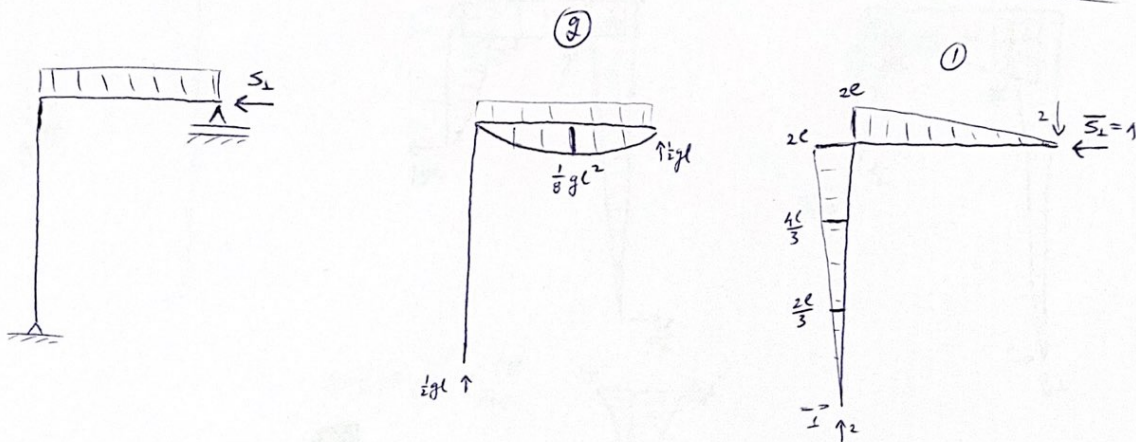
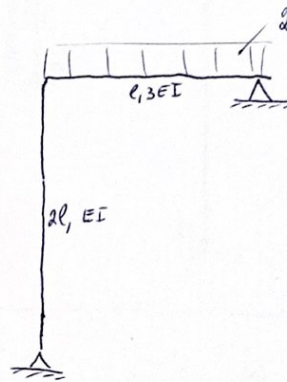


Zadaci - Versācin, statiski neodredati

- ① zadatak rešiti transformāciju ošņona B u horizontālo pārehi ošņonac



$$f_B = \alpha_{11} S_L + \Delta_1^q = 0$$

$$\alpha_{11} = \textcircled{1} \times \textcircled{1} = \frac{1}{3EI} \left\{ \frac{1}{2} 2c \cdot c \cdot \frac{4l}{3} \right\} + \frac{1}{EI} \left\{ \frac{1}{2} 2c \cdot 2c \cdot \frac{4l}{3} \right\} = \frac{1}{EI} \left\{ \frac{4c^3}{9} + \frac{8c^3}{3} \right\} = \frac{28}{9} \frac{c^3}{EI}$$

$$\Delta_1^q = \textcircled{1} \times \textcircled{2} = \frac{1}{3EI} \left\{ -2 \cdot \frac{2}{3} \cdot \frac{1}{8} ql^2 \cdot \frac{c}{2} \right\} = -\frac{1}{36} \frac{ql^4}{EI}$$

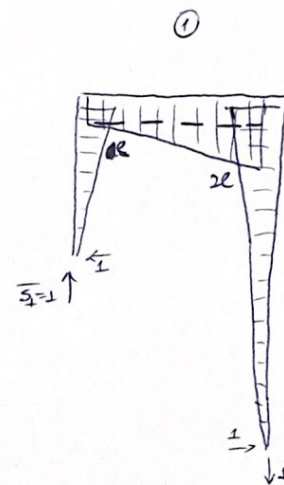
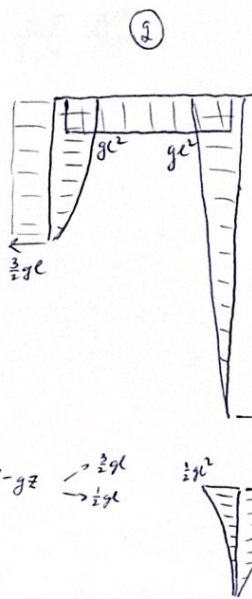
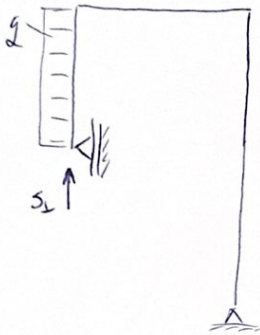
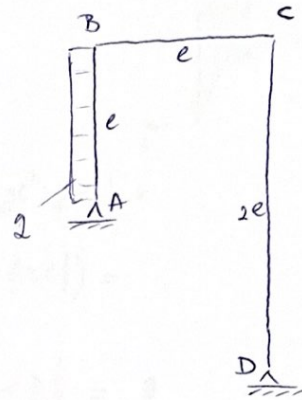
$$\frac{28}{9} \frac{qc^3}{EI} S_L - \frac{1}{36} \frac{ql^4}{EI} = 0 \quad | \cdot 36$$

$$112 S_L = ql$$

$$\boxed{S_L = \frac{1}{112} ql}$$

② rešiti nřak statiki neodreduh velicina horizontalno pomeraye tačke C?

$$EI = \text{const.}$$



$$f_A^{\text{vert}} = \alpha_{11} \cdot S_1 + \Delta_1^{\text{rot}} = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e - \frac{3}{2} e + \frac{1}{2} e \cdot e - \frac{5}{3} e + \frac{1}{2} 2e \cdot 2e \cdot \frac{4}{3} e \right\} = \frac{e^3}{EI} \left\{ \frac{1}{3} + \frac{2}{3} + \frac{5}{6} + \frac{8}{3} \right\} = \frac{16}{3} \frac{e^3}{EI}$$

$$\Delta_1^{\text{rot}} = \frac{1}{EI} \left\{ \frac{1}{2} gl^2 \cdot 2e \cdot \frac{4}{3} e + gl^2 \cdot e \cdot \frac{3}{2} e - \frac{1}{3} \frac{10}{2} gl^2 \cdot e \cdot \frac{3}{4} e + \frac{1}{2} \frac{3}{2} gl^2 \cdot e \cdot \frac{2}{3} e \right\} =$$

$$\Delta_1^{\text{rot}} = \frac{gl^4}{EI} \left\{ \frac{4}{3} + \frac{3}{2} - \frac{1}{8} + \frac{1}{2} \right\} = \frac{77}{24} \frac{gl^4}{EI}$$

$$\frac{16}{3} \frac{e^3}{EI} S_1 + 77 gl = 0$$

$$\underline{\underline{S_1 = -\frac{77}{128} gl}}$$

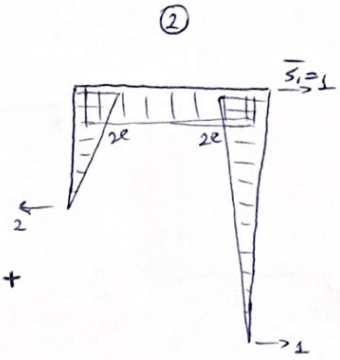
$$f_c^{hor} = \Delta_2^g + S_1 \cdot \textcircled{1} \cdot \textcircled{2}$$

$$f_o^{hor} = \frac{1}{EI} \left\{ \frac{1}{2} g l^2 \cdot 2e \cdot \frac{2}{3} 2e + g l^2 \cdot e \cdot 2e + \frac{1}{2} \frac{3}{2} g l^2 \cdot e \cdot \frac{2}{3} 2e - \frac{1}{3} \frac{1}{2} g l^2 \cdot \frac{3}{4} 2e \right\} +$$

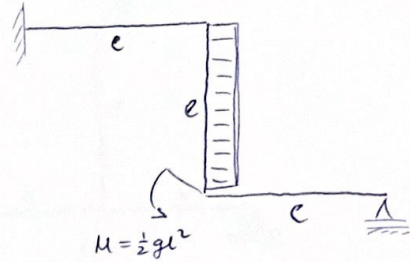
$$+ S_1 \left(\frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e + \frac{e+2e}{2} e \cdot 2e + \frac{1}{2} e \cdot e \cdot \frac{2}{3} 2e \right\} \right) =$$

$$f_c^{hor} = \frac{g l^4}{EI} \left\{ \frac{4}{3} + 2 + 1 - \frac{1}{4} \right\} - \frac{77}{128} \frac{g l^4}{EI} \left\{ \frac{8}{3} + 3 + \frac{2}{3} \right\} =$$

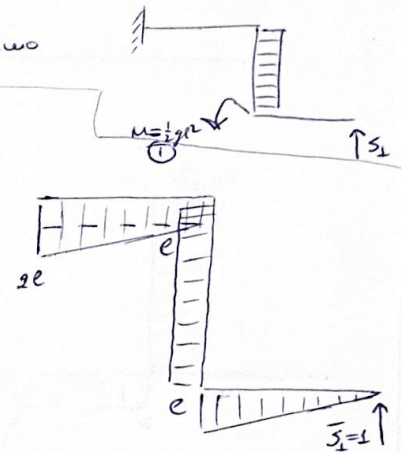
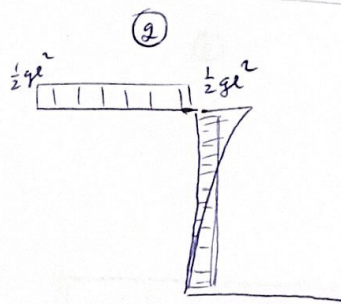
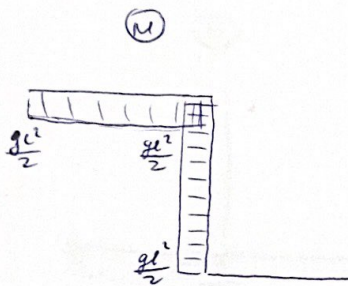
$$f_c^{hor} = \frac{g l^4}{EI} \left(\frac{45}{12} - \frac{77}{128} \cdot \frac{19}{3} \right) = \frac{105}{384} \frac{g l^4}{EI}$$



③ $EI = \text{const.}$



preporuka: kada imamo okretanje, da ga sačuvamo



$$\int_{\text{prot}} \delta = d_{11} S_1 + \Delta_1^M + \Delta_1^q = 0$$

$$d_{11} = \frac{1}{EI} \left\{ e \cdot e \cdot \frac{3}{2}e + \frac{1}{2}e \cdot e \cdot \left(e + \frac{2}{3}e\right) + e \cdot e \cdot e + \frac{1}{2}e \cdot e \cdot \frac{2}{3}e \right\} = \frac{e^3}{EI} \left(\frac{3}{2} + \frac{5}{6} + 1 + \frac{1}{3} \right) = \frac{11}{3} \frac{e^3}{EI}$$

$$\Delta_1^M = \frac{1}{EI} \left\{ \frac{1}{2}qe^2 \cdot e \cdot \frac{3}{2}e + \frac{1}{2}qe^2 \cdot e \cdot e \right\} = \frac{qe^4}{EI} \left(\frac{3}{4} + \frac{1}{2} \right) = \frac{5}{4} \frac{qe^4}{EI}$$

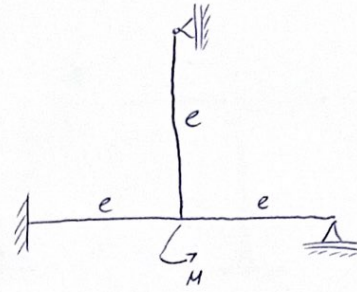
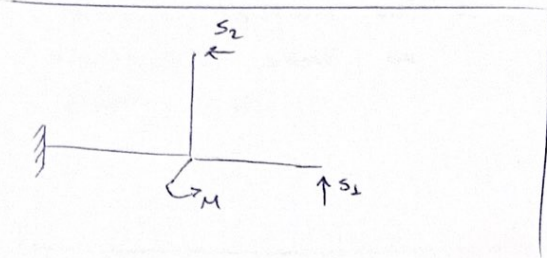
$$\Delta_1^q = \frac{1}{EI} \left\{ -\frac{1}{2}qe^2 \cdot e \cdot \frac{3}{2}e - \frac{1}{3} \frac{1}{2}qe^2 \cdot e \cdot e \right\} = \frac{qe^4}{EI} \left(-\frac{3}{4} - \frac{1}{6} \right) = -\frac{11}{12} \frac{qe^4}{EI}$$

$$\frac{11}{3} \frac{e^3}{EI} \cdot S_1 + \frac{5}{4} \frac{qe^4}{EI} - \frac{11}{12} \frac{qe^4}{EI} = 0 \quad / \cdot \frac{3EI}{e^3}$$

