarrow \quad \frac{dz_1}{dt} = -\frac{1}{4}\ell\cos\varphi\frac{d\varphi}{dt}$$

$$\frac{d'A}{dt} = \frac{1}{4} mg\ell\cos\varphi\cdot\omega + M\omega$$

$$\frac{1}{3} m\ell^2\alpha\varepsilon = \frac{1}{4} mg\ell\cos\varphi\,\alpha + M\alpha$$

$$\boxed{\varepsilon = \frac{3}{4}\frac{g}{\ell}\cos\varphi + \frac{3M}{m\ell^2}}$$

[redacted]

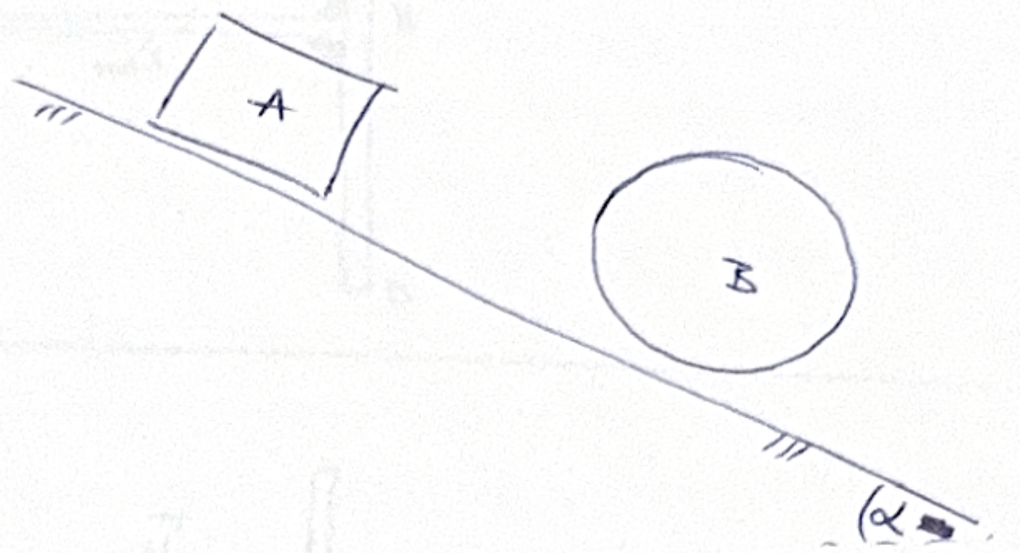
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10. 8.

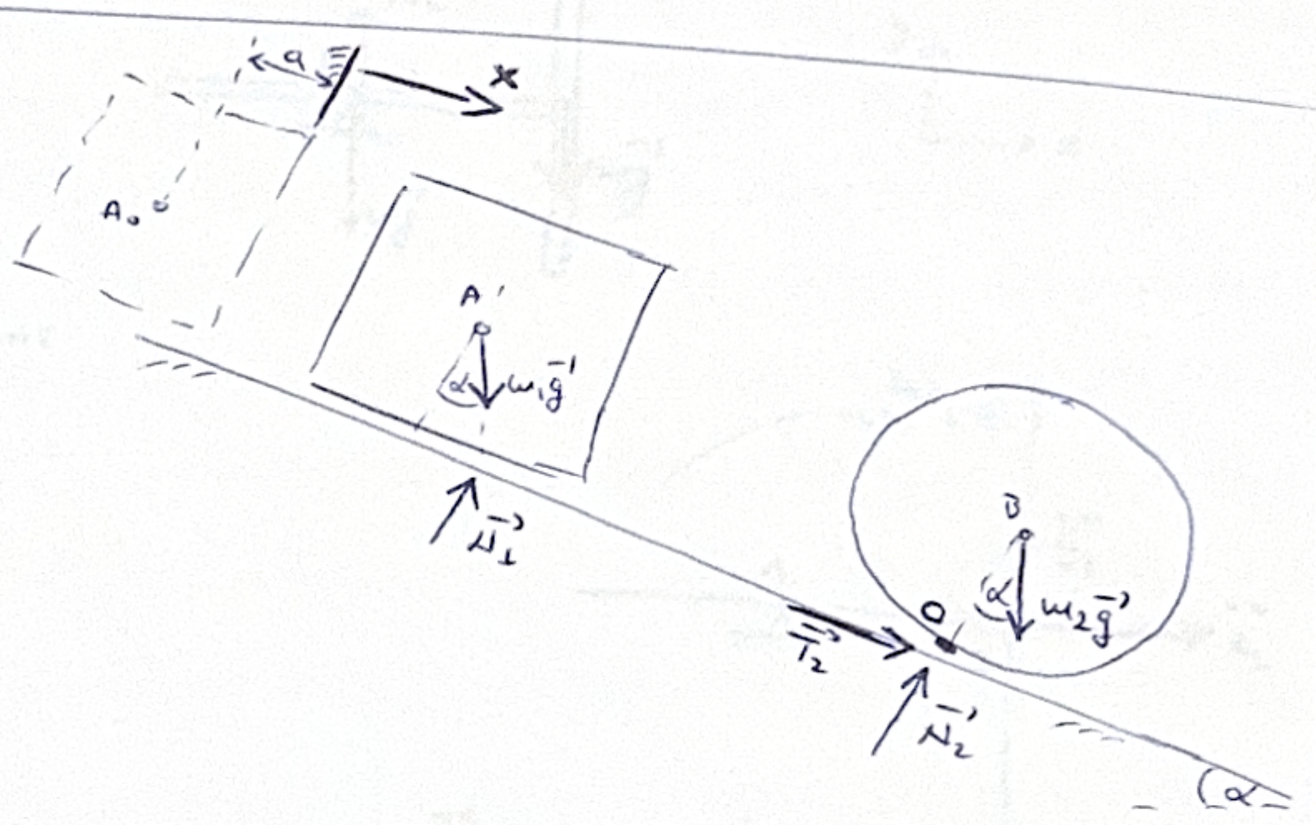
A: klizna bez trenja

B: kotrlja se bez klizanja (cilindar)

t_0: najkraće rastojanje između A i B je e



$t_1 = ?$ – trenutak kada A sustigne B



telo 1

$m_1 \vec{a}_1 = m_1 \vec{g} + \vec{N}_1$

x: $m_1 \ddot{x}_1 = m_1 g \sin\alpha \quad / \int dt$

$\dot{x}_1 = g \sin\alpha \cdot t + \cancel{c_1^0} \quad / \int dt$

$x_1 = \frac{1}{2} g \sin\alpha \cdot t^2 + \cancel{c^0}, \quad c_2 = -q$

$x_1 = \frac{1}{2} g \sin\alpha \cdot t^2 - q$

telo 2

$\frac{d\vec{L}_{O2}}{dt} = \sum \vec{M}_{O2}$ – na ovaj način ćemo izbeći pojavljivanje nepoznate sile T, da smo radili ovo za čvrsto telo onda bi se pojavila T_1 pa bi morali još jednu jednačinu da uvedemo (drugi Njutnov zakon)

$\sum M_{O2} = m_2 g \sin\alpha \cdot R$

$L_{O2} = R \cdot m_2 v_2 + J_2 \omega = R \cdot m_2 R\omega + m_2 R^2 \omega = 2 m_2 R^2 \omega$

$\frac{dL_{O2}}{dt} = 2 m_2 R^2 \varepsilon$

$2 \cancel{m_2} \cancel{R} \varepsilon = \cancel{m_2} g \sin\alpha \cdot \cancel{R}$

$2 \cdot \ddot{x}_2 = g \sin\alpha$

$\ddot{x}_2 = \frac{1}{2} g \sin\alpha \quad / \int dt$

$\dot{x}_2 = \frac{1}{2} g \sin\alpha \cdot t + \cancel{c_1^0} \quad / \int dt$

$x_2 = \frac{1}{4} g \sin\alpha \cdot t^2 + c_2, \quad c_2 = e + R$

$x_2 = \frac{1}{4} g \sin\alpha \cdot t^2 + e + R$

~~trenutak kada~~.

Za trenutak t_1 važi: $x_1(t_1) + q = x_2(t_1) - R$

$\frac{1}{2} g \sin\alpha \cdot t^2 - q + q = \frac{1}{4} g \sin\alpha \cdot t^2 + e + R - R$

$\frac{1}{4} g \sin\alpha \cdot t^2 = e$

$$t = \sqrt{\frac{4e}{g \sin\alpha}}$$

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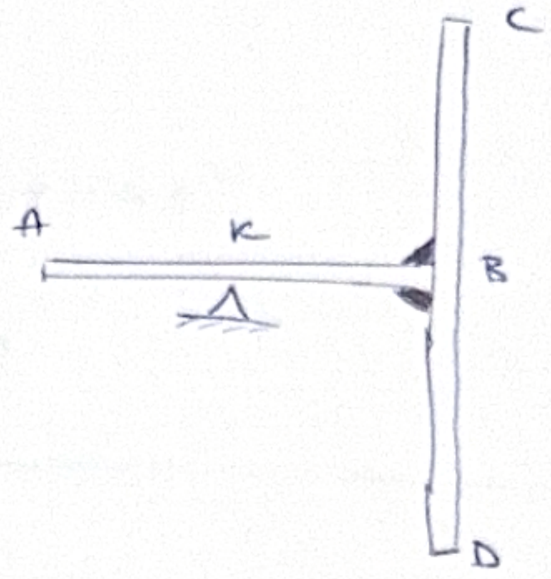
10.9.

AB: m, ℓ

CD: m, ℓ

t_0: AB je horizontalan sistem miruje

Štap AB je samo nasloncem na oslonac

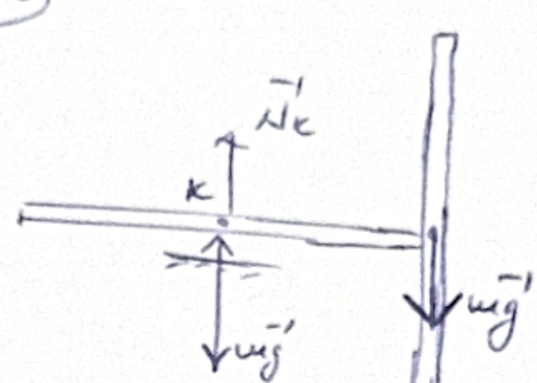


$N_K(t_0) = N_K = ?$

treće zanemarti

---

trenutak t_0



Posmatramo samo pokretni trenutak t_0:

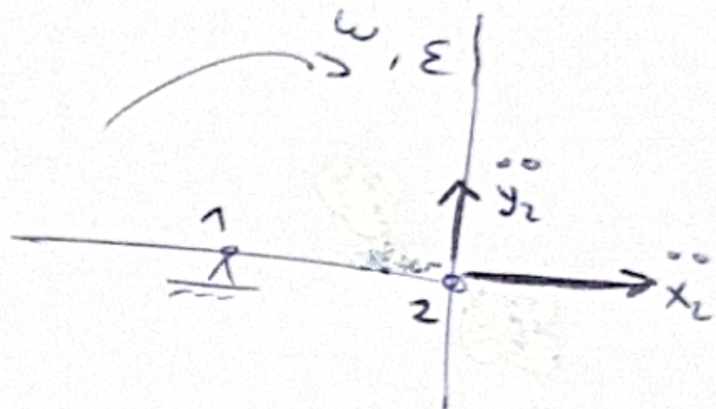
$$\underrightarrow{M\bar{a}_c' = m\bar{g}' + m\bar{g}' + \bar{N}_K'}$$

y: $M\ddot{y}_c = -2mg + N_K$

$m_1 \ddot{y}_1 + m_2 \ddot{y}_2 = -2mg + N_K$

$N_K = m_2 \ddot{y}_2 + 2mg$

$\underline{N_K = 2mg - \frac{1}{2} m\ell\varepsilon}$ (1) ($\mu_K = ?$, $\varepsilon = ?$)



$\ddot{x}_2 = -a_{2n} + \frac{d\vartheta_K}{dt} = \ldots$ (ne treba nam)

$\ddot{y}_2 = -a_{2t} = -\frac{1}{2}\ell\varepsilon$

$$\frac{d\vec{L}_{Kz}}{dt} = \sum \vec{M}_{Kz}$$

$\sum M_{Kz} = \frac{1}{2} mg\ell$

$L_{Kz} = [0 + J_1 \cdot \omega] + [\frac{1}{2}\ell \cdot m\omega\, \ell_2 + J_2\omega] = [\frac{1}{12} m\ell^2\omega] + [\frac{1}{2} m\ell \cdot \frac{\omega\ell}{2} + \frac{1}{12} m\ell^2\omega] = \frac{5}{12} m\ell^2\omega$

$\frac{dL_{Kz}}{dt} = \frac{5}{12} m\ell^2\varepsilon$

$\frac{5}{12} m\ell^2\varepsilon = \frac{1}{2} mg\ell$

$\underline{\varepsilon = \frac{6}{5}\frac{g}{\ell}}$

(1) $\Rightarrow$ $N_K = 2mg - \frac{1}{2} m\ell \cdot \frac{6}{5}\frac{g}{\ell}$

$\Rightarrow$ $\boxed{N_K = \frac{7}{5} mg}$

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(10.10)

Podatki: $m = 2\ kg$, $R = 0{,}5\ m$

$x_c = 5\cos 2t$, $y_c = 5\sin 2t$, $\varphi = 4t^2$

$x_R^S = ?$, $Y_R^S = ?$, $M_{Ct}^S = ?$

$\overrightarrow{m\,a_c = X_R^S + Y_R^S}$

x: $m\ddot{x}_c = X_R^S$

$m\cdot(-20\cdot\cos 2t) = X_R^S$

$$\boxed{X_R^S = -40\cos 2t \quad [N]}$$

y: $m\ddot{y}_c = Y_R^S$

$m(-20\sin 2t) = Y_R^S$

$$\boxed{Y_R^S = -40\sin 2t \quad [N]}$$

$\overrightarrow{\frac{dL_C}{dt} = \Sigma M_C}$

$\Sigma M_C = M_{Ct}^S$

$L_{Ct} = mR^2\cdot\omega$

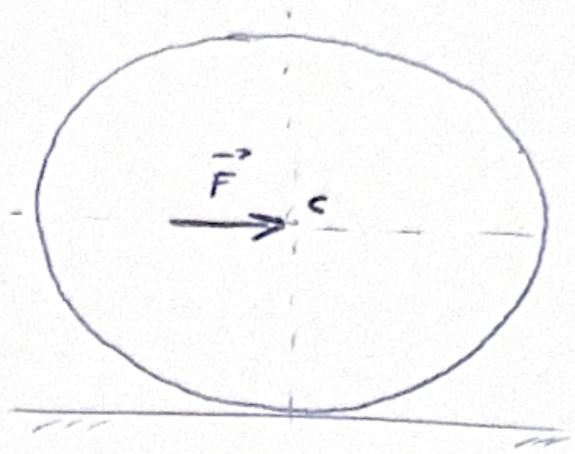
$\frac{dL_{Ct}}{dt} = mR^2\varepsilon$

$\Rightarrow mR^2\varepsilon = M_{Ct}$

$$\boxed{M_{Ct} = 4 \quad [Nm]}$$

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10. 11.



Preko: m, R

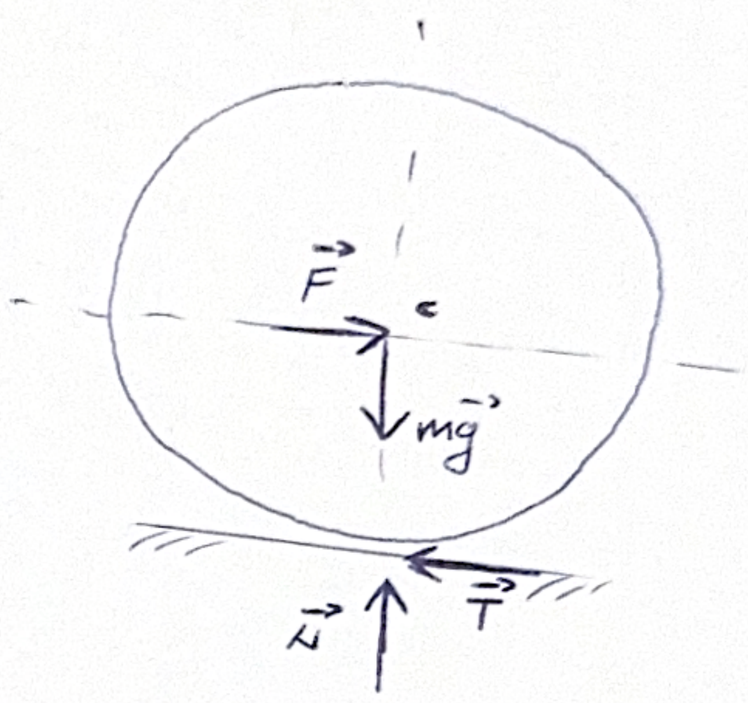
kotrljanje bez klizanja

$F = mg$

$t_0 = 0$: mirovanje

$x_c = ?$

$$m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N} + \vec{T}$$



x: $m\ddot{x}_c = F - T$

$m\ddot{x}_c = mg - T$ (1) $\left(\ddot{x}_c = ?,\ T = ?\right)$

y: ... (ne treba nam)

$$\frac{dL_{Cz}}{dt} = \Sigma M_{Cz}$$

$\Sigma M_{Cz} = T \cdot R$

$L_{Cz} = J\omega = mR^2\omega$

$\frac{dL_{Cz}}{dt} = mR^2\varepsilon$

$mR^2\varepsilon = T \cdot R$

$m\ddot{x}_c = T$

(1) $\Rightarrow$ $m\ddot{x}_c = mg - m\ddot{x}_c$

$\Rightarrow$ $\ddot{x}_c = \frac{1}{2}g$ //

$\Rightarrow$ $\dot{x}_c = \frac{1}{2}gt$ //

$\Rightarrow$ $\boxed{x_c = \frac{1}{4}gt^2}$

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10. 12

(nastavek na 11. zad)

$T = m\ddot{x}_c = \frac{1}{2} mg$ (1)

$T = \mu \cdot N$, $N = ?$ (2)

$m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N} + \vec{T}$

y: $0 = mg - N$

$N = mg$

(1) = (2)

$\frac{1}{2} mg = \mu \cdot N$

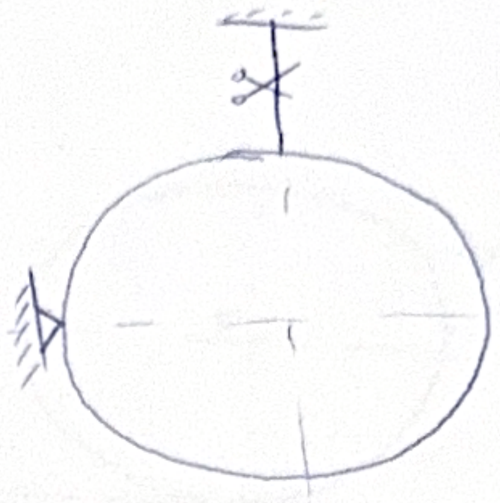
$\frac{1}{2} mg = \mu \cdot mg$

$\mu = \frac{1}{2}$ - graniční slučaj da ne dođe do proklizavanja

$\boxed{\mu_{min} = \frac{1}{2}}$

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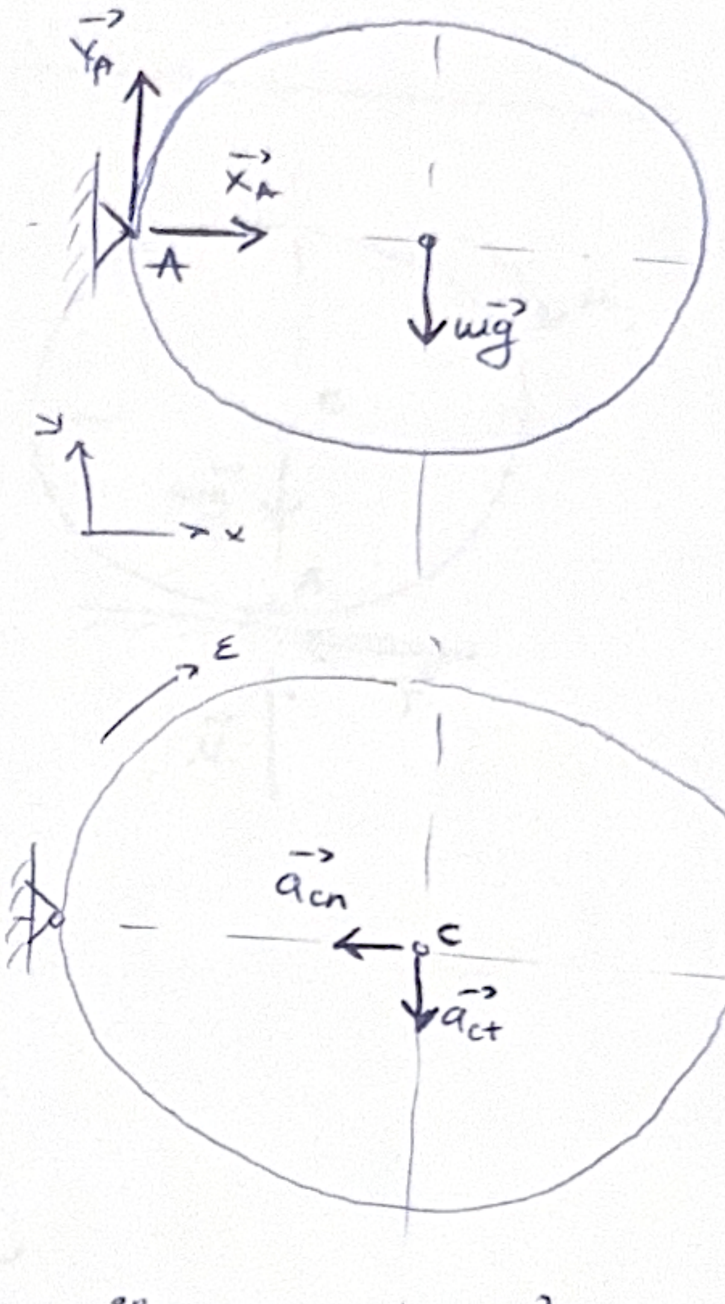
(10.13.)



disk: m, r

X_A? , Y_A? – neposredno nakon presecanja kanapa

Posmatramo sve za taj početni trenutak, kanap je presečen, a telo je mirovalo.



$$\vec{ma_c} = m\vec{g} + \vec{X_A} + \vec{Y_A}$$

x: $m\ddot{x}_c = X_A$, $\ddot{x}_c = 0$

$$\boxed{X_A = 0}$$

y: $m\ddot{y}_c = mg - Y_A$

$$Y_A = mg - mR\varepsilon \quad (1) \quad (\varepsilon = ?)$$

$$\frac{dL_{Az}}{dt} = \sum M_{Az}$$ – nećemo imati nepoznatih sila, a pojaviće se ε, pa ćemo ga izračunati. (mogli smo da raspišemo i za središte diska, dakle C_i i-nu su dve nepoznate, i sa prethodnom i-nom C_i to reši)

$$\sum M_{Az} = mg \cdot r$$

$$L_{Az} = r \cdot m v_c + J_c \omega = r m r \omega + \frac{1}{2} m r^2 \omega = \frac{3}{2} m r^2 \omega$$

$$\frac{dL_{Az}}{dt} = \frac{3}{2} m r^2 \varepsilon$$

$$\frac{3}{2} m r^2 \varepsilon = mgr$$

$$\varepsilon = \frac{2}{3} \frac{g}{r}$$

$$\ddot{x}_c = -a_{cn} = -\frac{v^2}{R} = 0$$

$$\ddot{y}_c = -a_{ct} = -\varepsilon \cdot R$$

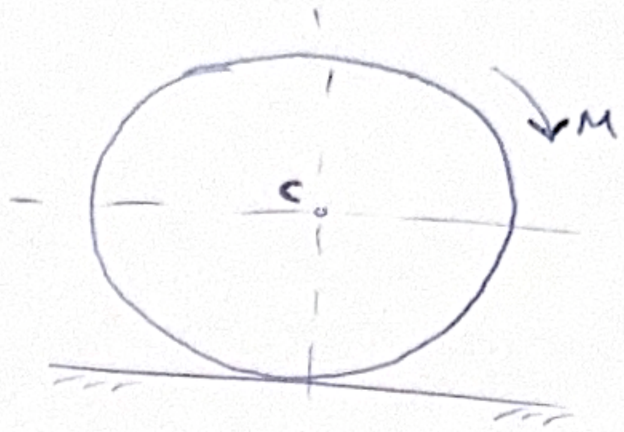
(1) $\Rightarrow$ $Y_A = mg - m \cdot \varepsilon \cdot r$

$\Rightarrow$ $Y_A = mg - \frac{2}{3} m \cdot \frac{g}{r} r$

$\Rightarrow$ $\boxed{Y_A = \frac{1}{3} mg}$

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(10. 14.)



disk: m, r, kotrljanje bez klizanja

$M = \text{const.}$

$t_0 = 0: \quad v_{c0} = 0$

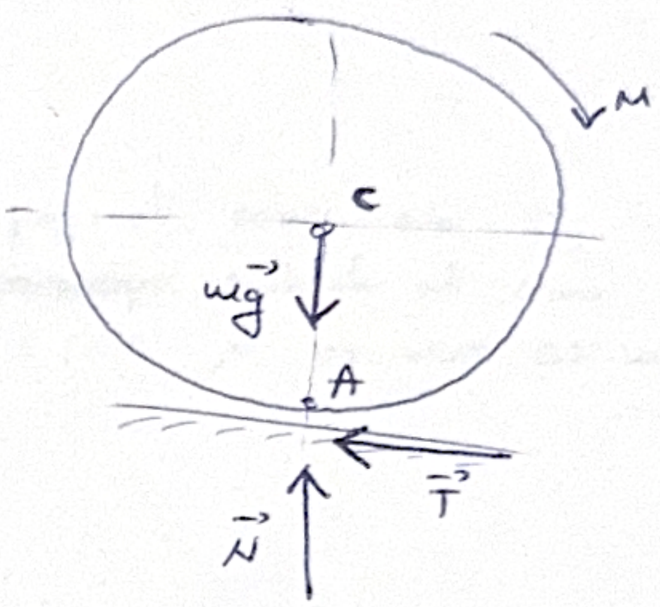
$x_c = ?$

Imamo dva načina:

1) $\frac{dL_{Cz}}{dt} = \sum M_{Cz}$, - pojaviće se dve reperuske (ε, T), pa ćemo moratí i drugi vektorski z. da raspišemo + još jedan i-ni

$+ \text{II Ni. Z.}$

2) $\frac{dL_{Az}}{dt} = \sum M_{Az}$ - pojaviće se samo jedna reperuska (ε)



Primetijemo da je drugi način brži:

$$\frac{dL_{Az}}{dt} = \sum M_{Az}$$

$$\sum M_{Az} = M$$

$$L_{Az} = r \cdot m v_c + J_c \cdot \omega = r \cdot \omega \cdot r\omega + \frac{1}{2} m r^2 \omega = \frac{3}{2} m r^2 \omega$$

$$\frac{dL_{Az}}{dt} = \frac{3}{2} m r^2 \varepsilon$$

$$\frac{3}{2} m r^2 \varepsilon = M$$

$$\varepsilon = \frac{2}{3} \frac{M}{m r^2}$$

$$\ddot{x}_c = r \cdot \varepsilon \Rightarrow \ddot{x}_c = \frac{2}{3} \frac{M}{mr} \quad //$$

$$\dot{x}_c = \frac{2}{3} \frac{M}{mr} t + \cancel{\varepsilon_L}^{\,0} \quad //$$

$$x_c = \frac{1}{3} \frac{M}{mr} t^2$$

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10.15

(nastavak na 14.)

$$m\vec{a}_c = m\vec{g} + \vec{N} + \vec{T}$$

y: $\quad 0 = mg - N$

$$N = mg$$

x: $\quad m\ddot{x}_c = -T$

$$\frac{2}{3}\frac{M}{r} = +\mu_0 N$$

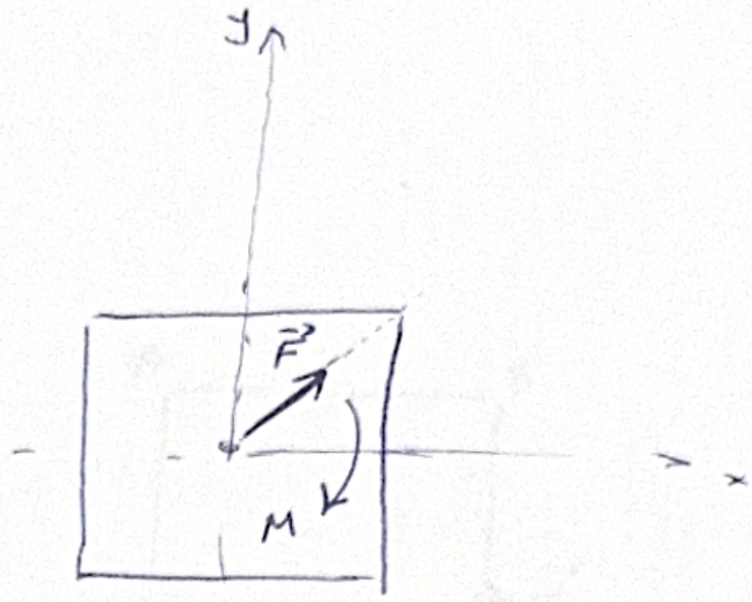
$$\frac{2}{3}\frac{M}{r} = \mu_0 mg$$

$$\boxed{\mu_{min} = \frac{2}{3}\frac{M}{mgr}}$$

- očigledno je da smo pogrešili smer sile T, pa ćemo nastaviti zadatak ali ćemo okrenuti smer (μ ne može biti negativno)

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(10.16.)



kvadratna ploča

$m = 1\ kg$, $\ell = 1\ m$ - dužina stranice

homogeno telo ravan

to: miruje, položaj na slici

$M = 2\ Nm$, $F = 5\sqrt{2}\ N$

$x_c(t_1)$? $y_c(t_1)$? $\varphi(t_1)$?

$t_1 = 2\ s$

$\underline{m\vec{a}_c = \vec{F}}$

x: $m\ddot{x}_c = F\frac{\sqrt{2}}{2}$

$1 \cdot \ddot{x}_c = 5\cancel{\sqrt{2}}\frac{\sqrt{2}}{2}$ /S

$\dot{x}_c = 5t + \cancel{C_1}^{0}$ /S

$x_c = 2{,}5t^2 + \cancel{C_2}^{0}$

$\boxed{x_c(t_1 = 2s) = 10\ [m]}$

---

y: $m\ddot{y}_c = F\frac{\sqrt{2}}{2}$

$\ddot{y}_c = \dot{y}_c = y_c = 0$

$\boxed{y_c(t_1 = 2s) = 10\ [m]}$

$\underline{\frac{d\vec{L}_{Cz}}{dt} = \Sigma\vec{M}_C}$

$\Sigma M_C = M$

$L_C = 0 + J_C\omega = \frac{1}{6}m\ell^2\omega$

$\frac{dL_C}{dt} = \frac{1}{6}m\ell^2\varepsilon$

$\frac{1}{6}m\ell^2\varepsilon = M$

$\varepsilon = \frac{6M}{m\ell^2} = \frac{6 \cdot 2}{1 \cdot 1^2} = 12$ /S

$\omega = 12t + \cancel{C_1}^{0}$ /S

$\varphi = 6t^2 + \cancel{C_2}^{0}$ $\Rightarrow$ $\boxed{\varphi(t_1 = 2s) = 24\ rad}$

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(10. 17.)

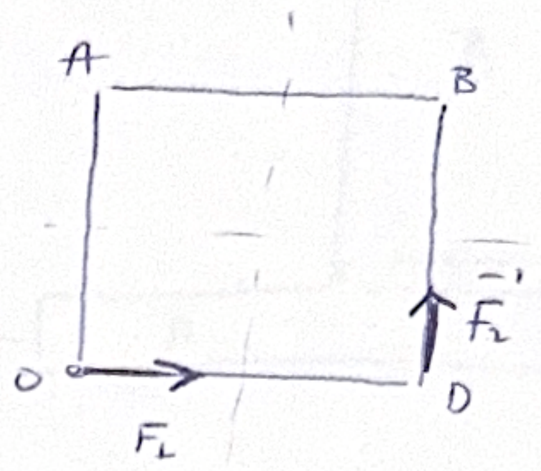
kvadratna ploča

m, a - stranica

F_1, F_2 - pravci se poklapaju sa stranicama OD i DB, respektivno (const. sile)

to: ploča miruje, položaj na slici

$\ddot{x}_c$? $\ddot{y}_c$?



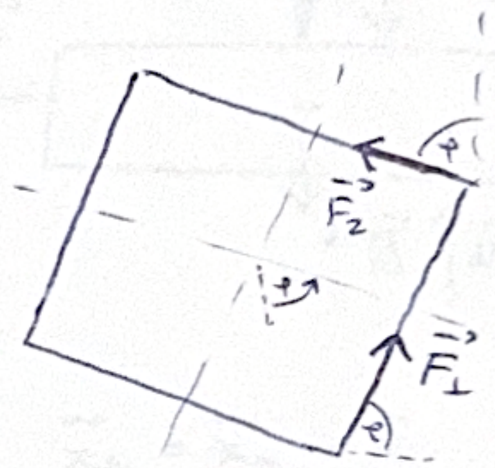
horiz. revan

$$m\vec{a}_c = \vec{F}_1 + \vec{F}_2$$

x: $m\ddot{x}_c = F_1 \cos\varphi - F_2 \sin\varphi$ (1)

y: $m\ddot{y}_c = F_1 \sin\varphi + F_2 \cos\varphi$ (2)

} +-?



$$\frac{d\vec{L}_{Ct}}{dt} = \sum \vec{M}_{Ct}$$

$$\sum M_{Ct} = F_1 \cdot \frac{1}{2}a + F_2 \cdot \frac{1}{2}a = \frac{1}{2}a(F_1 + F_2)$$

$$L_{Ct} = 0 + J_c\omega = \frac{1}{6}ma^2\omega$$

$$\frac{dL_{Ct}}{dt} = \frac{1}{6}ma^2\varepsilon$$

$$\frac{1}{6}ma^2\varepsilon = \frac{1}{2}a(F_1 + F_2)$$

$$\varepsilon = \frac{3(F_1 + F_2)}{ma} \quad //$$

$$\omega = \frac{3(F_1 + F_2)}{ma}t + C_1^{\,0} \quad //$$

$$\varphi = \frac{3}{2}\frac{(F_1 + F_2)}{ma}t^2 + C_2^{\,0}$$

(1) =>
$$\ddot{x}_c = \frac{1}{m}\left(F_1 \cos\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right) - F_2 \sin\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right)\right)$$

(2) =>
$$\ddot{y}_c = \frac{1}{m}\left(F_1 \sin\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right) + F_2 \cos\left(\frac{3}{2}\frac{F_1+F_2}{ma}t^2\right)\right)$$

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10. 20

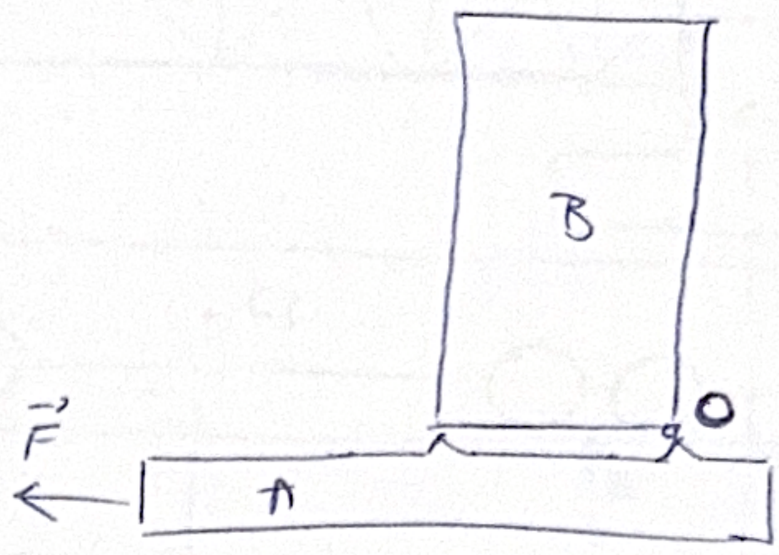
A: $2m$

B: m, $a \times 2a$

$F = kt$

t_0: sistem miruje, stranica a je horiz.

$t_1 = ?$ – trenutak kada B počinje da se obrće i-u kretanja tela A do t_1?



uslov za t_1: $\underline{N_1(t_1) = 0}$

=> posmatramo opšti trenutak t, nađemo $N_1(t)$, izjednačimo sa nulom : dobijemo t_1

$\underline{m\vec{a}_B = m\vec{g} + \vec{N}_1 + \vec{X}_1 + \vec{Y}_1}$

x: $\underline{m\ddot{x} = X_1}$ (1)

y: $\underline{0 = -mg + N_1 + Y_1}$ (2)

$\underline{\frac{dL_{Oz}}{dt} = \Sigma M_{Ot}}$

$\Sigma M_{Ot} = N_1 \cdot a - mg \cdot \frac{1}{2}a$

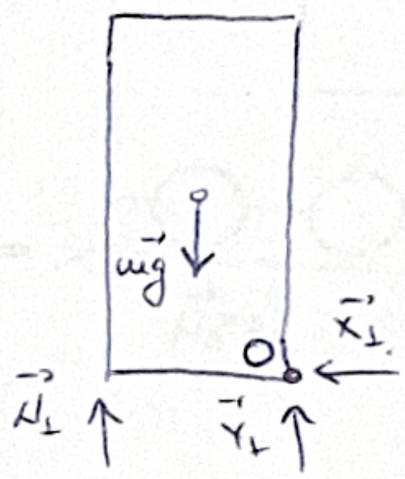
$L_{Ot} = a \cdot m\dot{x} + 0$

$\frac{dL_{Ot}}{dt} = -ma\ddot{x}$

$-ma\ddot{x} = N_1 \cdot a - \frac{1}{2}mga$

$\underline{N_1 = \frac{1}{2}mg - m\ddot{x}}$, $\ddot{x}(t) = ?$

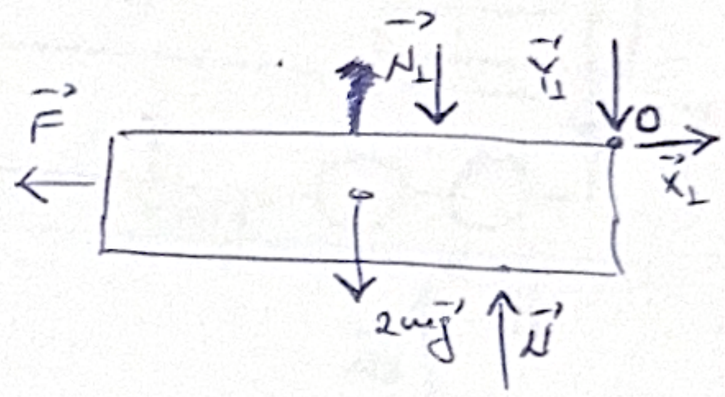
telo B



kinematska veza: $\underline{\ddot{x}_A = \ddot{x}_B = \ddot{x}}$

(t)

telo A



(3) => $N_1 = \frac{1}{2}mg - \frac{1}{3}kt$

$N_1(t_1) = \frac{1}{2}mg - \frac{1}{3}kt_1 = 0$

=> $\boxed{t_1 = \frac{3}{2}\frac{mg}{k}}$

$\underline{2m\vec{a}_A = 2m\vec{g} + \vec{F} + \vec{N} + \vec{N}_1 + \vec{X}_1 + \vec{Y}_1}$

y: – — (nepotrebno)

x: $2m\ddot{x} = F - X_1$

$2m\ddot{x} = kt - m\ddot{x}$

$\underline{\ddot{x} = \frac{1}{3}\frac{k}{m}t}$ //

$\dot{x} = \frac{1}{6}\frac{k}{m}t^2 + \cancel{c_1}^{\,0}$ //

$\boxed{x = \frac{1}{18}\frac{k}{m}t^3}$

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(10.21)

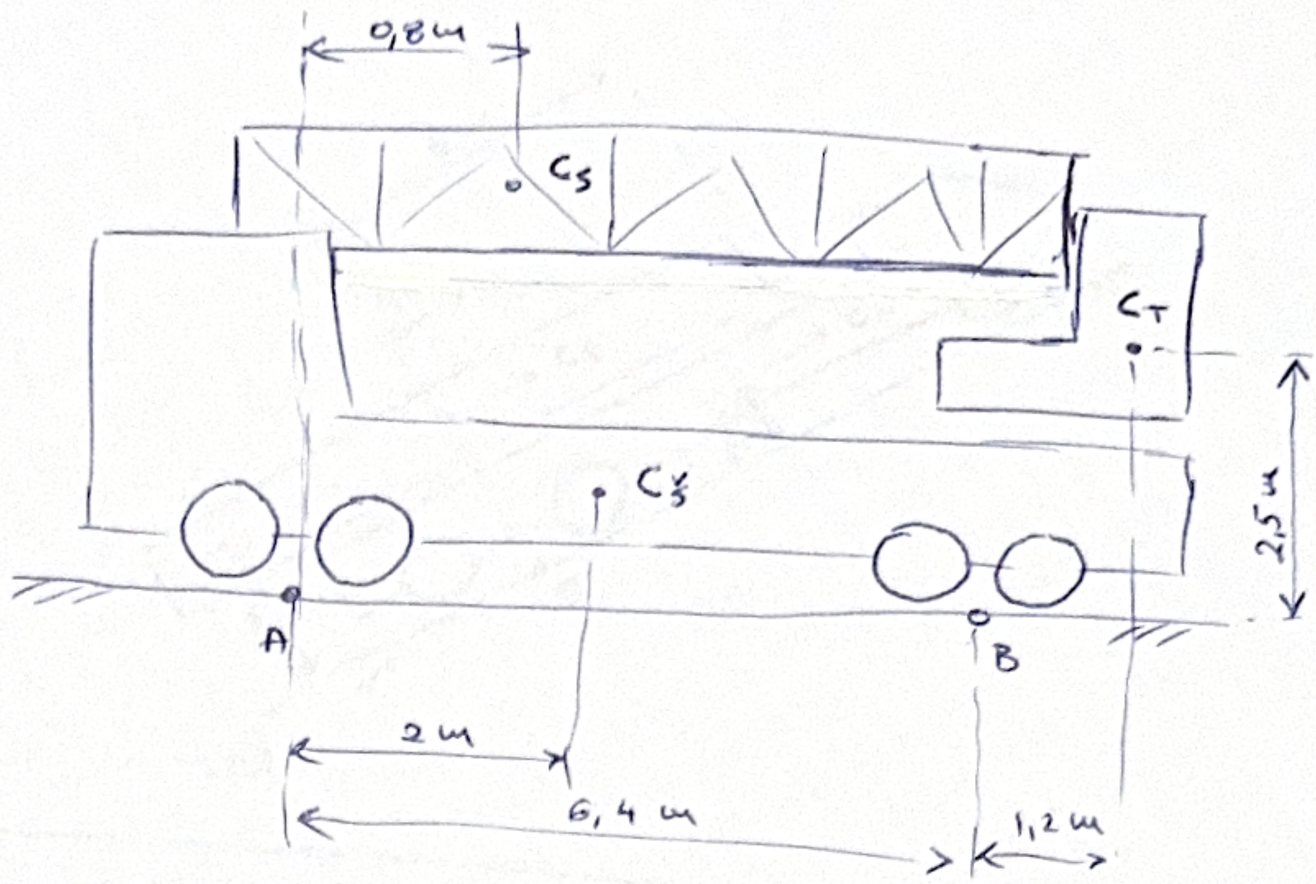
$M_t = ?$ - da se nađe dodatni moment prevrtanja

$m_s = 4\,t$

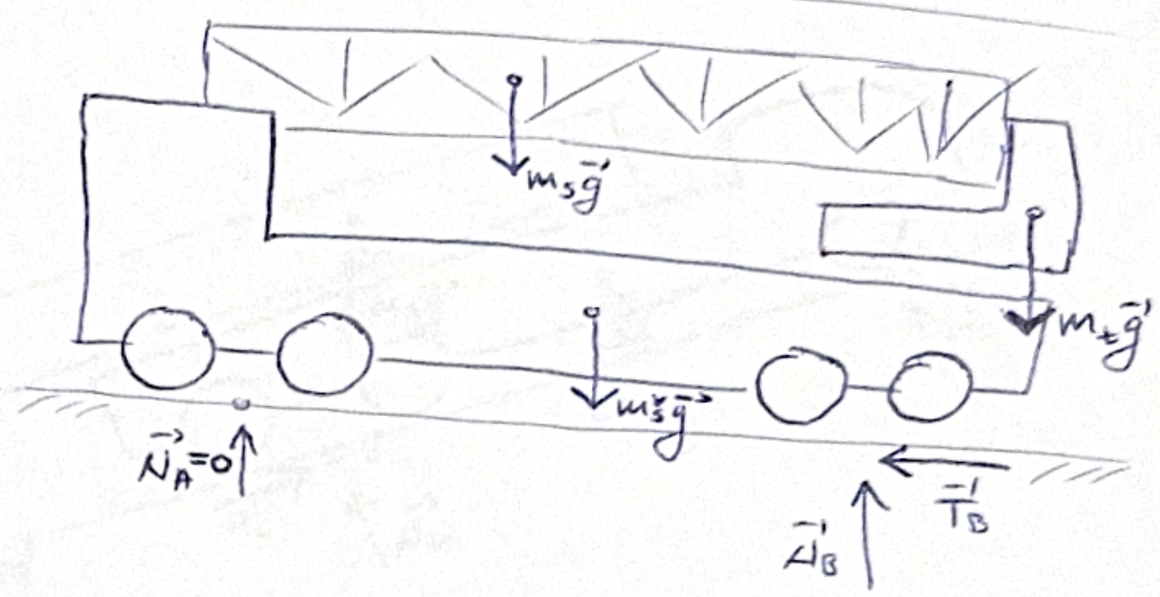
$m_s^{\check{z}} = 8\,t$

$a_{max} = 2\,m/s$

Pogon na zadnjim točkama



uslov da dodje do odvajanja: $N_A = 0$



$$\frac{d\vec{L}_B}{dt} = \sum \vec{M}_B$$

$$\sum M_B = \cancel{N_A \cdot 6{,}4}^{\,0} - m_s \cdot g \cdot 5{,}6 - m_s^{\check{z}} \cdot g \cdot 4{,}4 + m_t \cdot g \cdot 1{,}2 =$$

$$\sum M_B = + (-4 \cdot 5{,}6 - 8 \cdot 4{,}4 + m_t \cdot 1{,}2)\, g$$

$$\sum M_B = -(57{,}6 - m_t \cdot 1{,}2)\, g$$

$$L_B = [-3{,}8 \cdot m_s \cdot v + J_s \omega] + [-1{,}1 \cdot m_s^{\check{z}} \cdot v + J_s^{\check{z}} \cdot \omega] + [-2{,}5 \cdot m_t \cdot v + J_t \cdot \omega]$$

$$\frac{dL_B}{dt} = [-3{,}8 \cdot 4 \cdot a + \cancel{J_s \cdot \varepsilon}^{\,0}] + [-1{,}1 \cdot 8 \cdot a + \cancel{J_s^{\check{z}} \cdot \varepsilon}^{\,0}] + [-2{,}5\, m_t \cdot a + \cancel{J_t \cdot \varepsilon}^{\,0}]$$

$$\frac{dL_B}{dt} = -(24 + 2{,}5 \cdot m_t) \cdot a = -(24 + 2{,}5\, m_t) \cdot 2 = -(48 + 5\, m_t)$$

$$-(48 + 5\, m_t) = -(57{,}6 - m_t \cdot 1{,}2)\, g$$

$$48 + 5\, m_t = 57{,}6 \cdot g - m_t \cdot g \cdot 1{,}2$$

$$m_t (5 + 1{,}2 \cdot g) = 57{,}6 \cdot g - 48$$

$$m_t = \frac{57{,}6 \cdot g - 48}{5 + 1{,}2 \cdot g}$$

$$\boxed{m_t = 30\,829\ kg}$$

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(10.22.)

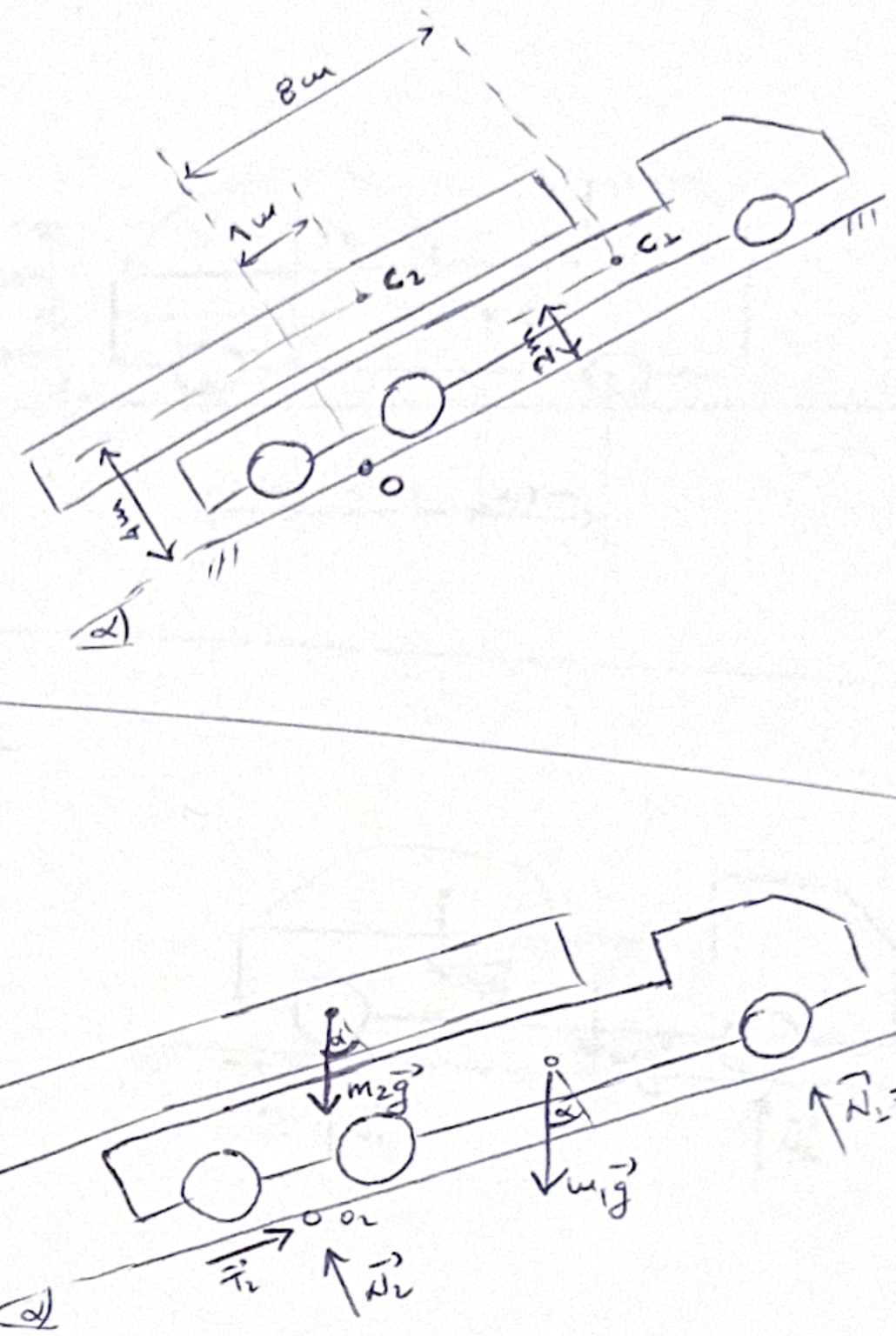
$m_1 = 25\ t$

$m_2 = 18\ t$

$\alpha = 15°$

pogon na zadnjim točkovima

$a_{max} = ?$ – da ne dođe do prevrtanja



uslov: $\ddot{\varphi}_1 = 0$

$$\frac{dL_{Oiz}}{dt} = \sum M_{Oiz}$$

$$\sum M_{Oiz} = -m_2 g \cos\alpha \cdot 1 + m_2 g \sin\alpha \cdot 4 - m_1 g \cos\alpha \cdot 8 + m_1 g \sin\alpha \cdot 2 =$$

$$\sum M_{Oiz} = -1740,716$$

$$L_{O2z} = [-4 \cdot m_2 v + J_2 \omega] + [-2 m_1 v + J_1 \omega]$$

$$\frac{dL_{Oiz}}{dt} = [-4 m_2 a_{max} + J_2 \varepsilon^{0}] + [-2 m_1 a_{max} + J_1 \varepsilon^{0}] =$$

$$\frac{dL_{Oiz}}{dt} = -122 \cdot a_{max}$$

$$-122 \cdot a_{max} = -1740,716$$

$$\boxed{a_{max} = 14,27\ [m/s^2]}$$

— približno tako računaj u zbirci

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(10.23.)

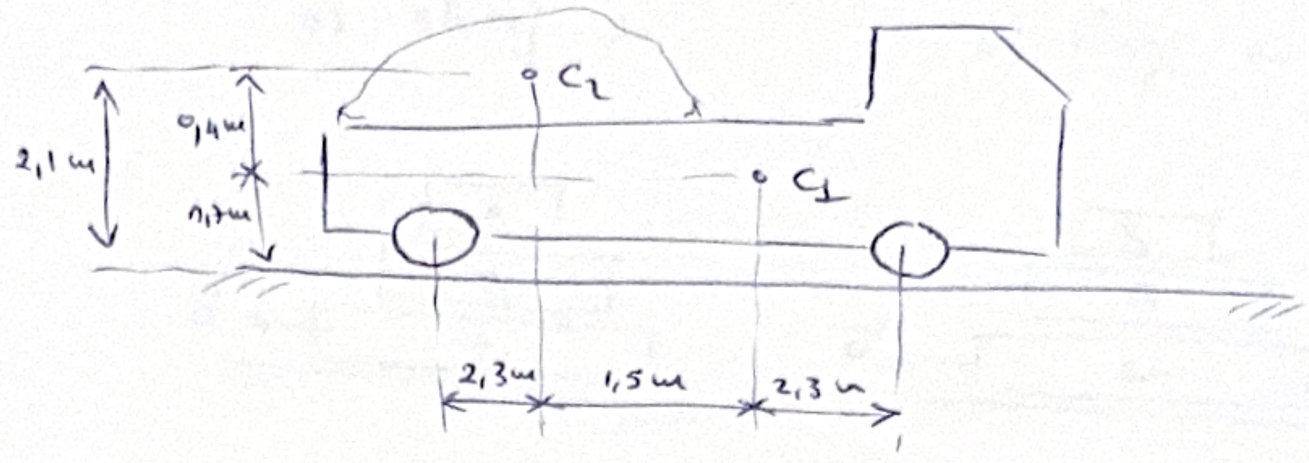
$a_{max} = ?$

m_1

$m_2 = m_1$

$\mu = 0,6$

pogon na zadnjim točkovima

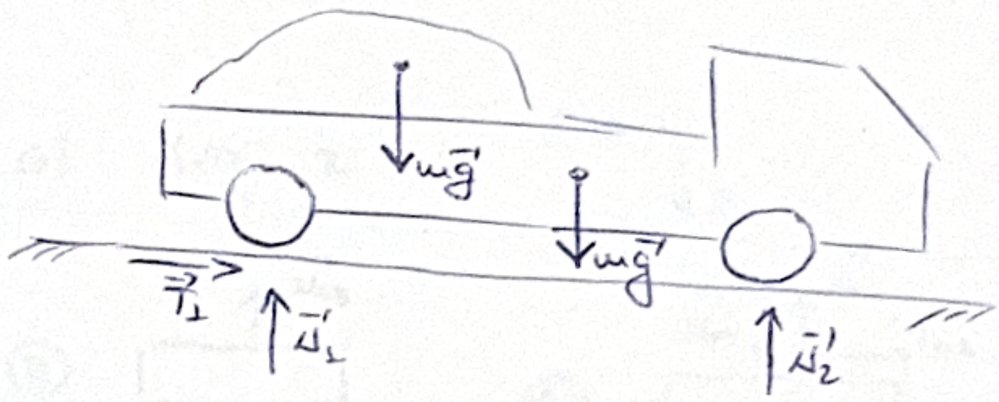


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granični uslov: $T_1 = N_1 \cdot \mu$ – tada je a_{max}

(kada je $a < a_{max}$, onda je $T_1 < N_1 \cdot \mu$)

$$m\vec{a}_c = \vec{N}_1 + \vec{T}_1 + \vec{N}_2 + m\vec{g} + m\vec{g}$$



x: $2m\ddot{x}_c = T_1$

$2m a_{max} = \mu \cdot N_1$ (1) $\begin{pmatrix} N_1 = ? \\ a_{max} = ? \end{pmatrix}$

y: $0 = N_1 + N_2 - mg - mg$ (2) $\begin{pmatrix} N_1 = ! \\ N_2 = ! \end{pmatrix}$

$$\frac{d\vec{L}_{1z}}{dt} = \sum \vec{M}_{1z}$$

$\sum M_{1z} = mg \cdot 2,3 + mg \cdot 3,8 - N_2 \cdot 6,1 = 6,1(mg - N_2) = 6,1(mg - 2mg + N_1) = 6,1(N_1 - mg)$

$L_{1z} = [1,7\, m v + J_1 \omega] + [2,1\, m v + J_2 \omega]$

$\frac{dL_{1z}}{dt} = 3,8 \cdot m \cdot a_{max} + J_1 \varepsilon^{0} + J_2 \varepsilon^{0}$

$3,8 \cdot m \cdot a_{max} = 6,1\,(N_1 - mg)$

$3,8\, m\, a_{max} = 6,1\left(\frac{2m a_{max}}{\mu} - mg\right) \quad / : m$

$3,8\, m\, a_{max} = \frac{6,1 \cdot 2}{0,6} \cdot a_{max} - 6,1 \cdot g$

$a_{max}\left(\frac{12,2}{0,6} - 3,8\right) = 6,1 \cdot g$

$a_{max} = \frac{6,1 \cdot g}{\frac{12,2}{0,6} - 3,8}$

$\boxed{a_{max} = 3,62\,[m/s^2]}$ – približno rešenje u zbirci

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10.25

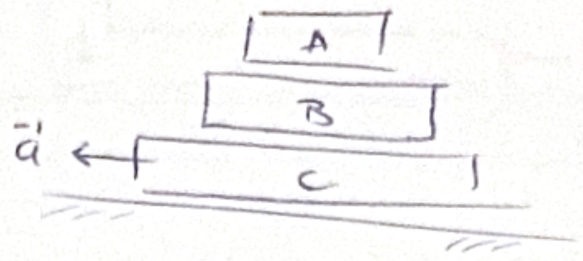
$a_{max} = ?$ - iz uslova da A : B miruju u odnosu na C

$\mu_{AB} = 0,4$, $\mu_{BC} = 0,3$, $\mu_{AC} = 0,5$

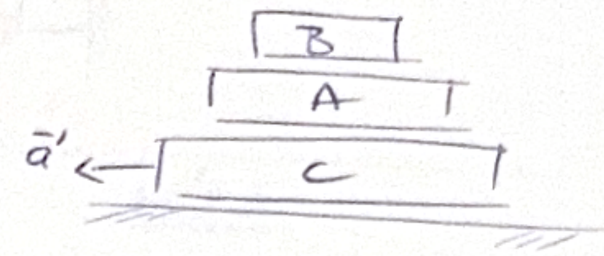
a) slučaj 1

b) slučaj 2

a) slučaj 1

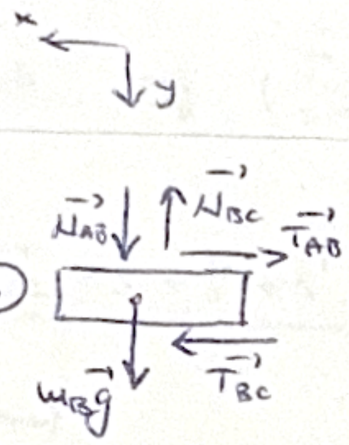


b) slučaj 2



kinematska veza: $a_A = a_B = a_C = a$

a) slučaj 1



$m_A \vec{a}_A = m_A\vec{g} + \vec{N}_{AB} + \vec{T}_{AB}$

y: $0 = m_A g - N_{AB} \Rightarrow N_{AB} = m_A g$

$\Rightarrow T_{AB} = \mu_{AB} \cdot m_A g$

x: $m_A a = T_{AB}$

$m_A a = \mu_{AB} m_A g$

$a_{max\,1} \leq \mu_{AB} \cdot g$

$a_{max\,1} \leq 0,4\, g$

$m_B \vec{a}_B = m_B\vec{g} + \vec{N}_{AB} + \vec{N}_{BC} + \vec{T}_{AB} + \vec{T}_{BC}$

y: $0 = m_B g + N_{AB} - N_{BC}$

$0 = m_B g + m_A g - N_{BC} \Rightarrow N_{BC} = (m_A + m_B) g$

$\Rightarrow T_{BC} = \mu_{BC}(m_A + m_B) g$

x: $m_B a = -T_{AB} + T_{BC}$

$m_B a = -m_A \cdot a + \mu_{BC}(m_A + m_B) g$

$(m_A + m_B) a = \mu_{BC}(m_A + m_B) g$

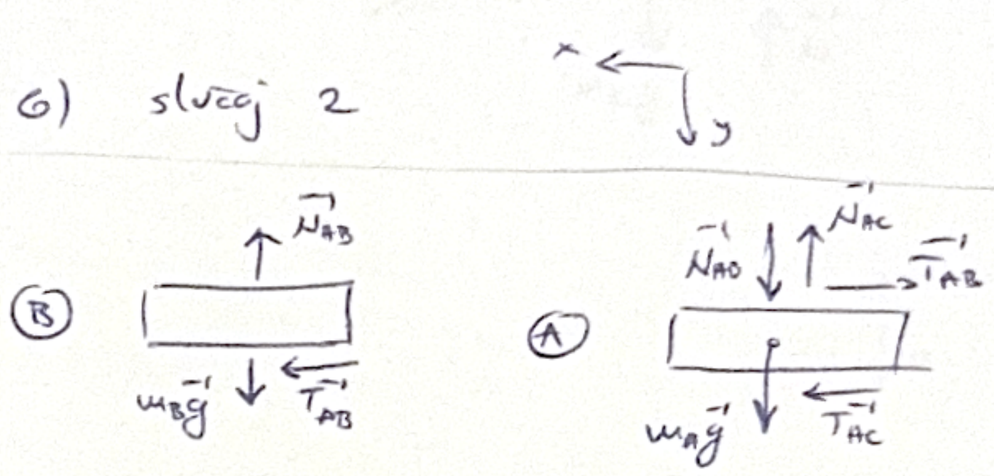
$a_{max\,2} \leq \mu_{BC} \cdot g$

$a_{max\,2} \leq 0,3\, g$

$a_{max} = \min(a_{max\,1}, a_{max\,2})$

$\boxed{a_{max} = 0,3\, g}$

b) slučaj 2



$m_B \vec{a}_B = m_B\vec{g} + \vec{N}_{AB} + \vec{T}_{AB}$

y: $0 = m_B g - N_{AB} \Rightarrow N_{AB} = m_B g$

$\Rightarrow T_{AB} = \mu_{AB} \cdot m_B g$

x: $m_B a = T_{AB}$

$m_B a = \mu_{AB} \cdot m_B \cdot g$

$a_{max\,1} = \mu_{AB} \cdot g$

$a_{max\,1} = 0,4 \cdot g$

$m_A \vec{a}_A = m_A\vec{g} + \vec{N}_{AB} + \vec{N}_{AC} + \vec{T}_{AB} + \vec{T}_{AC}$

y: $0 = m_A g + N_{AB} - N_{AC}$

$0 = m_A g + m_B g - N_{AC} \Rightarrow N_{AC} = (m_A + m_B) g$

$\Rightarrow T_{AC} = \mu_{AC}(m_A + m_B) g$

x: $m_A a = -T_{AB} + T_{AC}$

$m_A a = -m_B a + \mu_{AC}(m_A + m_B) g$

$(m_A + m_B) a = \mu_{AC}(m_A + m_B) g$

$a_{max\,2} \leq \mu_{AC} \cdot g$

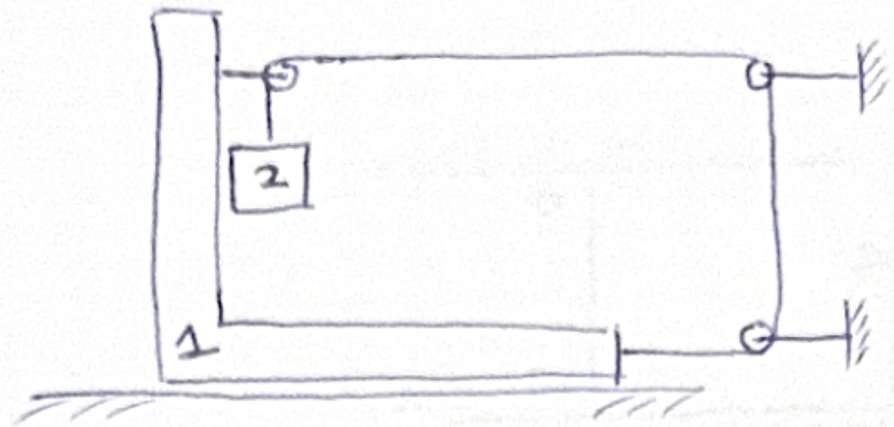
$a_{max\,2} = 0,5 \cdot g$

$a_{max} = \min(a_{max\,1}, a_{max\,2})$

$\boxed{a_{max} = 0,5\, g}$

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10.29



1: m_1

2: m_2

a_1 ? S?

---

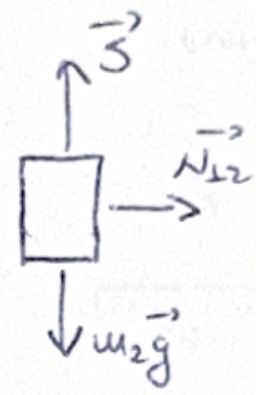
telo 2

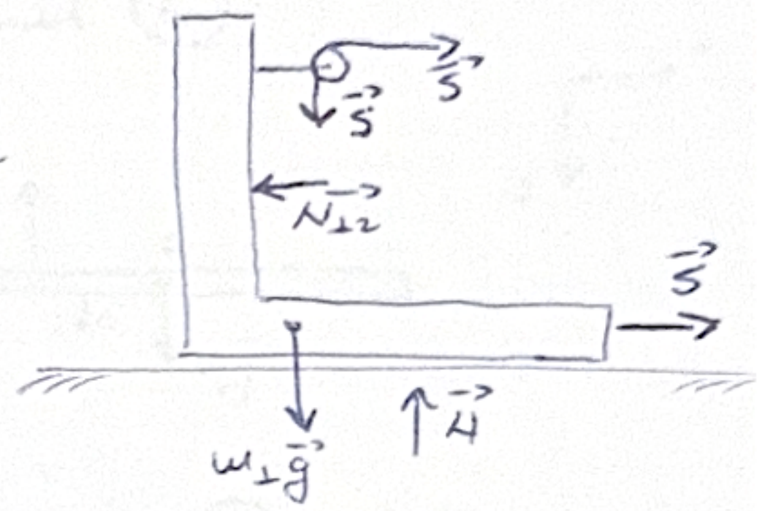
$\vec{m_2 a_2} = m_2\vec{g} + \vec{S} + \vec{N_{12}}$

x: $m_2 \ddot{x}_2 = N_{12}$, $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

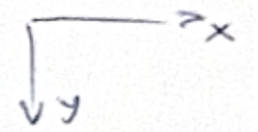
$m_2 \ddot{x} = N_{12}$ (1) $(\ddot{x}=?,\ N_{12}=?)$

y: $m_2 \ddot{y}_2 = m_2 g - S$ (2) $(\ddot{y}_2=?,\ S=?)$





$\vec{m_1 a_1} = m_1\vec{g} + \vec{N} + \vec{N_{12}} + \vec{S} + \vec{S} + \vec{S}'$



y: — — (nepotrebno)

x: $m_1 \ddot{x} = -N_{12} + S + S$

$m_1 \ddot{x} = -m_2 \ddot{x} = 2S$

$(m_1 + m_2)\ddot{x} = 2S$ (3)

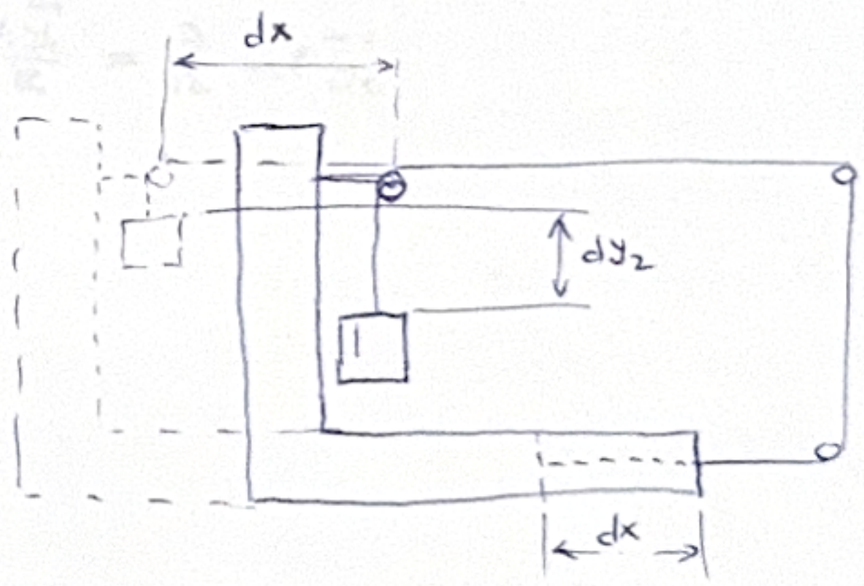
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kinematska veza:

$e = \text{const.}$

$e - dx - dx + dy_2 = \text{const.}$ $\quad / \cdot \frac{1}{dt} \quad / \cdot \frac{d}{dt}$

$2\ddot{x} = \ddot{y}_2$ (3)



---

(2) $\Rightarrow$ $m_2 \cdot 2\ddot{x} = m_2 g - \frac{m_1 + m_2}{2}\ddot{x}$ $\quad / \cdot 2$

$4 m_2 \ddot{x} = 2 m_2 g - m_1 \ddot{x} - m_2 \ddot{x}$

$\ddot{x}(5m_2 + m_1) = 2 m_2 g$

$$\boxed{\ddot{x} = \frac{2m_2}{m_1 + 5m_2} g}$$

(1) $\Rightarrow$ $S = \frac{m_1 + m_2}{2}\ddot{x} = \frac{m_1 + m_2}{2} \cdot \frac{2m_2}{m_1 + 5m_2} g$

$$\boxed{S = \frac{m_1 + m_2}{m_1 + 5m_2} m_2 g}$$

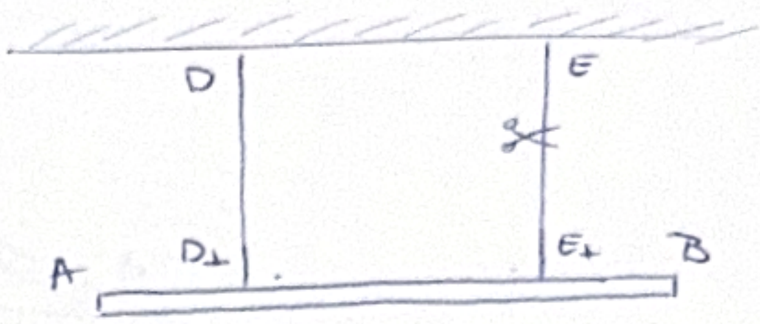
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10. 50.

AB: Q, ℓ

$\overline{AD_1} = \overline{BE_1} = \frac{1}{4}\ell$

$S = S_{DD_1} = ?$ – neposredno nekom preseczno kaugno EEI



Posmatramo i napisemo jedne z) trenutke t_0:

$$m\vec{a}_c = m\vec{g} + \vec{S}$$

x: – – – (nista)

y: $m\ddot{y}_c = mg - S$

$$S = m(g - \ddot{y}_c) \quad (1)$$

$$\frac{d\vec{L}_{Az}}{dt} = \sum \vec{M}_{Az}$$

$$\sum M_{Az} = \frac{1}{4} mg\ell$$

$$L_{Az} = \frac{1}{4}\ell \cdot m v_c + J_c \omega$$

$$\frac{dL_{Az}}{dt} = \frac{1}{4}\ell \cdot m a_{ct} + \frac{1}{12} m\ell^2 \varepsilon = \frac{1}{4}\ell \cdot m\ddot{y}_c + \frac{1}{12} m\ell^2 \cdot \frac{4\ddot{y}_c}{\ell} = \frac{7}{12} m\ell \ddot{y}_c$$

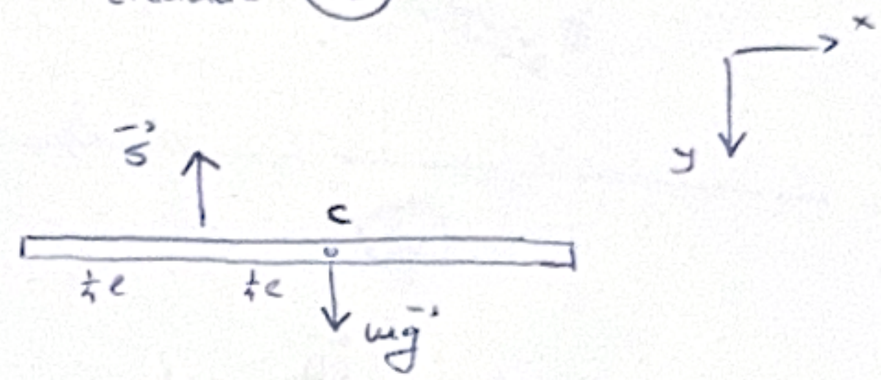
$$\frac{7}{12} m\ell \ddot{y}_c = \frac{1}{4} mg\ell$$

$$\ddot{y}_c = \frac{3}{7} g$$

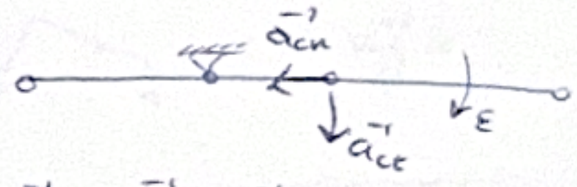
(1) $\Rightarrow$ $S = m\left(g - \frac{3}{7}g\right) = \frac{Q}{g} \cdot \frac{4}{7} g$

$$\boxed{S = \frac{4}{7} Q}$$

trenutak t_0



trenutak t_0 – ubrzanja



$\vec{a}_c = \vec{a}_{cn} + \vec{a}_{ct}$

x: $\ddot{x}_c = -a_{cn} = -\omega^2 \cdot \frac{1}{4}\ell = 0$

y: $\ddot{y}_c = +a_{ct} = +\varepsilon \cdot \frac{1}{4}\ell$

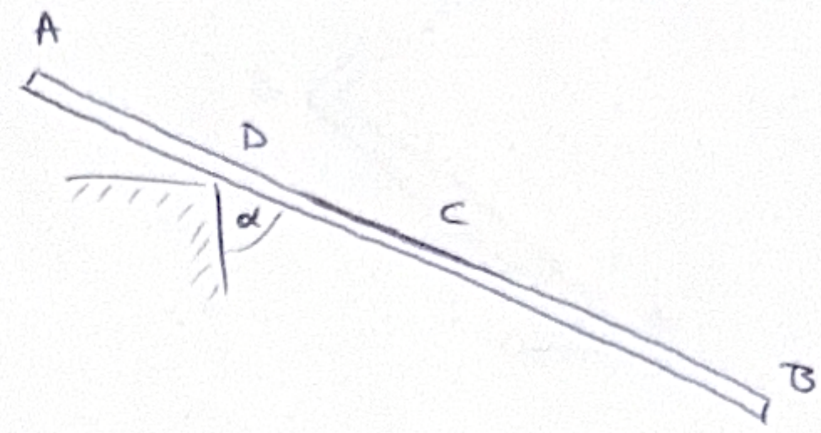
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(10. 31)

AD: Q, e

$\overline{CD} = a$

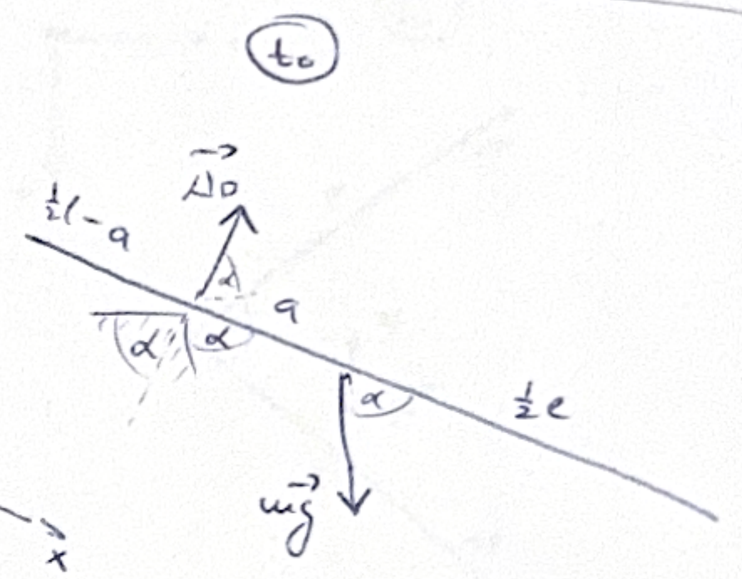
$t_0 = 0$: štap je mirovao ugao α



$N_D = N_D(t_0) = ?$

$a_c = a_c(t_0) = ?$

Posmatramo (t_0) raspisujemo jednačine za) trenutak (t_0):



$\rightarrow \quad m\vec{a}_c = m\vec{g} + \vec{N}_D$

y: $\quad m\ddot{y}_c = mg\sin\alpha - N_D$

$$N_D = m(g\sin\alpha - \ddot{y}_c) \quad (1)$$

$\rightarrow \quad \frac{d\vec{L}_{C\tau}}{dt} = \Sigma \vec{M}_{C\tau}$

$\Sigma M_C = N_D \cdot a$

$L_{C\tau} = J_C\omega = \frac{1}{12}ml^2\omega$

$\frac{dL_C}{dt} = \frac{1}{12}ml^2\varepsilon = \frac{1}{12}ml^2\frac{\ddot{y}_c}{a}$

$\rightarrow \quad \frac{1}{12}ml^2\frac{\ddot{y}_c}{a} = N_D \cdot a$

$$N_D = \frac{1}{12}m\frac{l^2}{a^2}\ddot{y}_c \quad (2)$$

(1) = (2)

$\frac{1}{12}\frac{l^2}{a^2}\ddot{y}_c = g\sin\alpha - \ddot{y}_c \quad / \cdot 12a^2$

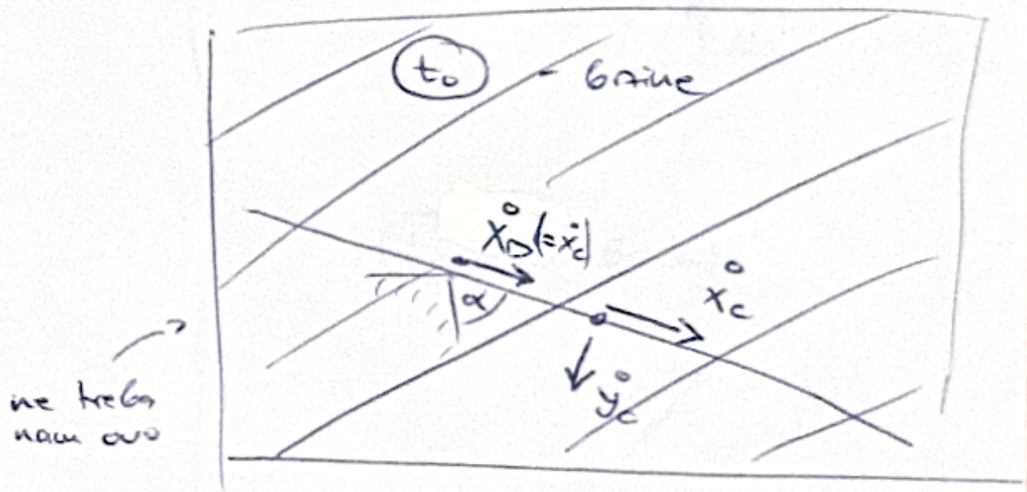
$\ddot{y}_c \cdot l^2 + \ddot{y}_c \cdot 12a^2 = g \cdot 12a^2\sin\alpha$

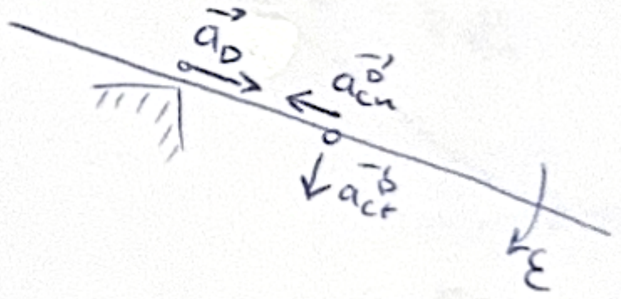
$$\ddot{y}_c = \frac{12a^2\sin\alpha}{12a^2 + l^2}g$$

$m\vec{a}_c' = m\vec{g}' + \vec{N}_D'$

x: $\quad m\ddot{x}_c = mg\cos\alpha$

$$a_c = \sqrt{\ddot{x}_c^2 + \ddot{y}_c^2} = \dots$$





$\vec{a}_c' = \vec{a}_D' + \vec{a}_{cn}^D + \vec{a}_{c\tau}^D$

x: $\quad \ddot{x}_c = \ddot{x}_D - a_{cn}^D = \ddot{x}_D - \omega^2 a^0 = \ddot{x}_D$

y: $\quad \ddot{y}_c = a_{c\tau} = \varepsilon \cdot a$

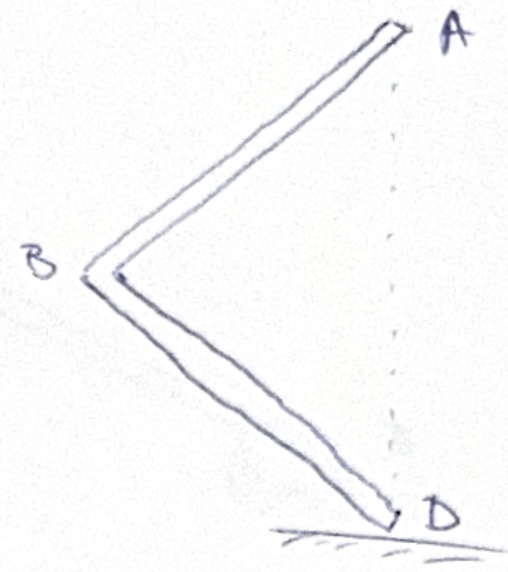
(1) $\Rightarrow \quad N_D = m\left(g\sin\alpha - \frac{12a^2\sin\alpha}{12a^2 + e^2}g\right)$

$N_D = mg\left(\frac{12a^2\sin\alpha + l^2\sin\alpha - 12a^2\sin\alpha}{12a^2 + l^2}\right)$

$$\boxed{N_D = Q \cdot \frac{l^2\sin\alpha}{12a^2 + l^2}}$$

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10. 32.