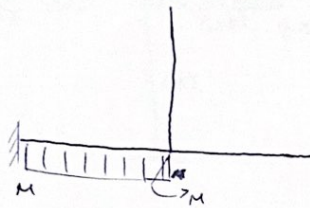
$$11 S_1 + \frac{4}{4} qe = 0$$

$$\underline{\underline{S_1 = -\frac{1}{11} qe}}$$

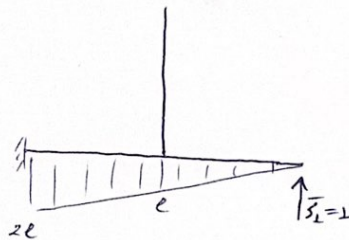
(4) $EI = \text{const.}$



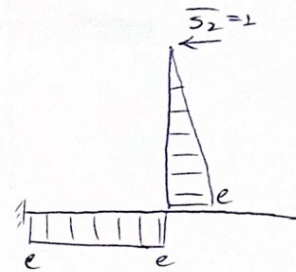
(M)



(1)



(2)



$$(1) \int_D^{vert} = \alpha_{11} S_1 + \alpha_{22} S_2 + \Delta_1^M = 0$$

$$(2) \int_B^{hor} = \alpha_{21} S_1 + \alpha_{22} S_2 + \Delta_2^M = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} 2e \right\} = \frac{8}{3} \frac{e^3}{EI}$$

$$\alpha_{22} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e \cdot e \right\} = \frac{4}{3} \frac{e^3}{EI}$$

$$\alpha_{12} = \alpha_{21} = \frac{1}{EI} \left\{ \frac{2e+e}{2} e \cdot e \right\} = \frac{3}{2} \frac{e^3}{EI}$$

$$\Delta_1^M = \frac{1}{EI} \left\{ \frac{2e+e}{2} e \cdot M \right\} = \frac{3}{2} \frac{Me^2}{EI}$$

$$\Delta_2^M = \frac{1}{EI} \{ M \cdot e \cdot e \} = \frac{Me^2}{EI}$$

$$(1) = \frac{8}{3} \frac{e^3}{EI} S_1 + \frac{3}{2} \frac{e^3}{EI} S_2 + \frac{3}{2} \frac{Me^2}{EI} = 0 \quad | \cdot \frac{6EI}{e^3}$$

$$(2) = \frac{3}{2} \frac{e^3}{EI} S_1 + \frac{4}{3} \frac{e^3}{EI} S_2 + \frac{Me^2}{EI} = 0 \quad | \cdot \frac{6EI}{e^3}$$

$$16 S_1 e + 9 S_2 e + 9 M = 0$$

$$9 S_1 e + 8 S_2 e + 6 M = 0$$

$$\Rightarrow S_1 = -\frac{9}{16} S_2 - \frac{9}{16} \frac{M}{e}$$

$$\Rightarrow 9 \left(-\frac{9}{16} S_2 - \frac{9}{16} \frac{M}{e} \right) e + 8 S_2 e + 6 M = 0$$

$$-\frac{81}{16} S_2 e - \frac{81}{16} M + 8 S_2 e + 6 M = 0$$

$$\frac{47}{16} S_2 e + \frac{15}{16} M = 0 \Rightarrow \underline{\underline{S_2 = -\frac{15}{47} \frac{M}{e}}}$$

$$S_1 = -\frac{9}{16} \left(-\frac{15}{47} \frac{M}{e} \right) - \frac{9}{16} \frac{M}{e} = \left(\frac{135}{752} - \frac{423}{752} \right) \frac{M}{e}$$

$$S_1 = -\frac{288}{752} \frac{M}{e} = 1 \quad \underline{\underline{S_2 = -\frac{18}{47} \frac{M}{e}}}$$

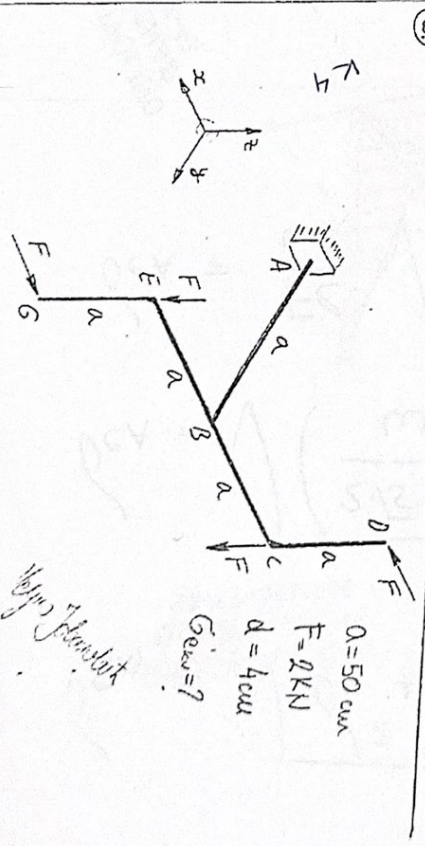
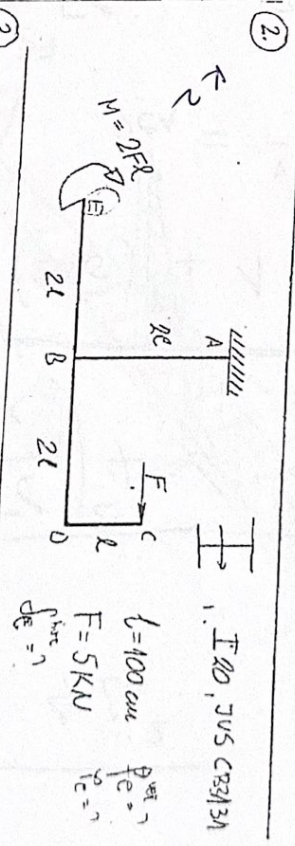
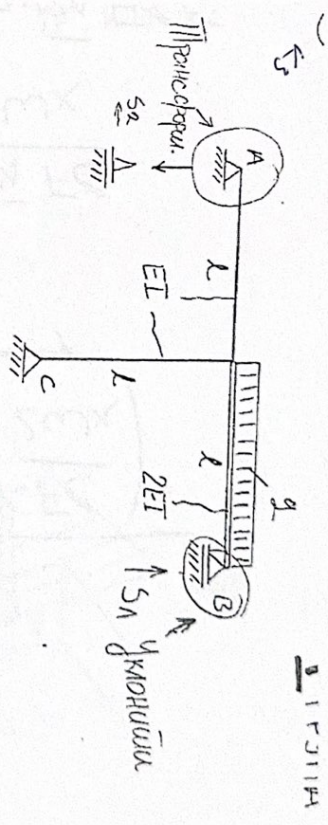
Писмени испит из **ООК**
 Септембар - Октобар 2010.

1. Статички неодређен равански носач променљиве крутости оптерећен је континуалним оптерећењем q према скици. Применајући методу сила - Верешалгинов поступак израчунајте статички непознате величине трансформисли носач у статички одређен на задати начин.
2. Равански носач константне крутости EI оптерећен је силом F и моментом M према скици.
 - Израчунајте тражена померања у функцији општих бројева.
 - Израчунајте максимални нормални напон у носачу користећи податке задате у скици.
3. Просторни носач константног кружног попречног пресека оптерећен је према скици.
 - Нацртајте статичке дијаграме.
 - Применом Хипотезе највећих напона смањена одређити еквивалентни напон у тачкама A у функцији F, a, W .
 - Израчунајте податке задате у скици. Применајући поступак сила, ако је задат дозволити одређивање нормалних напона.

Напомене:

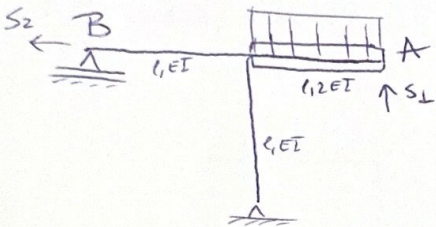
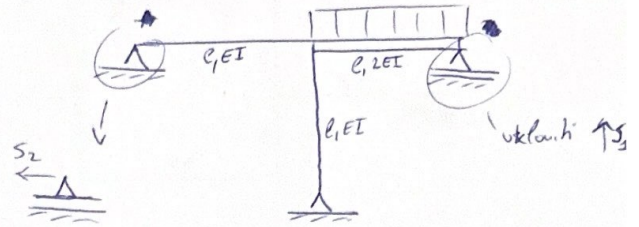
- Испит траје три сата.
- Дозвољена је само штампана литература - не збијке!

ČASOVNI POVOLJINO
 Profesor NENAD
 064/155-72-42

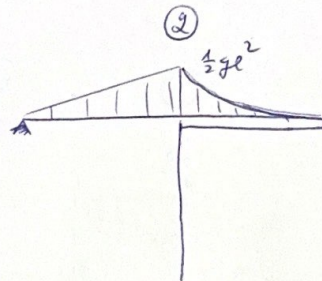
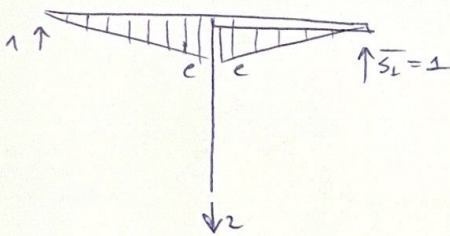


september 10

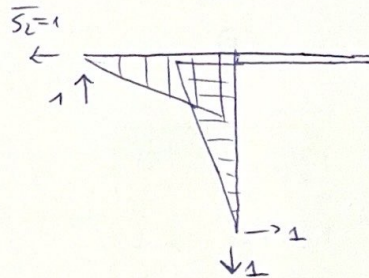
① izračunati nepoznate veličine transformirajući nosač na zadani način



①



②



$$1) f_A^{vert} = \alpha_{11} S_1 + \alpha_{12} S_2 + \Delta_1 = 0$$

$$2) f_B^{hor} = \alpha_{21} S_1 + \alpha_{22} S_2 + \Delta_2 = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \right\} + \frac{1}{2EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \right\} = \frac{1}{2} \frac{c^3}{EI}$$

$$\alpha_{12} = \frac{1}{EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \right\} = \frac{1}{3} \frac{c^3}{EI}$$

$$\alpha_{22} = \frac{1}{EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \cdot 2 \right\} = \frac{2}{3} \frac{c^3}{EI}$$

$$\Delta_1^Q = \frac{1}{EI} \left\{ -\frac{1}{2} \frac{1}{2} q l^2 \cdot c \cdot \frac{2}{3} c \right\} + \frac{1}{2EI} \left\{ -\frac{1}{2} \frac{1}{2} q l^2 \cdot c \cdot \frac{2}{3} c \right\} = -\frac{11}{48} \frac{q l^4}{EI}$$

$$\Delta_2^Q = \frac{1}{EI} \left\{ -\frac{1}{2} \frac{1}{2} q l^2 \cdot c \cdot \frac{2}{3} c \right\} = -\frac{1}{6} \frac{q l^4}{EI}$$

$$\frac{1}{2} \frac{c^3}{EI} S_1 + \frac{1}{3} \frac{c^3}{EI} S_2 - \frac{11}{48} \frac{q l^4}{EI} = 0 \quad | \cdot \frac{2EI}{c^3}$$

$$\frac{1}{3} \frac{c^3}{EI} S_1 + \frac{2}{3} \frac{c^3}{EI} S_2 - \frac{11}{24} \frac{q l^4}{EI} = 0 \quad | \cdot \frac{3EI}{c^3}$$

$$S_1 + \frac{2}{3} S_2 - \frac{11}{24} q l = 0 \Rightarrow S_1 = \frac{11}{24} q l - \frac{2}{3} S_2$$

$$\left(\frac{11}{24} q l - \frac{2}{3} S_2 \right) + 2 S_2 - \frac{1}{2} q l = 0$$