ABD: P, sastavljen od dva jednaka štapa

t_0: A i D su u vertikali

Cet. poč. Gotine

$N_D = N_D(t_0) = ?$

Posmatramo (i respektujemo jednačine za) trenutak t_0:

$$m\vec{a}_C = \vec{P} + \vec{N}_D$$

x: $-- = -- \quad (nema)$

y: $m\ddot{y}_C = P - N_D \quad (1)$

$$\frac{d\vec{L}_{Ct}}{dt} = \sum \vec{M}_{Ct}$$

$$\sum M_{Ct} = N_D \cdot \frac{1}{2} l \cos 45 = \frac{\sqrt{2}}{4} N_D \cdot l$$

$$L_{Ct} = J_{Ct} \cdot \omega = 2 \cdot \left[\frac{1}{12}\frac{m}{2} l^2 + \frac{m}{2}\left(\frac{1}{2} l \cos 45\right)^2\right]\omega$$

$$L_{Ct} = m\left[\frac{1}{12} l^2 + \frac{2}{16} l^2\right]\omega = \frac{5}{24} m l^2 \omega$$

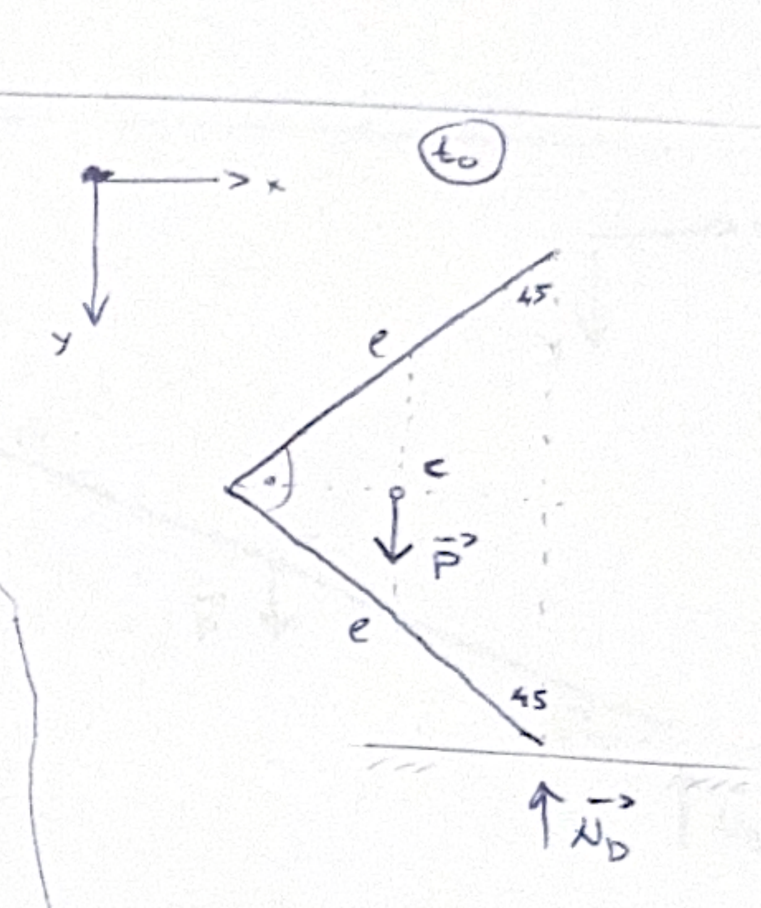
$$\frac{dL_{Ct}}{dt} = \frac{5}{24} m l^2 \varepsilon = \frac{5}{24} m l^2 \cdot 2\sqrt{2}\,\frac{\ddot{y}_C}{l} = \frac{5\sqrt{2}}{12} m l \ddot{y}_C$$

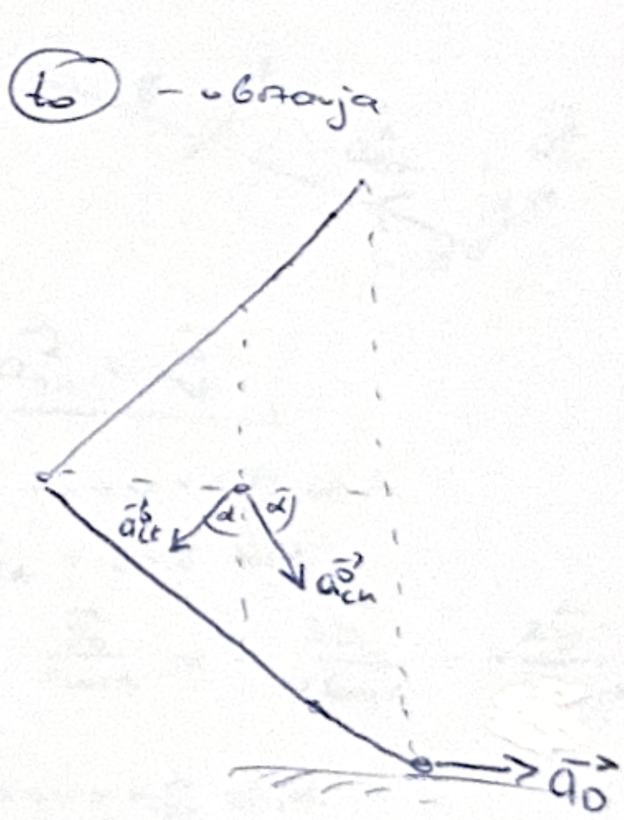
$$\frac{5\sqrt{2}}{12} m l \ddot{y}_C = \frac{\sqrt{2}}{4} N_D \cdot l$$

$$N_D = \frac{5}{3} m \ddot{y}_C = \frac{5}{3}(P - N_D)$$

$$\frac{8}{3} N_D = \frac{5}{3} P$$

$$\boxed{N_D = \frac{5}{8} P}$$





$$\vec{a}_C = \vec{a}_D + \vec{a}_{Cn} + \vec{a}_{Ct}$$

x: $\ddot{x}_C = -- \quad (nema)$

y: $\ddot{y}_C = 0 + 0 + a_{Ct} \cdot \cos\alpha$

$\ddot{y}_C = \varepsilon \cdot \overline{DC} \cdot \frac{1}{2}\cos 45$

$\ddot{y}_C = \frac{\sqrt{2}}{4} l \varepsilon$

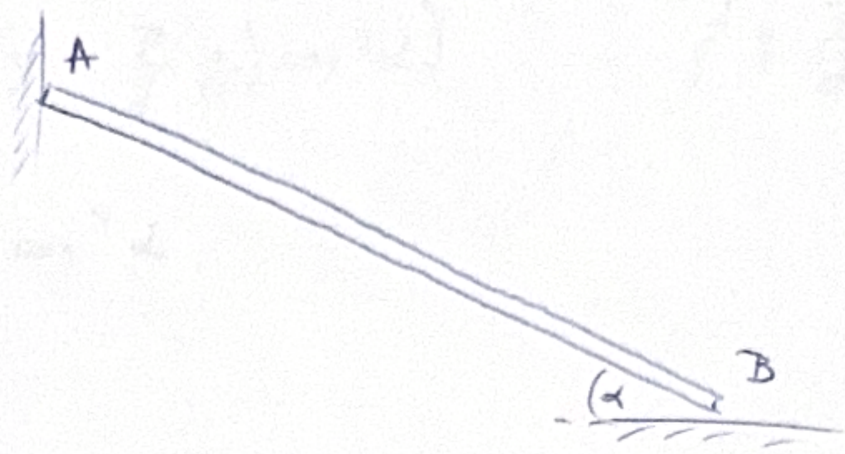
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(10. 33.)

AB : $\vec{P}$

$\perp$: miruvaye

ugao α

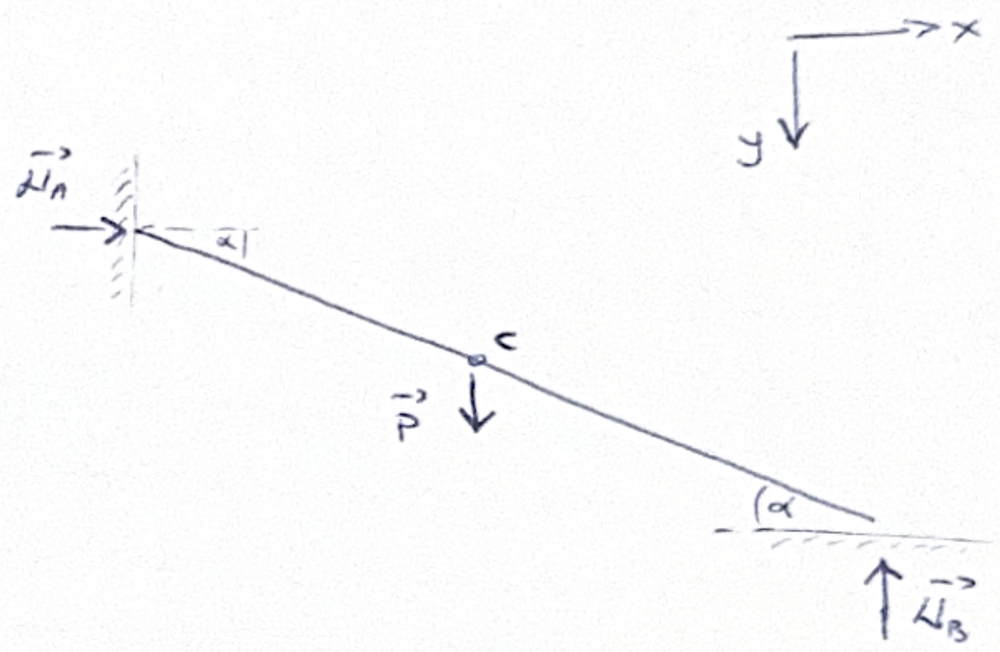


$N_A = N_A(\perp) = ?$

$N_B = N_B(\perp) = ?$

Postavimo teorem to:

$$\frac{P}{g}\vec{a}_c = \vec{P} + \vec{N}_A + \vec{N}_B$$

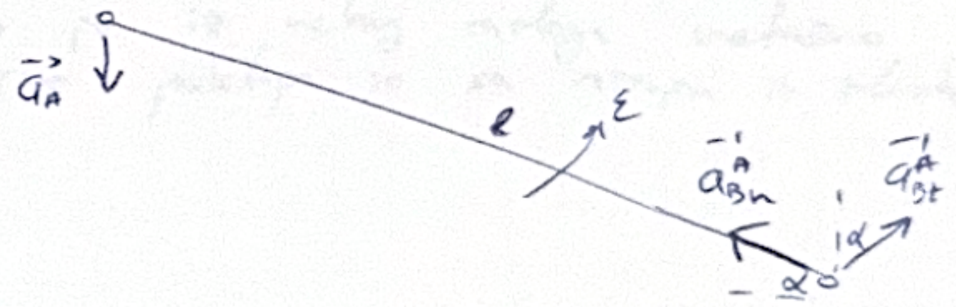


x: $\frac{P}{g}\ddot{x}_c = N_A$

$$N_A = \frac{P}{g}\ddot{y}\,tg\alpha \quad (1)$$

y: $\frac{P}{g}\ddot{y}_c = -P + N_B$

$$\frac{P}{g}\ddot{y} = -P + N_B \quad (2)$$



$$\frac{d\vec{L}_C}{dt} = \Sigma\vec{M}_C$$

$$\vec{a}_B = \vec{a}_A + \vec{a}^A_{Bn} + \vec{a}^A_{Bt}$$

y: $0 = -a_A + 0 + a^A_{Bt}\cos\alpha$

$0 = \ddot{y}_A + \varepsilon \cdot \ell\cos\alpha$

$\varepsilon = -\frac{\ddot{y}_A}{\ell\cos\alpha} = -\frac{2\ddot{y}_C}{\ell\cos\alpha} = -\frac{2\ddot{y}}{\ell\cos\alpha}$

x: $\ddot{x}_B = 0 + 0 + a^A_{Bt}\sin\alpha$

$\ddot{x}_B = \varepsilon\cdot\ell\cdot\sin\alpha = -\frac{2\ddot{y}}{\ell\cos\alpha}\cdot\ell\sin\alpha$

$\ddot{x}_B = -2\ddot{y}\,tg\alpha$

$\ddot{x}_C = \frac{1}{2}\ddot{x}_B = -\ddot{y}\,tg\alpha$

$\Sigma M_C = N_A\cdot\frac{1}{2}\ell\sin\alpha - N_B\cdot\frac{1}{2}\ell\cos\alpha$

$L_C = J_C\cdot\omega = \frac{1}{12}\frac{P}{g}\ell^2\omega$

$\frac{dL_C}{dt} = \frac{1}{12}\frac{P}{g}\ell^2\varepsilon = -\frac{1}{12}\frac{P}{g}\ell^2\frac{2\ddot{y}}{\ell\cos\alpha} = -\frac{1}{6}\frac{P}{g}\ell\frac{\ddot{y}}{\cos\alpha}$

$\frac{1}{6}\frac{P}{g}\ell\ddot{y}\cdot\frac{1}{\cos\alpha} = \frac{1}{2}\ell\,(N_A\sin\alpha - N_B\cos\alpha) \quad /\cdot\left(-\frac{6\cos\alpha}{\ell}\right)$

$$\frac{P}{g}\ddot{y} = 3(N_B\cos^2\alpha - N_A\sin\alpha\cos\alpha) \quad (3)$$

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(1) i (2) ubacimo u (3)

$$\frac{P}{g}\ddot{y} = 3\left(-\frac{P}{g}\ddot{y}\,tg\alpha\cdot\sin\alpha\cos\alpha - \left(\frac{P}{g}\ddot{y}+\frac{P}{g}g\right)\cos^2\alpha\right) \qquad /:\frac{P}{J}$$

$$\ddot{y}\left(1+3\sin^2\alpha+3\cos^2\alpha\right) = -3g\cos^2\alpha$$

$$4\ddot{y} = -3g\cos^2\alpha$$

$$\ddot{y} = -\frac{3}{4}g\cos^2\alpha$$

(1) $\Rightarrow$ $N_A = +\frac{P}{g}\cdot\frac{3}{4}g\cos^2\alpha\cdot tg\alpha$ ●

$$M_A = \frac{P}{g}\cdot\frac{3}{4}g\sin\alpha\cos\alpha\cdot 2\cdot\frac{1}{2}$$

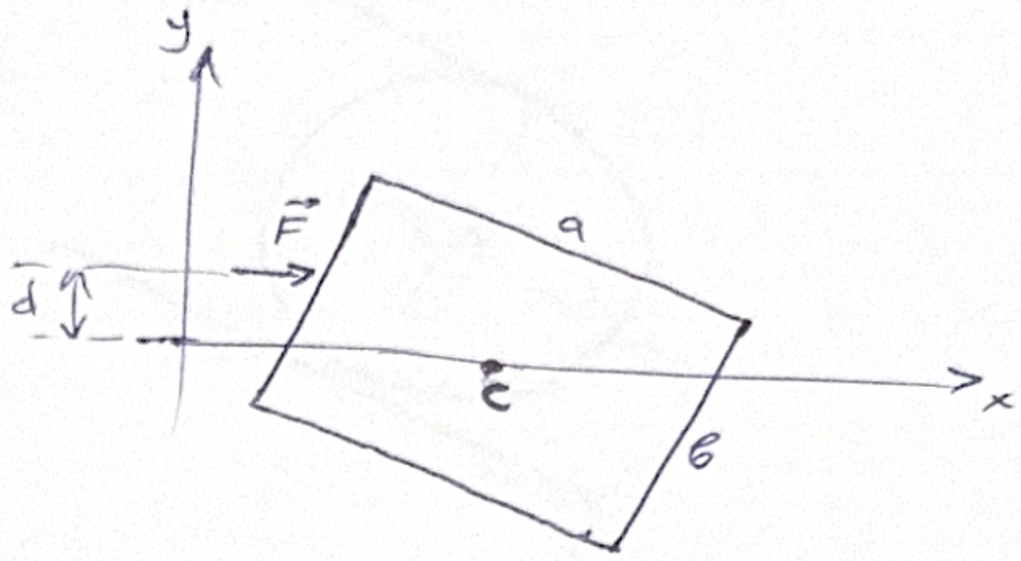
$$\boxed{N_A = \frac{3}{8}P\sin 2\alpha}$$

(2) $\Rightarrow$ $N_B = -\frac{P}{g}\cdot\frac{3}{4}g\cos^2\alpha + P$

$$\boxed{M_B = \frac{1}{4}P\left(4-3\cos^2\alpha\right)}$$ — ovo je iz nekog razloga netačno (ne poklapa se sa rešenjem iz zbirke)

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(10. 36.)



horizontalna ravan

ploča: Q, $a \times b$, $a > b$

$\vec{F} \parallel O_x$, $a > 2d$, $b > 2d$

t=0: ploča miruje

položaj: koord. poč, $a \parallel O_x$

j-ne kretanja?

$$m\vec{a}_c = \vec{F}$$

x: $m\ddot{x}_c = F$

$\ddot{x}_c = \frac{F}{m} = \frac{F}{Q} \cdot g$ //

$\dot{x}_c = \frac{F}{Q} \cdot g \cdot t + \cancel{C_1}^{=0}$ //

$x_c = \frac{1}{2} \frac{F}{Q} g t^2 + \cancel{C_2}^{=0}$

$$\boxed{x_c = \frac{1}{2} \frac{F}{Q} g t^2}$$

y: $\cancel{m}\ddot{y}_c = 0$ //

$\dot{y}_c = \cancel{C_1}^{=0} = 0$ //

$y_c = \cancel{C_2}^{=0} = 0$

$$\boxed{y_c = 0}$$

$$\frac{d\vec{L}_C}{dt} = \sum \vec{M}_C$$

$\sum M_C = F \cdot d$

$L_C = J_c \cdot \omega = m \frac{a^2+b^2}{6} \omega$

$\frac{dL_C}{dt} = \frac{1}{6} m (a^2+b^2) \varepsilon$

$\frac{1}{6} m (a^2+b^2) \varepsilon = F \cdot d$

$\varepsilon = \frac{6 \cdot F \cdot d}{m(a^2+b^2)}$ //

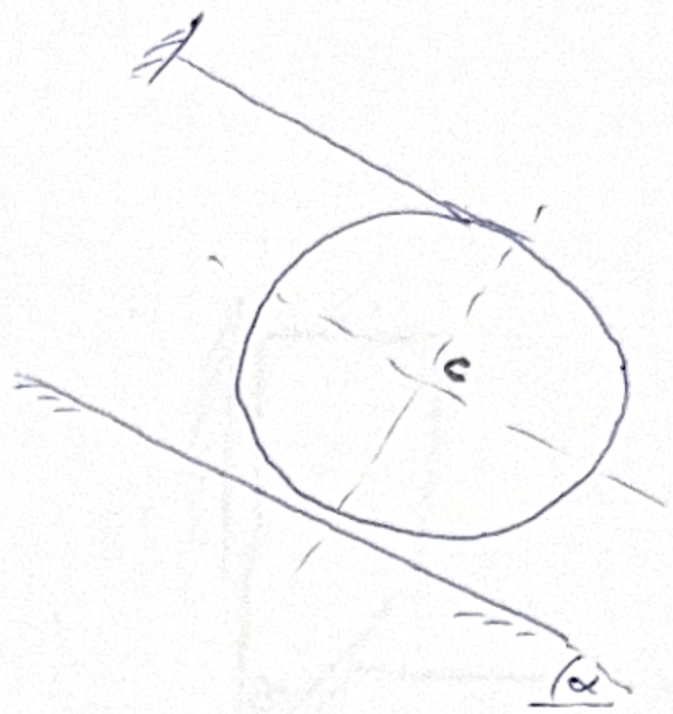
$\omega = \frac{6 \cdot F \cdot d}{m(a^2+b^2)} t + \cancel{C_1}^{=0}$ //

$\varphi = \frac{3Fd}{m(a^2+b^2)} t^2 + \cancel{C_2}^{=0}$

$$\boxed{\varphi = \frac{3Fdg}{Q(a^2+b^2)} t^2}$$

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10. 38.



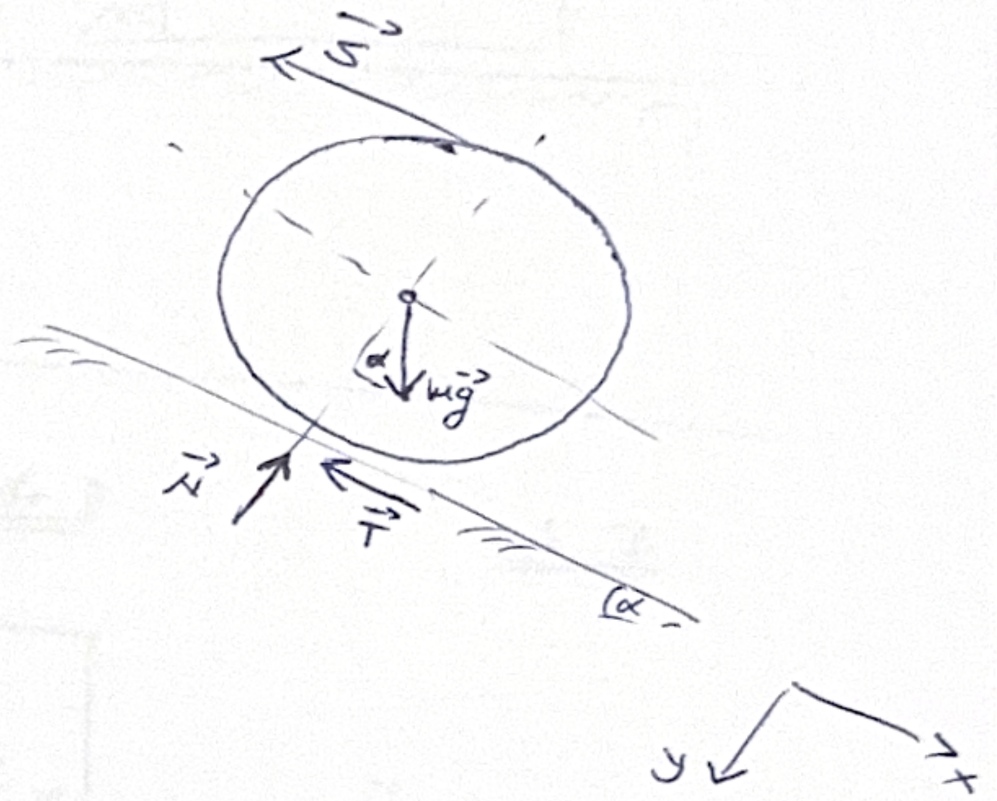
prsten: m, R

nagib α

μ - koef. trenja

a_c ? S?

$$m\vec{a}_c = m\vec{g} + \vec{N} + \vec{T} + \vec{S}$$



y: $0 = mg - N$

$$N = mg$$

$$T = \mu_0 mg$$

x: $m\ddot{x}_c = mg\sin\alpha - T - S$

$$S = mg\sin\alpha - \mu_0 mg\cos\alpha - m\ddot{x}_c \quad (1)$$

$$\frac{d\vec{L}_{C+}}{dt} = \vec{\Sigma M}_{C+}$$

$$\Sigma M_{C+} = S\cdot R - T\cdot R = S\cdot R - \mu\cdot mg\cos\alpha\cdot R$$

$$L_{C+} = J_{C+}\cdot\omega = mR^2\omega$$

$$\frac{dL_{C+}}{dt} = mR^2\varepsilon = mR^2\frac{\ddot{x}_c}{R}$$

$$S\cdot R - \mu\cdot mg\cos\alpha\cdot R = mR\ddot{x}_c$$

$$S = m\ddot{x}_c + \mu\cdot mg\cos\alpha \quad (2)$$

(1) = (2)

$$mg\sin\alpha - \mu\cdot mg\cos\alpha - m\ddot{x}_c = m\ddot{x}_c + \mu mg\cos\alpha$$

$$2m\ddot{x}_c = mg(\sin\alpha - \mu\cos\alpha - \mu\cos\alpha)$$

$$\boxed{\ddot{x}_c = \frac{1}{2}g(\sin\alpha - 2\mu\cos\alpha)}$$

(2) => $S = m\cdot\frac{1}{2}g(\sin\alpha - 2\mu\cos\alpha) + \mu mg\cos\alpha$

$$S = \frac{1}{2}mg\sin\alpha - \mu mg\cos\alpha + \mu mg\cos\alpha$$

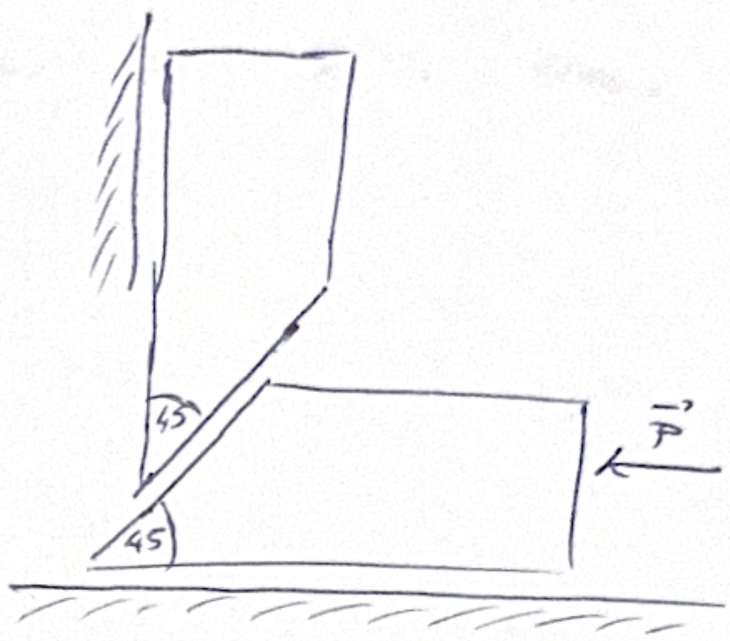
$$\boxed{S = \frac{1}{2}mg\sin\alpha}$$

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10.39.

A: ω, $45°$

B: ω, $45°$



i) na A deluje sila $\vec{P}$

to: sistem miruje

$\mu = \frac{1}{2}$

$P_{min} = ?$ – da bi se sistem pokrenuo u smeru dejstva sile P

ž) ako je $P = 2 \cdot P_{min}$

$\mu_2 = \frac{3}{4}\mu$

$a_B = ?$

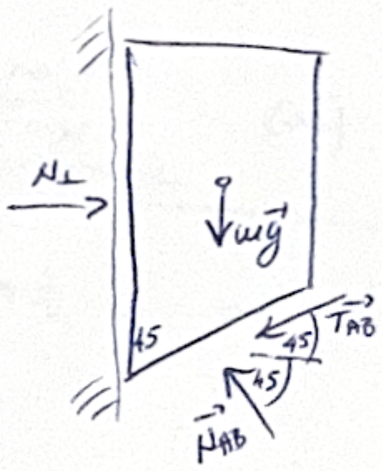
Kinematska veza:



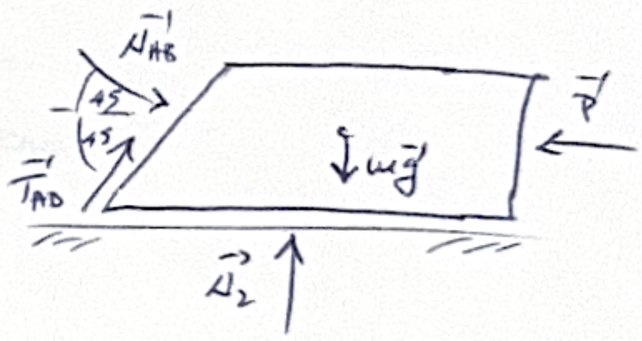
$dx_B = dy_A \quad / \frac{1}{dt} \cdot / \frac{d}{dt}$

$\ddot{x}_B = \ddot{y}_A = \ddot{x}$

Telo A



Telo B



---

d) uslov: $\ddot{x} = 0$, $T_{AB} = T_{AB,max}$ (sistem je na granici da se pokreće), $T_{AB} = \mu \cdot N_{AB}$

Neće pustati $\ddot{x} = 0$ i $\mu = \frac{1}{2}$ sve do pred kraj, jer želim da dođem jednačine koje važe i za 6)

$m_A \vec{a}_A = m_A \vec{g} + \vec{N}_1 + \vec{N}'_{AB} + \vec{T}_{AB}$

$m_B \vec{a}_B = m_B \vec{g} + \vec{P} + \vec{N}_2 + \vec{N}'_{AB} + \vec{T}_{AB}$

x: $0 = N_1 - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$0 = N_1 - N_{AB}\frac{\sqrt{2}}{2} - \mu \cdot N_{AB} \cdot \frac{\sqrt{2}}{2}$

$0 = N_1 - \frac{\sqrt{2}}{2} N_{AB}(1+\mu)$ (1)

y: $m\ddot{y}_A = -mg + N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = -mg + N_{AB}\frac{\sqrt{2}}{2} - \mu \cdot N_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = -mg + \frac{\sqrt{2}}{2} N_{AB}(1-\mu)$ (2)

y: $0 = -mg + N_2 - N_{AB}\frac{\sqrt{2}}{2} + T_{AB}\cdot\frac{\sqrt{2}}{2}$

$0 = -mg + N_2 - N_{AB}\frac{\sqrt{2}}{2} + \mu N_{AB}\frac{\sqrt{2}}{2}$

$0 = -mg + N_2 - \frac{\sqrt{2}}{2} N_{AB}(1-\mu)$ (3)

x: $m\ddot{x}_A = P - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = P - N_{AB}\frac{\sqrt{2}}{2} - \mu N_{AB}\frac{\sqrt{2}}{2}$

$m\ddot{x} = P - \frac{\sqrt{2}}{2} N_{AB}(1+\mu)$ (4)

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I za a) i za c) su dovoljne jne (2) i (4), jer su to 2 jne i 6 ice dve nepoznate: pod a) su to P i N_{AB}, jer je $\ddot{x}=0$ tad, a pod b) su to $\ddot{x}$ i N_{AB}.

(2) $\Rightarrow$ $\frac{\sqrt{2}}{2}(1-\mu)N_{AB} = m\ddot{x} + mg$

$$N_{AB} = \frac{(m\ddot{x}+mg)\sqrt{2}}{1-\mu}$$

(4) $\Rightarrow$ $m\ddot{x} = P - \frac{\sqrt{2}}{2}N_{AB}(1+\mu)$

$$m\ddot{x} = P - \frac{\sqrt{2}}{2}\frac{(m\ddot{x}+mg)\sqrt{2}}{(1-\mu)}\cdot(1+\mu)$$

$$m\ddot{x} = P - m\ddot{x}\cdot\frac{1+\mu}{1-\mu} - mg\cdot\frac{1+\mu}{1-\mu}$$

$$m\ddot{x}\left(1+\frac{1+\mu}{1-\mu}\right) = P - mg\cdot\frac{1+\mu}{1-\mu} \qquad (*)$$

2) postavimo da je $\ddot{x}=0$, $\mu=\frac{1}{2}$, pa dobijamo:

(*) $\Rightarrow$ $0 = P_{min} - mg\cdot\frac{1+\frac{1}{2}}{1-\frac{1}{2}}$

$$\boxed{P_{min} = 3mg}$$

b) postavimo da je $P = 2P_{min} = 6mg$, $\mu_1 = \frac{3}{4}\mu = \frac{3}{8}$, pa dobijamo:

(*) $\Rightarrow$ $m\ddot{x}\left(1+\frac{1+\frac{3}{8}}{1-\frac{3}{8}}\right) = 6mg - mg\frac{1+\frac{3}{8}}{1-\frac{3}{8}}$

$$m\ddot{x}\left(1+\frac{\frac{11}{8}}{\frac{5}{8}}\right) = 6mg - mg\frac{\frac{11}{8}}{\frac{5}{8}}$$

$$\frac{16}{5}m\ddot{x} = \frac{19}{5}mg$$

$$\boxed{\ddot{x} = \frac{19}{16}g}$$

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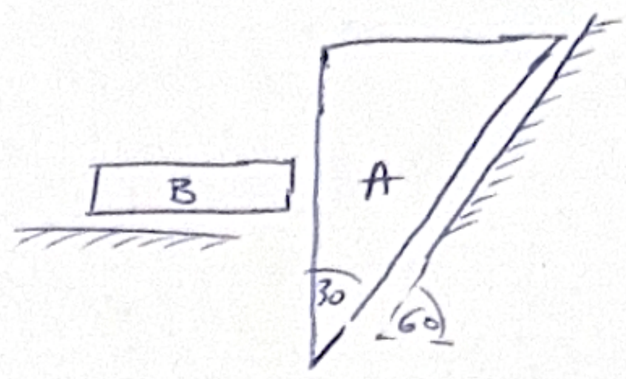
10.40.

A: m

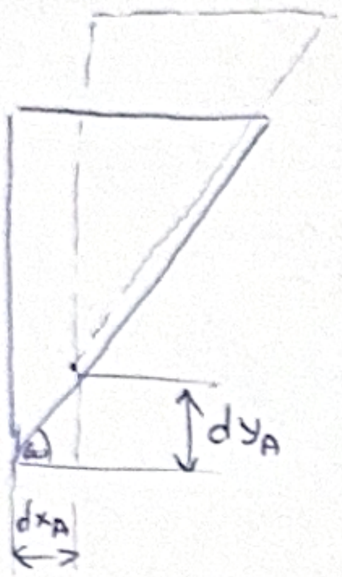
B: $2m$

$a_B = ?$

$N_B = ?$



kinematska veza:



$$tg\,60 = \frac{dy_A}{dx_A}$$

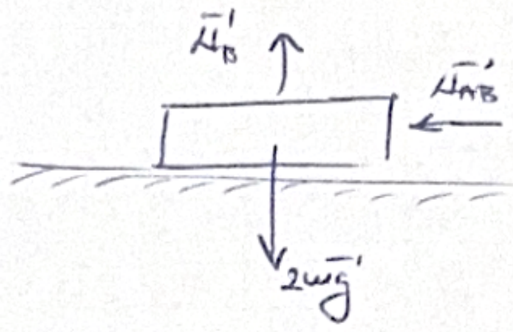
$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{dy_A}{dx_A}$$

$$dy_A = \sqrt{3}\, dx_A \quad / \frac{d}{dt} \; / \frac{d}{dt}$$

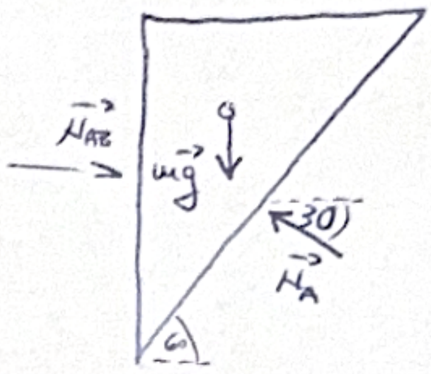
$$\ddot{y}_A = \sqrt{3}\, \ddot{x}_A = \sqrt{3}\, \ddot{x}$$

takoče važi: $\ddot{x}_B = \ddot{x}_A = \ddot{x}$

telo B



telo A



$m\vec{a}_A = m\vec{g} + \vec{N}_A + \vec{N}_{AB}$

x: $m\ddot{x}_A = N_A \cos 30 - N_{AB}$

$$m\ddot{x} = \frac{\sqrt{3}}{2} N_A - N_{AB} \quad (1)$$

y: $m\ddot{y} = mg - N_A \sin 30$

$$\sqrt{3}\, m\ddot{x} = mg - \frac{1}{2} N_A \quad (2)$$

$2m\vec{a}_B = m\vec{g}' + \vec{N}'_B + \vec{N}'_{AB}$

x: $2m\ddot{x}_B = N_{AB}$

$$2m\ddot{x} = N_{AB} \quad (3)$$

y: $0 = mg - N_B$

$$N_B = mg$$

(3) vbacimo v (1)

$$m\ddot{x} = \frac{\sqrt{3}}{2} N_A - 2m\ddot{x}$$

$$3m\ddot{x} = \frac{\sqrt{3}}{2} N_A$$

$$m\ddot{x} = \frac{\sqrt{3}}{6} N_A \quad - \text{ vbacimo v (2)}$$

(2) $-$, $\sqrt{3}\,\frac{\sqrt{3}}{6} N_A = mg - \frac{1}{2} N_A$

$$\frac{1}{2} N_A = mg - \frac{1}{2} N_A$$

$$N_A = mg$$

$\Rightarrow$, $m\ddot{x} = \frac{\sqrt{3}}{6} N_A$

$$\boxed{\ddot{x} = \frac{\sqrt{3}}{6} g}$$

(1) $\Rightarrow$ $N_{AB} = 2m \cdot \frac{\sqrt{3}}{6} g$

$$\boxed{N_{AB} = \frac{\sqrt{3}}{3} mg}$$

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(10.41)

A: G

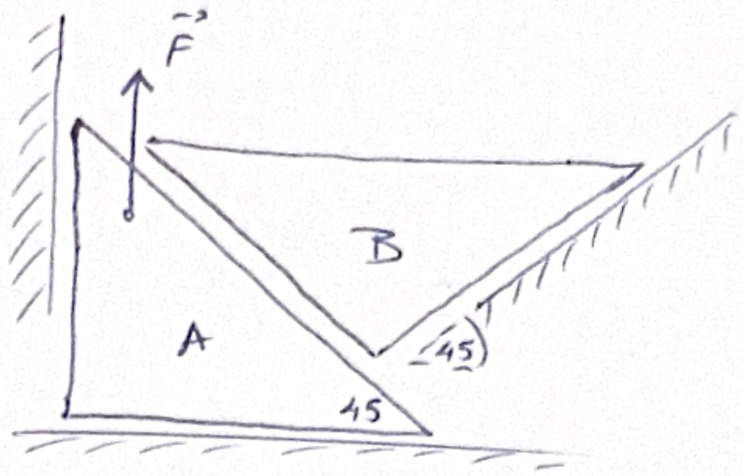
B: 2G

to: položaj na sliki

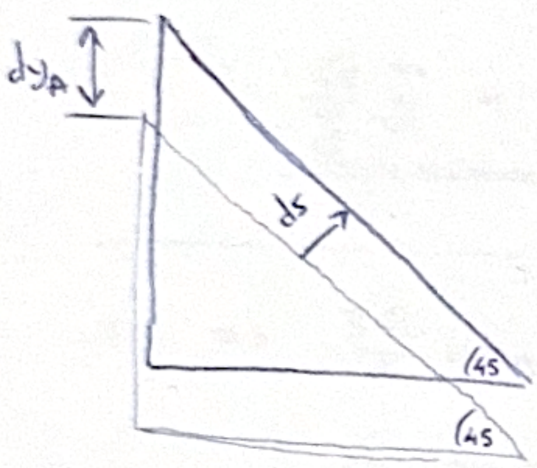
$F = 5G$

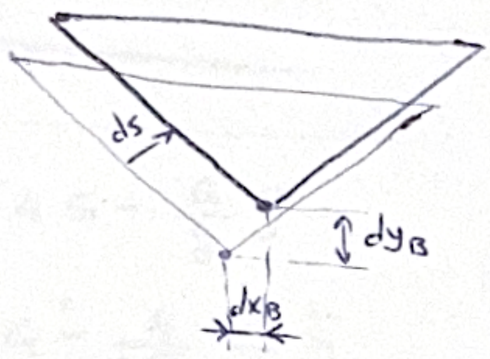
$\mu = 0{,}5$ – koef. trenja između ploča

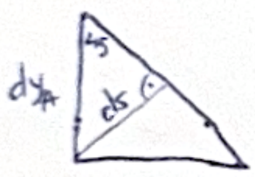
$a_A = ?$



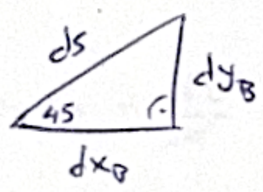
geometrijska veza







slika 1



slika 2

Posto cemo sve izraziti preko y_A, reći ćemo da je: $\ddot{y}_A = \ddot{y}$

Iz slike 1 zaključujemo:

$ds = dy_A \cos 45 = \frac{\sqrt{2}}{2} dy_A$

Iz slike 2 zaključujemo:

$dy_B = ds \cdot \cos 45 = \frac{\sqrt{2}}{2} dy_A \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} dy_A$

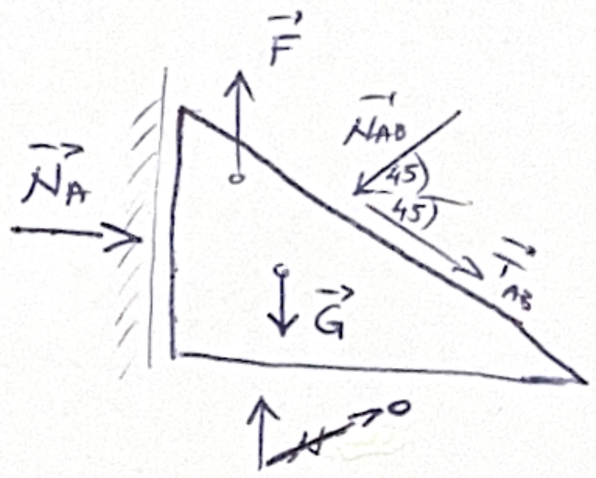
$dx_B = ds \cdot \cos 45 = \frac{\sqrt{2}}{2} dy_A \frac{\sqrt{2}}{2} = \frac{1}{2} dy_A$