$$\frac{4}{3} S_2 - \frac{1}{24} q l = 0 \Rightarrow S_2 = \frac{1}{32} q l$$

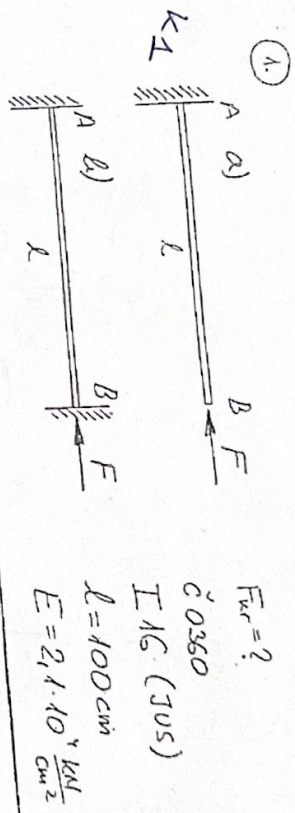
$$S_1 = \frac{11}{24} q l - \frac{2}{3} \frac{1}{32} q l = \frac{11}{24} q l - \frac{1}{48} q l =$$

$$S_1 = \frac{21}{48} q l$$

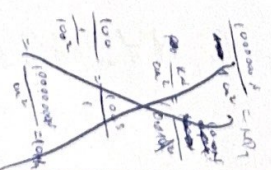
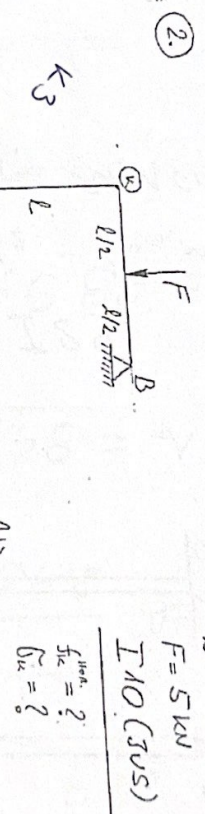
МАШИНСКИ ФАКУЛТЕТ УНИВЕРЗИТЕТА У БЕОГРАДУ
КАТЕДРА ЗА ОПОРНОСТ КОНСТРУКЦИЈА

ПИСМЕНИ ИСПИТ из Основа опрности конструкција
(05.09.2009 год.)

1. За приказане случајеве ослањања челичног штапа, израчунајте вредност критичне силе измицања. Користити податке дате уз скицу.



$E = 2,1 \cdot 10^4 \frac{kN}{cm^2}$
 $I = 16 (JUS)$

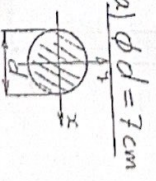
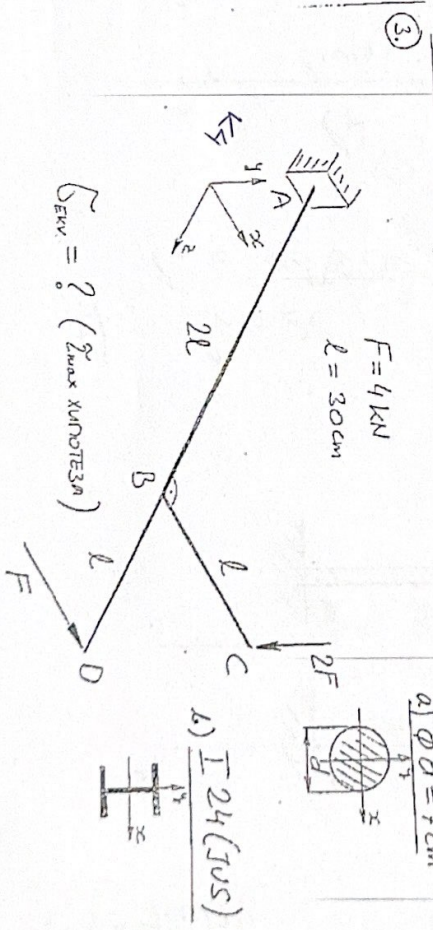


2. За конструкцију приказану на скици, користећи методу сила-Вешталингов поступак, услањањем ослона В израчунајте статички непознате величине и највећи дијаграм моментата савијања у функцији од оштрих брoјева. Користећи податке дате уз скицу израчунајте напон као и хоризонтално померање пресека К.

3. Челична конструкција задатог попречног пресека оптерећена је према скици. Користећи хитозезу максималних тангенцијалних напона израчунајте вредност највећег еквивалентног напона у конструкцији ако је попречни пресек:

- а) кружни ($d = 7 \text{ cm}$),
- б) стандардни профил (I 24).

Користити податке дате уз скицу.



$\sigma_{\text{equiv}} = ?$ ($\sigma_{\text{max}} \text{ хитозезом}$)

из Кабинета

Напомене:

- Дозвољена је употреба само штампане литературе
- Испит траје 3 сата.
- Резултати ће бити објављени у понедељак 07.09.2009. год.

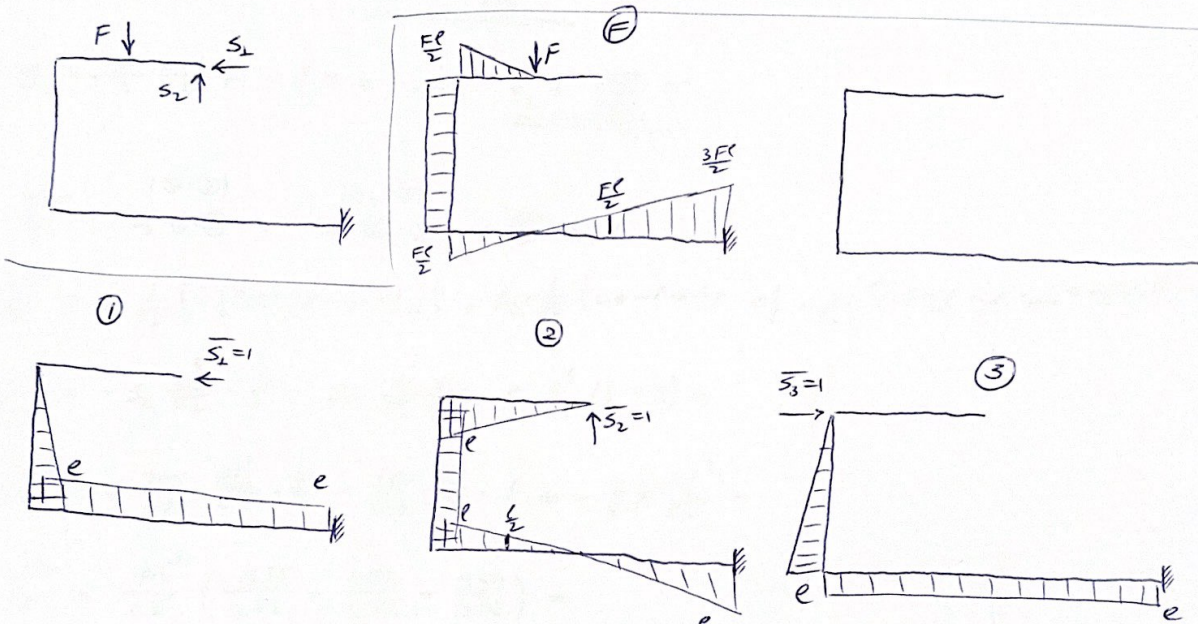
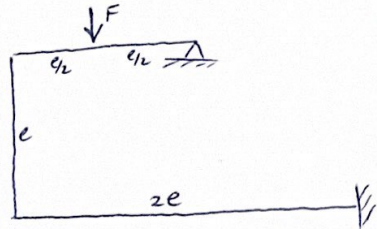
September 03

(2) uklanjajuće oslonca B

dijagrami uov. savijanja?
(u opštim koordinatama)

f_k^{hor} ? σ_k ?

$l = 100 \text{ cm}$, $F = 5 \text{ kN}$, $I_{10} \text{ (DUS)}$



$$f_B^{hor} = d_{11} S_1 + d_{12} S_2 + \Delta_1^F = 0$$

$$f_B^{vert} = d_{21} S_1 + d_{22} S_2 + \Delta_2^F = 0$$

$$d_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot 2e \cdot e \right\} = \frac{7}{3} \frac{e^3}{EI}$$

$$d_{12} = d_{21} = \frac{1}{EI} \left\{ 0 + \frac{1}{2} e \cdot e \cdot e + 0 \right\} = \frac{1}{2} \frac{e^3}{EI}$$

$$d_{22} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e \cdot e + 2 \cdot \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} = 2 \frac{e^3}{EI}$$

$$\Delta_1^F = \frac{1}{EI} \left\{ -\frac{Fl}{2} \cdot e \cdot \frac{1}{2} e - \frac{1}{2} \frac{Fl}{2} \cdot \frac{1}{2} \cdot e + \frac{1}{2} \frac{3Fl}{2} \cdot \frac{2}{3} e \cdot e \right\} = \frac{Fl^3}{EI} \left(-\frac{1}{4} - \frac{1}{8} + \frac{2}{3} \right) = \frac{3}{4} \frac{Fl^3}{EI}$$

$$\Delta_2^F = \frac{1}{EI} \left\{ -\frac{1}{2} \frac{Fl}{2} \cdot \frac{e}{2} \cdot \left(\frac{e}{2} + \frac{2}{3} \frac{e}{2} \right) - \frac{Fl}{2} \cdot e \cdot e - \frac{1}{2} \frac{Fl}{2} \cdot \frac{e}{2} \cdot \left(\frac{e}{2} + \frac{2}{3} \frac{e}{2} \right) + \frac{1}{2} \frac{Fl}{2} \cdot \frac{e}{2} \cdot \frac{1}{3} \frac{e}{2} - \frac{1}{2} e \cdot e \cdot \left(\frac{Fl}{2} + \frac{2}{3} Fl \right) \right\} =$$

$$= \frac{Fl^3}{EI} \left(-\frac{1}{8} \frac{5}{6} - \frac{1}{2} - \frac{1}{8} \frac{5}{6} + \frac{1}{48} - \frac{1}{2} \frac{7}{6} \right) = \frac{Fl^3}{EI} \left(-\frac{5}{48} - \frac{24}{48} - \frac{5}{48} + \frac{1}{48} - \frac{28}{48} \right) = -\frac{61}{48} \frac{Fl^3}{EI}$$

$$\frac{7}{3} \frac{e^3}{EI} S_1 + \frac{1}{2} \frac{e^3}{EI} S_2 + \frac{3}{4} \frac{Fl^3}{EI} = 0 \quad | \cdot 2 \frac{EI}{e^3}$$

$$\frac{1}{2} \frac{e^3}{EI} S_1 + 2 \frac{e^3}{EI} S_2 + - \frac{61}{48} \frac{Fl^3}{EI} = 0 \quad | \cdot \frac{e^3}{EI}$$

$$\frac{14}{3} S_1 + S_2 + \frac{3}{2} F = 0 \quad \Rightarrow \quad S_2 = -\frac{14}{3} S_1 - \frac{3}{2} F$$

$$\frac{1}{2} S_1 + 2 \left(-\frac{14}{3} S_1 - \frac{3}{2} F \right) - \frac{61}{48} F = 0$$

$$\frac{1}{2} S_1 - \frac{28}{3} S_1 - 3F - \frac{61}{48} F = 0$$

$$-\frac{53}{6} S_1 - \frac{205}{48} F = 0$$

$$S_1 = -\frac{205 \cdot 6}{48 \cdot 53} F \quad \Rightarrow \quad \underline{\underline{S_1 = -\frac{205}{424} F}}$$

$$S_2 = +\frac{14}{3} \frac{205}{424} F - \frac{3}{2} F \quad \Rightarrow \quad \underline{\underline{S_2 = \frac{481}{636} F}}$$

$$f_k^{hor} = \overset{(S_1 \cdot 3)}{S_1 \cdot 0 \cdot 3} + \overset{(S_2 \cdot 3)}{S_2 \cdot 2 \cdot 3} + \overset{(F \cdot 3)}{F \cdot 3} =$$

$$f_k^{hor} = S_1 \frac{1}{EI} \left\{ -\frac{1}{2} e \cdot e \cdot \frac{1}{3} e - e \cdot 2e \cdot e \right\} + S_2 \frac{1}{EI} \left\{ 0 - e \cdot e \cdot \frac{1}{2} e + 0 \right\} + \frac{1}{EI} \left\{ \frac{Fl}{2} \cdot e \cdot \frac{1}{2} e + \dots - e \cdot e \cdot \left(\frac{Fl}{2} + \frac{Fl}{2} \right) \right\} =$$

$$f_k^{hor} = -S_1 \frac{e^3}{EI} \cdot \frac{7}{3} - S_2 \frac{e^3}{EI} \frac{1}{2} + F \cdot \frac{e^3}{EI} \left(\frac{1}{4} - 1 \right) =$$

$$f_k^{hor} = + \frac{205}{424} \frac{Fl^3}{EI} \cdot \frac{7}{3} - \frac{481}{636} \frac{Fl^3}{EI} \cdot \frac{1}{2} - \frac{3}{4} \frac{Fl^3}{EI} =$$