Kada prethodna dva izraza pomnožimo sa $\frac{1}{dt} \cdot \frac{d}{dt}$, dobijamo:

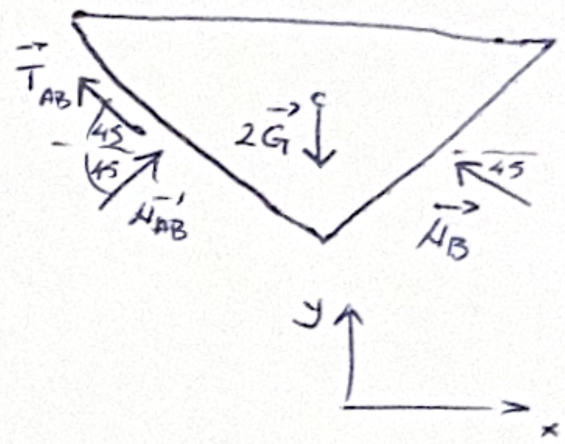
$\ddot{y}_B = \frac{1}{2} \ddot{y}_A = \frac{1}{2} \ddot{y}$

$\ddot{x}_B = \frac{1}{2} \ddot{y}_A = \frac{1}{2} \ddot{y}$

telo A



telo B



veza: $T_{AB} = \mu \cdot N_{AB} = \frac{1}{2} N_{AB}$

predpostavka: $N = 0$ (ako se dobije $\ddot{y}_A < 0$, onda je $N \neq 0$, a sistem će miovati)

$\frac{2G}{g} \vec{a}_B = 2\vec{G} + \vec{N}_B + \vec{N}_{AB} + \vec{T}_{AB}$

x: $\frac{2G}{g} \ddot{x}_B = -2G + N_B \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{2} + T_{AB} \frac{\sqrt{2}}{2}$

$\frac{2G}{g} \frac{1}{2} \ddot{y} = -2G + N_B \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{4}$

$\frac{G}{g} \ddot{y} = -2G + N_B \frac{\sqrt{2}}{2} + N_{AB} \cdot \frac{3\sqrt{2}}{4}$ (1)

y: $\frac{2G}{g} \ddot{y}_B = -N_B \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{2} - T_{AB} \frac{\sqrt{2}}{2}$

$\frac{2G}{g} \frac{1}{2} \ddot{y} = -N_B \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{2} - \frac{1}{2} N_{AB} \frac{\sqrt{2}}{4}$

$\frac{G}{g} \ddot{y} = -N_B \frac{\sqrt{2}}{2} + N_{AB} \frac{\sqrt{2}}{4}$ (2)

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$$\frac{G}{g}\vec{a}_A = \vec{G} + \vec{F} + \vec{N}_A + \vec{N}_{AB} + \vec{T}_{AB}$$

x: $\quad 0 = N_A - N_{AB}\frac{\sqrt{2}}{2} + T_{AB}\frac{\sqrt{2}}{2}$

$$0 = N_A - N_{AB}\frac{\sqrt{2}}{2} + N_{AB}\frac{\sqrt{2}}{4}$$

$$0 = N_A - N_{AB}\frac{\sqrt{2}}{4} \qquad (3)$$

y: $\quad \frac{G}{g}\ddot{y}_A = -G + F - N_{AB}\frac{\sqrt{2}}{2} - T_{AB}\frac{\sqrt{2}}{2}$

$$\frac{G}{g}\ddot{y} = -G + 5G - N_{AB}\frac{\sqrt{2}}{2} - N_{AB}\frac{\sqrt{2}}{4}$$

$$\frac{G}{g}\ddot{y} = 4G - N_{AB}\frac{3\sqrt{2}}{4} \qquad (4)$$

---

(4) $\Rightarrow$ $\quad \frac{3\sqrt{2}}{4}N_{AB} = 4G - \frac{G}{g}\ddot{y}$

$$N_{AB} = \frac{16}{3\sqrt{2}}G - \frac{4}{3\sqrt{2}}\frac{G}{g}\ddot{y}$$ - убацујемо у (3)

(3) $\Rightarrow$ $\quad N_A = \frac{\sqrt{2}}{4}\frac{16}{3\sqrt{2}}G - \frac{\sqrt{2}}{4}\frac{4}{3\sqrt{2}}\frac{G}{g}\ddot{y}$

$$N_A = \frac{4}{3}G - \frac{1}{3}\frac{G}{g}\ddot{y}$$ - убацујемо у (2)

(2) $\Rightarrow$ $\quad \frac{G}{g}\ddot{y} = -N_B\frac{\sqrt{2}}{2} + \frac{4G}{3} - \frac{1}{3}\frac{G}{g}\ddot{y}$

$$N_B\frac{\sqrt{2}}{2} = \frac{4}{3}G - \frac{4}{3}\frac{G}{g}\ddot{y} \qquad /\cdot\sqrt{2}$$

$$N_B = \frac{4\sqrt{2}}{3}G - \frac{4\sqrt{2}}{3}\frac{G}{g}\ddot{y}$$

(1) $\Rightarrow$ $\quad \frac{G}{g}\ddot{y} = -2G + \frac{\sqrt{2}}{2}\frac{4\sqrt{2}}{3} - \frac{\sqrt{2}}{2}\frac{4\sqrt{2}}{3}\frac{G}{g}\ddot{y} + 4G - \frac{G}{g}\ddot{y}$

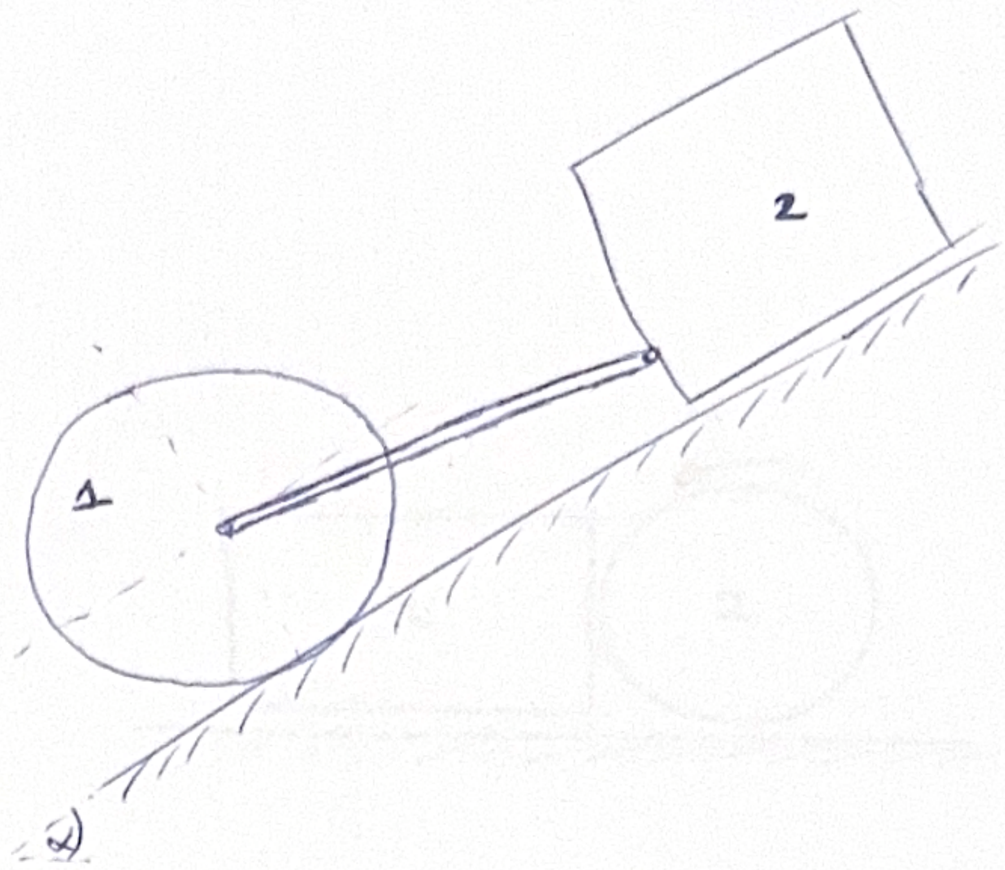
$$\frac{G}{g}\ddot{y}\left(1 + \frac{4}{3} + 1\right) = G\left(-2 + \frac{4}{3} + 4\right)$$

$$\frac{10}{3}\frac{G}{g}\ddot{y} = \frac{10}{3}G$$

$$\boxed{\ddot{y} = g}$$

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(10. 42.)



1: m, r

2: m

k - krak otpora kotrljanju kotrljaye bez klizanja

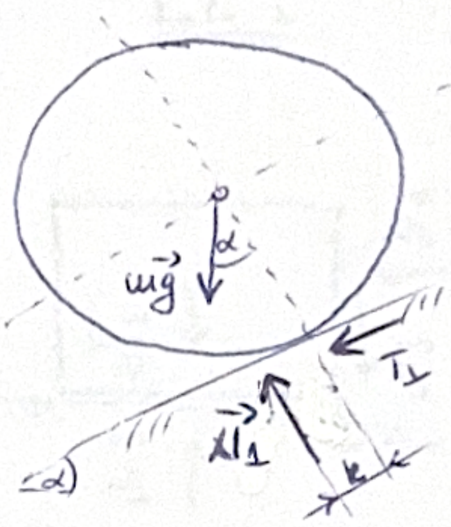
μ - koef. trenja klizanja između tela 2 i tla

$\alpha = ?$ - da bi sila u štapu bila jednaka nuli

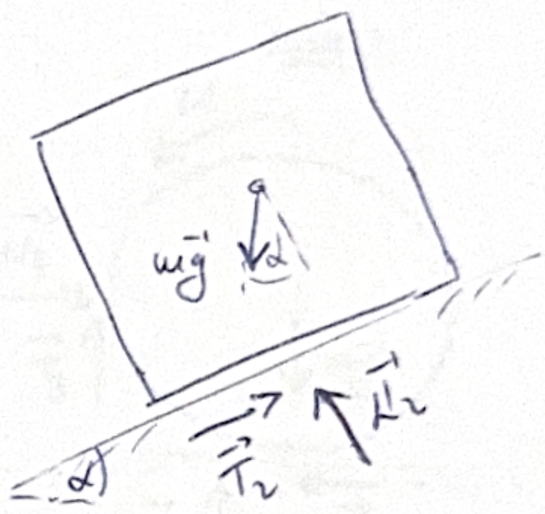
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kinematska veza: $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

telo 1



telo 2



$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{T}_1$

y: $0 = mg\cos\alpha - N_1$

$N_1 = mg\cos\alpha$

x: $m\ddot{x} = mg\sin\alpha + T_1$ (1) $\left(\begin{matrix} T_1 = ? \\ \ddot{x} = ? \\ \alpha = ? \end{matrix}\right)$

$\frac{d\vec{L}_{C_1}}{dt} = \sum \vec{M}_{C_1}$

$\sum M_{C_1} = -N_1 \cdot k - T_1 \cdot r = -mg\cos\alpha \cdot k - T_1 \cdot r$

$L_{C_1} = J_{C_1} \cdot \omega = \frac{1}{2} m r^2 \frac{\dot{x}}{r}$

$\frac{dL_{C_1}}{dt} = \frac{1}{2} m r \ddot{x}$

$\frac{1}{2} m r \ddot{x} = -mg\cos\alpha \cdot k - T_1 \cdot r$ (2)

$m\vec{a}_2 = m\vec{g} + \vec{N}_2 + \vec{T}_2$

y: $0 = mg\cos\alpha - N_2$

$N_2 = mg\cos\alpha$

$T_2 = \mu \cdot N_2 = \mu \cdot mg\cos\alpha$

x: $m\ddot{x} = mg\sin\alpha - T_2$

$m\ddot{x} = mg\sin\alpha - \mu mg\cos\alpha$ (3)

(1) $\Rightarrow$ $T_1 = m\ddot{x} - mg\sin\alpha$

(2) $\Rightarrow$ $\frac{1}{2} m r \ddot{x} = -mg\cos\alpha \cdot k - m\ddot{x} \cdot r + mg\sin\alpha \cdot r$

$\frac{3}{2} m r \ddot{x} = -mg\cos\alpha \cdot k + mg\sin\alpha \cdot r$ (*)

(3) ubacimo u (*)

(*) $\Rightarrow$ $\frac{3}{2} r(mg\sin\alpha - \mu mg\cos\alpha) = -mg\cos\alpha \cdot k + mg\sin\alpha \cdot r$

$\sin\alpha\left(\frac{3}{2} r \cdot mg - mg \cdot r\right) = \cos\alpha\left(-mg \cdot k + \frac{3}{2}\mu mg \cdot r\right)$

$tg\alpha = \frac{\frac{3}{2}\mu \cdot mg \cdot r - mg \cdot k}{\frac{1}{2} mg r}$

$$\boxed{tg\alpha = 3\mu - 2\frac{k}{r}}$$

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(10.43.)

1: m, klizi bez trenja

2: m, r, kotrlja se bez klizanja

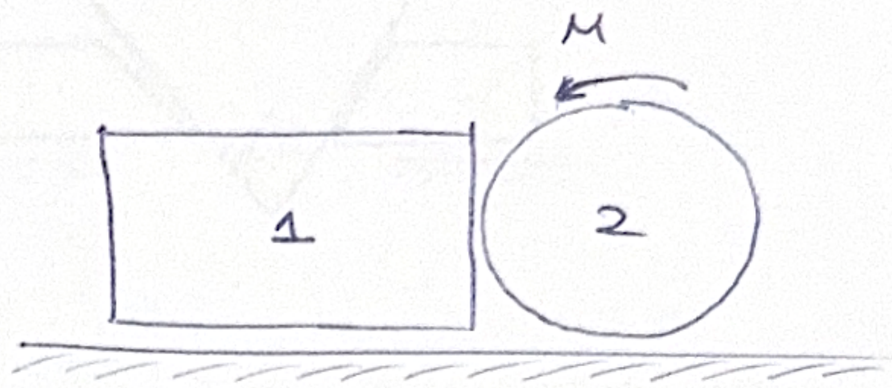
$\mu = \frac{1}{2}$ – koef. trenja između 1 i 2

$k = \frac{1}{10} r$ – krak otpora kotrljanju

to: sistem miruje

M – moment

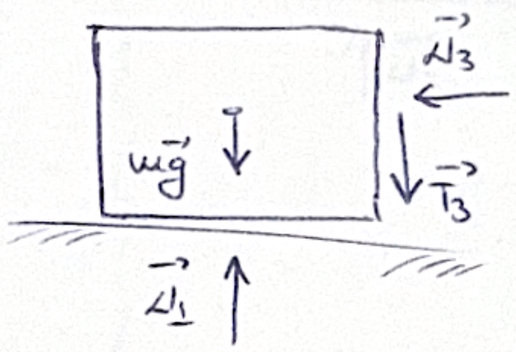
a_1? N_1? N_2? T_2?



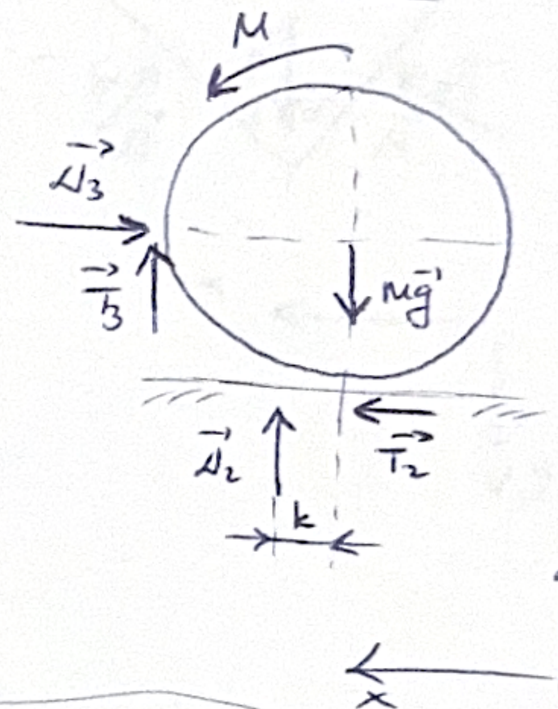
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kinematska veza: $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

telo 1



telo 2



$T_2 = ?$ $\quad T_3 = \mu \cdot N_3$

$m\vec{a}_2 = m\vec{g} + \vec{N}_2 + \vec{T}_2 + \vec{N}_3 + \vec{T}_3$

y: $\quad 0 = -mg + N_2 + \mu \cdot N_3 \quad (1) \quad (N_2 = ?,\ N_3 = ?)$

x: $\quad m\ddot{x} = T_2 - N_3 \quad (2) \quad (T_2 = ?,\ \ddot{x} = ?)$

$\frac{dL_{C+}}{dt} = \sum M_{C+}$ $\quad$ (+ ↺)

$\sum M_{C+} = M - T_3 \cdot r - N_2 \cdot k - T_2 \cdot r$

$\sum M_{C+} = M - \mu \cdot N_3 \cdot r - \frac{1}{10} N_2 \cdot r - T_2 \cdot r$

$L_{C+} = J_2 \omega = \frac{1}{2} m r^2 \cdot \frac{\dot{x}}{r}$

$\frac{dL_{C+}}{dt} = \frac{1}{2} m r \ddot{x}$

$\frac{1}{2} m r \ddot{x} = M - \mu \cdot N_3 \cdot r - \frac{1}{10} N_2 \cdot r - T_2 \cdot r \quad (3)$

$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{N}_3 + \vec{T}_3$

y: $\quad 0 = -mg + N_1 - \mu \cdot N_3 \quad (4) \quad (N_1 = ?)$

x: $\quad m\ddot{x} = N_3 \quad (5)$

(1) $\Rightarrow$ $N_2 = mg - \mu N_3$

(4) $\Rightarrow$ $N_1 = mg + \mu \cdot N_3$

(5) $\Rightarrow$ $m\ddot{x} = N_3$

(2) $\Rightarrow$ $N_3 = T_2 - N_3 \Rightarrow T_2 = 2 \cdot N_3$

(3) $\Rightarrow \frac{1}{2} r \cdot N_3 = M - \mu \cdot r N_3 - \frac{1}{10} r (mg - \mu \cdot N_3) - 2 N_3 \cdot r$

$N_3 \left(\frac{1}{2} r + \mu r - \frac{1}{10} r \mu + 2r\right) = M - \frac{1}{10} mgr$

$N_3 \left(\frac{5}{2} r + \frac{1}{2} r - \frac{1}{20} r\right) = M - \frac{1}{10} mgr$

$N_3 = \frac{M - \frac{1}{10} mgr}{\frac{59}{20} r} \Rightarrow N_3 = \frac{2(10M - mgr)}{59 r}$

$T_2 = 2 N_3 \Rightarrow \boxed{T_2 = \frac{4(10M - mgr)}{59 r}}$

$N_1 = mg + \frac{1}{2} N_3 = \frac{59 mgr}{59 r} + \frac{10M - mgr}{59 r} \Rightarrow \boxed{N_1 = \frac{10M - 58 mgr}{59 r}}$

$N_2 = mg - \mu \cdot N_3 = \frac{59 mgr}{59 r} - \frac{10M - mgr}{59 r} \Rightarrow \boxed{N_2 = \frac{60 mgr - 10M}{59 r}}$

$\boxed{\ddot{x} = \frac{2(10M - mgr)}{59 mr}}$

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10.44.

klin: P, 2α

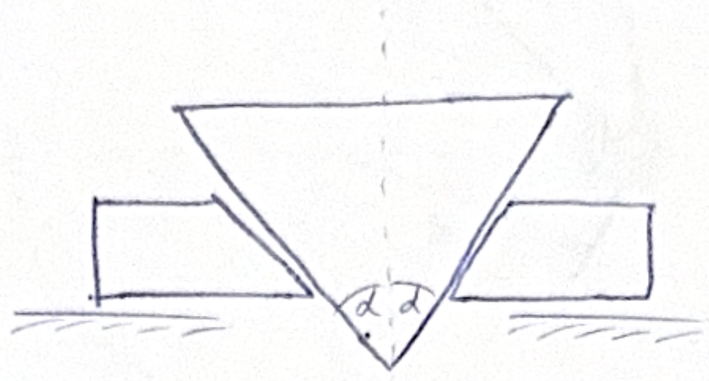
ploča: po P_1 svaka

to: miruvaat

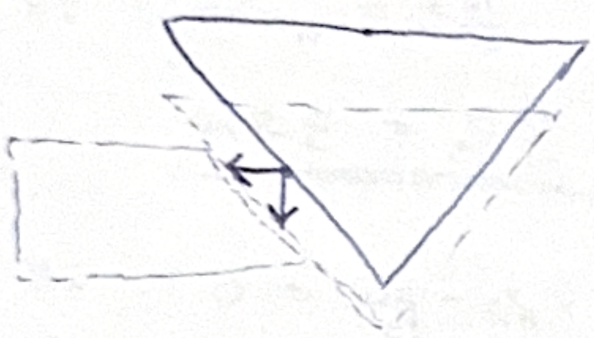
$x = ?$ - horiz. pomestuvanje ploča

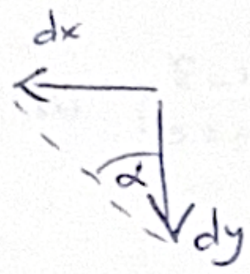
$y = ?$ - vertikalno pomestuvanje ploča

$N = ?$ - pritisok klina na ploča



Kinematska reta:

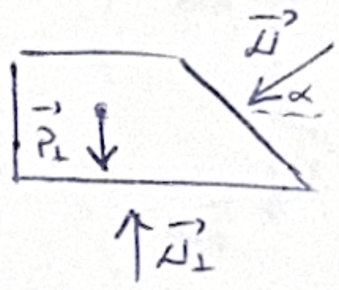




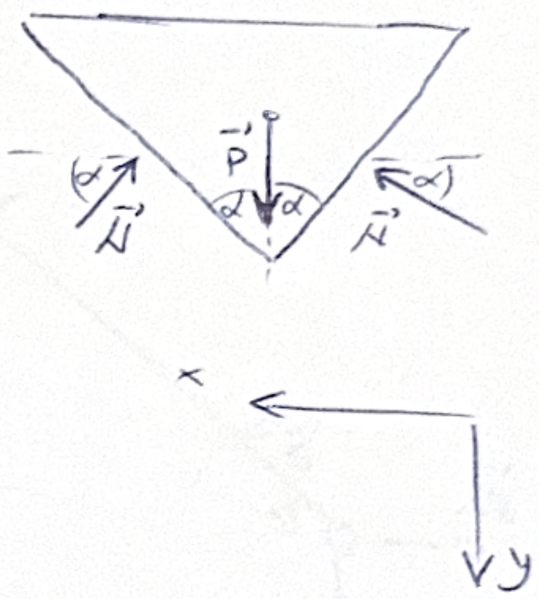
$$tg\alpha = \frac{dx}{dy} \Rightarrow dx = dy\, tg\alpha \quad /\frac{1}{dt} \quad /\frac{d}{dt}$$

$$\ddot{x} = \ddot{y}\, tg\alpha$$

Telo 1



Telo 2



$$m_2 \bar{a}_2 = \bar{P} + \bar{N}' + \bar{N}'$$

y: $\frac{P}{g}\ddot{y} = P - 2N\sin\alpha \quad /\cdot\frac{2}{P}$

$$\ddot{y} = g - 2\frac{g}{P}N\sin\alpha \quad (1) \quad (\ddot{y}=?,\ N=?)$$

x: ——— (nema)

$$m_1 \bar{a}_1 = \bar{P}_1 + \bar{N}_1' + \bar{N}'$$

y: $0 = P_1 - N_1 - N\sin\alpha$

$$N_1 = P_1 - N\sin\alpha \quad (2) \quad (N_1 = ?)$$

x: $\frac{P_1}{g}\ddot{x} = N\cos\alpha$

$$\frac{P_1}{g}\ddot{y}\, tg\alpha = N\cos\alpha \quad (3)$$

(1) obratno vo (3)

$$\frac{P_1}{g}\left(g - \frac{2g}{P}N\sin\alpha\right) tg\alpha = N\cos\alpha$$

$$P_1 tg\alpha - \frac{2P_1}{P}N\sin\alpha\, tg\alpha = N\cos\alpha$$

$$N\left(\cos\alpha + \frac{2P_1}{P}\sin\alpha\, tg\alpha\right) = P_1 tg\alpha$$

$$N = \frac{P_1 tg\alpha}{\cos\alpha + \frac{2P_1}{P}\sin\alpha tg\alpha} = \frac{P\cdot P_1 tg\alpha}{P\cos\alpha + 2P_1\sin\alpha tg\alpha}$$

$$\boxed{N = \frac{P\cdot P_1 tg\alpha}{\cos\alpha(P + 2P_1 tg^2\alpha)}}$$

vrakjame se vo (3)

$$\frac{P_1}{g}\ddot{x} = \frac{P P_1 tg\alpha}{\cos\alpha(P + 2P_1 tg^2\alpha)}\cos\alpha \quad /\cdot g \quad /\iint dt\,dt$$

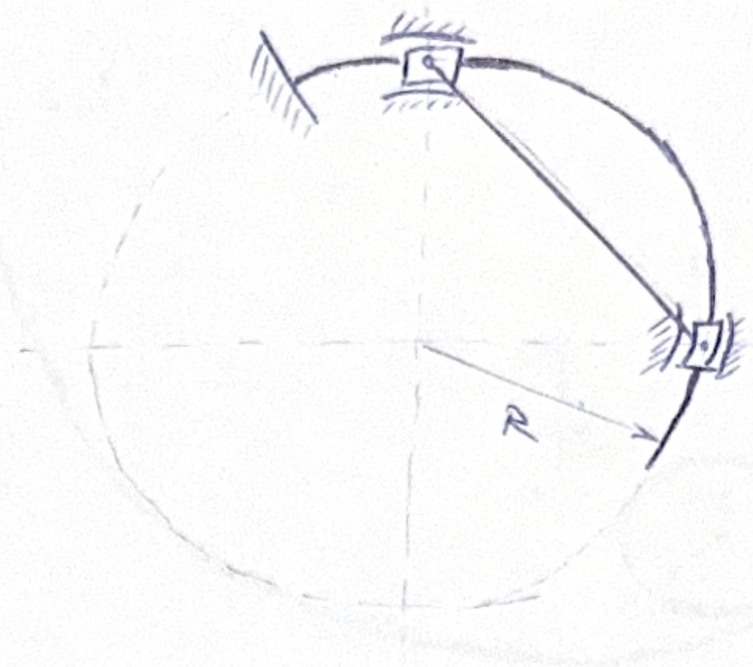
$$\boxed{x = \frac{P g\, tg\alpha}{P + 2P_1 tg^2\alpha}\frac{t^2}{2}}$$

$$\frac{P_1}{g}\ddot{y}\, tg\alpha = \frac{P\cdot P_1 tg\alpha}{\cos\alpha(P + 2P_1 tg^2\alpha)}\cos\alpha \quad /\cdot g \quad /\iint dt\,dt$$

$$\boxed{y = \frac{Pg}{P + 2P_1 tg^2\alpha}\frac{t^2}{2}}$$

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10.45



A: u

B: u

Štap: $\emptyset$

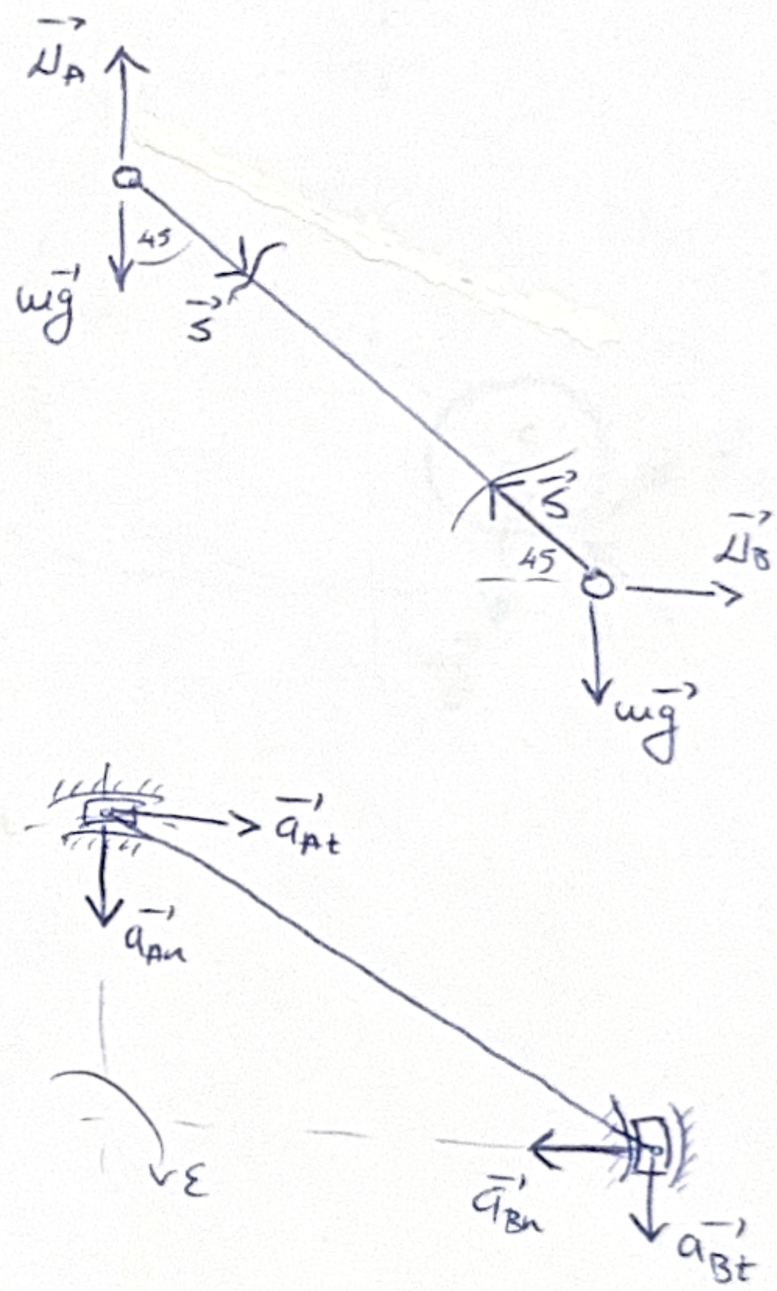
To: položaj na slici miruanje

$\omega_A = \omega_A(t_0) = ?$

$\omega_B = \omega_B(t_0) = ?$

Posmatrano translatorno to:

$$m\vec{a}_A = m\vec{g} + \vec{N}_A + \vec{S}$$



x: $m\ddot{x}_A = S\frac{\sqrt{2}}{2}$

$mR\varepsilon = S\frac{\sqrt{2}}{2}$ (1) $(\varepsilon = ?,\ S = ?)$

y: $0 = mg - N_A + S\frac{\sqrt{2}}{2}$

$N_A = mg + S\frac{\sqrt{2}}{2}$ (2) $(N_A = ?)$

$$m\vec{a}_B = m\vec{g} + \vec{N}_B + \vec{S}$$

x: $0 = N_B - S\frac{\sqrt{2}}{2}$

$N_B = S\frac{\sqrt{2}}{2}$ (3) $(N_B = ?)$

y: $m\ddot{y}_B = mg - S\frac{\sqrt{2}}{2}$

$mR\varepsilon = mg - S\frac{\sqrt{2}}{2}$ (4)

$\ddot{x}_A = a_{At} = \varepsilon R$

$\ddot{y}_A = a_{An} = \frac{v_A^2}{R} = 0$

$\ddot{x}_B = -a_{Bn} = -\frac{v_B^2}{R} = 0$

$\ddot{y}_B = a_{Bt} = \varepsilon R$

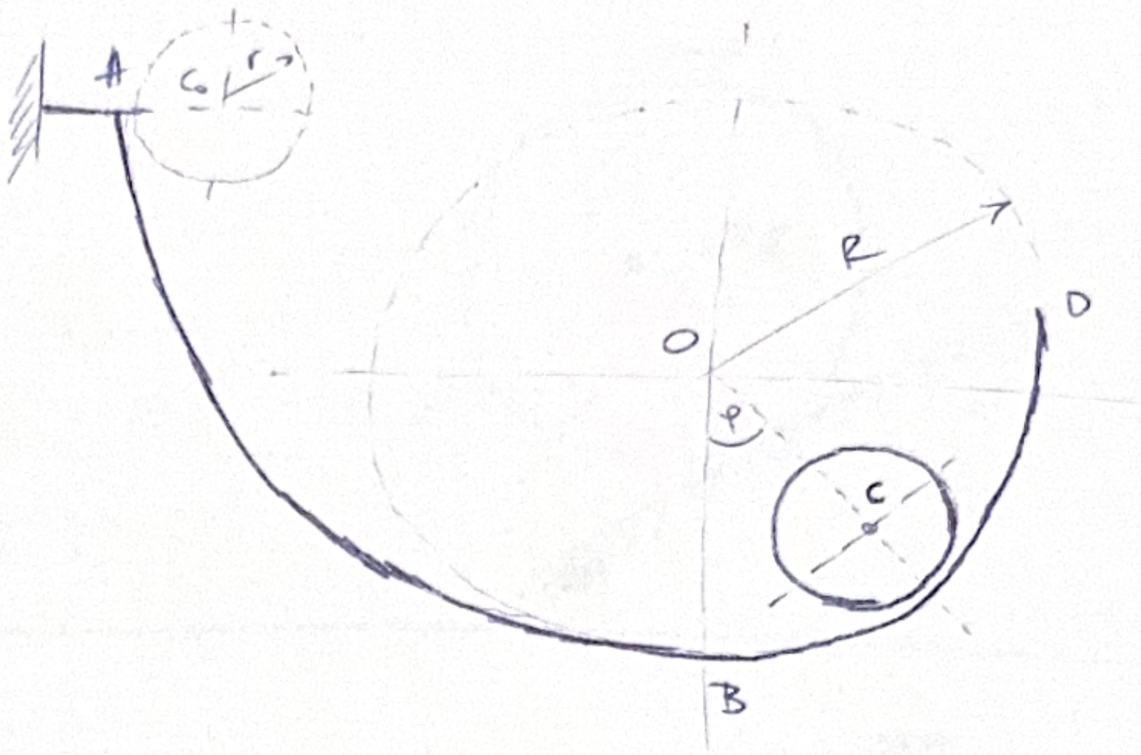
(1) = (4) $\Rightarrow$ $S\frac{\sqrt{2}}{2} = mg - S\frac{\sqrt{2}}{2} \Rightarrow S\sqrt{2} = mg \Rightarrow \underline{S = \frac{\sqrt{2}}{2}mg}$

(2) $\Rightarrow$ $N_A = mg + mg\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} \Rightarrow$ $\boxed{N_A = \frac{3}{2}mg}$

(3) $\Rightarrow$ $N_B = mg\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2} \Rightarrow$ $\boxed{N_B = \frac{1}{2}mg}$

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10.46.



disk: m, r

kotrljanje bez klizanja

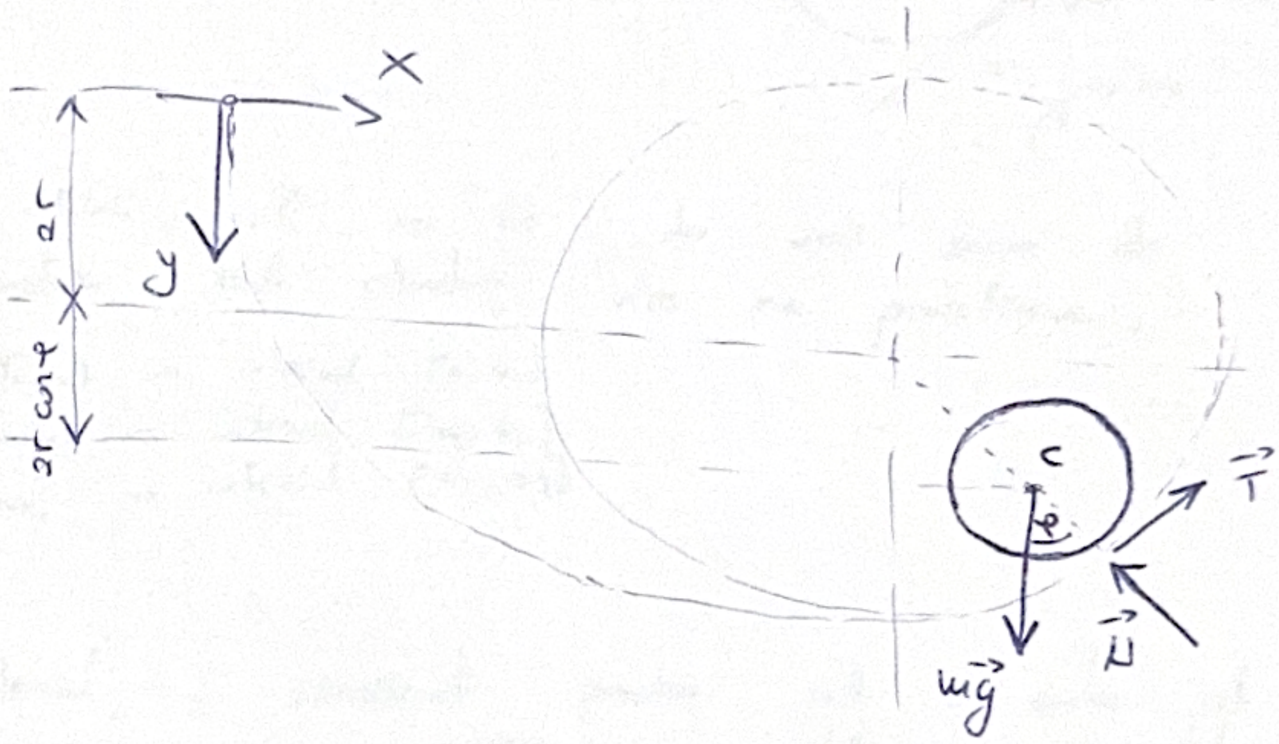
$t_0 = 0$: stanje mirovanja

$H_{C0} = 5r$

$R = 3r$

$\omega(t) = ?$

$\vec{ma_c} = \vec{mg} + \vec{N} + \vec{T}$



n: $m\frac{v^2}{2r} = -mg\cos\varphi + N$ (1) $(v=?, \omega=?)$

t: $m\ddot{s} = -mg\sin\varphi + T$

$mr\ddot{\varphi} = -mg\sin\varphi + T$ (1)

$T - T_0 = A$

$T_0 = 0$

$T = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}mv_c^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2\frac{v_c^2}{r^2} = \frac{3}{4}mv_c^2$

$A = \int m\vec{g}\,d\vec{r}_c = \int_0^{2r+2r\cos\varphi} mg\,dy_c = 2mgr(1+\cos\varphi)$

$\frac{3}{4}mv_c^2 = 2mgr(1+\cos\varphi)$

$v_c^2 = \frac{8}{3}gr(1+\cos\varphi)$

(1) $\Rightarrow N = mg\cos\varphi + \frac{m}{2r}\cdot\frac{8gr}{3}(1+\cos\varphi) = mg\cos\varphi + \frac{4}{3}mg + \frac{4}{3}mg\cos\varphi$

$$\boxed{N = \frac{1}{3}mg(4+7\cos\varphi)}$$

$\frac{d\vec{L}_C}{dt} = \Sigma\vec{M}_C$

$\Sigma M_C = -T\cdot r$

$L_C = J_C\cdot\omega = \frac{1}{2}mr^2\omega$

$\frac{dL_C}{dt} = \frac{1}{2}mr^2\varepsilon$

$\frac{1}{2}mr^2\ddot{\varphi} = -T\cdot k$ /:2

$mr\ddot{\varphi} = -2T$ (3)

(2) = (3)

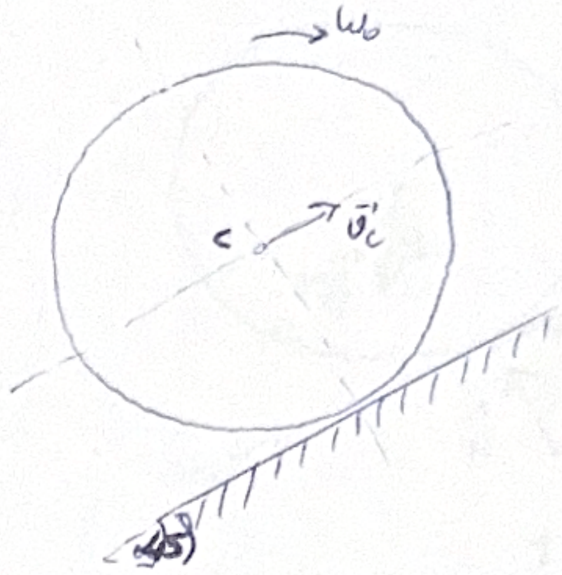
$-mg\sin\varphi + T = -2T$

$3T = mg\sin\varphi$

$$\boxed{T = \frac{1}{3}mg\sin\varphi}$$

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(10.47.)



tanki cilindar: m, r

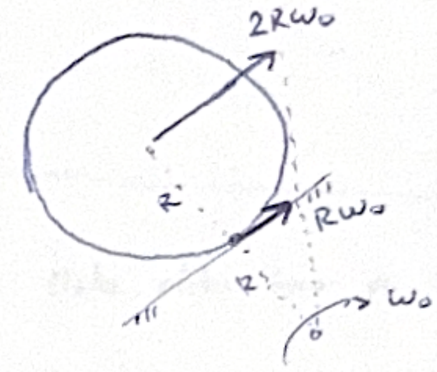
to: poč. ugaona brzina $\omega_0 = \sqrt{g/R}$

poč. brzina $v_C = 2R\omega_0$

$\mu = \frac{1}{3}$

$\alpha_{max} = ?$

Da bi znali u koju stranu deluje sila $\vec{T}$, moramo znati u kojoj strani disk proklizava, a da bi to znali, nacrtaćemo trenutak (t_0) i vektor brzine:



$\overline{CP_v} = \frac{v_0}{\omega_0}$

$\overline{CP_v} = \frac{2R\omega_0}{\omega_0}$

Sila $\vec{T}$ deluje suprotno od brzine $R\omega_0$, ali to će da vazi samo do trenutka t_1 - a to je trenutak kada cilindar više ne proklizava.