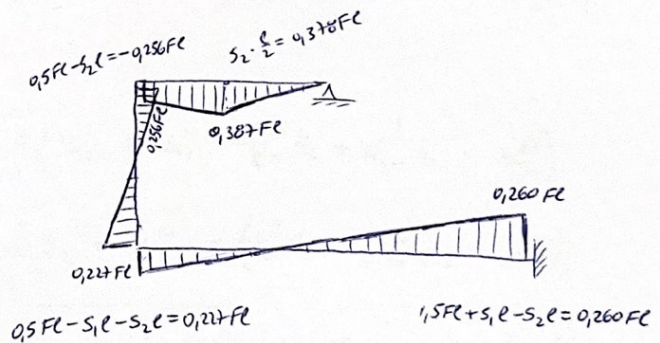
$$f_k^{hor} = \frac{Fl^3}{EI} \left(\frac{1435}{1272} - \frac{481}{1272} - \frac{954}{1272} \right) =$$

$$\underline{\underline{f_k^{hor} = 0}}$$

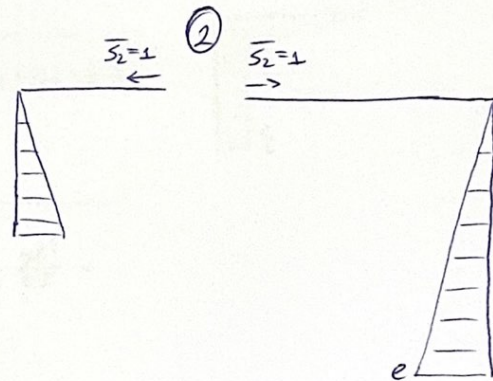
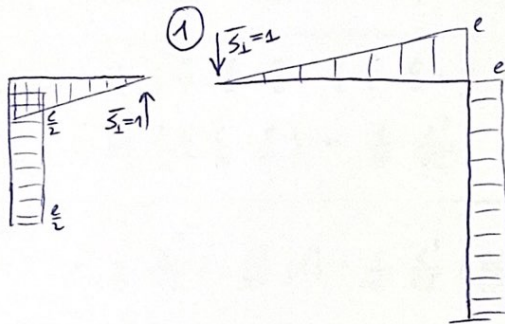
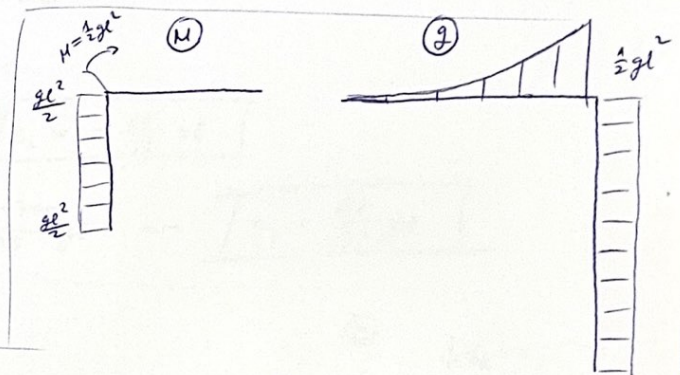
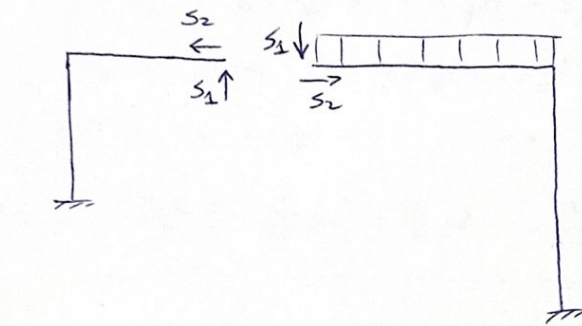
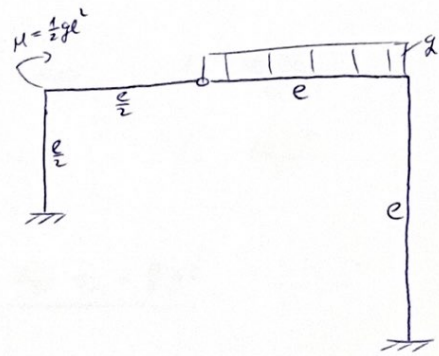
$$W_x = 34,2 \text{ cm}^3 \quad - \quad \text{za I 10 (SUS)}$$

$$\sigma_k = \frac{M_k}{W_x} = \frac{9256 \text{ FP}}{34,2} = \frac{9256 \cdot 5 \cdot 100}{34,2} =$$

$$\underline{\underline{\sigma_k = 3,74 \frac{\text{kN}}{\text{cm}^2}}}$$



5) stat. uap. vel. rine?
 $f_{G,rec} = ?$
 dijagram waw. sawijanja?



$$(1) f_{G,rec}^{vert} = \alpha_{11} S_1 + \alpha_{12} S_2 + \Delta_1^M + \Delta_1^Q = 0$$

$$(2) f_{G,rec}^{hor} = \alpha_{21} S_1 + \alpha_{22} S_2 + \Delta_2^M + \Delta_2^Q = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} \frac{e}{2} \frac{e}{2} \cdot \frac{2}{3} \frac{e}{2} + \frac{e}{2} \frac{e}{2} \frac{e}{2} + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e \cdot e \right\} = \frac{e^3}{EI} \left(\frac{1}{24} + \frac{1}{8} + \frac{1}{3} + 1 \right) = \frac{3}{2} \frac{e^3}{EI}$$

$$\alpha_{12} = \frac{1}{EI} \left\{ \frac{e}{2} \cdot \frac{1}{2} \cdot \frac{e}{4} - e \cdot e \cdot \frac{e}{2} \right\} = \frac{e^3}{EI} \left(\frac{1}{16} - \frac{1}{2} \right) = -\frac{7}{16} \frac{e^3}{EI}$$

$$\alpha_{22} = \frac{1}{EI} \left\{ \frac{1}{2} \frac{e}{2} \frac{e}{2} \cdot \frac{2}{3} \frac{e}{2} + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} = \frac{e^3}{EI} \left(\frac{1}{24} + \frac{1}{3} \right) = \frac{3}{8} \frac{e^3}{EI}$$

$$\Delta_1^M = \frac{1}{EI} \left\{ -\frac{g l^2}{2} \cdot \frac{e}{2} \cdot \frac{e}{2} \right\} = -\frac{1}{8} \frac{g l^4}{EI}$$

$$\Delta_1^Q = \frac{1}{EI} \left\{ \frac{1}{3} \frac{g l^2}{EI} e \cdot \frac{2}{3} e + \frac{g l^2}{2} \cdot e \cdot e \right\} = \frac{5}{8} \frac{g l^4}{EI}$$

$$\Delta_2^M = \frac{1}{EI} \left\{ -\frac{g l^2}{2} \cdot \frac{e}{2} \cdot \frac{1}{4} \right\} = -\frac{1}{16} \frac{g l^4}{EI}$$

$$\Delta_2^Q = \frac{1}{EI} \left\{ -\frac{g l^2}{2} \cdot e \cdot \frac{e}{2} \right\} = -\frac{1}{4} \frac{g l^4}{EI}$$

$$\frac{3}{2} \frac{e^3}{EI} S_1 - \frac{7}{16} \frac{e^3}{EI} S_2 - \frac{1}{8} \frac{gl^4}{EI} + \frac{5}{8} \frac{gl^4}{EI} = 0 \quad | \cdot \frac{16 EI}{e^3}$$

$$-\frac{7}{16} \frac{e^3}{EI} S_1 + \frac{3}{8} \frac{e^3}{EI} S_2 - \frac{1}{16} \frac{gl^4}{EI} - \frac{1}{4} \frac{gl^4}{EI} = 0 \quad | \cdot \frac{16 EI}{e^3}$$

$$24 S_1 - 7 S_2 + 8 gl = 0 \quad \Rightarrow \quad \underline{S_2 = \frac{24}{7} S_1 + \frac{8}{7} gl}$$

$$-7 S_1 + 6 S_2 - 5 gl = 0$$

$$-7 S_1 + 6 \left(\frac{24}{7} S_1 + \frac{8}{7} gl \right) - 5 gl = 0 \quad | \cdot 7$$

$$-49 S_1 + 144 S_1 + 48 gl - 35 gl = 0$$

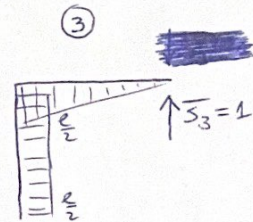
$$95 S_1 + 13 gl = 0 \quad \Rightarrow \quad \boxed{S_1 = -\frac{13}{95} gl}$$

$$S_2 = \frac{24}{7} \left(-\frac{13}{95} gl \right) + \frac{8}{7} gl = -\frac{312 + 760}{7 \cdot 95} gl \quad \Rightarrow \quad \boxed{S_2 = \frac{64}{95} gl}$$

$$f_G^{vert} = \alpha_{31} S_1 + \alpha_{32} S_2 + \Delta_3^M + \Delta_3^Q$$

$$\alpha_{31} = \frac{1}{EI} \left\{ \frac{e}{2} \cdot \frac{e}{2} \cdot \frac{e}{2} + \frac{1}{2} \frac{e}{2} \cdot \frac{e}{2} \cdot \frac{2}{3} \frac{e}{2} \right\} = \frac{e^3}{EI} \left(\frac{1}{8} + \frac{1}{24} \right) = \frac{e^3}{6 EI}$$

$$\alpha_{32} = \frac{1}{EI} \left\{ \frac{e}{2} \frac{e}{2} \cdot \frac{e}{4} \right\} = \frac{1}{16} \frac{e^3}{EI}$$

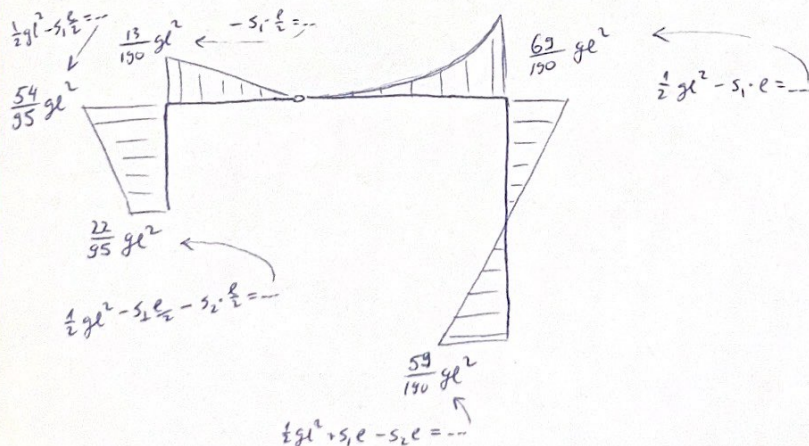


$$f_G^{vert} = \frac{1}{6} \frac{e^3}{EI} \left(-\frac{13}{95} gl \right) + \frac{1}{16} \frac{e^3}{EI} \left(\frac{64}{95} gl \right) - \frac{1}{8} \frac{gl^4}{EI} =$$