Kretanje: 1) proklizavanje naviše $(T = T_{max})$ - interval $[0, t_1]$
2) Bez proklizavanja $(T = [0, T_{max}))$ - interval $[t_1, t_2]$
3) proklizavanje naviše $(T = T_{max})$ - interval $[t_2, t_3]$

Sada ćemo analitički interval po interval, i izračunati preteći put u svakom od njih, da bi na kraju dobili visinu penjanja. Treba primetiti da trenutak zaustavljanja (trenutak t_3) može da se desi i pre t_2, ili čak i t_1, pa bi u tom slučaju imali samo dva ili čak jedan interval kretanja.

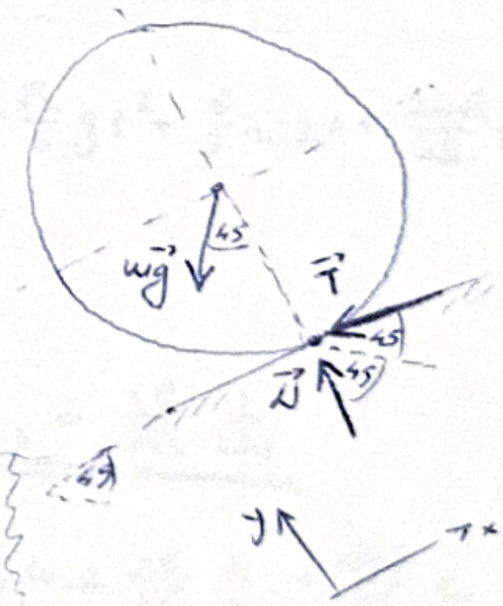
---

interval $t_0 - t_1$

opis kretanja: proklizavanje naviše, sve vreme deluje $T = T_{max}$, ne važi da je $v = r\omega$

cilj: naći $x(t_1)$?

uslov za trenutak t_1: $v(t_1) = R\omega(t_1)$



$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$

y: $0 = -mg\frac{\sqrt{2}}{2} + N$

$N = mg\frac{\sqrt{2}}{2}$

$T = \mu \cdot mg\frac{\sqrt{2}}{2} = mg\frac{\sqrt{2}}{6}$

x: $m\ddot{x} = -mg\frac{\sqrt{2}}{2} - T$

$m\ddot{x} = -mg\frac{\sqrt{2}}{2} - mg\frac{\sqrt{2}}{6}$

$\ddot{x} = -\frac{2\sqrt{2}}{3}g$ $\quad / \int dt$

$\dot{x} = -\frac{2\sqrt{2}}{3}gt + C_1 = -\frac{2\sqrt{2}}{3}gt + 2R\omega_0$ $\quad / \int dt$

$x = -\frac{\sqrt{2}}{3}gt^2 + 2R\omega_0 t + C_2^{\,0}$, $\quad x(t_1 = ?)$

$\frac{d\vec{L}_{CT}}{dt} = \sum \vec{M}_{CT}$

$\sum M_{CT} = T \cdot R = mgR\frac{\sqrt{2}}{6}$

$L_{CT} = J_{CT}\omega = mR^2\omega$

$\frac{dL_{CT}}{dt} = mR^2\varepsilon$

$mgR\frac{\sqrt{2}}{6} = mR^2\varepsilon$

$\varepsilon = \frac{g}{R}\frac{\sqrt{2}}{6}$ $\quad / \int dt$

$\omega = \frac{g}{R}\frac{\sqrt{2}}{6}t + \omega_0$

za t_1 važi:

$\dot{x}(t_1) = R\omega(t_1)$

$-\frac{2\sqrt{2}}{3}gt_1 + 2R\omega_0 = \frac{\sqrt{2}}{6}\frac{g}{R}t_1 \cdot R + \omega_0 R$

$-\frac{2\sqrt{2}}{3}gt_1 - \frac{\sqrt{2}}{6}gt_1 = -\omega_0 R$

$\frac{5\sqrt{2}}{6}gt_1 = \omega_0 R \Rightarrow t_1 = \frac{6}{5\sqrt{2}}\frac{\omega_0 R}{g} = \frac{3\sqrt{2}}{5}\frac{v_0}{\omega_0^2}$

$x(t_1) = -\frac{\sqrt{2}}{3}g \cdot \frac{9 \cdot 2}{25}\frac{1}{\omega_0^2} + 2R\omega_0 \cdot \frac{3\sqrt{2}}{5}\frac{1}{\omega_0}$

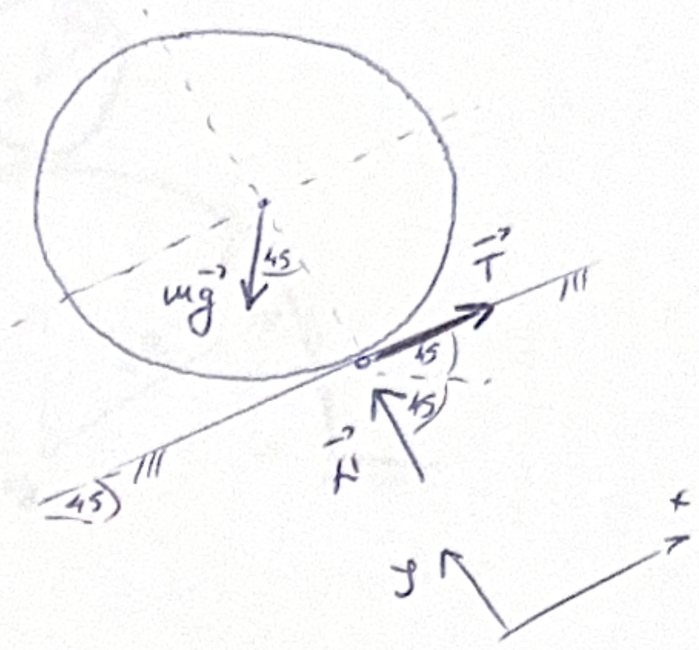
$x(t_1) = -\frac{6\sqrt{2}}{25}g\frac{R}{g} + \frac{6\sqrt{2}}{5}R$

$$\boxed{x(t_1) = \frac{24\sqrt{2}}{25}R}$$

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interval $t_1 - t_2$

opis kretanja: sila $\vec{T}$ je svojim delovanjem odmila ok, viče nema proklizavanja i ona je u trenutku t_1 jednaka nuli. Ona međutim vremenom raste, kako bi spreala proklizavanje, sve dok ne dostigne max. vr. i taj trenutak ćemo obeležiti sa t_2 - nakon njega počinje proklizavanje, samo u suprotnu stranu



cilj: naći $x(t_2)$?

uslov za nalazak t_2: $T(t_2) = T_{max} = \mu \cdot N$

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{T}$$

y: $0 = -mg\frac{\sqrt{2}}{2} + N$

$$N = mg\frac{\sqrt{2}}{2}$$

$T = ?$

x: $m\ddot{x} = -mg\frac{\sqrt{2}}{2} + T$ (1) $\left(\ddot{x} = ?,\ T = ?\right)$

$$\frac{d\vec{L}_{C_t}}{dt} = \sum \vec{M}_{C_t}$$

$\sum M_{C_t} = -T \cdot R$

$L_{C_t} = J_{C_t} \cdot \omega = mR^2 \frac{\dot{x}}{R}$

$\frac{dL_{C_t}}{dt} = mR\ddot{x}$

$mR\ddot{x} = -T \cdot R$

$m\ddot{x} = -T$ (2)

(1) = (2)

$-mg\frac{\sqrt{2}}{2} + T = -T$

$T = mg\frac{\sqrt{2}}{4} = const.$ - za sve vreme kretanja

s obzirom da je $T_{max} = \mu \cdot N = mg\frac{\sqrt{2}}{6}$, zaključujemo da se cilindar uopšte neće kotrljati bez proklizavanja, tako da odmah prelazimo na interval $t_2 - t_3$

$\left(x_{C_2}(t_2) = x(t_1) = \frac{24\sqrt{2}}{25} R\right)$

interval $t_2 - t_3$

(ista slika kao za interval $t_1 - t_2$)

cilj: naći $x(t_3)$

uslov za nalazak t_3: $\dot{x}(t_3) = 0$

$$m\vec{a}' = m\vec{g}' + \vec{N}' + \vec{T}'$$

y: $0 = -mg\frac{\sqrt{2}}{2} + N$

$$N = mg\frac{\sqrt{2}}{2}$$

$$T = mg\frac{\sqrt{2}}{6}$$

x: $m\ddot{x} = -mg\frac{\sqrt{2}}{2} + T = -mg\frac{\sqrt{2}}{2} + mg\frac{\sqrt{2}}{6} = -mg\frac{\sqrt{2}}{3}$ /$\int dt$

$\dot{x} = -\frac{\sqrt{2}}{3} g + C_1 = -\frac{\sqrt{2}}{3} gt + \frac{6}{5}\omega_0 R$ /$\int$

$C_1 = \dot{x}(t_1) = -\frac{2\sqrt{2}}{3} g \cdot \frac{3\sqrt{2}}{5} \frac{1}{\omega_0} + 2R\omega_0 = -\frac{4}{5}\omega_0 R \cdot \frac{1}{\omega_0} + 2R\omega_0$

ovo smo mogli lako što smo se vratili u interval $t_0 - t_1$

$x = -\frac{\sqrt{2}}{6} gt^2 + \frac{6}{5}\omega_0 R t + C_2 = -\frac{\sqrt{2}}{6} gt^2 + \frac{6}{5}\omega_0 R t + \frac{24\sqrt{2}}{25} R$

uslov za t_3: $\dot{x}(t_3) = 0$

$-\frac{\sqrt{2}}{3} g t_3 + \frac{6}{5}\omega_0 R = 0$

$+\frac{\sqrt{2}}{3}\omega_0^2 R t_3 = \frac{6}{5}\omega_0 R \Rightarrow t_3 = \frac{18}{5\sqrt{2}}\frac{1}{\omega_0}$

$x(t_3) = -\frac{\sqrt{2}}{6} g \cdot \frac{324}{50}\frac{1}{\omega_0^2} + \frac{6}{5}\omega_0 R \frac{18}{5\sqrt{2}}\frac{1}{\omega_0} + \frac{24\sqrt{2}}{25} R$

$x(t_3) = -\frac{27\sqrt{2}}{25} R \frac{1}{\omega_0^2}\omega_0^2 + \frac{54\sqrt{2}}{25} R + \frac{24\sqrt{2}}{25} R$

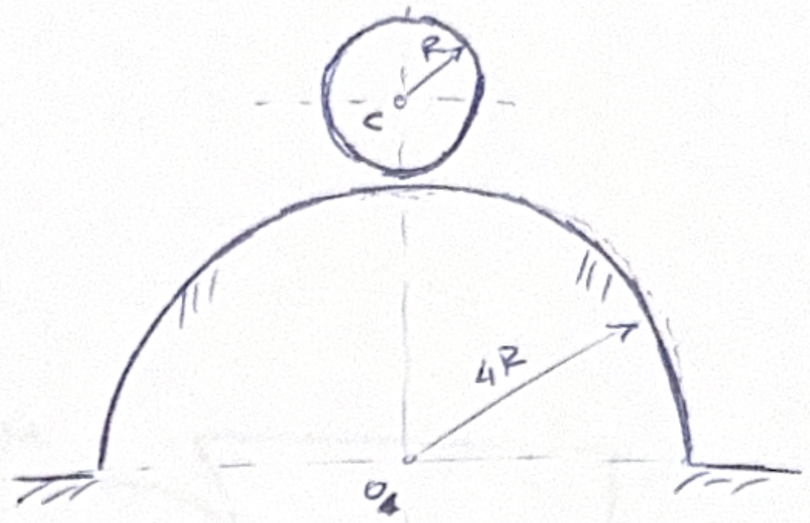
$x(t_3) = \frac{51\sqrt{2}}{25} R$

$H = x(t_3) \cdot \frac{\sqrt{2}}{2}$

$$\boxed{H = \frac{51}{25} R}$$

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10.4B



tanki cilindar: m, R

kotrljanje bez klizanja

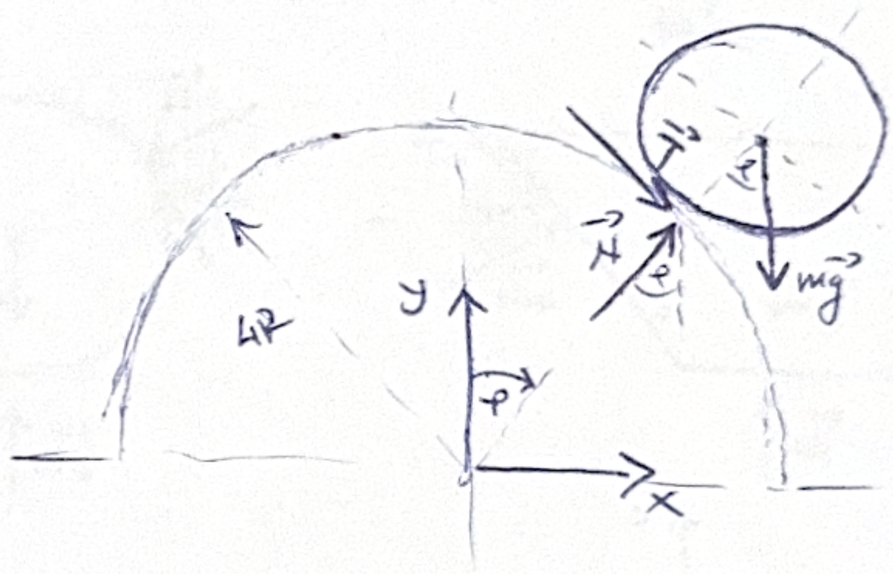
to: cilindar u najnižem položaju
stanje mirovanja (početno stanje)

odrediti položaj u kojem će doći
do odvajanja cilindra od podloge ?

---

ideja: naći $N(\varphi)$, pa onda postaviti uvjet: $\underline{\underline{N(\varphi_1)=0}}$
(odatle tek naći φ_1)

$$\underrightarrow{m\vec{a} = m\vec{g} + \vec{N} + \vec{T}}$$



n: $m \cdot \frac{v^2}{5R} = mg\cos\varphi - N$

$$\underline{\underline{N = -m\frac{v^2}{5R} + mg\cos\varphi}} \quad (1) \qquad (v(\varphi)=?)$$

t: ... (ne treba mi)

$$\underrightarrow{T - T_0 = A}$$

$T_0 = 0$

$T = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\frac{v^2}{R^2} = mv^2$

$A = \int m\vec{g}\,d\vec{r}_c = \int_{5R}^{5R\cos\varphi} -mg\,dy = 5mgR(1-\cos\varphi)$

$$\underline{\underline{mv^2 = 5mgR(1-\cos\varphi)}} \qquad (2)$$

(2) ubacimo u (1)

$$N = -m\frac{5gR(1-\cos\varphi)}{5R} + mg\cos\varphi$$

$$N = -mg + mg\cos\varphi + mg\cos\varphi$$

$$\underline{\underline{N = mg(2\cos\varphi - 1)}}$$

postavljamo uvjet: $\underline{\underline{N(\varphi_1)=0}}$

$mg(2\cos\varphi_1 - 1) = 0$

$\cos\varphi_1 = \frac{1}{2}$

$$\boxed{\varphi_1 = 60^\circ}$$

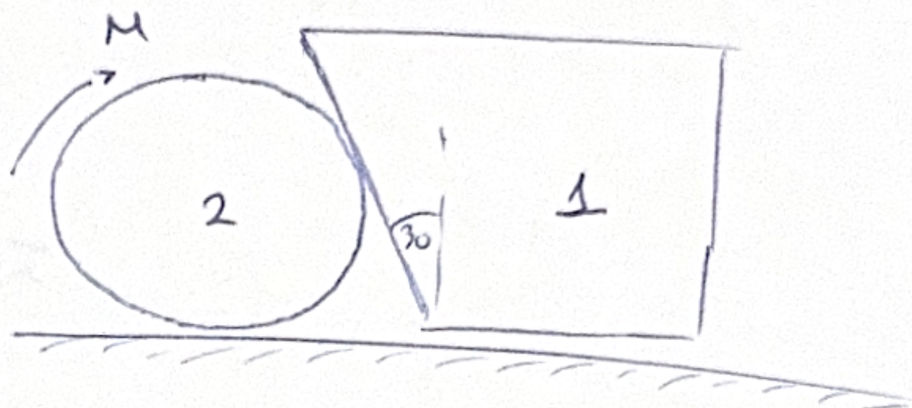
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(10.49)

2: $2w$, r,
kotrljanje bez klizanja

M, $\mu = \frac{\sqrt{3}}{4}$ - koef. trenja između prizme i ~~ploče~~ valjka

1: w
kliza bez trenja



$\ddot{x}_1 = ?$ $\quad N_1 = ?$ $\quad N_2 = ?$ $\quad T_2 = ?$

$N_{12} = ?$ $\quad T_{12} = ?$

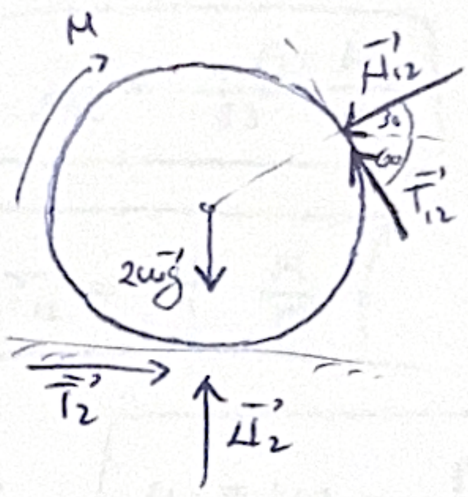
t: miruvanje

---

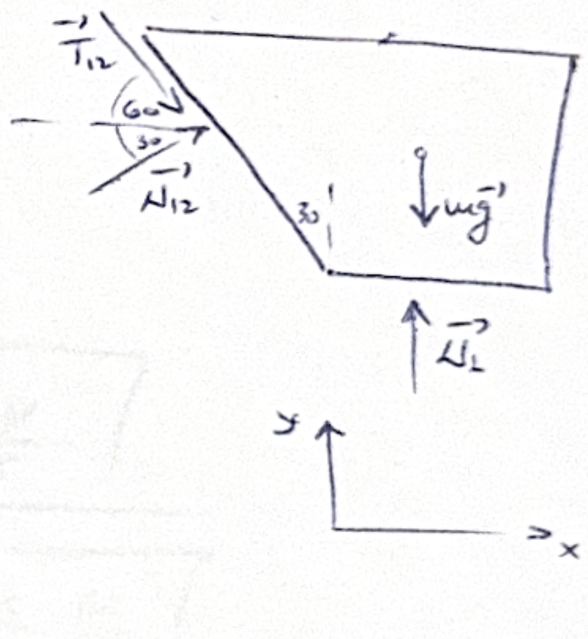
kinematska veza: $\rightarrow$ $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

veza: $\rightarrow$ $T_{12} = \mu \cdot N_{12} = \frac{\sqrt{3}}{4} N_{12}$

$m\vec{a}_1 = m\vec{g} + \vec{N}_1 + \vec{N}_{12} + \vec{T}_{12}$



telo 1



y: $0 = -mg + N_1 + N_{12}\sin 30 - T_{12}\cos 30$

$0 = -mg + N_1 + \frac{1}{2}N_{12} - \frac{\sqrt{3}}{4}\frac{\sqrt{3}}{2}N_{12}$

$0 = -mg + N_1 + \frac{1}{8}N_{12}$ (1) $\quad (N_1 = ?,\ N_{12} = ?)$

x: $m\ddot{x} = N_{12}\cos 30 + T_{12}\sin 30$

$m\ddot{x} = N_{12}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}N_{12}\cdot\frac{1}{2}$

$m\ddot{x} = \frac{5\sqrt{3}}{8}N_{12}$ (2) $\quad (\ddot{x} = ?)$

$\frac{d\vec{L}_{CT}}{dt} = \Sigma\vec{M}_{CT}$

$\Sigma M_{CT} = M - T_2\cdot R - T_{12}\cdot R = M - T_2\cdot R - \frac{\sqrt{3}}{4}N_{12}\cdot R$

$L_{CT} = J_{CT}\cdot\omega = \frac{1}{2}2mR^2\omega = mR^{\cancel{2}}\frac{\dot{x}}{\cancel{R}}$

$\frac{dL_A}{dt} = mR\ddot{x}$

$mR\ddot{x} = M - T_2\cdot R - \frac{\sqrt{3}}{4}N_{12}\cdot R$ (5)

$m\vec{a}_2 = 2m\vec{g} + \vec{N}_2' + \vec{T}_2 + \vec{N}_{12}' + \vec{T}_{12}'$

y: $0 = -2mg + N_2 - N_{12}\sin 30 + T_{12}\cos 30$

$0 = -2mg + N_2 - \frac{1}{2}N_{12} + \frac{\sqrt{3}}{4}N_{12}\cdot\frac{\sqrt{3}}{2}$

$0 = -2mg + N_2 - \frac{1}{8}N_{12}$ (3) $\quad (N_2 = ?)$

x: $2m\ddot{x} = T_2 - N_{12}\cos 30 - T_{12}\sin 30$

$2m\ddot{x} = T_2 - \frac{\sqrt{3}}{2}N_{12} - \frac{\sqrt{3}}{4}N_{12}\cdot\frac{1}{2}$

$2m\ddot{x} = T_2 - \frac{5\sqrt{3}}{8}N_{12}$ (4)

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(2) $\Rightarrow \quad N_{12} = \frac{8}{5\sqrt{3}} m\ddot{x}$

(1) $\Rightarrow \quad N_1 = mg - \frac{1}{8} \frac{8}{5\sqrt{3}} m\ddot{x} = mg - \frac{1}{5\sqrt{3}} m\ddot{x}$

(3) $\Rightarrow \quad N_2 = 2mg + \frac{1}{8} \frac{8}{5\sqrt{3}} m\ddot{x} = 2mg + \frac{1}{5\sqrt{3}} m\ddot{x}$

(4) $\rightarrow \quad T_2 = 2m\ddot{x} + \frac{5\sqrt{3}}{8} \cdot \frac{8}{5\sqrt{3}} m\ddot{x} = 3m\ddot{x}$

(5) $\Rightarrow \quad M - 3m\ddot{x} \cdot R - \frac{\sqrt{3}}{4} \frac{8}{8\sqrt{3}} m\ddot{x} R = mR\ddot{x}$

$mR\ddot{x}\left(1 + 3 + \frac{2}{5}\right) = M$

$$\boxed{\ddot{x} = \frac{5}{22} \frac{M}{mR}}$$

$\Rightarrow \quad N_{12} = \frac{8}{5\sqrt{3}} m \frac{5}{22} \frac{M}{mR} \quad \Rightarrow \quad \boxed{N_{12} = \frac{4\sqrt{3}}{33} \frac{M}{r}}$

$\Rightarrow \quad T_{12} = \frac{\sqrt{3}}{4} \frac{4\sqrt{3}}{33} \frac{M}{r} \quad \Rightarrow \quad \boxed{T_{12} = \frac{1}{11} \frac{M}{r}}$

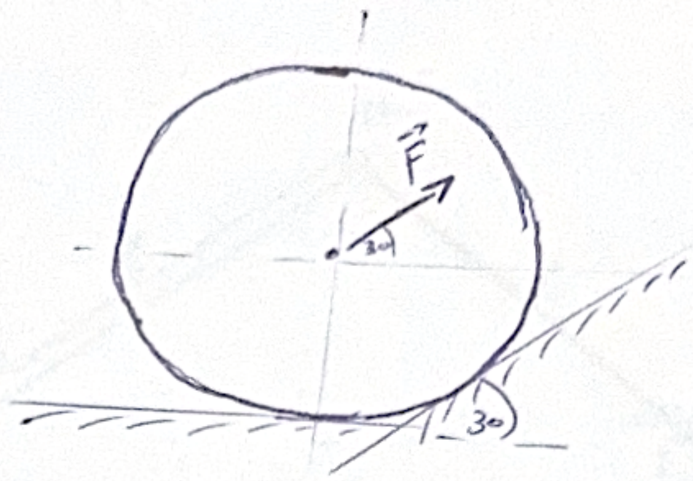
$\Rightarrow \quad N_1 = mg - \frac{1}{5\sqrt{3}} m \frac{5}{22} \frac{M}{mR} \quad \Rightarrow \quad \boxed{N_1 = mg - \frac{\sqrt{3}}{66} \frac{M}{r}}$

$\Rightarrow \quad N_2 = 2mg + \frac{1}{5\sqrt{3}} m \frac{5}{22} \frac{M}{mR} \quad \Rightarrow \quad \boxed{N_2 = 2mg + \frac{\sqrt{3}}{66} \frac{M}{R}}$

$\Rightarrow \quad T_2 = 3m \frac{5}{22} \frac{M}{mR} \quad \Rightarrow \quad \boxed{T_2 = \frac{15}{22} \frac{M}{mR}}$

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10.50.



poni cilindar: m, R

$F = \frac{mg}{4k} t$

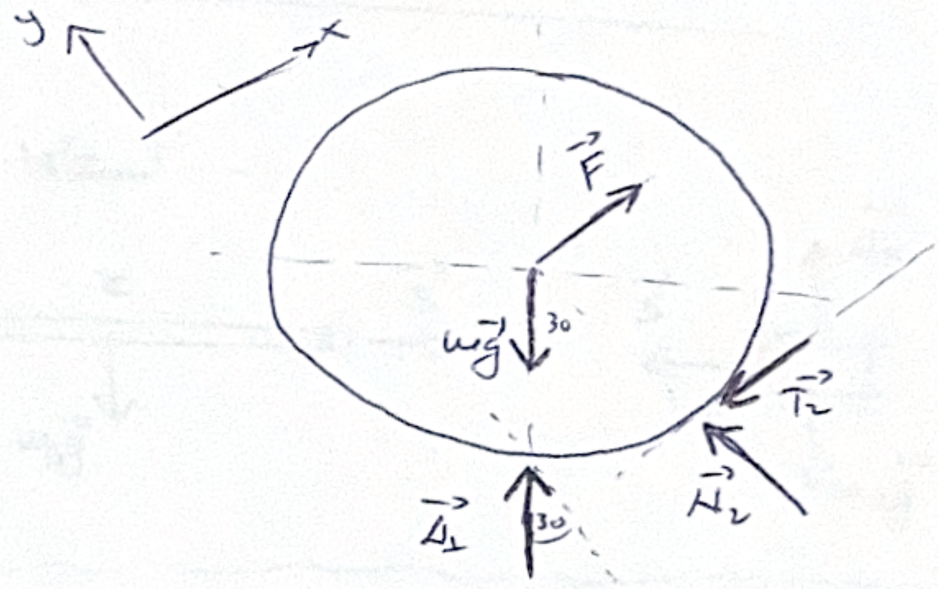
$\mu = \frac{\sqrt{3}}{3}$

t_0: miruje

$t_2 = ?$ - trenutak u kojem dolazi do proklizavanja?

Kretanje:

1) $t_0 - t_1$: cilindar još uvek nije započeo kretanje
2) $t_1 - t_2$: kretanje uz stalnu ravan uz prisustvo proklizavanja



interval $t_0 - t_1$:

cilj: naći t_1?

uslov: $N_1(t_1) = 0$

$m\vec{a} = m\vec{g}' + \vec{F}' + \vec{N}_1' + \vec{N}_2' + \vec{T}_2' \to 0$

x: $0 = -mg \sin 30 + F + N_1 \sin 30$

$0 = -\frac{1}{2} mg + \frac{mg}{4k} t + \frac{1}{2} N_1$

$N_1 = \frac{1}{2} mg - \frac{mg}{4k} t$

$N_1(t_1) = 0$

$\frac{1}{2} mg - \frac{mg}{4k} t_1 = 0$

$\frac{1}{2} = \frac{t_1}{4k}$

$t_1 = 2k$

interval $t_1 - t_2$

cilj: naći t_2

uslov: $T_2(t_2) = T_{2,max} = \mu \cdot N_2$, $N_1 = 0$

$m\vec{a} = m\vec{g}' + \vec{F}' + \vec{N}_2' + \vec{T}_2'$

y: $0 = -mg \cos 30 + N_2$

$N_2 = \frac{\sqrt{3}}{2} mg$

x: $m\ddot{x} = -mg \sin 30 + F - T_2$

$m\ddot{x} = -\frac{1}{2} mg + \frac{mg}{4k} t - T_2$ (1)

$\frac{d\vec{L}_{C+}}{dt} = \Sigma \vec{M}_{C+}$

$\Sigma M_{C+} = T_2 \cdot R$

$L_{C+} = J_{C+} \cdot \omega = \frac{1}{2} mR^2 \omega = \frac{1}{2} mR^2 \frac{\dot{x}}{R}$

$\frac{dL_{C+}}{dt} = \frac{1}{2} mR\ddot{x}$

$\frac{1}{2} mR\ddot{x} = +T_2 \cdot R$

$m\ddot{x} = 2T_2$ (2)

(1) = (2)

$2 \cdot T_2(t) = -\frac{1}{2} mg + \frac{mg}{4k} t - T_2(t)$

$T_2(t) = \frac{mg}{12k} t - \frac{1}{6} mg$

$T_2(t_2) = \frac{mg}{12k} t_2 - \frac{1}{6} mg$

$T_2(t_2) = \mu \cdot N_2 = \frac{\sqrt{3}}{3} \frac{\sqrt{3}}{2} mg = \frac{1}{2} mg$

$\Rightarrow \frac{mg}{12k} t_2 - \frac{1}{6} mg = \frac{1}{2} mg \quad / \cdot \frac{12k}{mg} \Rightarrow \boxed{t_2 = 8k}$

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(10.51.)

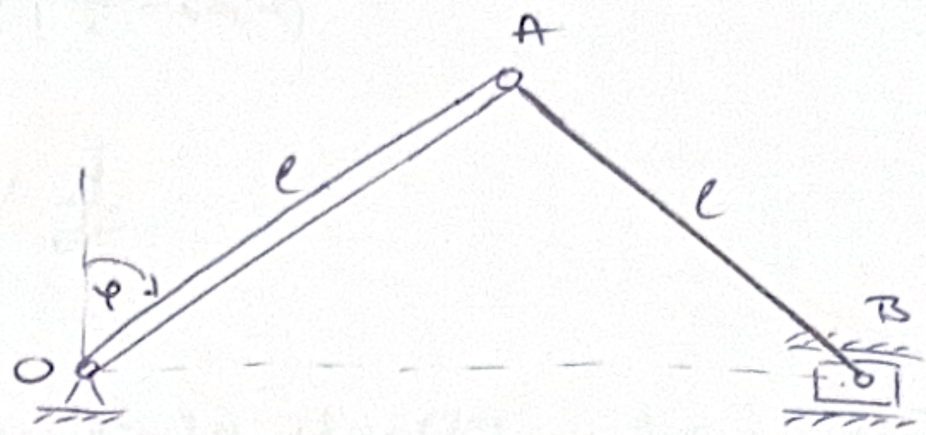
OA: m_1, ℓ

AB: $\emptyset$, ℓ

B: m_2

t_0: $\varphi_0 = 30^\circ$

sistem miruje



$\bar{N}_0'(t_1) = ?$

t_1 - trenutek v katerem je $\varphi(t_1) = 90^\circ$

Posmatramo sve za trenutak t_1:

Kinematske veze:

$\vec{a}_C' = \vec{a}_{Cn}' + \vec{a}_{Ct}'$

x: $\ddot{x}_C = -a_{Cn} = -\omega_1^2 \cdot \frac{1}{2}\ell = -\frac{1}{2}\ell\omega_1^2$

y: $\ddot{y}_C = -a_{Ct} = -\varepsilon_1 \cdot \frac{1}{2}\ell = -\frac{1}{2}\ell\varepsilon_1$

$\vec{a}_A' = \vec{a}_{An}' + \vec{a}_{At}'$

$a_{An} = \ell\omega_1^2$

$a_{At} = \ell\varepsilon_1$

$\vec{a}_B' = \vec{a}_{An}' + \vec{a}_{At}' + \vec{a}_{BA_n}^{A} + \vec{a}_{BA_t}^{A}$

$a_{Bn}^{A} = \omega_{AB_1}^2 \cdot \ell = \ell\omega_1^2$

$a_{Bt}^{A} = \varepsilon_{AB} \cdot \ell$

y: $0 = 0 - a_{At} + 0 + a_{Bt}^{A}$

$a_{Bt}^{A} = a_{At}$

$\varepsilon_{AB} \cdot \ell = \varepsilon_1 \cdot \ell$

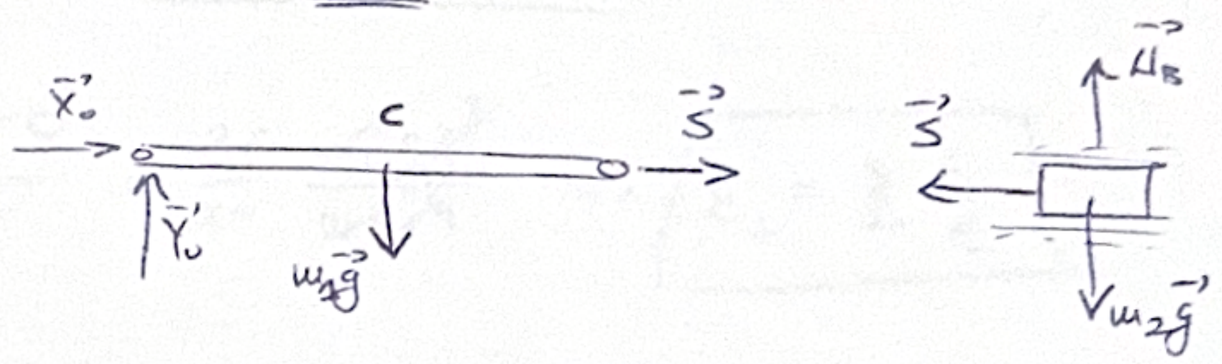
x: $\ddot{x}_B = -a_{An} + a - a_{Bn}^{A} + 0$

$\ddot{x}_B = -\ell\omega_1^2 - \ell\omega_1^2$

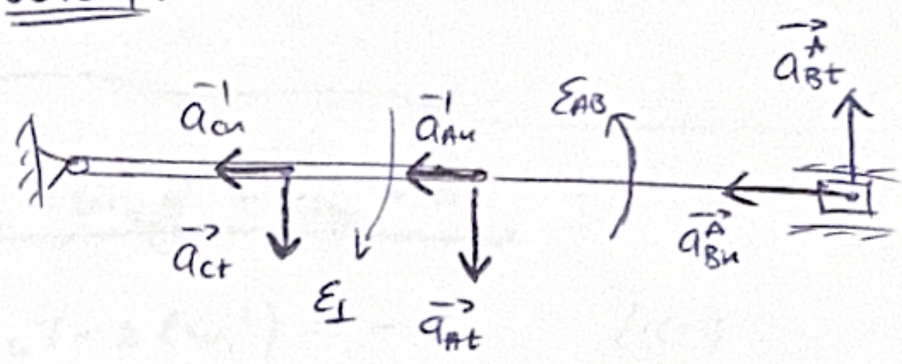
$\ddot{x}_B = -2\ell\omega_1^2$

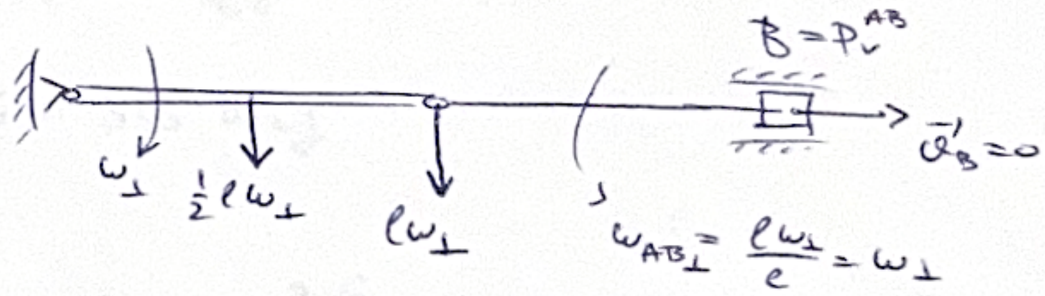
$\ddot{y}_B = 0$

Telo 1



Ubrzanja





$T_1 - T_0 = A$

$T_0 = 0$

$T = \left[\frac{1}{2}m_1 v_c^2 + \frac{1}{2}J_c\omega^2\right] + \left[\frac{1}{2}m_2 v_B^2 + 0\right]$



$x_C = \frac{1}{2}\ell\sin\varphi \Rightarrow \dot{x}_C = \frac{1}{2}\ell\omega\cos\varphi$

$y_C = \frac{1}{2}\ell\cos\varphi \Rightarrow \dot{y}_C = -\frac{1}{2}\ell\omega\sin\varphi$

$\Rightarrow v_C^2 = \dot{x}_C^2 + \dot{y}_C^2$

$x_B = 2\ell\sin\varphi \Rightarrow \dot{x}_B = 2\ell\omega\cos\varphi$

$y_B = 0 \Rightarrow \dot{y}_B = 0$

$\Rightarrow v_B^2 = 4\ell^2\omega^2\cos^2\varphi$

$T = \left[\frac{1}{2}m_1\frac{\ell^2\omega^2}{4} + \frac{1}{2}\frac{1}{12}m_1\ell^2\omega^2\right] + \left[\frac{1}{2}m_2 \cdot 4\ell^2\omega^2\cos^2\varphi\right]$

$T = \frac{1}{6}m_1\ell^2\omega^2 + 2m_2\ell^2\omega^2\cos^2\varphi$

$A = \int m_1\vec{g}\,d\vec{r}_C = \int_{\frac{1}{2}\ell\cos 30^\circ}^{\frac{1}{2}\ell\cos\varphi} -m_1 g\,dy = \frac{1}{2}m_1 g\ell\left(\frac{\sqrt{3}}{2} - \cos\varphi\right)$

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$T - T_0 = A$

$\frac{1}{6} m_1 \ell^2 \omega^2 + 2 m_2 \ell^2 \omega^2 \cos^2\varphi = \frac{1}{2} m_1 g \ell \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)$

$$\omega = \sqrt{\frac{\frac{1}{2} m_1 g \ell \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)}{\frac{1}{6} m_1 \ell^2 + 2 m_2 \ell^2 \cos^2\varphi}} \qquad / \frac{d}{dt}$$

$$\varepsilon = \frac{1}{2\sqrt{\ldots}} \cdot \frac{\frac{1}{2} m_1 g \ell \cdot \omega \sin\varphi \left(\frac{1}{6} m_1 \ell^2 + 2 m_2 \ell^2 \cos^2\varphi\right) - \left(\frac{1}{2} m_1 g \ell \left(\frac{\sqrt{3}}{2} - \cos\varphi\right)\right) \cdot \left(-2 m_2 \ell^2 \cdot 2 \cos\varphi \cdot \sin\varphi \cdot \omega\right)}{\left(\frac{1}{6} m_1 \ell^2 + 2 m_2 \ell^2 \cos^2\varphi\right)^2}$$

$\omega_1 = \omega(\varphi = 90) = \sqrt{\frac{\frac{1}{2} m_1 g \ell \cdot \frac{\sqrt{3}}{2}}{\frac{1}{6} m_1 \ell^2}} \Rightarrow \boxed{\omega_1 = \frac{3\sqrt{3}}{2} \frac{g}{\ell}}$

$\varepsilon_1 = \varepsilon(\varphi = 90) = \frac{1}{2} \frac{\frac{1}{2} m_1 g \ell \cdot \left(-\frac{1}{6} m_1 \ell^2 + 0\right) - 0}{\left(\frac{1}{6} m_1 \ell^2 + 0\right)^2} = \frac{36}{24} \frac{m_1^2 g \ell^3}{m_1^2 \ell^4} \Rightarrow \boxed{\varepsilon_1 = \frac{3}{2} \frac{g}{\ell}}$

$m_1 \vec{a}_C = m_1 \vec{g}' + \vec{X}_0' + \vec{Y}_0' + \vec{S}'$

$x: \quad m_1 \cdot \left(-\frac{1}{2} \ell \omega_1^2\right) = X_0 + S$

$-\frac{1}{2} m_1 \ell \frac{3\sqrt{3} g}{2\ell} = X_0 + S$

$-\frac{3\sqrt{3}}{4} m_1 g = X_0 + S \qquad (1)$

$y: \quad m_1 \left(-\frac{1}{2} \ell \varepsilon_1\right) = -m_1 g + Y_0$

$-\frac{1}{2} m_1 \ell \cdot \frac{3g}{2\ell} = -m_1 g + Y_0$

$-\frac{3}{4} m_1 g = -m_1 g + Y_0$

$\boxed{Y_0 = \frac{1}{4} m_1 g}$

$m_2 \vec{a}_B' = m_2 \vec{g}' + \vec{A}_B' + \vec{S}'$

$x: \quad m_2 \left(-2 \ell \omega_1^2\right) = -S \qquad / \cdot (-1)$

$2 m_2 \ell \cdot \frac{3\sqrt{3} g}{2\ell} = S$

$S = 3\sqrt{3}\, m_2 g$

(1) $\Rightarrow \quad X_0 = -\frac{3\sqrt{3}}{4} m_1 g - 3\sqrt{3}\, m_2 g$

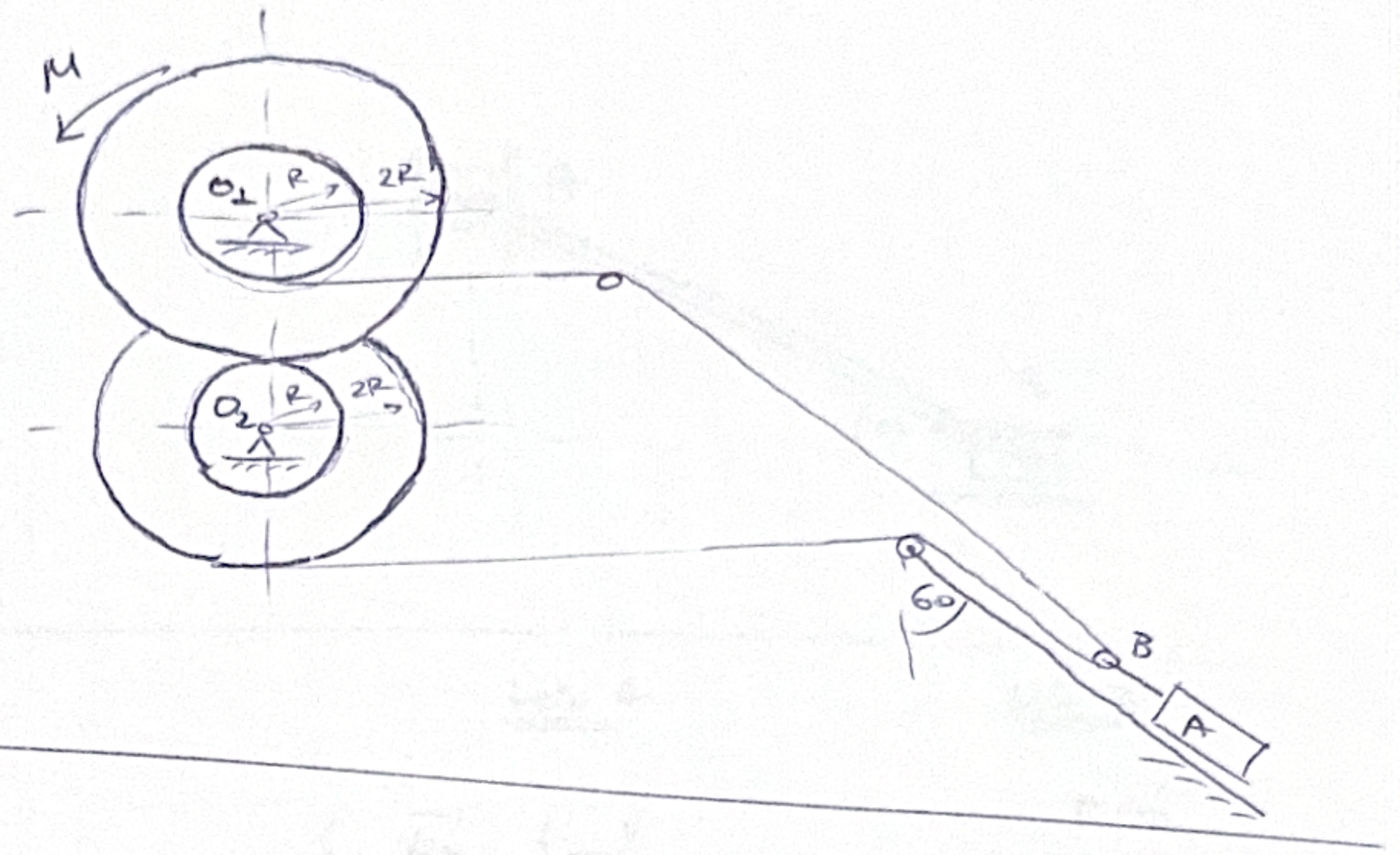
$\boxed{X_0 = -\frac{3\sqrt{3}}{4} g (m_1 + 4 m_2)}$

$\frac{d\vec{L}_C}{dt} = \vec{\Sigma M}_C$

000 (ne dolijan vista)

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10.52.



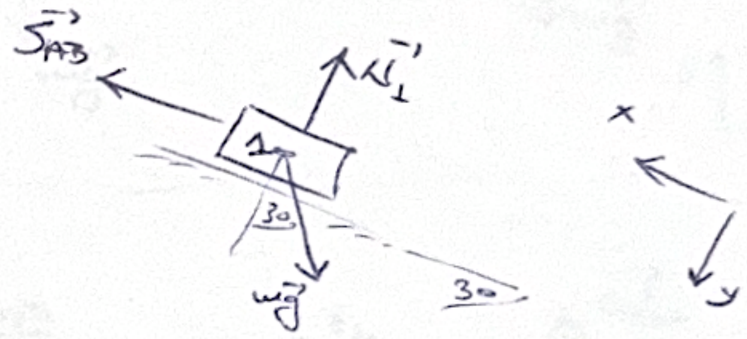
A: ω, podići se

zupčanici, koturovi, užad: ϕ

$M = ?$ - da bi $S_{AB} = \omega g$

$\underline{S_{AB} = \omega g}$ - uslov zadatka

$\underline{m\vec{a}_1 = m\vec{g} + \vec{N} + \vec{S}_{AB}}$



y: --- (nepotrebno)

x: $m\ddot{x}_1 = -mg\sin 30 + S_{AB}$

$m\ddot{x}_1 = -\frac{1}{2}mg + mg$

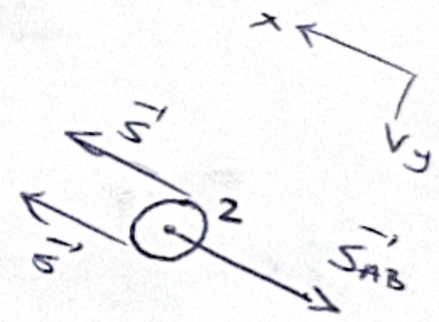
$\underline{\ddot{x}_1 = \frac{1}{2}g}$

$\underline{m\vec{a}_2 = \vec{S}'_{AB} + \vec{S}' + \vec{S}}$

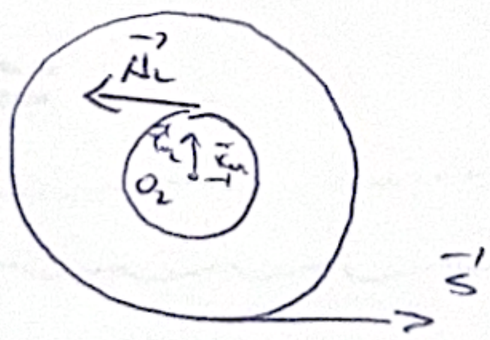
x: $0 = -S_{AB} + 2S$

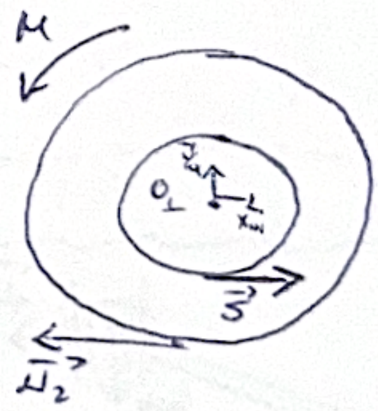
$S = \frac{1}{2}S_{AB}$

$\underline{S = \frac{1}{2}mg}$



$\underline{\frac{d\vec{L}_{O_2}}{dt} = \Sigma \vec{M}_{O_2}}$





$\Sigma M_{O_2} = N_2 \cdot R - S \cdot 2R$

$L_{O_2} = 0 \Rightarrow \frac{dL_{O_2}}{dt} = 0$

$0 = N_2 \cdot R - \frac{1}{2}mg \cdot 2R$

$\underline{N_2 = mgR}$

$\underline{\frac{d\vec{L}_{O_1}}{dt} = \Sigma \vec{M}_{O_1}}$

$\Sigma M_{O_1} = M + S \cdot R - N_2 \cdot 2R$

$L_{O_1} = 0 \Rightarrow \frac{dL_{O_1}}{dt} = 0$

$0 = M + \frac{1}{2}mgR - 2mgR$

$\boxed{M = \frac{3}{2}mgR}$

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(10.53.)

A: u

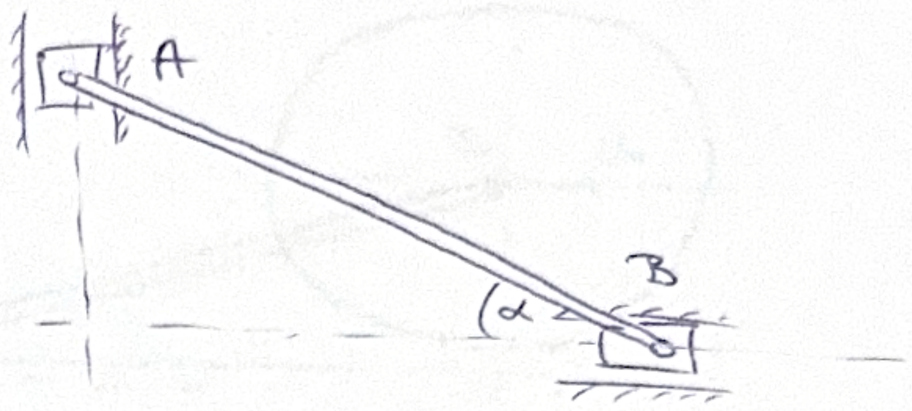
B: u

AB: $\emptyset$, ℓ

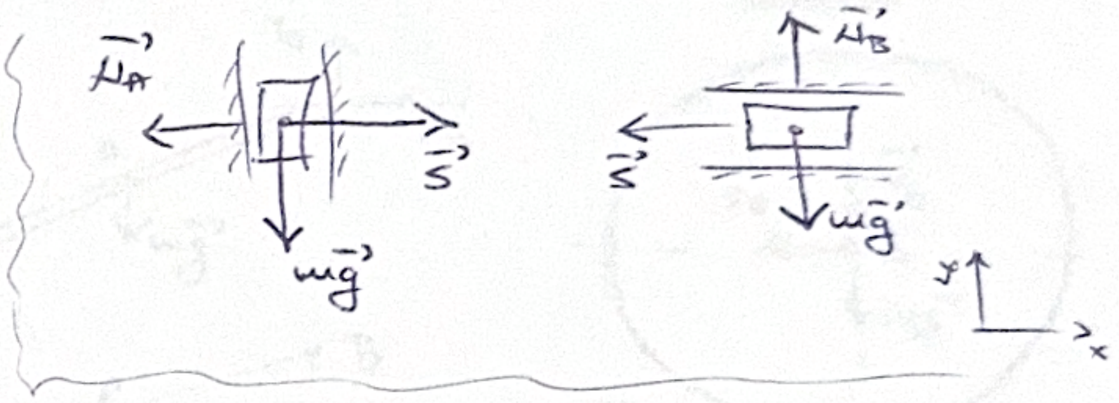
t_0: sistem miruje

$\alpha_0 = 30°$

$S(t_1) = ?$, t_1 - AB se horizontalan



Posmatramo sve za trenutak t_1:



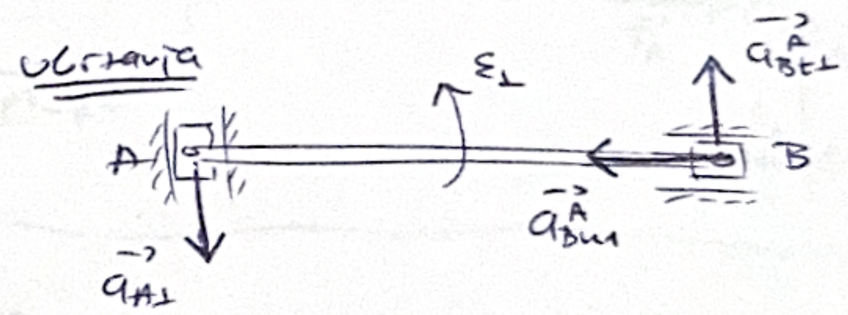
kinematska veza:

$\vec{a}_{A\perp} = ?$

x: $\ddot{x}_{A\perp} = 0$

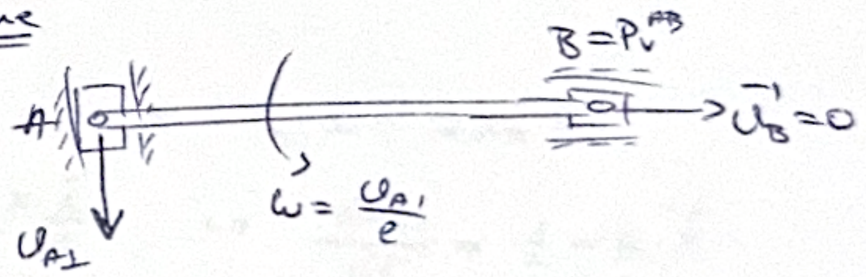
y: $\ddot{y}_{A\perp} = -a_{A\perp} = ?$

$\vec{a}_{B1} = \vec{a}_{A\perp} + \vec{a}^{A}_{Bn1} + \vec{a}^{A}_{Bt1}$



x: $\ddot{x}_{B1} = 0 - a^{A}_{Bn1} + 0$

$\ddot{x}_{B1} = -\omega_1^2 \ell$, $\ddot{y}_{B1} = 0$



y: $0 = -a_{A\perp} + 0 + a^{A}_{Bt1}$

$\ddot{y}_{A1} = -\varepsilon_1 \cdot \ell$, $\ddot{x}_{A1} = 0$

$m\vec{a}_{A1} = m\vec{g} + \vec{N}_A + \vec{S}'$

x: $0 = S - N_A$

$S = N_A$ (1)

y: $m\ddot{y}_{A1} = -mg$ /·(-1)

$-\varepsilon_1 \cdot \ell = -g$

$\varepsilon_1 = \frac{g}{\ell}$

$m\vec{a}_{B1} = m\vec{g}' + \vec{N}_B' + \vec{S}'$

x: $m\ddot{x}_{B2} = -S$

$-m\omega_1^2 \ell = -S$

$S = m\ell\omega_1^2$ (2)

cilj: naći ω_1!

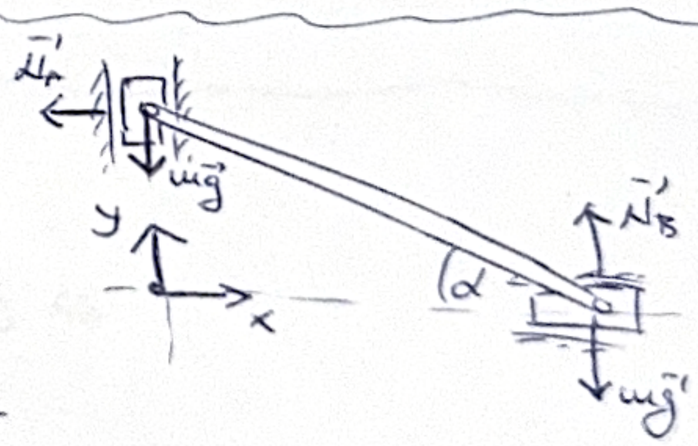
$T_1 - T_0 = A_{01}$

$T_0 = 0$

$T_1 = \left[\frac{1}{2} m v_{A1}^2 + 0\right] + \left[\frac{1}{2} v_{B1}^2 \cdot m + 0\right] =$

$T_1 = \frac{1}{2} m \ell^2 \omega_1^2 + 0 = \frac{1}{2} m \ell^2 \omega_1^2$

$A = \int_{\text{putanja}} m\vec{g}\, d\vec{r}_A = \int_0^{\ell \sin 30} -mg\, dy_A = \frac{1}{2} mg\ell$



$\frac{1}{2} m \ell^2 \omega_1^2 = \frac{1}{2} mg\ell$

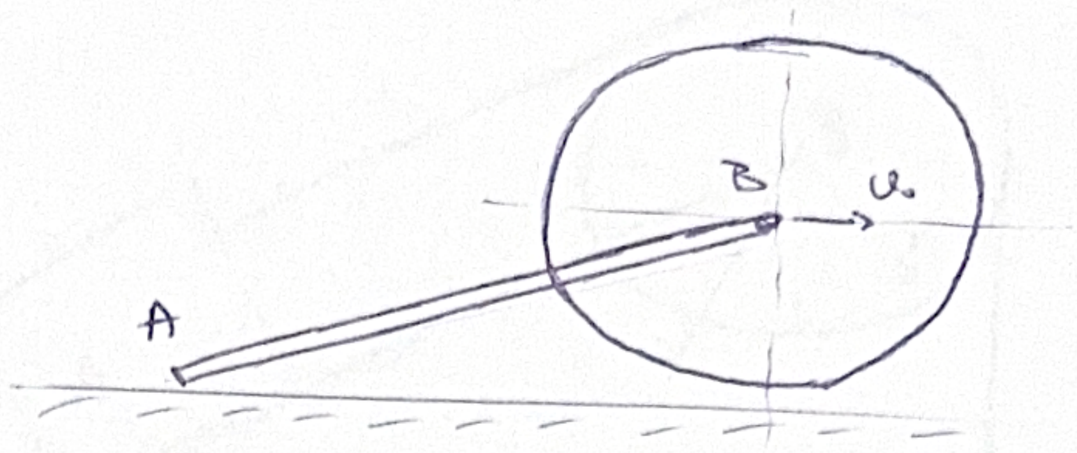
$\omega_1^2 = \frac{g}{\ell}$

(2) $\Rightarrow$ $S = m\ell \frac{g}{\ell}$

$\boxed{S = mg}$

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(10.54.)



disk: m, R

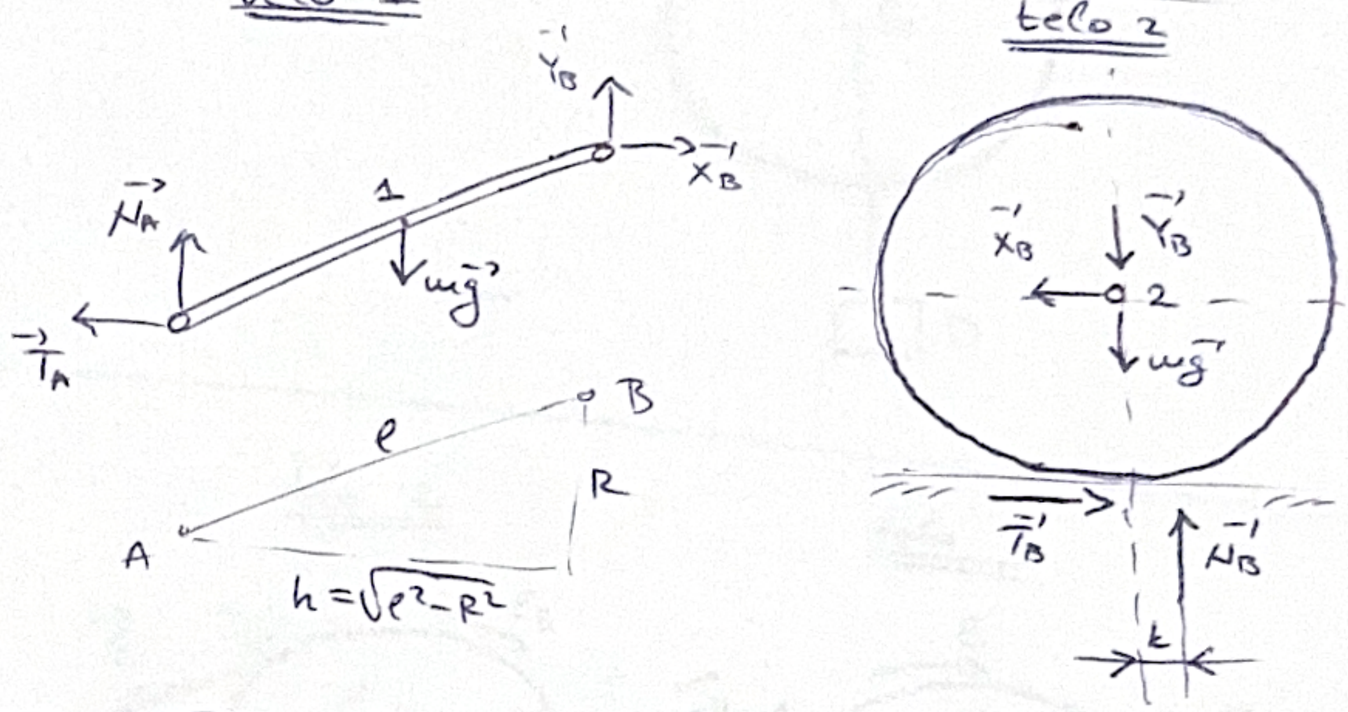
AB: m

$\mu = 0,1$

$k = \frac{1}{10} R$

t_0: $v_c(t_0) = v_0$

$s = ?$ - prevtni put diska do zaustavljanja



veza: $T_A = \mu \cdot N_A = \frac{1}{10} N_A$

kinematske veze: $\ddot{x}_1 = \ddot{x}_2 = \ddot{x}$

$m\vec{a}_1 = m\vec{g} + \vec{N}_A + \vec{T}_A + \vec{X}_B + \vec{Y}_B$

x: $m\ddot{x} = -T_A + X_B$

$m\ddot{x} = -\frac{1}{10} N_A + X_B \quad (1)$

y: $0 = N_A - mg + Y_B \quad (2)$

$\frac{d\vec{L}_{1z}}{dt} = \sum \vec{M}_{1z}$

$\sum M_{1z} = N_A \cdot \frac{1}{2} h + T_A \cdot \frac{1}{2} R + X_B \cdot \frac{1}{2} R - Y_B \cdot \frac{1}{2} h$

$L_{1z} = 0 \Rightarrow \frac{dL_{1z}}{dt} = 0$

$0 = \frac{1}{2}\left(N_A \cdot h + \frac{1}{10} N_A \cdot R + X_B \cdot R - Y_B \cdot h\right) \quad (3)$

$m\vec{a}_2 = m\vec{g} + \vec{N}_B + \vec{T}_B + \vec{X}_B + \vec{Y}_B$

x: $m\ddot{x} = -X_B + T_B \quad (4)$

y: $0 = -mg - Y_B + N_B \quad (5)$

$\frac{d\vec{L}_{2z}}{dt} = \sum \vec{M}_{2z}$

$\sum M_{2z} = -T_B \cdot R - N_B \cdot k = -T_B \cdot R - \frac{1}{10} N_B R$

$L_{2z} = J_2 \omega = \frac{1}{2} m R^2 \frac{\dot{x}}{R} \Rightarrow \frac{dL_{2z}}{dt} = \frac{1}{2} m R \ddot{x}$

$\frac{1}{2} m R \ddot{x} = -T_B \cdot R - \frac{1}{10} N_B \cdot R \quad / \cdot \frac{2}{R}$

$m\ddot{x} = -2T_B - \frac{1}{5} N_B \quad (6)$

(4) $\Rightarrow T_B = m\ddot{x} + X_B$

(5) $\Rightarrow N_B = mg + Y_B$

(1) $\Rightarrow X_B = m\ddot{x} + \frac{1}{10} N_A$

(2) $\Rightarrow N_A = mg - Y_B$

(6) $\Rightarrow m\ddot{x} = -2\left(m\ddot{x} + m\ddot{x} + \frac{1}{10}(mg - Y_B)\right) - \frac{1}{5}(mg + Y_B)$

$m\ddot{x} = -4m\ddot{x} - \frac{1}{5} mg + \frac{1}{5} Y_B - \frac{1}{5} mg - \frac{1}{5} Y_B$

$5m\ddot{x} = -\frac{2}{5} mg \quad / : 5m \quad / \int dt$

$\dot{x} = -\frac{2}{25} g t + v_0 \quad / \int dt$

$x = -\frac{1}{25} g t^2 + v_0 t$

$\dot{x}(t_1) = 0 \Rightarrow -\frac{2}{25} g t_1 + v_0 = 0 \Rightarrow t_1 = \frac{25}{2} \frac{v_0}{g}$

$s = x(t_1) = -\frac{1}{25} g \frac{25^2}{4} \frac{v_0^2}{g^2} + v_0 \cdot \frac{25}{2} \frac{v_0}{g} = \frac{v_0^2}{g}\left(-\frac{25}{4} + \frac{25}{2}\right)$

$$\boxed{s = \frac{25}{4} \frac{v_0^2}{g}}$$

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(10. 55)

A: G_1, μ

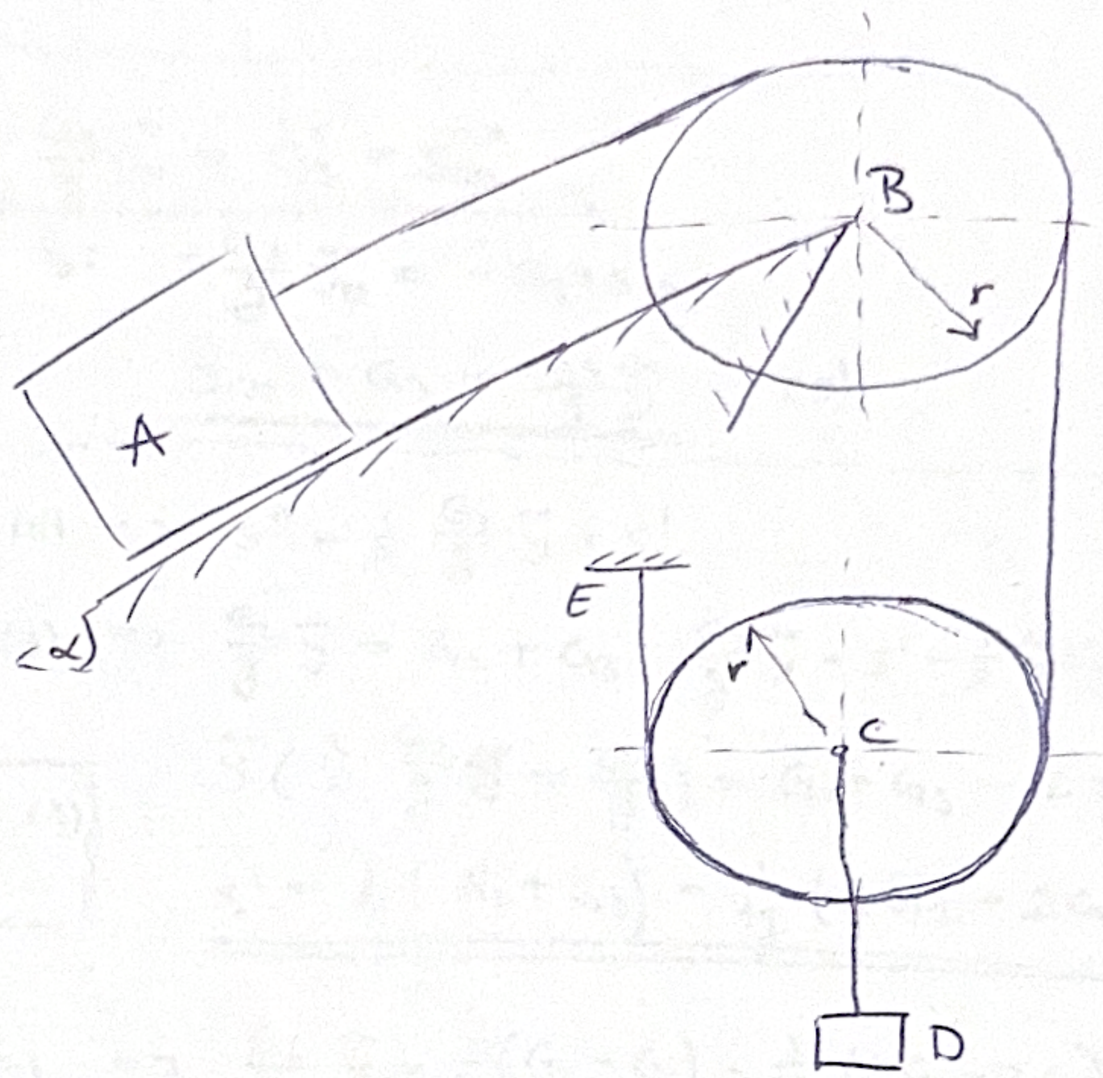
B: G_2, μ

C: G_2, μ

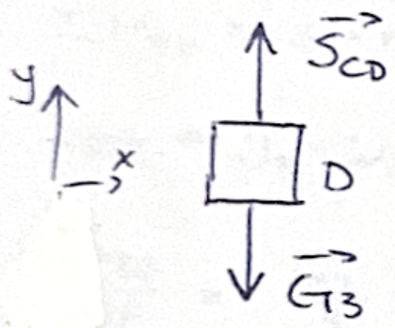
D: G_3

$a_D = ?$ $\quad S_{AB} = ?$

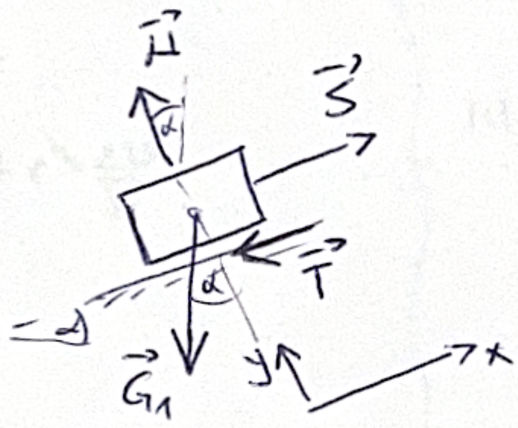
(Telo se kreće uz strmu ravan)



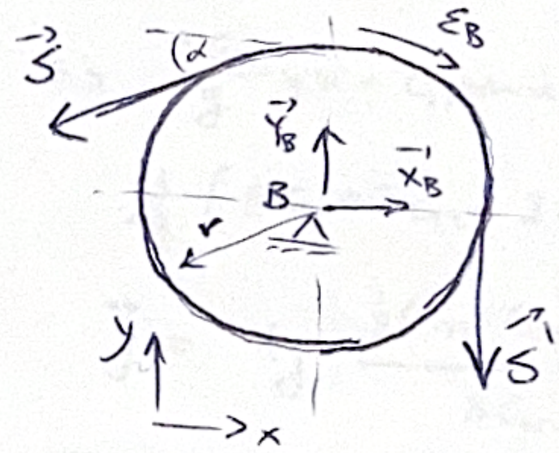
telo 4



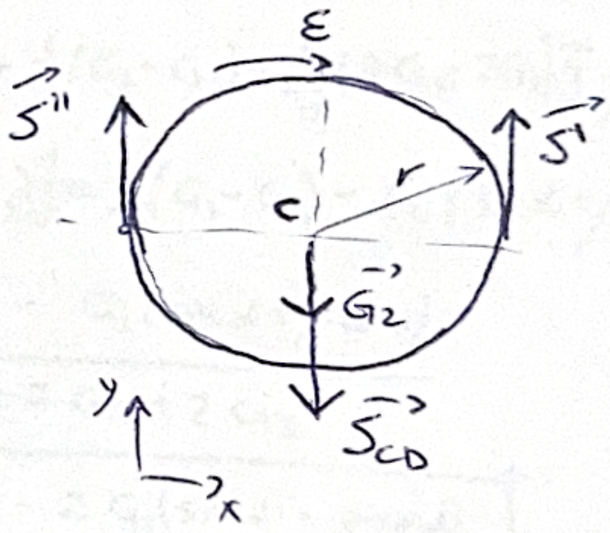
telo 1



telo 2

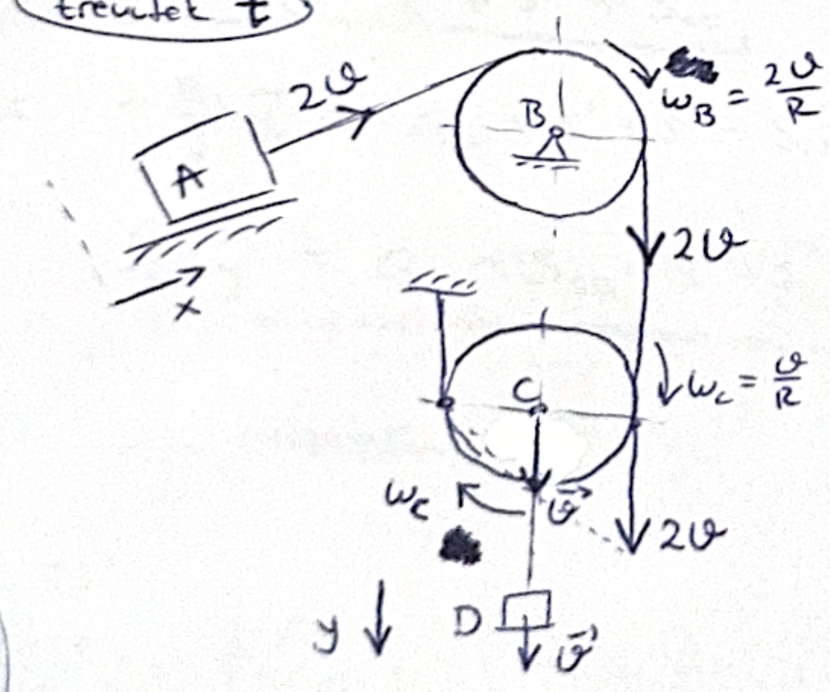


telo 3



kinematske veze:

trenutak t



(posmatramo opšti trenutak t, da bi mogli da diferenciramo)

izrazićemo sve preko brzine (i ubrzanja) tačke D:

neka je $\dot{y}_D = v_D$, i $\ddot{y}_D = \frac{dv}{dt} = \ddot{y}$,

pa odatle dobijamo:

$\dot{x}_A = 2v \Rightarrow \quad \ddot{x}_A = 2\ddot{y}$

$\omega_B = \frac{2v}{r} \Rightarrow \quad \varepsilon_B = \frac{2\ddot{y}}{r}$

$\omega_C = \frac{2v}{2r} \Rightarrow \quad \varepsilon_C = \frac{\ddot{y}}{r}$

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$$M\vec{a}_A = \vec{G}_1 + \vec{S} + \vec{N} + \vec{T}$$

y: $0 = -G_1\cos\alpha + N$

$$N = G_1\cos\alpha$$

$$T = \mu_0 G_1\cos\alpha$$

x: $\frac{G_1}{g}\ddot{x}_A = -G_1\sin\alpha + S - T$

$$\frac{2G_1}{g}\ddot{y} = -G_1\sin\alpha + S - \mu G_1\cos\alpha \quad (1)$$

$$\frac{dL_{Bz}}{dt} = \Sigma M_{Bz}$$

$\Sigma M_{Bz} = S'\cdot r - S\cdot r$

$L_{Bz} = J_B\cdot\omega_B = \frac{1}{2}\frac{G_2}{g}r^2\frac{v}{r}$

$\frac{dL_{Bz}}{dt} = \frac{G_2}{g}r\ddot{y}$

$$\frac{G_2}{g}\cdot r\cdot\ddot{y} = S'\cdot r - S\cdot r \quad (2)$$

$$\frac{G_2}{g}\vec{a}_C = \vec{G}_2 + \vec{S}' + \vec{S}'' + \vec{S}_{CD}$$

y: $-\frac{G_2}{g}\ddot{y}_C = -G_2 + S' + S'' - S_{CD}$

$$\frac{G_2}{g}\ddot{y} = G_2 + S_{CD} - S' - S'' \quad (3)$$

x: -- (nepotrebno)

$$\frac{dL_{Ct}}{dt} = \Sigma M_{Ct}$$

$\Sigma M_{Ct} = (S'' - S')\cdot r$

$L_{Ct} = J_C\omega_C = \frac{1}{2}\frac{G_2}{g}r^2\frac{v}{r}$

$\frac{dL_{Ct}}{dt} = \frac{1}{2}\frac{G_2}{g}r\ddot{y}$

$$\frac{1}{2}\frac{G_2}{g}r\ddot{y} = (S'' - S')\cdot r \quad (4)$$

$$\frac{G_3}{g}\vec{a}_D = \vec{G}_3 + \vec{S}_{CD}$$

y: $-\frac{G_3}{g}\ddot{y}_D = -G_3 + S_{CD}$

$$S_{CD} = G_3 - \frac{G_3}{g}\ddot{y} \quad (5)$$

(4) => $S'' = \frac{1}{2}\frac{G_2}{g}\ddot{y} + S'$

(3) => $\frac{G_2}{g}\ddot{y} = G_2 + G_3 - \frac{G_3}{g}\ddot{y} - S' - \frac{1}{2}\frac{G_2}{g}\ddot{y} - S'$

$\ddot{y}\left(\frac{3}{2}\frac{G_2}{g} + \frac{G_3}{g}\right) = G_2 + G_3 - 2S'$

$$S' = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(3G_2 + 2G_3)\ddot{y}$$

(2) => $\frac{G_2}{g}\ddot{y} = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(3G_2 + 2G_3)\ddot{y} - S$

=> $S = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(7G_2 + 2G_3)\ddot{y}$

(1) => $\frac{2G_1}{g}\ddot{y} = -G_1\sin\alpha + \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(7G_2 + 2G_3)\ddot{y} - \mu G_1\cos\alpha$

$\frac{1}{4g}(8G_1 + 7G_2 + 2G_3)\ddot{y} = \frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)$

$\ddot{y} = 4g\,\frac{\frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$

$$\ddot{y} = 2g\,\frac{(G_2 + G_3) - 2G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$$

↑ ovo je dobro (kao u zbirci)

=> $S = \frac{1}{2}(G_2 + G_3) - \frac{1}{4g}(7G_2 + 2G_3)\cdot 4g\,\frac{\frac{1}{2}(G_2 + G_3) - G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$

$$S_{AB} = S = \frac{1}{2}(G_2 + G_3) - \frac{1}{2}(7G_2 + 2G_3)\,\frac{(G_2 + G_3) - 2G_1(\sin\alpha + \mu\cos\alpha)}{8G_1 + 7G_2 + 2G_3}$$

↑ ne poklapa se baš sa rešenjem, možda samo ulazim sredio?

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(10.56.)

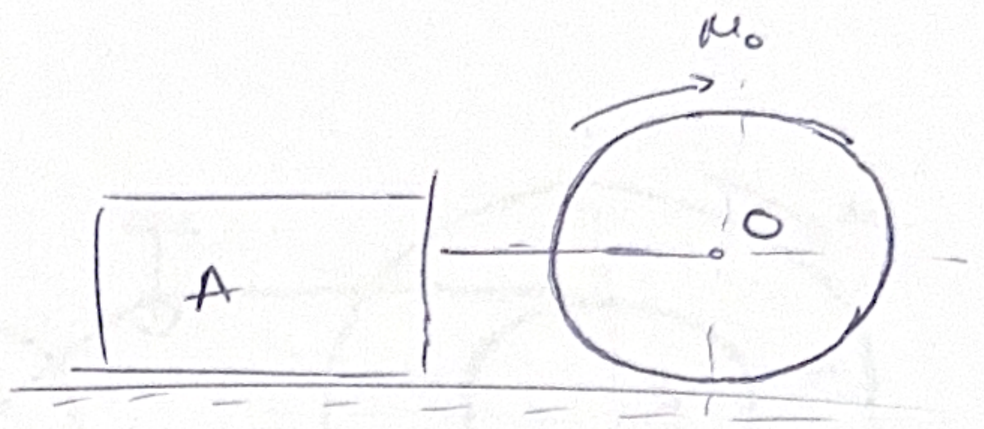
O: G_1, R, kotrljanje, ček klizanja, k - krak otpora kotrljanja, cilindar

A: G_2, μ - koef. trenja klizanja

M_0 - moment

to: sistem je mirovao

$a_A = ?$ $\quad S = ?$



Kinematska veza: $\ddot{x}_O = \ddot{x}_A = \ddot{x}$

$\frac{G_1}{g}\vec{a}_1 = \vec{G}_1 + \vec{N}_1 + \vec{T}_1 + \vec{S}$

y: $0 = -G_1 + N_1$

$N_1 = G$

x: $\frac{G_1}{g}\ddot{x} = T_1 - S \quad (1)$

$\frac{d\vec{L}_{1z}}{dt} = \sum \vec{M}_{1z}$

$\sum M_{1z} = M_0 - T_1 \cdot R - N_1 \cdot k$

$L_{1z} = J_1 \cdot \omega = \frac{G_1}{g} r^2 \frac{\dot{x}}{r}$

$\frac{dL_{1z}}{dt} = \frac{G_1}{g} r \ddot{x}$

$\frac{G_1}{g} R \ddot{x} = M_0 - T_1 \cdot R - G_1 \cdot k \quad (2)$

$\frac{G_2}{g}\vec{a}_2 = \vec{G}_2 + \vec{N}_2 + \vec{T}_2 + \vec{S}$

y: $0 = -G_2 + N_2$

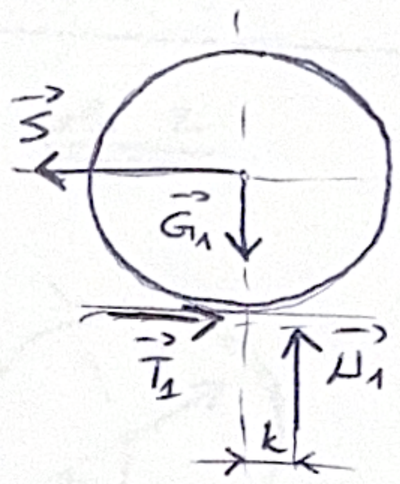
$N_2 = G_2$

$T_2 = \mu \cdot G_2$

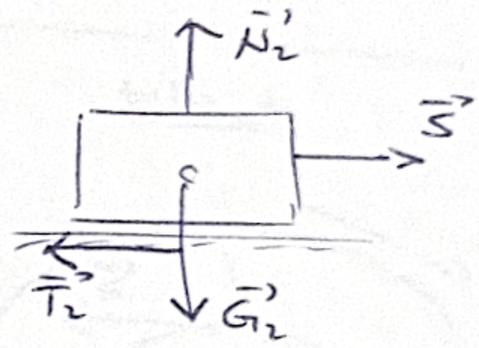
x: $\frac{G_2}{g}\ddot{x} = -T_2 + S$

$\frac{G_2}{g}\ddot{x} = S - \mu G_2 \quad (3)$

telo 1



telo 2



(3) $\Rightarrow$ $S = \frac{G_2}{g}\ddot{x} + \mu G_2$

(1) $\Rightarrow$ $T_1 = \frac{G_1}{g}\ddot{x} + \frac{G_2}{g}\ddot{x} + \mu G_2$

(2) $\Rightarrow$ $\frac{G_1}{g} R \ddot{x} = M_0 - \frac{G_1}{g} R \ddot{x} - \frac{G_2}{g} R \ddot{x} - \mu G_2 R - G_1 k$

$\frac{1}{g} r \ddot{x} (2G_1 + G_2) = M_0 - \mu G_2 R - G_1 k$

$$\ddot{x} = \frac{g}{R} \frac{M_0 - \mu G_2 R - G_1 k}{2G_1 + G_2}$$

↑ ovo se poklapa s rešenjem

$\Rightarrow$ $S = \frac{G_2}{g} \frac{g}{R} \frac{M_0 - \mu G_2 R - G_1 k}{2G_1 + G_2} + \mu G_2$

$$S = \frac{G_2}{R} \frac{M_0 - \mu G_2 R - G_1 k}{2G_1 + G_2} + \mu G_2$$

↑ ovo se verovatno ne poklapa, vrlo verovatno jer nisam sredio!