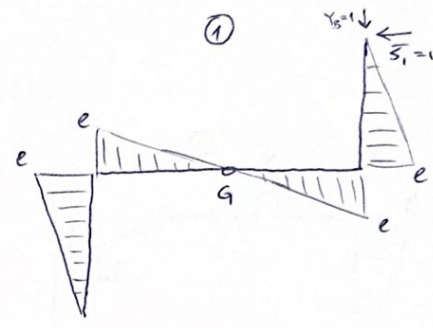
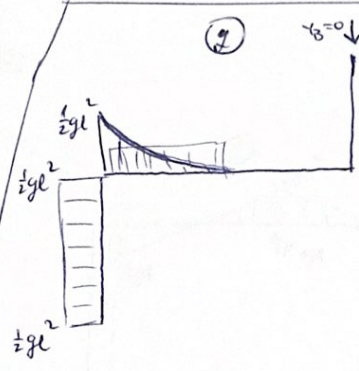
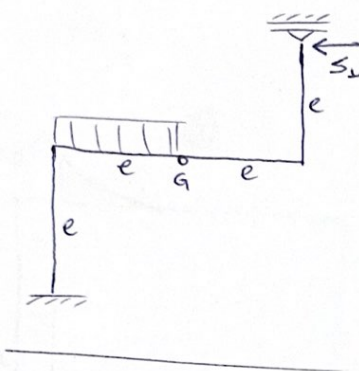
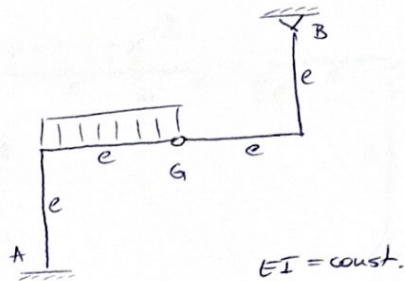
$$f_G^{vert} = \frac{gl^4}{EI} \left(-\frac{13}{570} + \frac{24}{570} - \frac{285}{2280} \right) =$$

$$f_G^{vert} = -\frac{241}{2280} \frac{gl^4}{EI}$$

ZGimi dijagram uov. su: (4) + (9) + S1 · (1) + S2 · (2)



6) nacrtati zbirni dijagram uočavajući fleksije



$$d_{11} \cdot S_1 + \Delta_1^q = 0$$

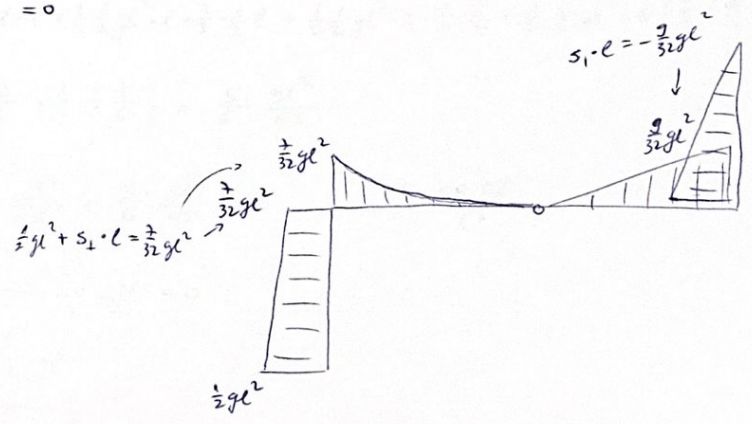
$$d_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} \cdot 4 = \frac{4}{3} \frac{e^3}{EI}$$

$$\Delta_1^q = \frac{1}{EI} \left\{ \frac{1}{2} q l^3 \cdot e \cdot \frac{1}{2} + \frac{1}{3} \frac{1}{2} q l^2 \cdot e \cdot \frac{3}{4} e \right\} = \frac{q l^4}{EI} \left(\frac{1}{4} + \frac{1}{8} \right) = \frac{3}{8} \frac{q l^4}{EI}$$

$$\frac{4}{3} \frac{e^3}{EI} S_1 + \frac{3}{8} \frac{q l^4}{EI} = 0 \quad | \cdot \frac{24 EI}{e^3}$$

$$32 S_1 + 9 q l = 0$$

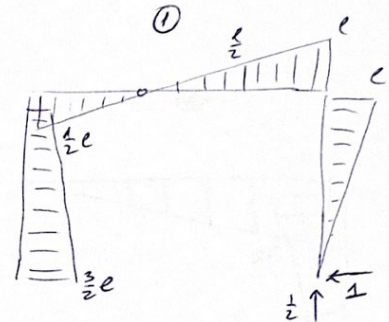
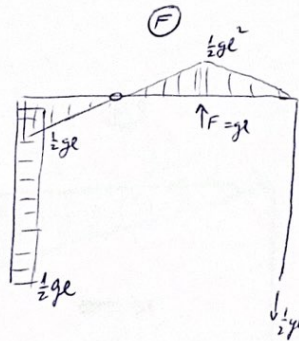
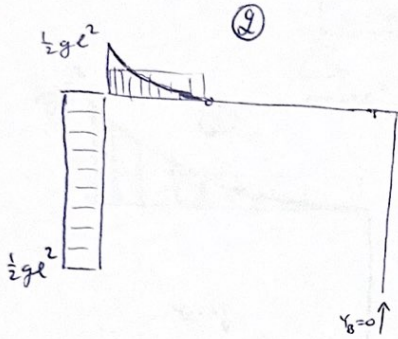
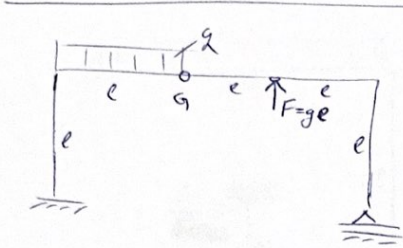
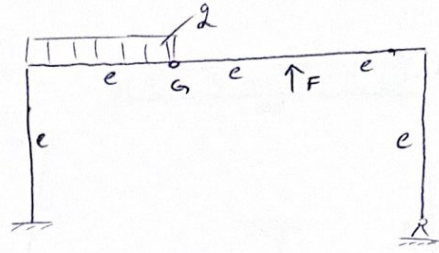
$$S_1 = -\frac{9}{32} q l$$



7

$$F = gl$$

$$EI = \text{const.}$$



$$\alpha_{11} S_1 + \Delta_1^g + \Delta_1^F = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot e + \frac{1}{2} e \cdot e \cdot \left(\frac{1}{2} e + \frac{2}{3} e \right) + \frac{1}{2} \frac{1}{2} e \cdot e \cdot \frac{2}{3} \frac{1}{2} e + \frac{1}{2} e \cdot 2e \cdot \frac{2}{3} e + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} =$$

$$\alpha_{11} = \frac{e^3}{EI} \left(\frac{1}{3} + \frac{1}{12} + \frac{2}{3} + \frac{1}{2} + \frac{2}{12} \right) = \frac{13}{6} \frac{e^3}{EI}$$

$$\Delta_1^g = \frac{1}{EI} \left\{ -\frac{1}{2} gl^2 \cdot e \cdot e - \frac{1}{3} \frac{1}{2} gl^2 \cdot e \cdot \frac{3}{4} \frac{1}{2} e \right\} = \frac{gl^4}{EI} \left\{ -\frac{1}{2} - \frac{1}{16} \right\} = -\frac{9}{16} \frac{gl^4}{EI}$$

$$\Delta_1^F = \frac{1}{EI} \left\{ \frac{1}{2} gl^2 \cdot e \cdot e + \frac{1}{2} \frac{1}{2} gl^2 \cdot e \cdot \frac{2}{3} \frac{1}{2} e + \frac{1}{2} \frac{1}{2} gl^2 \cdot e \cdot \frac{2}{3} \frac{1}{2} e + \frac{1}{2} \frac{1}{2} gl^2 \cdot e \left(\frac{2}{3} + \frac{2}{3} \frac{1}{2} \right) \right\} =$$

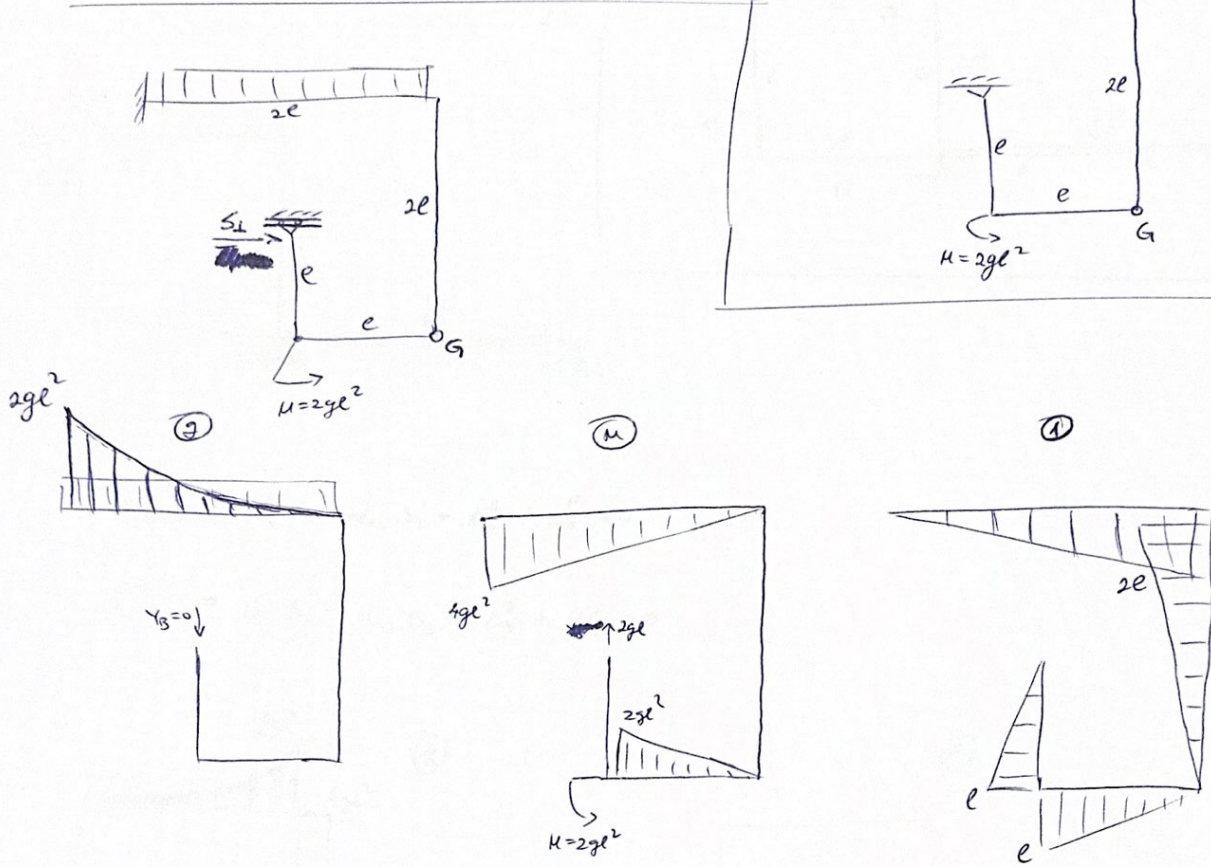
$$\Delta_1^F = \frac{gl^4}{EI} \left\{ \frac{1}{2} + \frac{1}{12} + \frac{1}{12} + \frac{1}{6} \right\} = \frac{5}{6} \frac{gl^4}{EI}$$

$$\frac{13}{6} \frac{e^3}{EI} S_1 - \frac{9}{16} \frac{gl^4}{EI} + \frac{5}{6} \frac{gl^4}{EI} = 0 \quad | \cdot \frac{48 EI}{e^3}$$

$$104 S_1 - 27 gl + 40 gl = 0$$

$$S_1 = -\frac{13}{104} gl = -\frac{1}{8} gl$$

⑧ $EI = \text{const.}$



$$\alpha_{11} S_L + \Delta_1^g + \Delta_1^m = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \right\} = \frac{e^3}{EI} \left(\frac{1}{3} + \frac{1}{3} + \frac{8}{3} + \frac{8}{3} \right) = 6 \frac{e^3}{EI}$$

$$\Delta_1^g = \frac{1}{EI} \left\{ -\frac{1}{3} 2gl^2 \cdot 2e \cdot \frac{1}{4} 2e \right\} = -\frac{2}{3} \frac{gl^4}{EI}$$

$$\Delta_1^m = \frac{1}{EI} \left\{ \frac{1}{2} 4gl^2 \cdot 2e \cdot \frac{1}{3} 2e - \frac{1}{2} 2gl^2 \cdot e \cdot \frac{2}{3} e \right\} = \frac{2l^4}{EI} \left(\frac{8}{3} - \frac{2}{3} \right) = 2 \frac{2l^4}{EI}$$

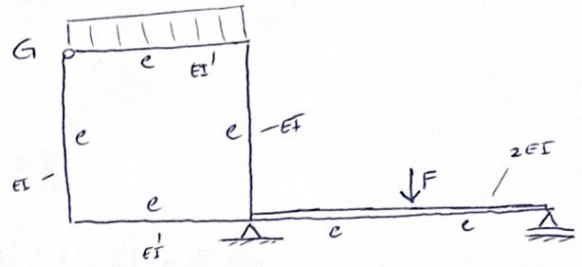
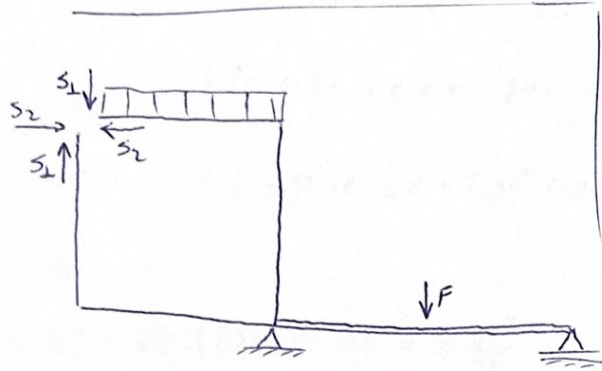
$$6 \frac{e^3}{EI} \cdot S_L - \frac{2}{3} \frac{gl^4}{EI} + 2 \frac{2l^4}{EI} = 0 \quad | \cdot 3 \frac{EI}{e^3}$$

$$18 S_L - 2gl + 6gl = 0$$

$$18 S_L + 4gl = 0$$

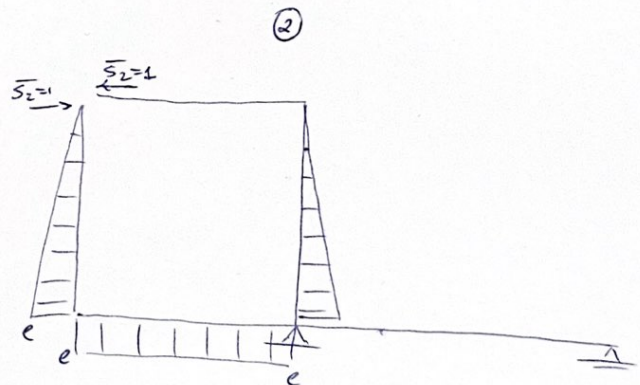
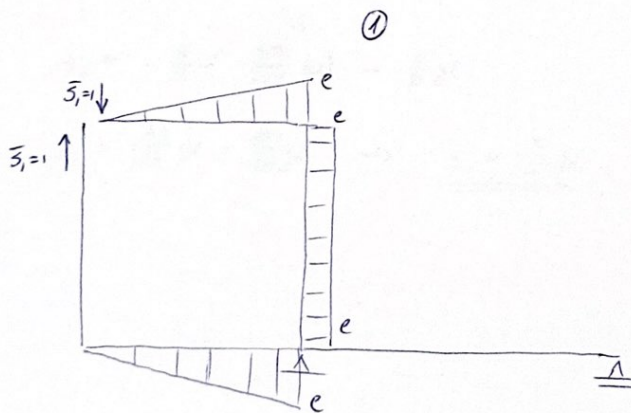
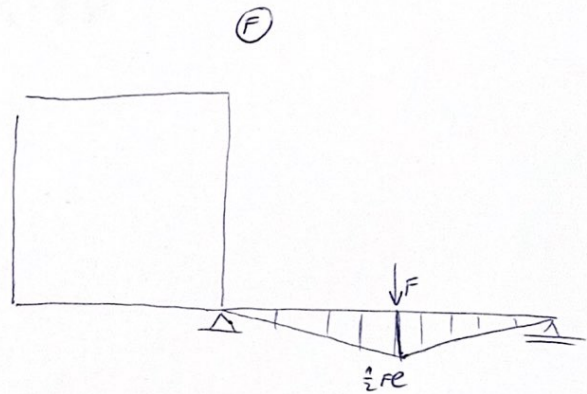
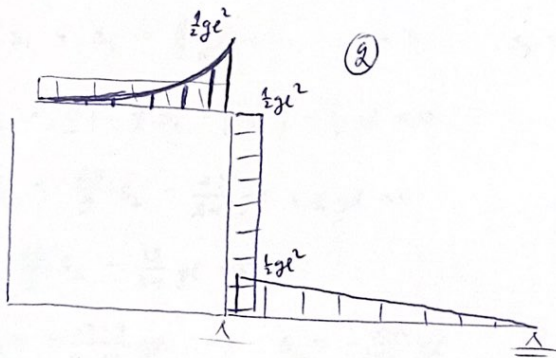
$$\underline{\underline{S_L = -\frac{2}{9} gl}}$$

9.



$$(1) f_{G, \text{vert}}^{\text{rel}} = \alpha_{11} S_1 + \alpha_{12} S_2 + \Delta_1^G + \Delta_1^F = 0$$

$$(2) f_{G, \text{hor}}^{\text{rel}} = \alpha_{21} S_1 + \alpha_{22} S_2 + \Delta_2^G + \Delta_2^F = 0$$



$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e \cdot e + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} = \frac{5}{3} \frac{e^3}{EI}$$

$$\alpha_{12} = \frac{1}{EI} \left\{ e \cdot e \cdot \frac{e}{2} + e \cdot e \cdot \frac{e}{2} \right\} = \frac{e^3}{EI}$$

$$\alpha_{22} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e + e \cdot e \cdot e + \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} = \frac{5}{3} \frac{e^3}{EI}$$

$$\Delta_1^q = \frac{1}{EI} \left\{ \frac{1}{3} \frac{1}{2} g e^2 \cdot e \cdot \frac{3}{4} e + \frac{1}{2} g e^2 \cdot e \cdot e \right\} = \frac{g e^4}{EI} \left(\frac{1}{8} + \frac{1}{2} \right) = \frac{5}{8} \frac{g e^4}{EI}$$

$$\Delta_1^F = 0$$

$$\Delta_2^q = \frac{1}{EI} \left\{ \frac{1}{2} g e^2 \cdot e \cdot \frac{e}{2} \right\} = \frac{1}{4} \frac{g e^4}{EI}$$

$$\Delta_2^F = 0$$

$$\frac{5}{3} \frac{e^3}{EI} s_1 + \frac{e^3}{EI} s_2 + \frac{5}{8} \frac{g e^4}{EI} = 0 \quad | \cdot \frac{EI}{e^3}$$

$$\frac{e^3}{EI} s_1 + \frac{5}{3} \frac{e^3}{EI} s_2 + \frac{1}{4} \frac{g e^4}{EI} = 0 \quad | \cdot \frac{EI}{e^3}$$

$$\frac{5}{3} s_1 + s_2 + \frac{5}{8} g e = 0 \quad \Rightarrow \quad s_2 = -\frac{5}{3} s_1 - \frac{5}{8} g e$$

$$s_1 + \frac{5}{3} \left(-\frac{5}{3} s_1 - \frac{5}{8} g e \right) + \frac{1}{4} g e = 0$$

$$s_1 - \frac{25}{9} s_1 - \frac{25}{24} g e + \frac{1}{4} g e = 0$$

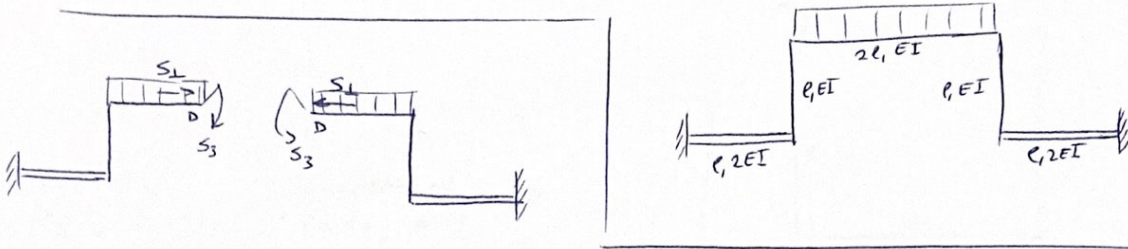
$$-\frac{16}{9} s_1 - \frac{19}{24} g e = 0$$

$$s_1 = -\frac{19 \cdot 9}{16 \cdot 24} g e = \underline{\underline{s_1 = -\frac{57}{128} g e}}$$

$$s_2 = -\frac{5}{3} \cdot \left(-\frac{57}{128} g e \right) - \frac{5}{8} g e$$

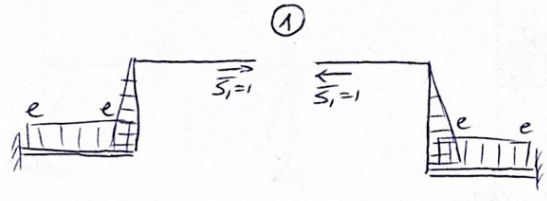
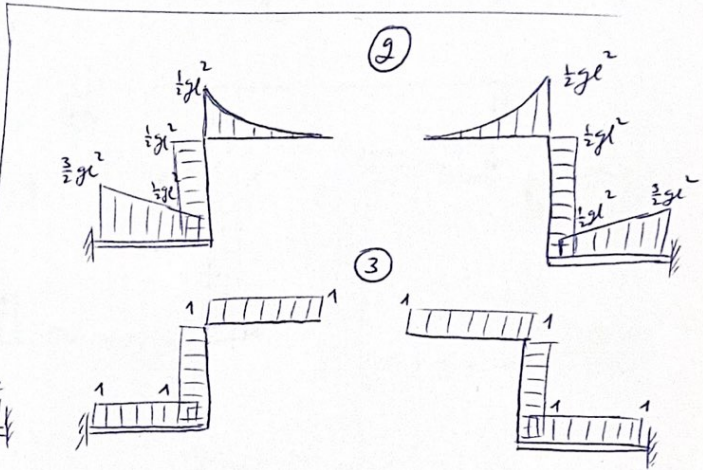
$$s_2 = \frac{95}{128} g e - \frac{80}{128} g e \quad \Rightarrow \quad \underline{\underline{s_2 = \frac{15}{128} g e}}$$

10



sim. konstrukcija, sim. opt. $\Rightarrow S_2 = 0$

(1) $f_{D,rec}^{hor} = \alpha_{11} S_1 + \alpha_{13} S_3 + \Delta_1^2 = 0$
 (2) $f_{D,rec}^{ver} = \alpha_{31} S_1 + \alpha_{33} S_3 + \Delta_3^2 = 0$



$$\alpha_{11} = \frac{1}{2EI} \{ e \cdot e \cdot e \} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \right\} \cdot 2 = \frac{5}{3} \frac{e^3}{EI}$$

$$\alpha_{13} = \frac{1}{2EI} \{ e \cdot e \cdot 1 \} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot 1 \right\} \cdot 2 = 2 \frac{e^3}{EI}$$

$$\alpha_{33} = \frac{1}{2EI} \{ 1 \cdot e \cdot 1 \} \cdot 2 + \frac{1}{EI} \{ 1 \cdot e \cdot 1 \cdot 2 \} \cdot 2 = 5 \frac{e}{EI}$$

$$\Delta_1^2 = \frac{1}{2EI} \left\{ \frac{\frac{3}{2} gl^2 + \frac{1}{2} gl^2}{2} \cdot e \cdot e \right\} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{2} gl^2 \cdot e \cdot \frac{e}{2} \right\} \cdot 2 = \frac{3}{2} \frac{gl^4}{EI}$$

$$\Delta_3^2 = \frac{1}{2EI} \left\{ \frac{\frac{3}{2} gl^2 + \frac{1}{2} gl^2}{2} \cdot e \cdot 1 \right\} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{2} gl^2 \cdot e \cdot 1 + \frac{1}{3} \frac{1}{2} gl^2 \cdot e \cdot 1 \right\} \cdot 2 = \frac{7}{3} \frac{gl^3}{EI}$$

$$\frac{5}{3} \frac{e^3}{EI} S_1 + 2 \frac{e^3}{EI} S_3 + \frac{3}{2} \frac{gl^4}{EI} = 0 \quad | \cdot \frac{EI}{2e^3}$$

$$2 \frac{e^2}{EI} S_1 + 5 \frac{e^3}{EI} S_3 + \frac{7}{3} \frac{gl^3}{EI} = 0 \quad | \cdot \frac{EI}{e}$$

$$S_2 = -\frac{5}{6} S_1 e - \frac{3}{4} gl^2$$

$$2 S_1 e + 5 \left(-\frac{5}{6} S_1 e - \frac{3}{4} gl^2 \right) + \frac{7}{3} gl^2 = 0$$

$$2 S_1 e - \frac{25}{6} S_1 e - \frac{15}{4} gl^2 + \frac{7}{3} gl^2 = 0$$

$$-\frac{13}{6} S_1 e + \frac{17}{12} gl^2 = 0$$

$$S_1 = \frac{17 \cdot 6}{12 \cdot 13} gl \Rightarrow \boxed{S_1 = \frac{17}{26} gl}$$

$$S_3 = -\frac{5}{6} \left(-\frac{17}{26} gl^2 \right) - \frac{3}{4} gl^2 = gl^2 \left(\frac{85}{156} - \frac{117}{156} \right) =$$