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(10.57.)

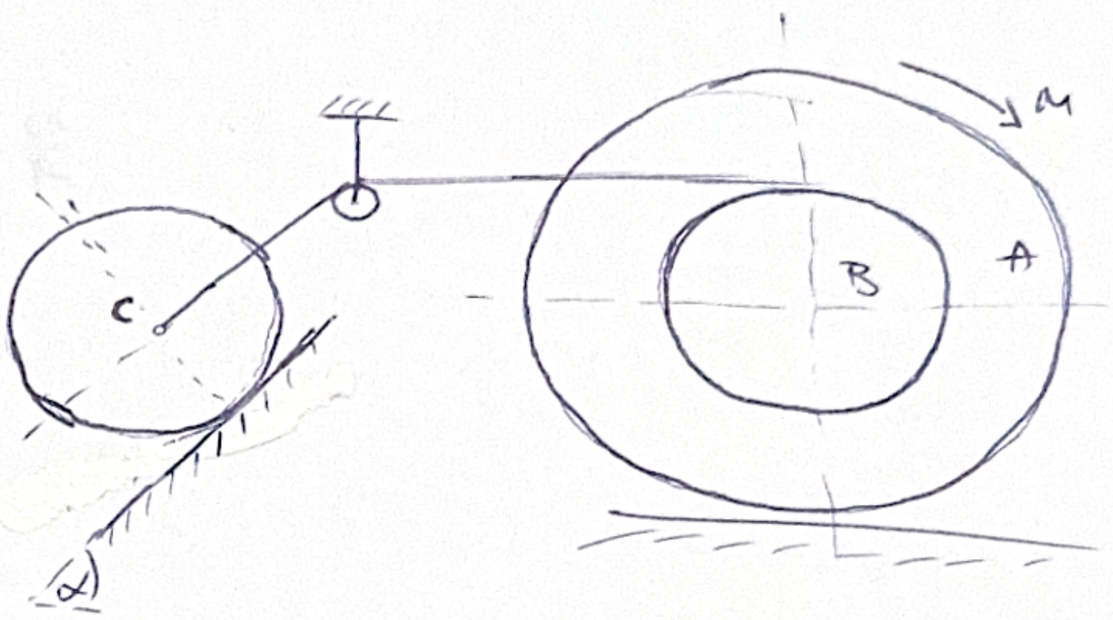
dva koaksijalna diska kako spojena:

A: $2R$
B: R
} G, $i = R\sqrt{2}$

C: R, G

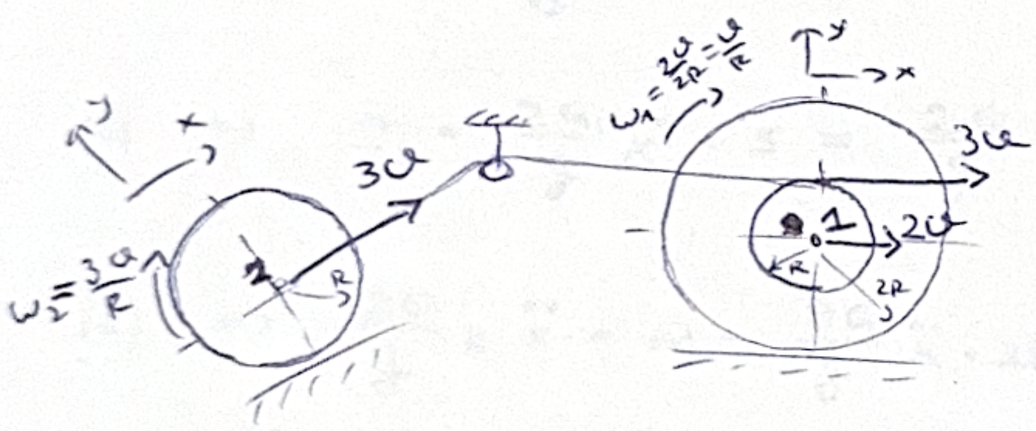
M, α

$S = ?$



Kinematska veza:

poseutrano opsti trenutak t
(da Gi wagi da diferenciramo)



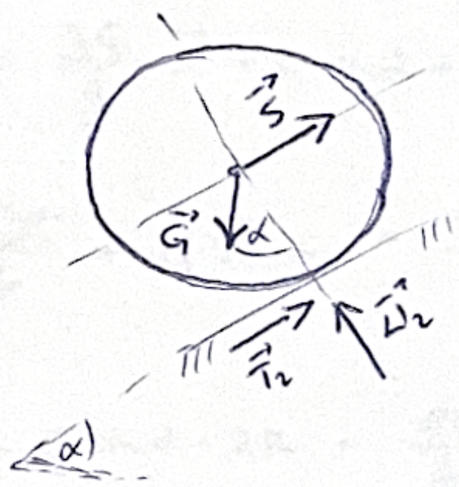
$\dot{x}_1 = 2v$, $\ddot{x}_1 = 2\ddot{x}$ (=?)

$\dot{x}_2 = 3v$, $\ddot{x}_2 = 3\ddot{x}$

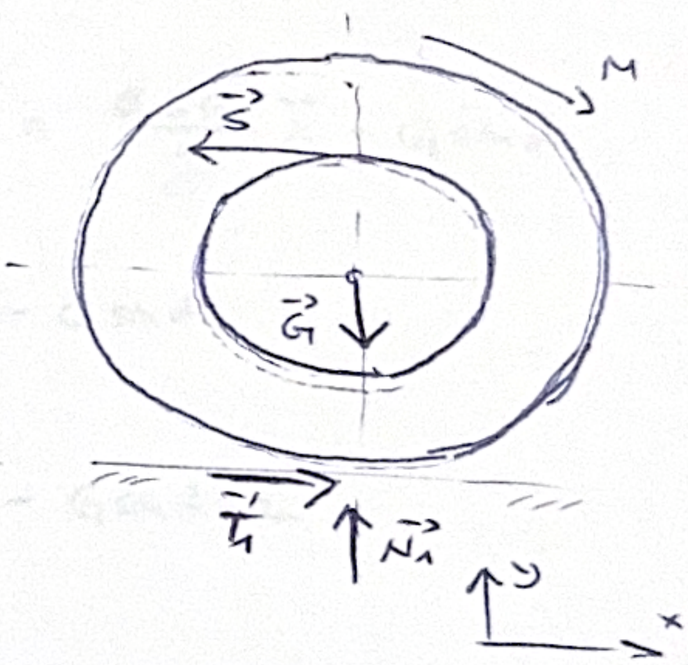
$\omega_1 = \frac{v}{R}$, $\varepsilon_1 = \frac{\dot{x}}{R}$

$\omega_2 = \frac{v}{R}$, $\varepsilon_2 = \frac{\dot{x}}{R}$

telo 2



telo 1



$\frac{G}{g}\bar{a}_2 = \bar{G} + \bar{N}_2 + \bar{T}_2 + \bar{S}$

y: $0 = -G\cos\alpha + N_2$

$N_2 = G\cos\alpha$

x: $\frac{G}{g}\ddot{x}_2 = -G\sin\alpha + T_2 + S$

$\frac{G}{g}\cdot 3\ddot{x} = -G\sin\alpha + T_2 + S$ (1)

$\frac{d\vec{L}_{2z}}{dt} = \sum \vec{M}_{2z}$

$\sum M_{2z} = -T_2 \cdot R$

$L_{2z} = J_2 \cdot \omega_2 = \frac{G}{g} R^2 \cdot \frac{3v}{R}$

$\frac{dL_{2z}}{dt} = \frac{3G}{g} R\ddot{x}$

$\frac{d\vec{L}_{2z}}{dt} = \sum \vec{M}_{2z}$

$\frac{3G}{g} R\ddot{x} = -T_2 \cdot R$

$\frac{G}{g}\bar{a}_1 = \bar{G} + \bar{N}_1 + \bar{T}_1 + \bar{S}$

y: $0 = -G + N_1$

$N_1 = G$

x: $\frac{G_1}{g}\ddot{x}_1 = T_1 - S$

$\frac{G_1}{g}\cdot 2\ddot{x} = T_1 - S$ (3)

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$$\frac{d\vec{L}_{1z}}{dt} = \Sigma \vec{M}_{1z}$$

$$\Sigma M_{1z} = M - T_1 \cdot 2R - S \cdot R$$

$$L_{1z} = J_1 \cdot \omega_1 = 2 \cdot \frac{G}{g} \cdot \frac{\varrho}{R} \cdot R^2 \cdot 2 \cdot \frac{G}{g} \cdot \frac{\varrho}{R}$$

$$\frac{dL_{1z}}{dt} = \frac{2G}{g} R \ddot{x}$$

$$\frac{2G}{g} R \ddot{x} = M - T_1 \cdot 2R - S \cdot R \qquad (4)$$

$(2) \Rightarrow \quad T_2 = -\frac{3G}{g} \ddot{x}$

$(1) \Rightarrow \quad S = \frac{3G}{g} \ddot{x} + G \sin\alpha - T_2 = \frac{3G}{g} \ddot{x} + G \sin\alpha + \frac{3G}{g} \ddot{x} = \frac{6G}{g} \ddot{x} + G \sin\alpha$

$(3) \Rightarrow \quad T_1 = \frac{2G_1}{g} \ddot{x} + S = \frac{2G_1}{g} \ddot{x} + \frac{6G_1}{g} \ddot{x} + G \sin\alpha = \frac{8G_1}{g} \ddot{x} + G \sin\alpha$

$(4) \Rightarrow \quad \frac{2G}{g} R \ddot{x} = M - \frac{8G}{g} \ddot{x} \cdot 2R - G \sin\alpha \cdot 2R - \frac{6G}{g} \ddot{x} R - G \sin\alpha \cdot R$

$$\frac{G}{g} R \ddot{x} \cdot 24 = M - 3 G R \sin\alpha$$

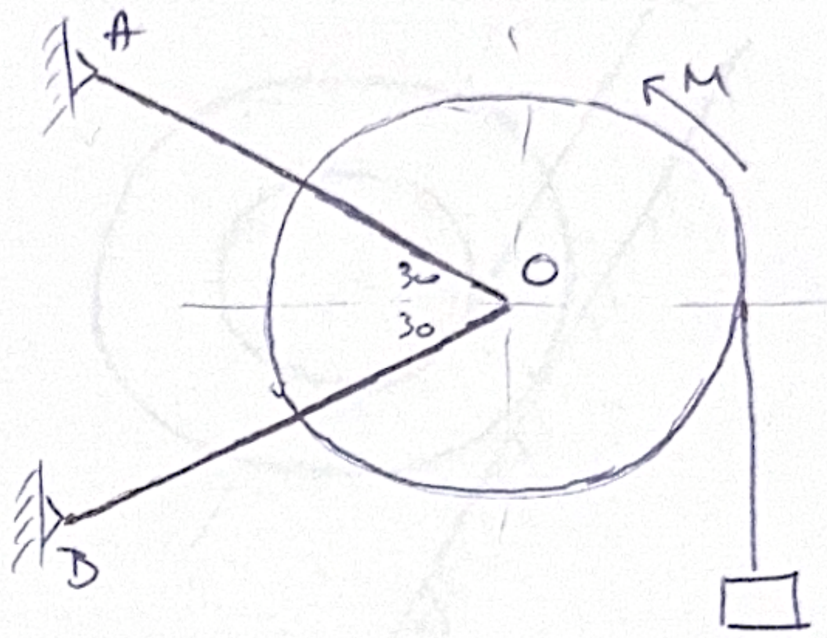
$$\ddot{x} = \frac{1}{24} \frac{g}{G} \left(\frac{M}{R} - 3 G \sin\alpha \right)$$

$(1) \Rightarrow \quad S = \frac{6G}{g} \cdot \frac{1}{24} \frac{g}{G} \left(\frac{M}{R} - 3 G \sin\alpha \right) + G \sin\alpha = \frac{1}{4} \frac{M}{R} - \frac{3}{4} G \sin\alpha + G \sin\alpha$

$$\boxed{S = \frac{1}{4} \frac{M}{R} + \frac{1}{4} G \sin\alpha}$$

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(10.58.)



A0: $\emptyset$
B0: $\emptyset$
} isht dutina

dobor: G_2, r

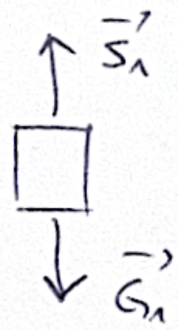
$M = 4 r G_1$

test: $G_1 = 3 G_2$

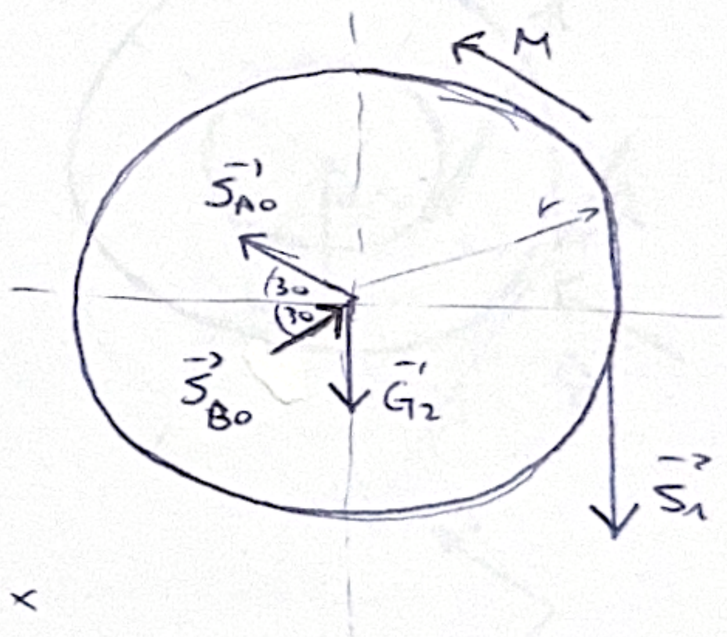
sile u stepovima ?

---

telo 1



telo 2



$\frac{G_1}{g}\vec{a}_1 = \vec{G}_1' + \vec{S}_1'$

x: — — (uvrte)

y: $\frac{3G_2}{g} = -3G_2 + S_1$ (1)

$\frac{G_2}{g}\vec{a}_2 = \vec{G}_2' + \vec{S}_1' + \vec{S}_{A0}' + \vec{S}_{B0}'$

x: $0 = -S_{A0}\cos 30 + S_{B0}\cos 30$

$S_{A0} = S_{B0} = S$

y: $0 = -G_2 + S\sin 30 + S\sin 30 - S_1$

$0 = -G_2 + S - S_1$ (2)

$\frac{dL_{2z}}{dt} = \sum M_{2z}$

$\sum M_{2z} = M - S_1 \cdot R$

$L_{2z} = J_2 \cdot \omega_2 = \frac{1}{2}\frac{G_1}{g} R^2 \omega_2$

$\frac{dL_{2z}}{dt} = \frac{1}{2}\frac{G_2}{g} R^2 \varepsilon_2 = \frac{1}{2}\frac{G_2}{g} R^2 \frac{\ddot{y}}{R}$

$\frac{1}{2}\frac{G_2}{g} R\ddot{y} = M - S_1 \cdot R$ (3)

(1) $\Rightarrow$ $S_1 = \frac{3G_2}{g}\ddot{y} + 3G_2$

(2) $\Rightarrow$ $\frac{1}{2}\frac{G_2}{g} R\ddot{y} = M - \frac{3G_1}{g}\ddot{y}\cdot R - 3G_2 \cdot R$

$\frac{7}{2}\frac{G_2}{g} R\ddot{y} = M - 3G_2 R$

$\frac{G_2}{g}\ddot{y} = \frac{2}{7}\frac{1}{R}(M - 3G_2 R)$

(1) $\Rightarrow$ $S_1 = 3\cdot\frac{2}{7}\cdot\frac{1}{R}(M - 3G_2 R) + 3G_2$

$S_1 = \frac{6}{7}\frac{M}{R} - \frac{18}{7}G_2 + 3G_2$

$S_1 = \frac{6}{7}\frac{M}{R} + \frac{3}{7}G_2$

(2) $\Rightarrow$ $S = G_2 + \frac{6}{7}\frac{M}{R} + \frac{3}{7}G_2$

$S = \frac{10}{7}G_2 + \frac{6}{7}\frac{M}{R}$

$S = \frac{10}{7}G_2 + \frac{6}{7}\cdot\frac{4R\cdot 3G_2}{R}$

$\boxed{S = S_{A0} = S_{B0} = \frac{82}{7}G_2}$

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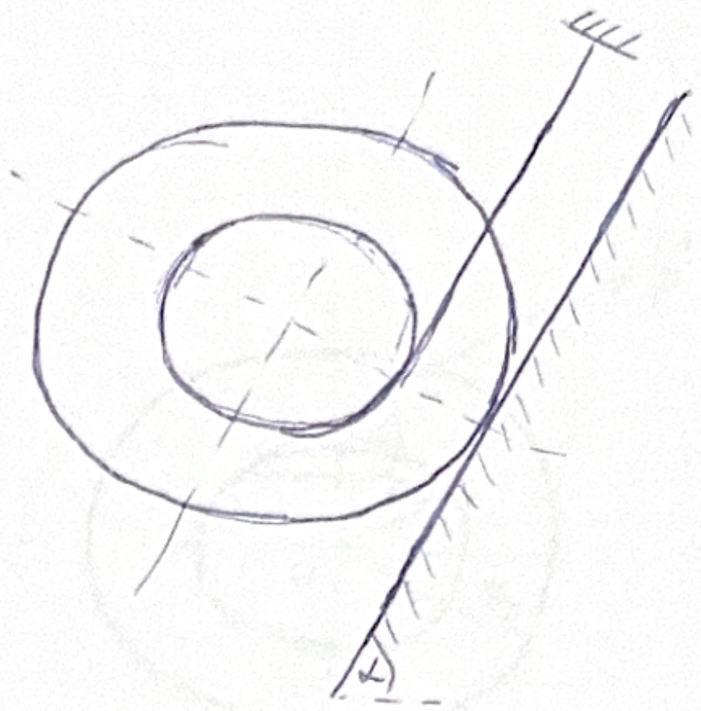
(10.61.)

α, μ

koaks. cilindar: m, R, $2R$, $r_0 = R$

to: miroranje

$S = ?$



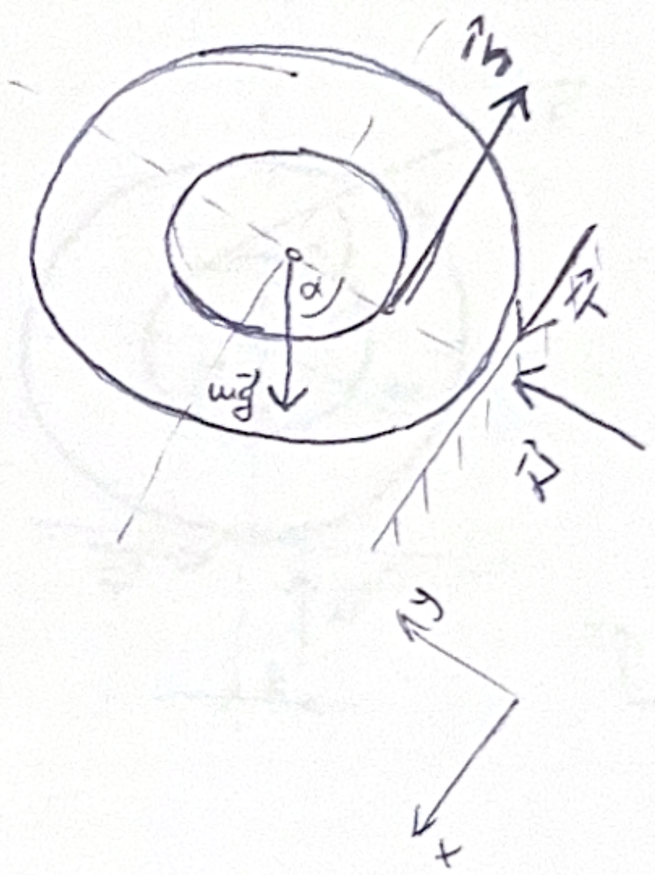
$\underrightarrow{m\vec{a} = m\vec{g} + \vec{N} + \vec{T} + \vec{S}}$

y: $0 = -mg\cos\alpha + N$

$N = mg\cos\alpha$

x: $m\ddot{x} = mg\sin\alpha + T - S$

$m\ddot{x} = mg\sin\alpha + \mu mg\cos\alpha - S$ (1)



$\underrightarrow{\frac{d\vec{L}_{CT}}{dt} = \sum \vec{M}_{CT}}$

$\sum M_{CT} = S \cdot R - T \cdot 2R = S \cdot R - \mu \cdot mg\cos\alpha \cdot 2R$

$L_{CT} = J_C \cdot \omega = R^2 \cdot m \cdot \omega$

$\frac{dL_{CT}}{dt} = mR^2\varepsilon$

$mR^2\varepsilon = S \cdot R - 2\mu mg\cos\alpha \cdot R$

$mR\varepsilon = S - 2\mu mg\cos\alpha$ (2)

(1) => $mR\varepsilon = mg\sin\alpha + \mu mg\cos\alpha - S$ (*)

(*) = (2)

$mg\sin\alpha + \mu mg\cos\alpha - S = S - 2\mu mg\cos\alpha$

$2S = mg\sin\alpha + 3\mu mg\cos\alpha$

$\boxed{S = \frac{1}{2} mg(\sin\alpha + 3\mu\cos\alpha)}$

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Zd. 62.

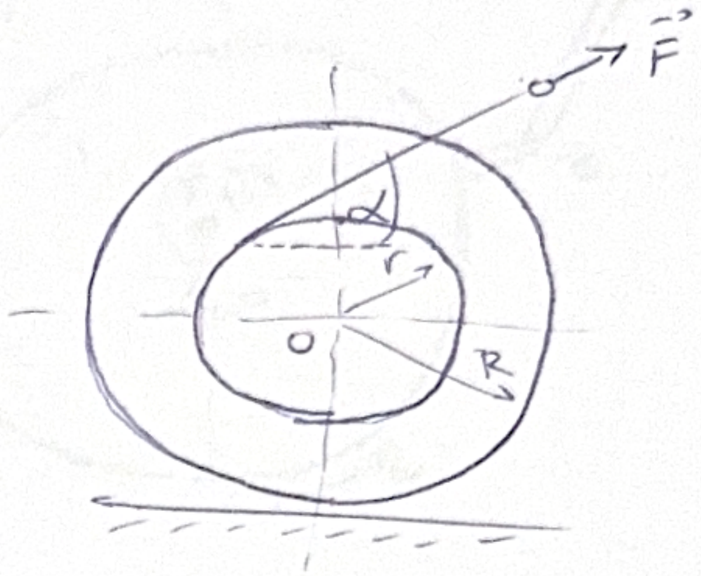
$F = const.$

$\alpha,\ k$

kocks. oliudri: $G,\ r, R,\ i$
kotrljanje bez klizanja

t_0: sistem miruje

$v_0 = ?\quad S = ?$

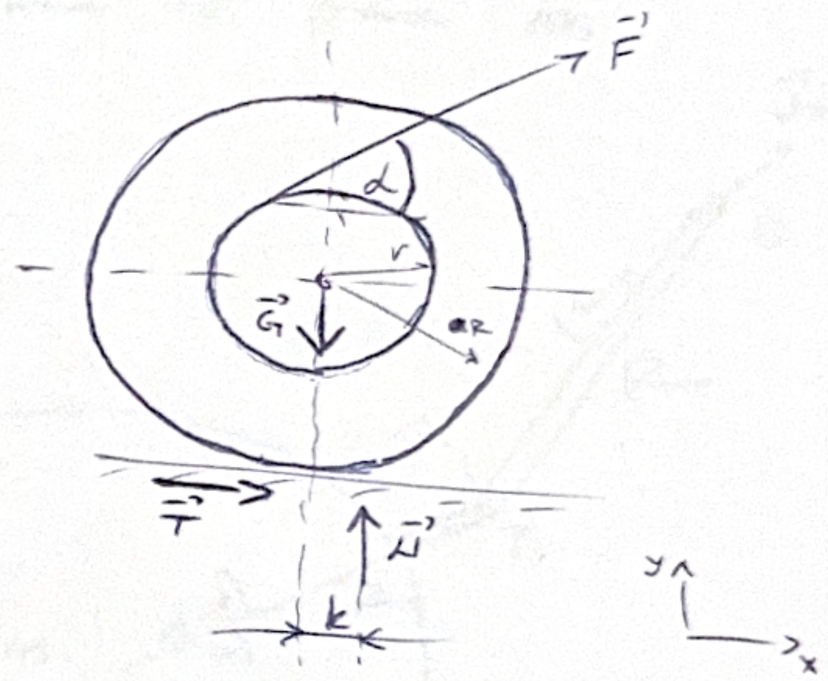


$$\frac{G}{g}\vec{a} = \vec{G} + \vec{F} + \vec{N} + \vec{T}$$

x: $\frac{G}{g}\ddot{x} = F\cos\alpha + T \quad (1)$

y: $0 = -G + F\sin\alpha + N$

$N = G - F\sin\alpha$



$$\frac{d\vec{L}_{Oz}}{dt} = \Sigma \vec{M}_{Oz}$$

$\Sigma M_{Oz} = F\cdot r - T\cdot R - N\cdot k = F\cdot r - T\cdot R - G\cdot k + F\sin\alpha\cdot k$

$L_{Oz} = R\cdot\frac{G}{g}v_0 + J_0\omega = R\cdot\frac{G}{g}R\omega + z^2\frac{G}{g}\omega = \frac{G}{g}(R^2+z^2)\omega$

$\frac{dL_{Oz}}{dt} = \frac{G}{g}(R^2+z^2)\varepsilon$

$\frac{G}{g}(R^2+z^2)\varepsilon = F\cdot r - T\cdot R - G\cdot k + F\sin\alpha\cdot k \quad (2)$

(1) $\Rightarrow$ $\frac{G}{g}R\varepsilon = F\cos\alpha + T \Rightarrow T = \frac{G}{g}R\varepsilon - F\cos\alpha$

(2) $\Rightarrow$ $\frac{G}{g}(R^2+z^2)\varepsilon = F\cdot r - \frac{G}{g}R\varepsilon\cdot R + F\cos\alpha\cdot R - G\cdot k + F\sin\alpha\cdot k$

$\frac{G}{g}(2R^2+z^2)\varepsilon = F\cdot r + F\cos\alpha\cdot R - G\cdot k + F\sin\alpha\cdot k$

$\frac{G}{g}(2R^2+z^2)\frac{\ddot{x}}{R} = F(r + R\cos\alpha + k\sin\alpha) - G\cdot k$

$\ddot{x} = \frac{gR\,(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{G(2R^2+z^2)}$ / $\int dt$

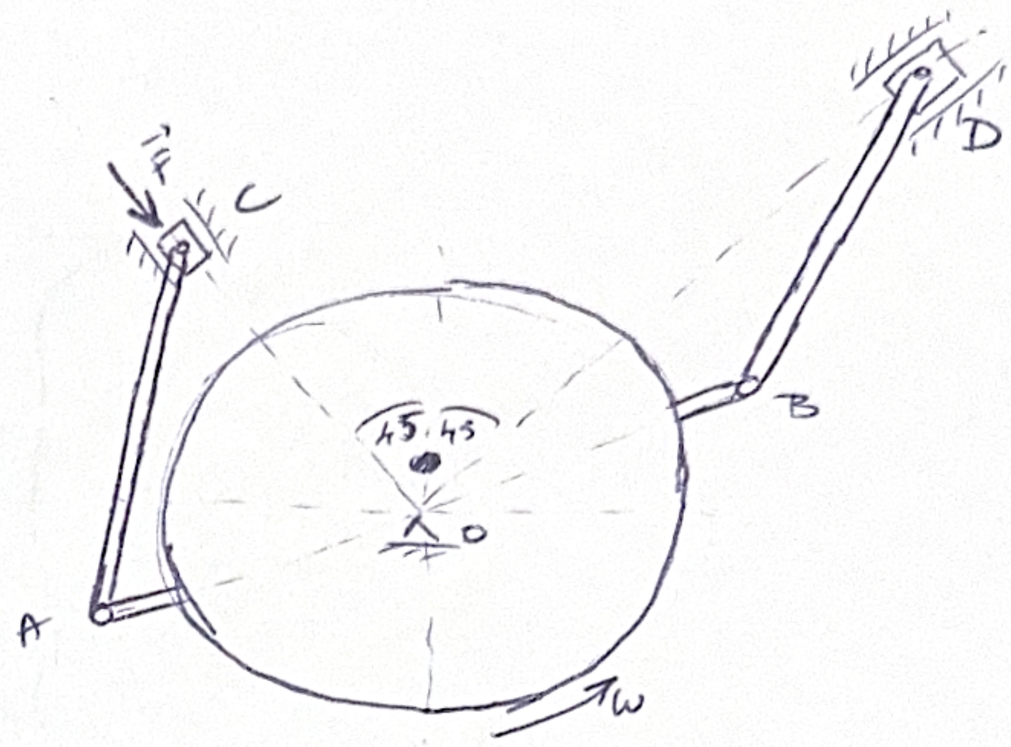
$v = \dot{x} = \frac{gRt\,(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{G(2R^2+z^2)}$ / $\int dt$

$s = x = \frac{gRt^2(F(r+R\cos\alpha+k\sin\alpha) - G\cdot k)}{2G(2R^2+z^2)}$

← u rešenju je (R^2+z^2) umesto $(2R^2+z^2)$

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(10.63.)



disk: J

štapovi: ∅

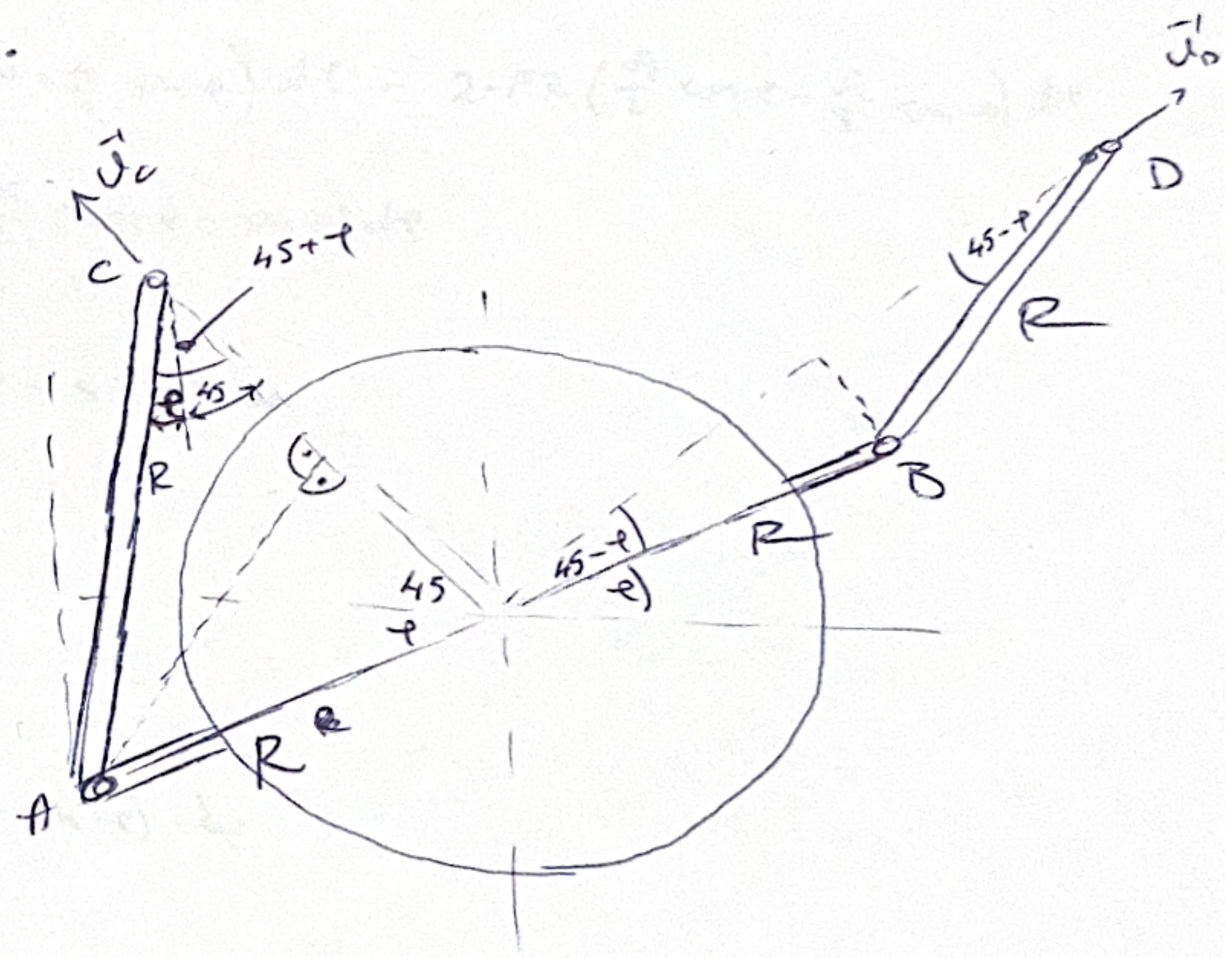
klizači: m

$\overline{OA} = \overline{AC} = \overline{OB} = \overline{BD} = R$

$F(\varphi) = ?$

uslov: $\omega = const.$

Da bi popednostavili sliku, ugao φ ćemo da merimo od pravca AB kada je on bio horizontalan.



$$\frac{dT}{dt} = \frac{dA}{dt}$$

$$T = \left[\tfrac{1}{2} m v_c^2 + 0\right] + \left[\tfrac{1}{2} m v_0^2 + 0\right] + \left[0 + \tfrac{1}{2} J\omega^2\right]$$

$r_C = \overline{OC} = 2R\cos(\varphi+45)$

$v_C = \frac{d(\overline{OC})}{dt} = -2R\omega\sin(\varphi+45)$

$r_D = \overline{OD} = 2R\cos(45-\varphi)$

$v_D = \frac{d(\overline{OD})}{dt} = +2R\omega\sin(45-\varphi)$

$$T = \left[\tfrac{1}{2} m \cdot 4R^2 \sin^2(\varphi+45)\cdot\omega^2\right] + \left[\tfrac{1}{2} m 4R^2 \sin^2(45-\varphi)\omega^2\right] + \left[\tfrac{1}{2} J\omega^2\right]$$

$$T = 2mR^2\omega^2\left(\sin^2(\varphi+45) + \sin^2(45-\varphi)\right) + \tfrac{1}{2} J\omega^2$$

$$T = 2mR^2\omega^2\left(\left(\sin\varphi\cos 45 + \cos\varphi\sin 45\right)^2 + \left(\sin 45\cos\varphi - \sin\varphi\cos 45\right)^2\right) + \tfrac{1}{2} J\omega^2$$

$$T = 2mR^2\omega^2\left(\left(\tfrac{1}{2}\sin^2\varphi + \tfrac{1}{2}\cos^2\varphi + \sin\varphi\cos\varphi\right) + \left(\tfrac{1}{2}\sin^2\varphi + \tfrac{1}{2}\cos^2\varphi - \sin\varphi\cos\varphi\right)\right) + \tfrac{1}{2} J\omega^2$$

$$T = 2mR^2\omega^2\left(\sin^2\varphi + \cos^2\varphi\right) + \tfrac{1}{2} J\omega^2$$

$$T = 2mR^2\omega^2 + \tfrac{1}{2} J\omega^2$$

$$\frac{dT}{dt} = 0 \qquad (\text{jer je } \omega = const.)$$

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$$\underline{d'A} = -mg\,dy_c - mg\,dy_D - F\cdot dr_D$$

$$\frac{dy_c}{dt} = \dot{y}_c = v_c\frac{\sqrt{2}}{2} = -\sqrt{2}\,R\sin(\varphi+45)\frac{d\varphi}{dt}$$

$$\frac{dy_D}{dt} = \dot{y}_D = v_D\frac{\sqrt{2}}{2} = \sqrt{2}\,R\sin(45-\varphi)\frac{d\varphi}{dt}$$

$$\frac{dr_D}{dt} = v_D = 2R\sin(45-\varphi)\frac{d\varphi}{dt}$$

$$d'A = mg\cdot\sqrt{2}R\sin(\varphi+45)\,d\varphi - mg\sqrt{2}\cdot R\sin(45-\varphi)\,d\varphi - F\cdot 2R\sin(45-\varphi)\,d\varphi$$

$$d'A = \sqrt{2}\,mgR\left(\sin(\varphi+45) - \sin(45-\varphi)\right)d\varphi - 2\cdot F\cdot R\cdot\sin(45-\varphi)\,d\varphi$$

$$d'A = \sqrt{2}\,mgR\left(\sin\varphi\frac{\sqrt{2}}{2} + \cos\varphi\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cos\varphi + \frac{\sqrt{2}}{2}\sin\varphi\right)d\varphi - 2\cdot FR\left(\frac{\sqrt{2}}{2}\cos\varphi - \frac{\sqrt{2}}{2}\sin\varphi\right)d\varphi$$

$$d'A = \sqrt{2}\,mgR\sin\varphi\frac{\sqrt{2}}{2}\cdot 2\,d\varphi - 2\cdot F\cdot R\frac{\sqrt{2}}{2}(\cos\varphi - \sin\varphi)\,d\varphi$$

$$\frac{d'A}{dt} = 2mgR\sin\varphi\cdot\omega - \sqrt{2}FR(\cos\varphi - \sin\varphi)\,\omega$$

$$\frac{dT}{dt} = \frac{d'A}{dt}$$

$$0 = 2mgR\sin\varphi\cdot\omega - \sqrt{2}FR(\cos\varphi - \sin\varphi)\cdot\omega$$

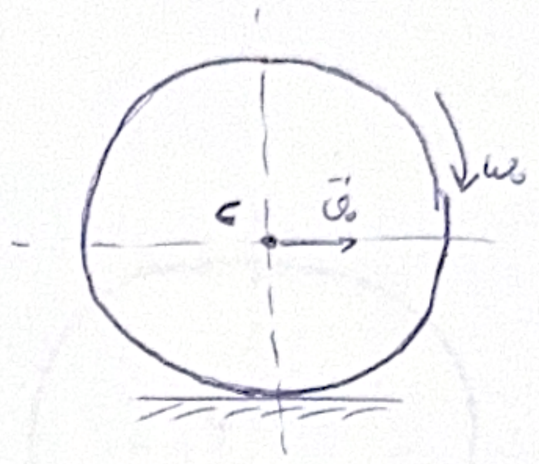
$$F = \frac{2\,mg\sin\varphi}{\sqrt{2}(\cos\varphi - \sin\varphi)}$$

$$\boxed{F = \frac{\sqrt{2}\,mg\sin\varphi}{\cos\varphi - \sin\varphi}}$$

↑ ujima se u računu $-(\cos\varphi+\sin\varphi)$ umesto mg $+(\cos\varphi-\sin\varphi)$

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10.64.