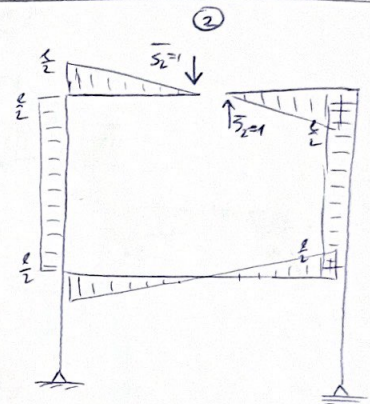
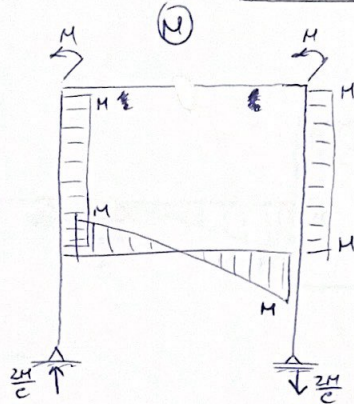
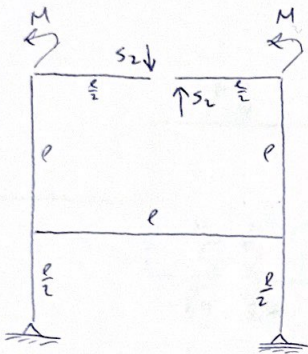
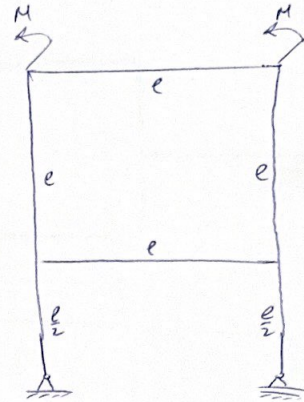
$$S_3 = -\frac{32}{156} gl^2 \Rightarrow \boxed{S_3 = -\frac{8}{39} gl^2}$$

①

$EI = \text{const.}$

sim. konst. , aufwachen opt. \Rightarrow
 $(\alpha_{12}=0, \alpha_{32}=0)$, $(\Delta_1=0, \Delta_3=0)$

$\Rightarrow \alpha_{22} S_2 + \Delta_2 = 0$



$\alpha_{22} = \textcircled{2} \times \textcircled{2} = \frac{1}{EI} \left\{ \frac{1}{2} \frac{e}{2} \frac{e}{2} \frac{2}{3} \frac{e}{2} \cdot 4 + \frac{e}{2} \frac{e}{2} \cdot 2 \right\} = \frac{e^3}{EI} \left(\frac{1}{6} + \frac{1}{2} \right) = \frac{2}{3} \frac{e^3}{EI}$

$\Delta_2 = \textcircled{M} \times \textcircled{2} = \frac{1}{EI} \left\{ -M \frac{e}{2} \cdot 2 - \frac{1}{2} M \frac{e}{2} \frac{2}{3} \frac{e}{2} \cdot 2 \right\} = \frac{M e^2}{EI} \left\{ -1 - \frac{1}{6} \right\} = -\frac{7}{6} \frac{M e^2}{EI}$

$\frac{2}{3} \frac{e^3}{EI} S_2 - \frac{7}{6} \frac{M e^2}{EI} = 0 \quad | \cdot \frac{6EI}{e^2}$

$4 S_2 e - 7 M = 0$

$S_2 = \frac{7}{4} \frac{M}{e}$

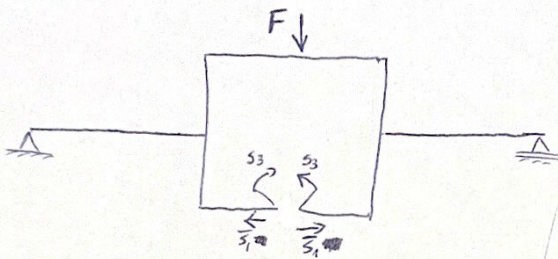
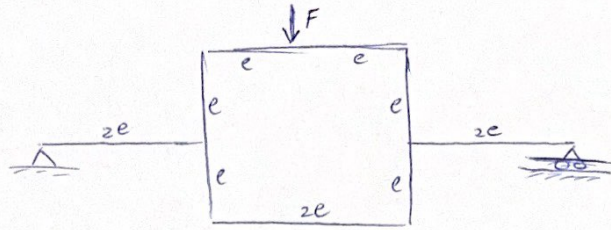
⑫ $EI = \text{const}$

st. konstr. , st. opt.
 $(\alpha_{12}=0 \quad \alpha_{32}=0)$, $(\Delta_2=0)$
 $(\alpha_{21}=0 \quad \alpha_{13}=0)$

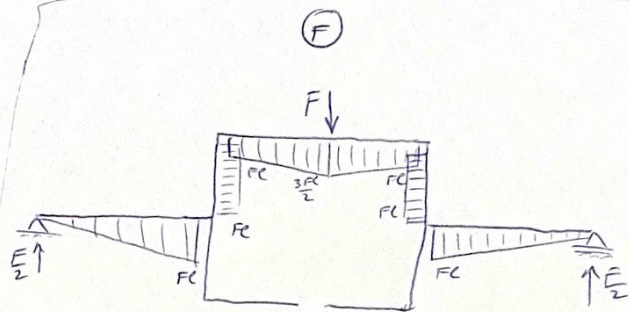
$\Rightarrow S_2 = 0$

$\alpha_{11} S_1 + \alpha_{31} S_3 + \Delta_1 = 0$

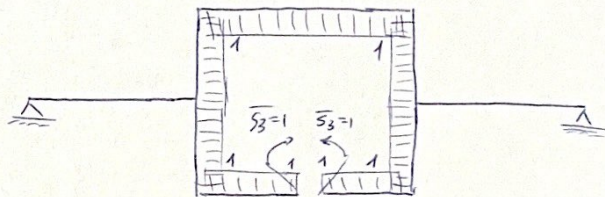
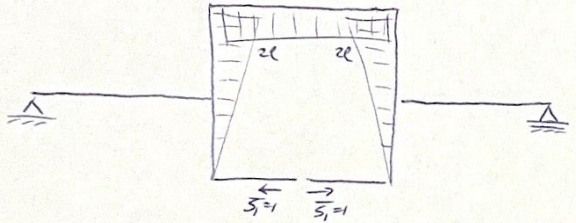
$\alpha_{31} S_1 + \alpha_{33} S_3 + \Delta_3 = 0$



①



②



$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \cdot 2 + 2e \cdot 2e \cdot 2e \right\} = \frac{c^3}{EI} \left\{ \frac{16}{3} + 8 \right\} = \frac{40}{3} \frac{c^3}{EI}$

$\alpha_{12} = \alpha_{21} = \frac{1}{EI} \left\{ 1 \cdot 2e \cdot c \cdot 2 + 1 \cdot 2e \cdot 2e \right\} = 8 \frac{c^2}{EI}$

$\alpha_{22} = \frac{1}{EI} \left\{ 1 \cdot c \cdot 1 \cdot 8 \right\} = 8 \frac{c}{EI}$

$\Delta_1 = \frac{1}{EI} \left\{ Fc \cdot c \left(e + \frac{1}{2}e \right) \cdot 2 + 2e \cdot e \left(Fc + \frac{1}{4}Fc \right) \cdot 2 \right\} = \frac{Fc^3}{EI} (3 + 5) = 8 \frac{Fc^3}{EI}$

$\Delta_2 = \frac{1}{EI} \left\{ Fc \cdot c \cdot 1 \cdot 2 + \frac{Fc + \frac{3}{2}Fc}{2} \cdot e \cdot 1 \cdot 2 \right\} = \frac{Fc^2}{EI} \left(2 + \frac{5}{2} \right) = \frac{9}{2} \frac{Fc^2}{EI}$

$\frac{40}{3} \frac{c^3}{EI} S_1 + 8 \frac{c^2}{EI} S_3 + 8 \frac{Fc^3}{EI} = 0 \quad | \cdot \frac{EI}{8c^2}$

$8 \frac{c^2}{EI} S_1 + 8 \frac{1}{EI} S_3 + \frac{9}{2} \frac{Fc^2}{EI} = 0 \quad | \cdot \frac{EI}{c}$

$\frac{5}{3} S_1 c + S_3 + Fc = 0 \Rightarrow S_3 = -Fc - \frac{5}{3} S_1 c$

$8 S_1 c + 8 \left(-Fc - \frac{5}{3} S_1 c \right) + \frac{9}{2} Fc = 0$

$-\frac{16}{3} S_1 c - \frac{7}{2} Fc = 0 \Rightarrow S_1 = -\frac{21}{32} F$

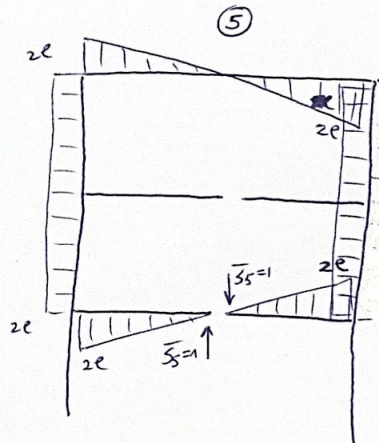
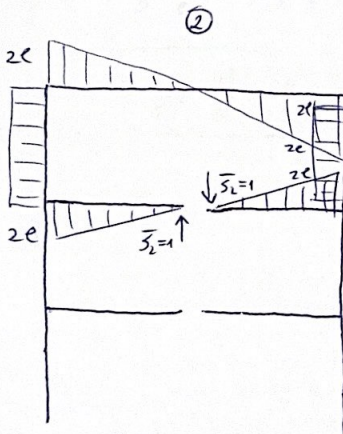
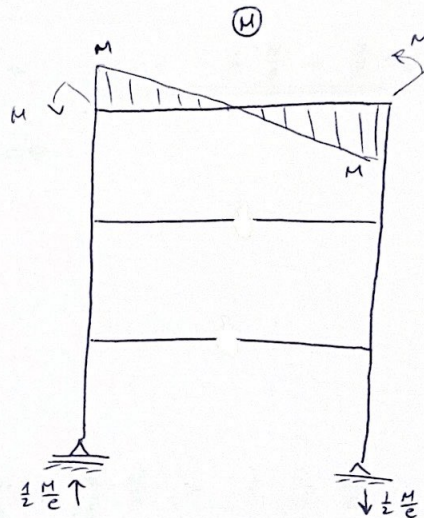
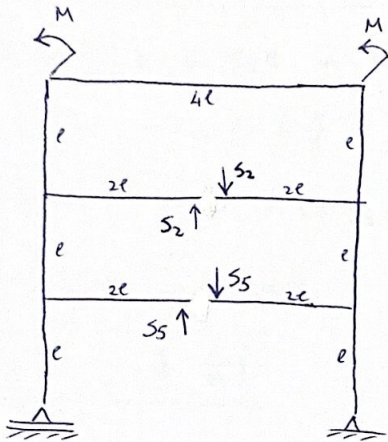
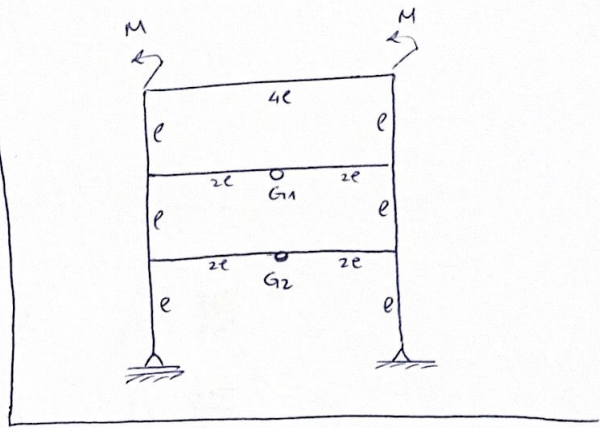
$S_3 = -Fc + \frac{5}{3} \frac{21}{32} Fc = -Fc + \frac{35}{32} Fc = -\frac{1}{32} Fc$