C: P, r

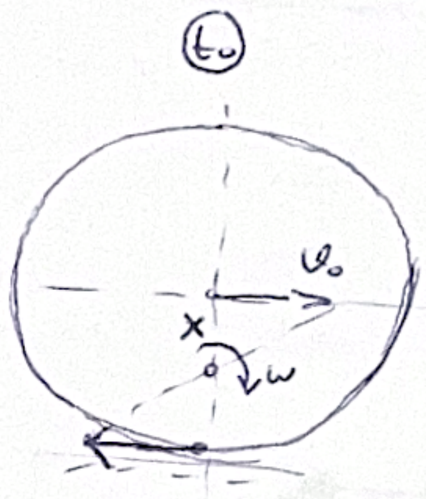
t_0: v_0, ω_0, gde je $r\omega_0 > v_0$

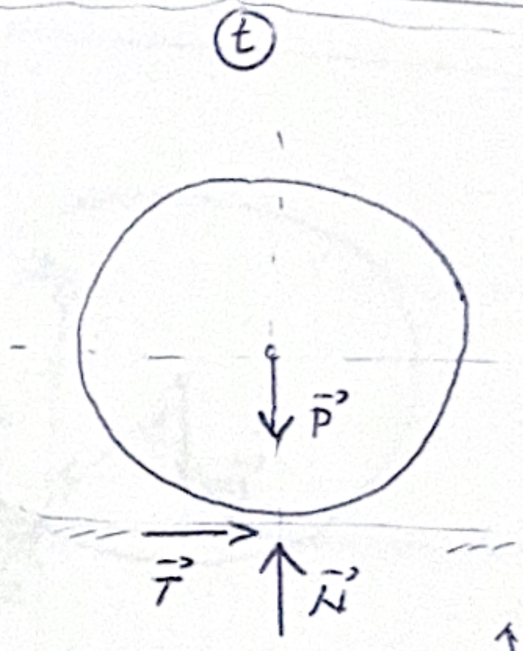
μ

$t_k = ?$ - trenutak kada će centar cilindra C preći da se obrće ravnom. kretanjem

$v_C(t_k) = ?$

Da bi cilindar krenuo u desno sila $\vec{T}$, moramo znati u kojoj strani prolizava cilindar, pa zato najpre pretpostavimo trenutak to:





$$\frac{P}{g}\vec{a} = \vec{P} + \vec{N} + \vec{T}$$

y: $0 = P - N$

$\underline{N = P}$

x: $\frac{P}{g}\ddot{x} = T$

$$\frac{d\vec{L}_C}{dt} = \Sigma \vec{M}_C$$

$\Sigma M_C = -T \cdot r$

$L_C = J_C \cdot \omega = \frac{P}{g} r^2 \cdot \omega$

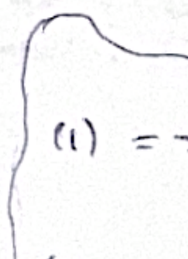
$\frac{dL_C}{dt} = \frac{P}{g} r^2 \varepsilon$

$\Rightarrow \underline{\frac{P}{g} r^2 \varepsilon = -T} \quad (2)$

za t_1 važi $\ddot{x}(t_1) = 0$

$\Rightarrow T(t_1) = 0$ $\Rightarrow$ eliminišemo je prolizavanje

$\Rightarrow \underline{\dot{x}(t_1) = r\omega(t_1)}$



(1) $\Rightarrow \frac{P}{g}\ddot{x} = \mu \cdot P \quad \Rightarrow \quad \underline{\dot{x} = \mu g t + v_0}$

(2) $\Rightarrow \frac{P}{g} r\varepsilon = -\mu P \quad \Rightarrow \quad \underline{\omega = -\frac{\mu g}{r} t + \omega_0}$

$\dot{x}(t_1) = r\omega(t_1)$

$\mu g t_1 + v_0 = -\mu g t_1 + \omega_0 r$

$2\mu g t_1 = r\omega_0 - v_0$

$$\boxed{t_1 = \frac{\omega_0 r - v_0}{2\mu g}}$$

$v(t_1) = \dot{x}(t_1) = \mu g \cdot \frac{\omega_0 r - v_0}{2\mu g} + v_0$

$$\boxed{v(t_1) = \frac{1}{2}(v_0 + \omega_0 r)}$$

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10.66.

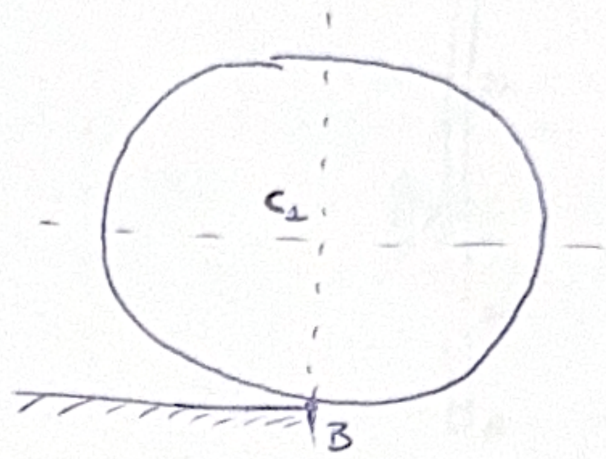
cilindar C: R

t_0: zanemarljivo mala brzina

centar pada s ivice bez početne brzine

t_1: odvaja se od podloge

$\alpha_1 = ?$ $\omega_1 = ?$



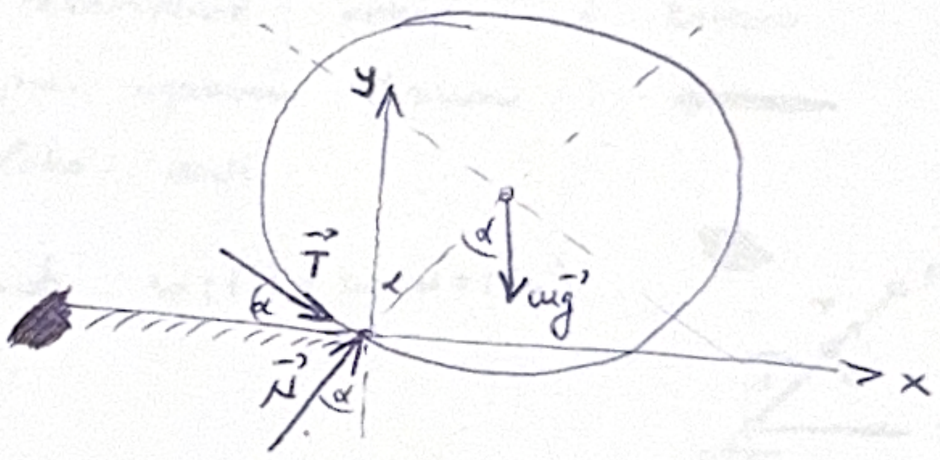
---

trenutak t_1: $N(t_1) = 0$, $T(t_1) = 0$

(odvajanje od podloge)

ideja: izraziti N u f-ji α / ω, pustiti da je $\alpha_1 = 0$; odatle naći α_1 / ω_1

(moramo naći vezu između α_1 i ω_1, da kad nađemo jedno da mogu naći i drugo)



$T - T_0 = A$

$T_0 = 0$

$T = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2 = \frac{1}{2} m R^2 \omega^2 + \frac{1}{2} m R^2 \omega^2 = m R^2 \omega^2$

$A = \int_R^{R\cos\alpha} -mg\, dy_c = mgR(1 - \cos\alpha)$

$m R^2 \omega^2 = mgR(1 - \cos\alpha)$

$\omega^2 = \frac{g(1-\cos\alpha)}{R}$

$m\vec{a} = m\vec{g}' + \vec{N} + \vec{T}'$

t: --- (nećemo)

n: $m\frac{v^2}{R} = mg\cos\alpha - N$

$N = -m\frac{R^2\omega^2}{R} + mg\cos\alpha = mg\cos\alpha - mR \cdot \frac{g(1-\cos\alpha)}{R} = mg\cos\alpha - mg + mg\cos\alpha = mg(2\cos\alpha - 1)$

$N_1 = mg(2\cos\alpha_1 - 1) = 0 \Rightarrow 2\cos\alpha_1 = 1 \Rightarrow \cos\alpha_1 = \frac{1}{2} \Rightarrow \boxed{\alpha_1 = 60°}$

$\omega_1^2 = \frac{g(1-\cos\alpha_1)}{R} = \frac{g(1-\frac{1}{2})}{R} = \frac{\frac{1}{2}g}{R} = \frac{g}{2R} \Rightarrow \boxed{\omega_1 = \frac{g}{2R}}$

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(10. 67.)

Štap: $M = 3m$, $2a$

A: m

t_0: nalazi se u vertikalnom položaju zanemarljivo mala poč. brzina

t_1: nalazi se u vertikalnom položaju uklanja se zglob C



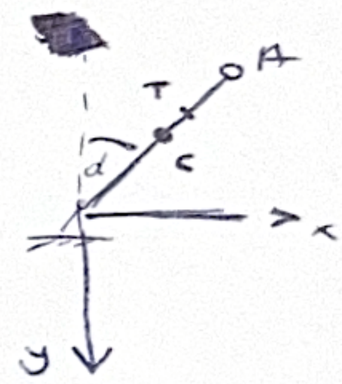
odrediti j-ne kretanja štapa nakon t_1 ?

---

ideja: - da nađemo $\omega(t_1)$; da onda ovaj resetujemo vreme i kažemo da je štap započeo kretanje tom poč. ugaonom brzinom ; [redacted] poč. brzinom čestica koje sada leže na

- najbolje je naći: $\omega(\alpha)$, jer je onda $\omega(t_1) = \omega(\alpha = 180°)$

[redacted]



$\underline{T_1 - T_0 = A_{01}}$

$T_0 = 0$

$T_1 = \left[\frac{1}{2} 3m\, v_{C1}^2 + \frac{1}{2} J_C \omega_{01}^2\right] + \left[\frac{1}{2} m v_{A1}^2 + 0\right] =$

$T_1 = \left[\frac{5}{2} m R^2 \omega_1^2 + \frac{1}{2}\frac{1}{12} 4a^2 \cdot 3m \cdot \omega_1^2\right] + \left[\frac{1}{2} m \cdot 4a^2 \cdot \omega_1^2\right] =$

$T_1 = 4 m R^2 \omega_1^2$

$A_{01} = \int_{-a}^{a} 3mg\, dy_C + \int_{-2a}^{2a} mg\, dy_A = 3mg \cdot 2a + mg \cdot 4a = 10\, mga$

$4 m a \omega_1^2 = 10 mga \Rightarrow \omega_1^2 = \frac{5}{2}\frac{g}{a} \Rightarrow \underline{\omega_1 = \sqrt{\frac{5}{2}\frac{g}{a}}}$, $\underline{\dot{x}_{C1} = -\sqrt{\frac{5}{2} ga}}$, $\underline{\dot{y}_{C1} = 0}$

(ovo su poč. uslovi za novo (slobodno) kretanje štapa)

$4m x_T = 3m x_C + m x_A$

$4m x_T = 3m x_C + 2m x_C$

$4m x_T = 5m x_C$

$\underline{x_T = \frac{5}{4} x_C}$ $\underline{y_T = \frac{5}{4} y_C}$

(na slican način se dobija) 5

$\underline{4m \vec{a}_T = m\vec{g}' + 3m\vec{g}'}$

x: $4m \ddot{x}_T = 0$ /∫dt

$\dot{x}_T = const = -\frac{5}{4}\sqrt{\frac{5}{2} ga}$ /∫dt

$\boxed{x_T = -\frac{5}{4}\sqrt{\frac{5}{2} ga} \cdot t}$

y: $4m \ddot{y}_T = -mg - 3mg$

$\ddot{y}_T = -g$ /∫dt

$\dot{y}_T = -gt$ /∫dt

$\boxed{y_T = -\frac{1}{2} g t^2 - \frac{5}{4} a}$

$\underline{\frac{d\vec{L}_{Cz}}{dt} = \Sigma \vec{M}_{Cz}}$

$\Sigma M_{Cz} = -mg \cdot \frac{3}{4} a \sin\varphi + 3mg \cdot \frac{1}{4} a \sin\varphi = 0$

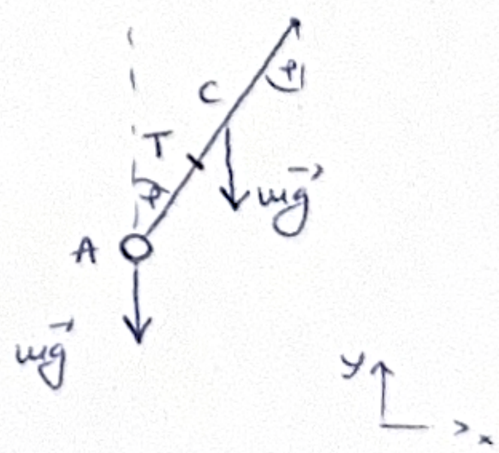
$L_{Cz} = J\omega = \ldots$

$\frac{dL_{Cz}}{dt} = J\varepsilon = \ldots$

$J\varepsilon = 0$ /∫dt

$\omega = \omega_1 = \sqrt{\frac{5}{2}\frac{g}{a}}$ /∫dt

$\boxed{\varphi = \sqrt{\frac{5}{2}\frac{g}{a}}\, t}$



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10.68.

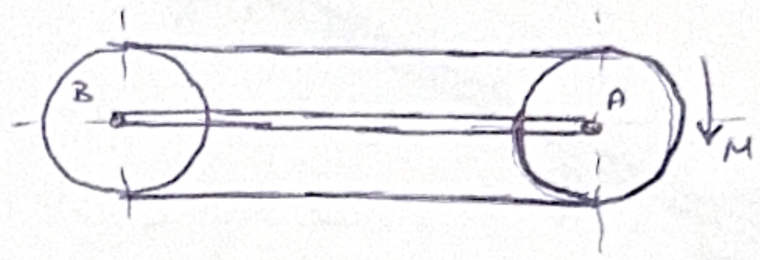
gusenica: m, zanemarljiva debljina

diskovi: m, r

AB: $2m$, e

$M = \text{const.}$

$\varepsilon = ?$



$$\frac{dT}{dt} = \frac{d'A}{dt}$$

Brzina donjeg dela gusenice je jednaka nuli

Brzina gornjeg dela gusenice

$$T = 2\left[\frac{1}{2}m v_A^2 + \frac{1}{2}J\omega^2\right]_D + \left[\frac{1}{2}2m v_{AB}^2 + 0\right]_{AD} + \left[\frac{1}{2}\cdot\frac{1}{2}m\cdot 0 + 0\right] + \left[\frac{1}{2}\cdot\frac{1}{2}m\, v_{GG}^2 + 0\right]$$

$$T = \left[mR^2\omega^2 + \frac{1}{2}mR^2\omega^2\right] + \left[mR^2\omega^2\right] + \left[\frac{1}{4}m\left(\frac{1}{2}R\omega\right)^2\right]$$

$$T = \frac{7}{2}mR^2\omega^2$$

$$\frac{dT}{dt} = 7mR^2\omega\varepsilon$$

$$d'A = M\,d\varphi$$

$$\frac{d'A}{dt} = M\omega$$

$$7mR^2\omega\varepsilon = M\omega$$

$$\boxed{\varepsilon = \frac{M}{7mR^2}}$$

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(10.70.)

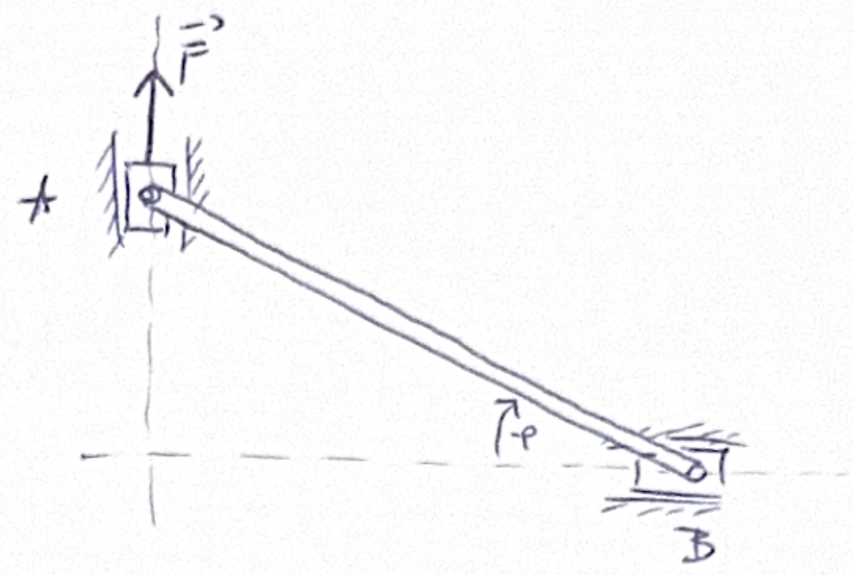
AB: $m, 2\ell$

A: $\emptyset$

$F = const. = ?$

to: mirovanje u horiz. položaju

uvjet: kraj B se odvaja od podloge pod uglom $\alpha = 30°$



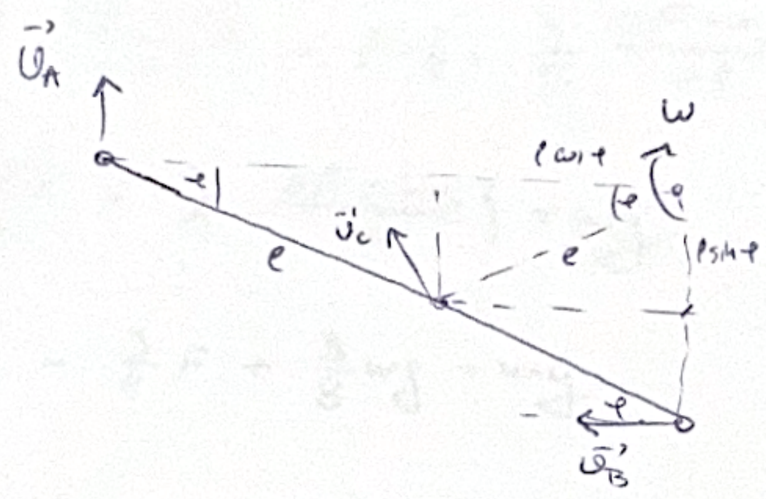
kinematska veza:

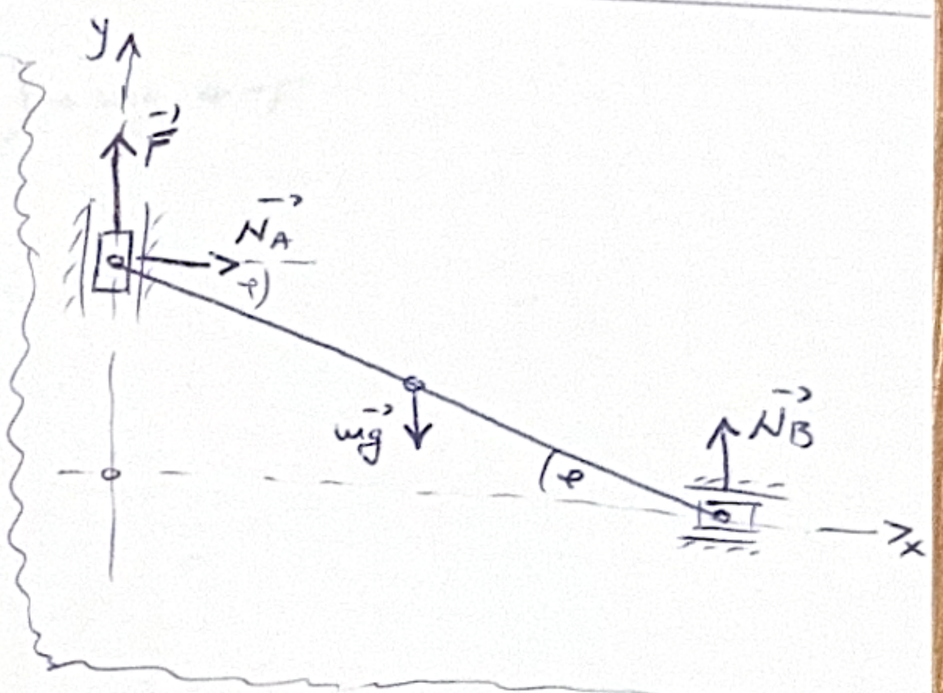
$\dot{x}_c = \ell \sin\varphi \cdot \omega$

$\dot{y}_c = \ell \cos\varphi \cdot \omega$

$\ddot{x}_c = \ell \cos\varphi \cdot \omega^2 + \ell \sin\varphi \cdot \varepsilon$

$\ddot{y}_c = -\ell \sin\varphi \cdot \omega^2 + \ell \cos\varphi \cdot \varepsilon$





ideja:

- izraziti M_B u f-iji φ i F kao jedinih nepoznatih, zatim reći da je $M_1 = 0$, $\varphi_1 = 30$, pa odatle naći: F
  (slično M_B i φ neće trebati kao nepoznate)
- za sad imamo 3-jine : 4 nepoznate ($\omega, \varepsilon, F, N_A$)
- preko promjene kin. energije moći ćemo izraziti ω i ε u f-iji ugla φ

$T - T_0 = A$

$T_0 = 0$

$T = \frac{1}{2} m v_c^2 + \frac{1}{2} J \omega^2 = \frac{1}{2} m (\ell^2\omega^2\sin^2\varphi + \ell^2\omega^2\cos^2\varphi) + \frac{1}{2}\frac{1}{12} m 4\ell^2\omega^2 =$

$T = \frac{1}{2} m\ell^2\omega^2 + \frac{1}{6} m\ell^2\omega^2 = \frac{2}{3} m\ell^2\omega^2$

$A = \int_0^{\ell\sin\varphi} -mg\,dy_c + \int_0^{2\ell\sin\varphi} F\,dy_A = -mg\ell\sin\varphi + 2F\ell\sin\varphi$

$\frac{2}{3} m\ell^2\omega^2 = 2F\ell\sin\varphi - mg\ell\sin\varphi$

$\omega = \sqrt{\frac{3}{2}\frac{2F - mg}{m\ell}\sin\varphi}$ $\Rightarrow$ $\underline{\omega_1 = \sqrt{\frac{3}{4}\cdot\frac{2F - mg}{m\ell}}}$

$\varepsilon = \frac{1}{2\sqrt{\ldots}} \cdot \frac{3}{2}\frac{2F - mg}{m\ell}\cos\varphi \cdot \varphi$

$\varepsilon = \frac{3}{4}\frac{2F - mg}{m\ell}\cos\varphi$ $\Rightarrow$ $\underline{\varepsilon_1 = \frac{3\sqrt{3}}{8}\frac{2F - mg}{m\ell}}$

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$$\underline{m\vec{a}_c = m\vec{g} + \vec{F} + \vec{N}_A + \vec{N}_B}$$

y: $\quad m\ddot{y}_c = -mg + F + N_B$

$$m(\ell\cos\varphi\,\varepsilon - \ell\sin\varphi\,\omega^2) = -mg + F + N_B$$

$$N_B = m(\ell\cos\varphi\cdot\varepsilon - \ell\sin\varphi\cdot\omega^2) + mg - F$$

postavimo trenutak t_1 gde je: $N_{B1} = 0$, $\varphi_1 = 30$, $\varepsilon_1 = \frac{3\sqrt{3}}{8}\,\frac{2F-mg}{m\ell}$, $\omega_1 = \sqrt{\frac{3}{4}\,\frac{2F-mg}{m\ell}}$

$$0 = m\left(\ell\cdot\frac{\sqrt{3}}{2}\cdot\frac{3\sqrt{3}}{8}\,\frac{2F-mg}{m\ell} - \ell\cdot\frac{1}{2}\cdot\frac{3}{4}\,\frac{2F-mg}{m\ell}\right) + mg - F$$

$$F = m\left(\frac{9}{16}\,\frac{2F-mg}{m} - \frac{3}{8}\,\frac{2F-mg}{m}\right) + mg$$

$$F = \frac{9}{8}F - \frac{9}{16}mg - \frac{3}{4}F + \frac{3}{8}mg + mg$$

$$F\left(1 - \frac{9}{8} + \frac{3}{4}\right) = mg\left(-\frac{9}{16} + \frac{3}{8} + 1\right)$$

$$F\cdot\frac{5}{8} = mg\cdot\frac{13}{16} \qquad /\cdot\frac{8}{5}$$

$$\boxed{F = \frac{13}{10}mg}$$

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(10.71)

A: m

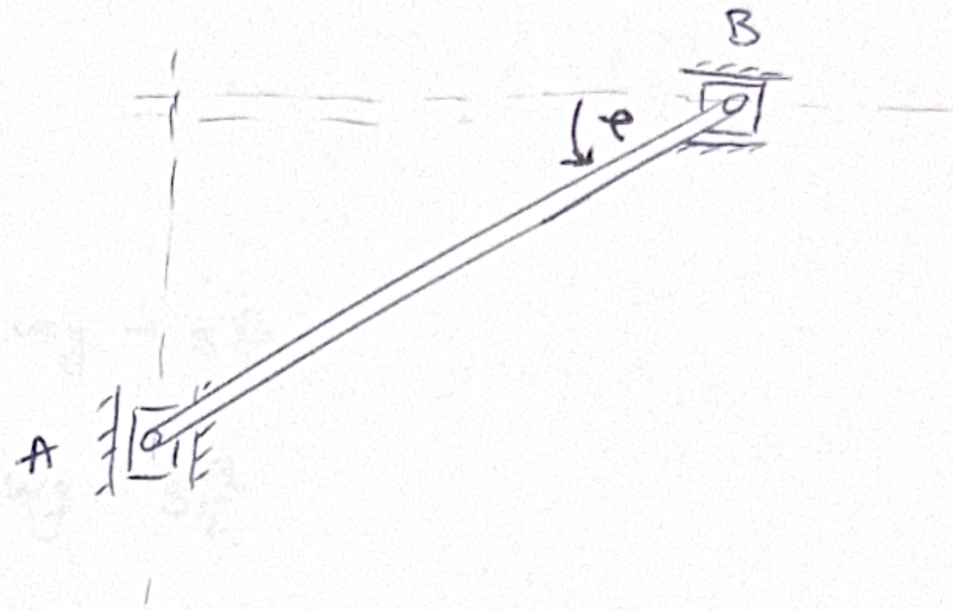
B: m

AB: $\varnothing$, ℓ

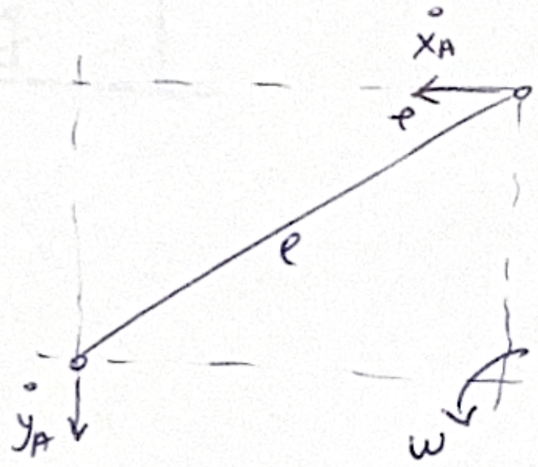
t_0: $\varphi_0 = 0$
mirovanje

t_1: $\varphi_1 = 45°$

$S(t_1) = ?$



kinematska veza:

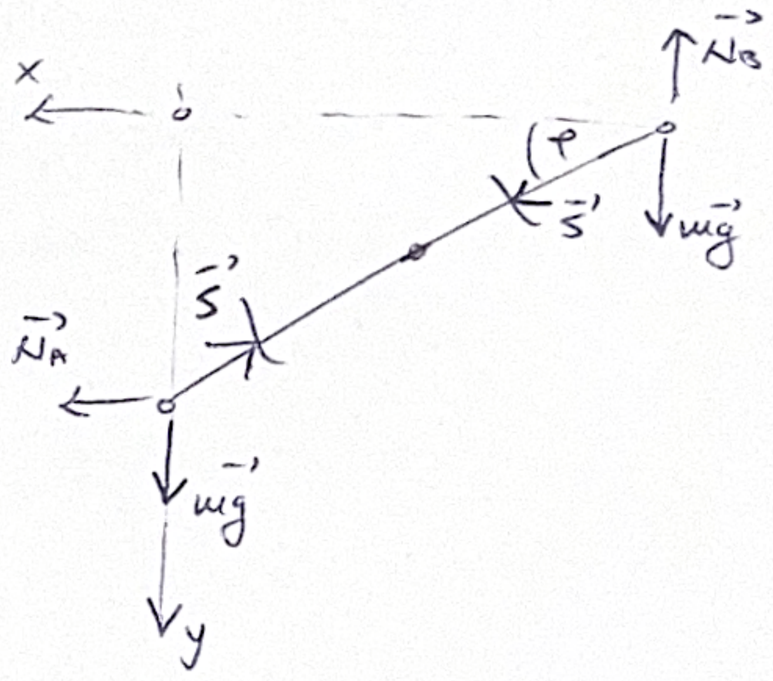


$\dot{x}_A = \ell \sin\varphi \cdot \omega$

$\dot{y}_A = \ell \cos\varphi \cdot \omega$

$\ddot{x}_A = \ell\omega^2 \cos\varphi + \ell\varepsilon \sin\varphi$

$\ddot{y}_A = -\ell\omega^2 \sin\varphi + \ell\varepsilon\cos\varphi$



ideja:
- imamo 4 j-ne i 5 nepoznanih ($\omega, \varepsilon, N_A, A_B, S$)
- dopisacemo još jedan j-nu za kinetičku energiju - to ćemo radi ω i ε

$T - T_0 = A$

$T_0 = 0$

$T = \left[\frac{1}{2} m v_A^2 + 0\right] + \left[\frac{1}{2} m v_B^2 + 0\right] = \frac{1}{2} m \ell^2 \omega^2 \sin^2\varphi + \frac{1}{2} m \ell^2 \omega^2 \cos^2\varphi = \frac{1}{2} m \ell^2 \omega^2$

$A = \int_0^{\ell\sin\varphi} mg\, dy_A = mg\ell\sin\varphi$

$\frac{1}{2} m\ell^2\omega^2 = mg\ell\sin\varphi$

$\omega = \sqrt{\frac{2g}{\ell}\sin\varphi} \quad \Rightarrow \quad \omega_1 = \sqrt{\frac{2g}{\ell}\frac{\sqrt{2}}{2}} = \sqrt{\frac{\sqrt{2}g}{\ell}}$

$\varepsilon = \frac{1}{2\sqrt{\ldots}} \cdot \frac{2g}{\ell}\cos\varphi \cdot (\ldots)$

$\varepsilon = \frac{g}{\ell}\cos\varphi \quad \Rightarrow \quad \varepsilon_1 = \frac{g}{\ell}\frac{\sqrt{2}}{2}$

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$$\underline{m\vec{a}_{A1} = m\vec{g} + \vec{N}_{A1} + \vec{S}}$$

x:

y: $\quad m\ddot{y}_{A1} = mg - S\sin 45$

$$m(-\ell \sin\varphi_1 \omega_1^2 + \ell\cos\varphi_1 \cdot \varepsilon_1) = mg - S\frac{\sqrt{2}}{2}$$

$$m\left(-\ell\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\frac{g}{\ell} + \ell\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\frac{g}{\ell}\right) = mg - S\frac{\sqrt{2}}{2}$$

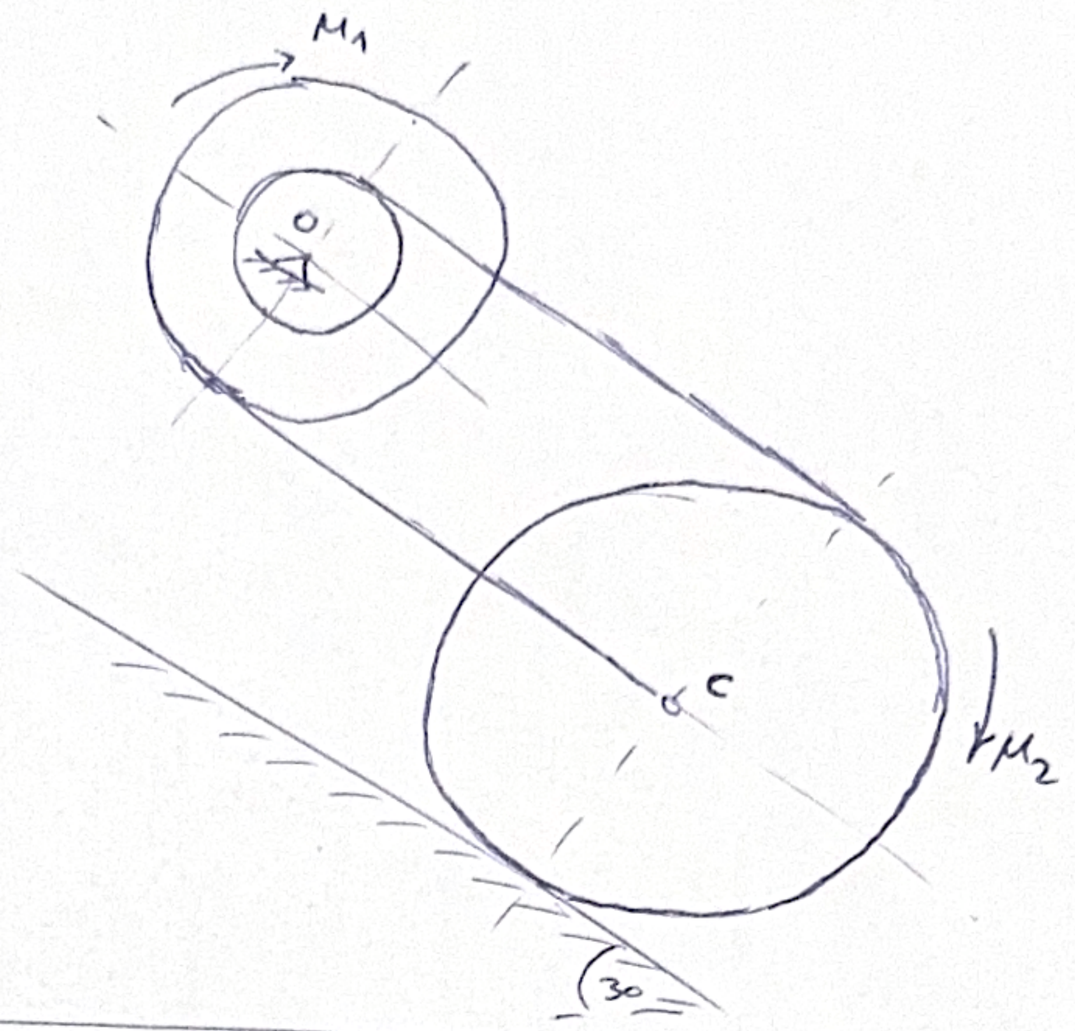
$$-\frac{1}{2}mg = mg - S\frac{\sqrt{2}}{2}$$

$$S\frac{\sqrt{2}}{2} = \frac{3}{2}mg$$

$$\boxed{S = \frac{3\sqrt{2}}{2}mg}$$

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10.72



1: m_1 R_1 $2R$, $r_0 = R\sqrt{2}$

2: m_1 $3R$

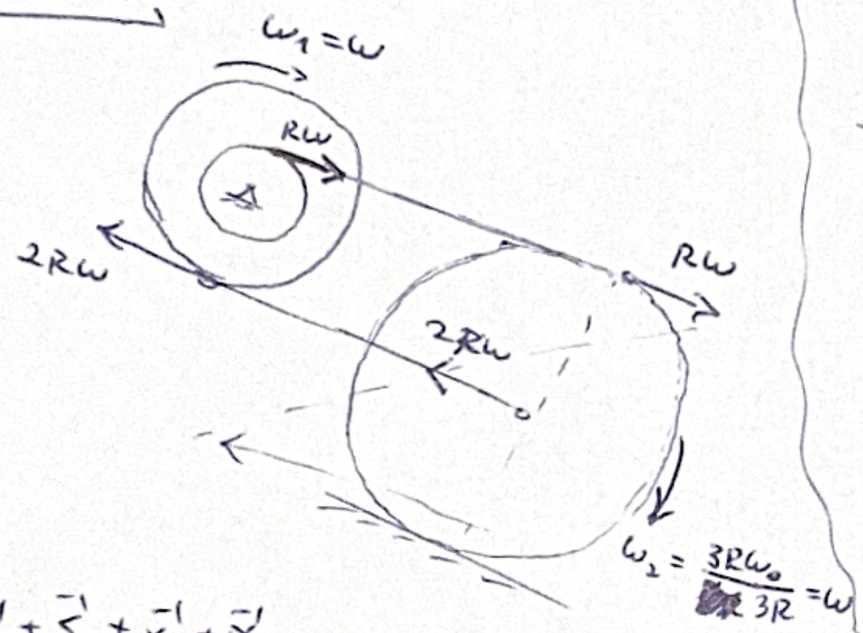
$\alpha = 30°$, $\mu = \frac{\sqrt{3}}{3}$

$M_1 = \text{const.} = ?$

$M_2 = \text{const.} = ?$

$S_1 = S_2 = mg$

kinematska veza:



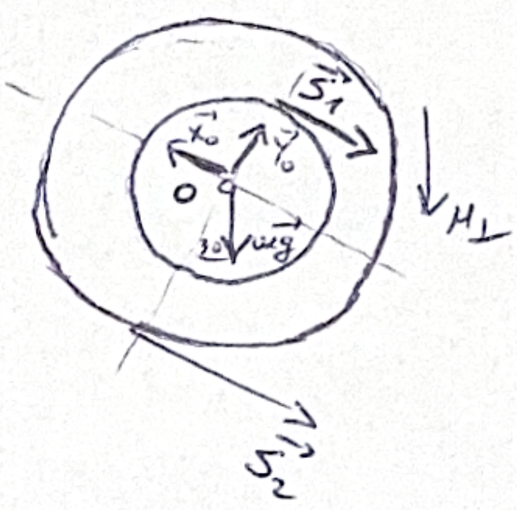
$\omega_1 = \omega$, $\varepsilon_1 = \varepsilon$

$\omega_2 = \omega$, $\varepsilon_2 = \varepsilon$

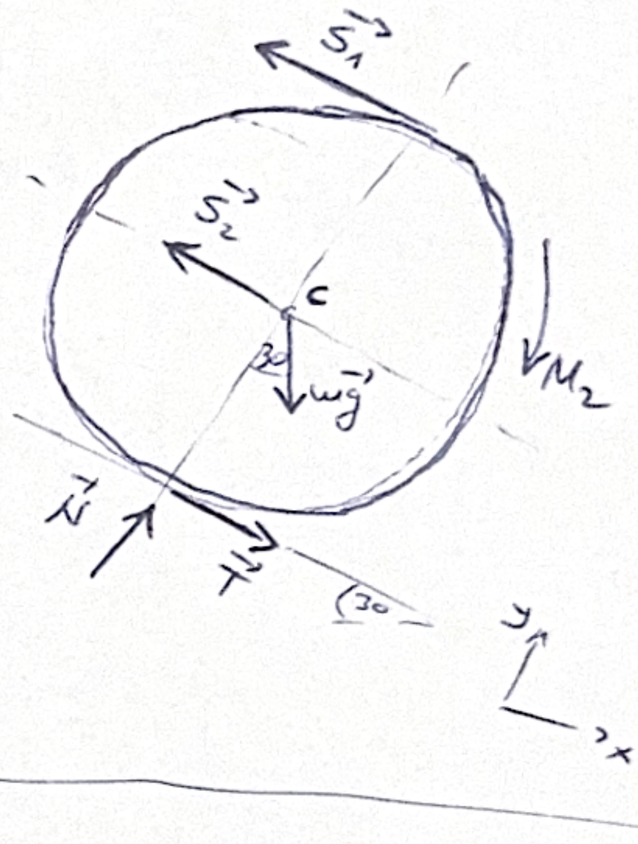
$\dot{x}_2 = -2R\omega$

$\ddot{x}_2 = -2R\varepsilon$

telo 1



telo 2



$m\vec{a}_1 = m\vec{g}_1 + \vec{S}_1' + \vec{S}_2' + \vec{x}_0' + \vec{y}_0'$

x: - (nepotrebno) (ker ne treba mi x_0)

y: ——— (nepotrebno) (ker ne treba mi y_0)

$\frac{d\vec{L}_{Ot}}{dt} = \sum \vec{M}_{Ot}$

$\sum M_{Ot} = M_1 + S_1 \cdot R - S_2 \cdot 2R$

$\sum M_{Ot} = M_1 - mgR$

$L_{Ot} = J_{Ot} \cdot \omega_1 = m \cdot R^2 \cdot 2 \cdot \omega$

$\frac{dL_{Ot}}{dt} = 2mR^2\varepsilon$

$M_1 - mgR = 2mR^2\varepsilon$

$m\vec{a}_2 = m\vec{g}_2 + \vec{S}_1' + \vec{S}_2' + \vec{N} + \vec{T}$

y: $0 = -mg\cos 30 + N$

$N = \frac{\sqrt{3}}{2}mg$, $T = \mu \cdot N = \frac{1}{2}mg$

x: $m\ddot{x}_2 = mg\sin 30 - S_1 - S_2 + T$

$-2mR\varepsilon = \frac{1}{2}mg - 2mg + \frac{1}{2}mg$

$\varepsilon = \frac{1}{2}\frac{g}{R}$

(1) => $M_1 = mgR + 2mR^2 \cdot \frac{1}{2}\frac{g}{R} = 2mgR$

$\boxed{M_1 = 2mgR}$

$\frac{d\vec{L}_{Ct}}{dt} = \sum \vec{M}_{Ct}$

$\sum M_{Ct} = M_2 - S_1 \cdot 3R - T \cdot 3R = M_2 - 3mgR - \frac{1}{2} \cdot 3mgR = M_2 - \frac{9}{2}mgR$

$L_{Ct} = J_{Ct} \cdot \omega_2 = \frac{1}{2}m \cdot 9R^2\omega$

$\frac{dL_{Ct}}{dt} = \frac{9}{2}mR^2\varepsilon = \frac{9}{2}mR^2 \cdot \frac{1}{2}\frac{g}{R} = \frac{9}{4}mgR$

$M_2 - \frac{9}{2}mgR = \frac{9}{4}mgR$

$\boxed{M_2 = \frac{27}{4}mgR}$

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