⑬ $EI = \text{const.}$

Saw. koush. , antiwzmiano opt. \Rightarrow
 \Rightarrow postaje sawo transv. sila

$$f_{G_1, \text{vert}}^{\text{vert}} = \alpha_{22} S_2 + \alpha_{25} S_5 + \Delta_2 = 0$$

$$f_{G_2, \text{vert}}^{\text{vert}} = \alpha_{52} S_2 + \alpha_{55} S_5 + \Delta_5 = 0$$



$$\alpha_{22} = \frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \cdot 4 + 2e \cdot e \cdot 2e \cdot 2 \right\} = \frac{e^3}{EI} \left\{ \frac{32}{3} + 8 \right\} = \frac{56}{3} \frac{e^3}{EI}$$

$$\alpha_{25} = \frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \cdot 2 + 2e \cdot e \cdot 2e \cdot 2 \right\} = \frac{e^3}{EI} \left\{ \frac{16}{3} + 8 \right\} = \frac{40}{3} \frac{e^3}{EI}$$

$$\alpha_{55} = \frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \cdot 4 + 2e \cdot 2e \cdot 2e \cdot 2 \right\} = \frac{e^3}{EI} \left\{ \frac{32}{3} + 16 \right\} = \frac{80}{3} \frac{e^3}{EI}$$

$$\Delta_2 = \frac{1}{EI} \left\{ \frac{1}{2} M \cdot 2e - \frac{2}{3} 2e \cdot 2 \right\} = \frac{8}{3} \frac{Me^2}{EI}$$

$$\Delta_5 = \frac{1}{EI} \left\{ \frac{1}{2} M \cdot 2e - \frac{2}{3} 2e \cdot 2 \right\} = \frac{8}{3} \frac{Me^2}{EI}$$

$$\frac{56}{3} \frac{e^3}{EI} S_2 + \frac{40}{3} \frac{e^3}{EI} S_5 + \frac{8}{3} \frac{Me^2}{EI} = 0 \quad | \cdot \frac{3EI}{8e^2}$$

$$\frac{40}{3} \frac{e^3}{EI} S_2 + \frac{80}{3} \frac{e^3}{EI} S_5 + \frac{8}{3} \frac{Me^2}{EI} = 0 \quad | \cdot \frac{3EI}{8e^2}$$

$$7 S_2 e + 5 S_5 e + M = 0 \quad \Rightarrow \quad \underline{S_5 = -\frac{1}{5} \frac{M}{e} - \frac{7}{5} S_2}$$

$$5 S_2 e + 10 \left(-\frac{1}{5} \frac{M}{e} - \frac{7}{5} S_2 \right) e + M = 0$$

$$5 S_2 e - 2 M - 14 S_2 e + M = 0$$

$$-9 S_2 e - M = 0$$

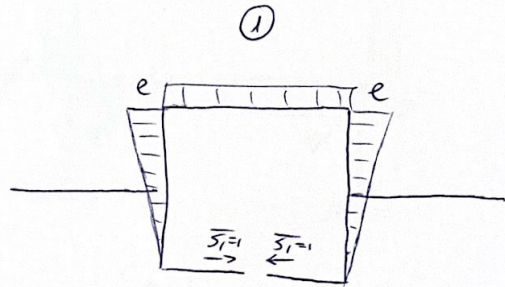
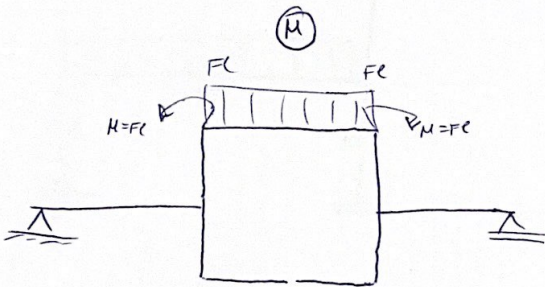
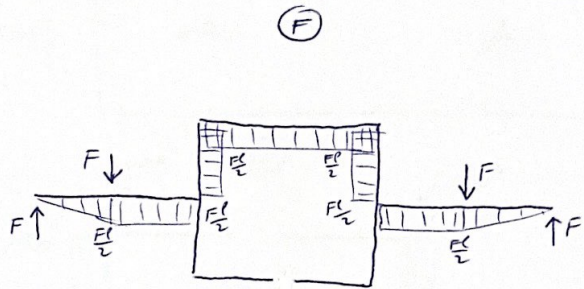
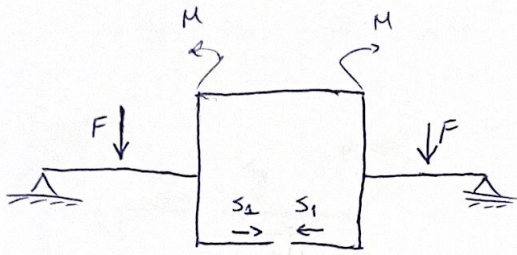
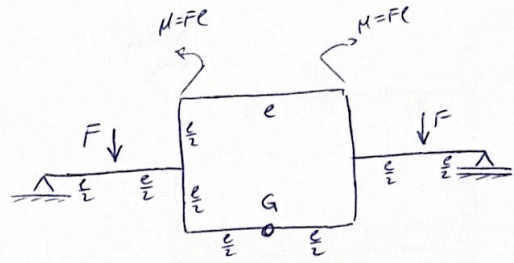
$$\boxed{S_2 = -\frac{1}{9} \frac{M}{e}}$$

$$S_5 = -\frac{1}{5} \frac{M}{e} - \frac{7}{5} \left(-\frac{1}{9} \frac{M}{e} \right) = -\frac{1}{5} \frac{M}{e} + \frac{7}{45} \frac{M}{e} = -\frac{2}{45} \frac{M}{e}$$

$$\boxed{S_5 = -\frac{2}{45} \frac{M}{e}}$$

⑭ $M = F \cdot e$
 $EI = \text{const.}$

sim. konst., sim. opt. \Rightarrow
 $\Rightarrow S_2 = 0$



$$f_{G, \text{res}}^{\text{hor}} = \alpha_{11} S_1 + \Delta_1^F + \Delta_1^M = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \cdot 2 + e \cdot e \cdot e \right\} = \frac{5}{3} \frac{e^3}{EI}$$

$$\Delta_1^F = \frac{1}{EI} \left\{ -\frac{F \cdot e}{2} \cdot \frac{1}{2} \left(\frac{e}{2} + \frac{e}{2} \right) \cdot 2 - \frac{F \cdot e}{2} \cdot e \cdot e \right\} = \frac{F \cdot e^3}{EI} \left(-\frac{3}{8} - \frac{1}{2} \right) = -\frac{7}{8} \frac{F \cdot e^3}{EI}$$

$$\Delta_1^M = \frac{1}{EI} (F \cdot e \cdot e \cdot e) = \frac{F \cdot e^3}{EI}$$

$$\frac{5}{3} \frac{e^3}{EI} S_1 - \frac{7}{8} \frac{F \cdot e^3}{EI} + \frac{F \cdot e^3}{EI} = 0 \quad | \cdot 24 \frac{EI}{e^3}$$

$$40 S_1 + 3 F = 0$$

$$\boxed{S_1 = -\frac{3}{40} F}$$

Писмени испит из
ОСНОВА ОТПОРНОСТИ КОНСТРУКЦИЈА
Јуни испитни доп. 2008.

1. Носач АВС константног попречног пресека оптерећен је континуираном силом F и континуалним оптерећењем $q = F/l$. Израдити дијаграм момената савијања.
2. Носач АВСД просторно је оптерећен према слици. Користећи Хипотезу максималних тангенцијалних напона одредити највећи резултујући напон у конструкцији ако је попречни пресеци:
а) пуни круг $\varnothing d = 9 \text{ cm}$,
б) стандардни профил I 26.

I група: $F=5 \text{ kN}$, $l=40 \text{ cm}$
II група: $F=7 \text{ kN}$, $l=30 \text{ cm}$

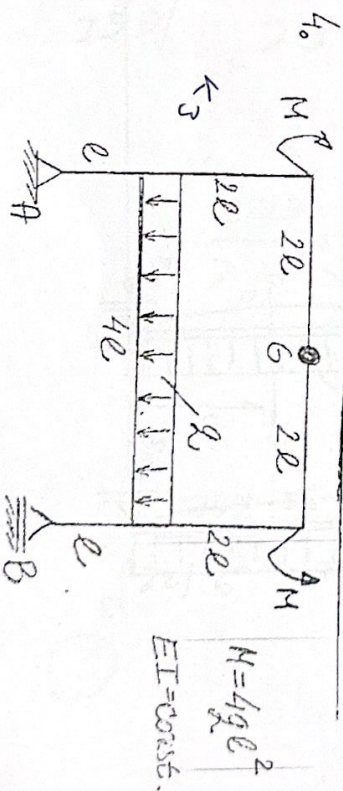
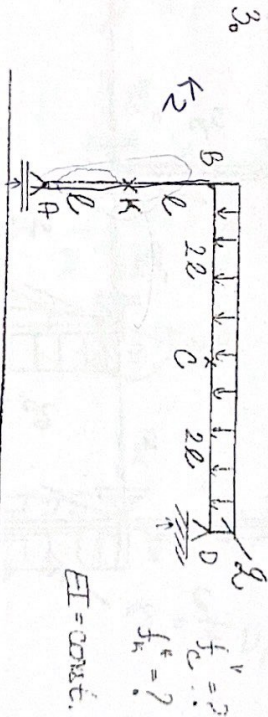
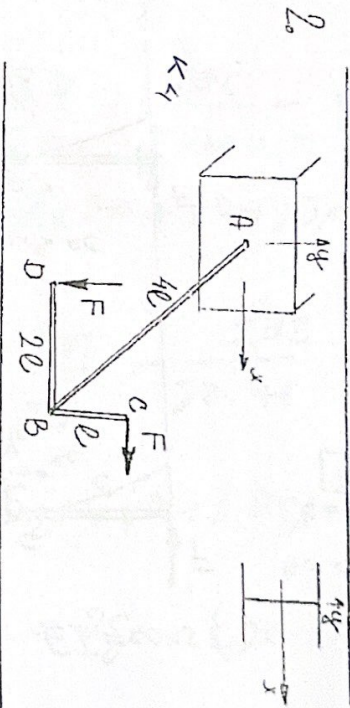
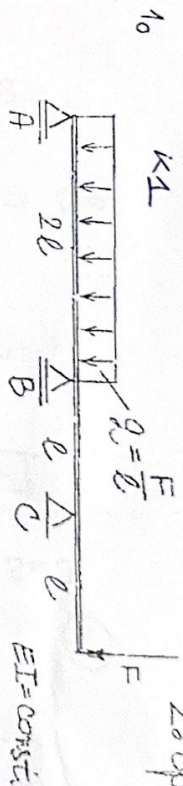
3. Равански статички одређен носач АВСД константног попречног пресека оптерећен је према слици. Одредити хоризонтално померање пресека К и вертикално померање пресека С.
4. Равански носач са зглобом С оптерећен је континуалним оптерећењем q и моментом M .
а) Решити носач и напратити дијаграм момената савијања.
б) Одредити максималну вредност момента M ако је дозвољени напон $\sigma_{\text{доп}} = 8 \text{ kN/cm}^2$, а попречни пресек стандардни профил I група: I 14,
II група: I 16.

Напомене:

- а) Испит траје три сата.
- б) Резултати ће бити објављени до петка 4. јула у 15 сати.
- в) Не користити свеске, збирке задатака, мобилне телефоне!

- у Београду, 01.07.2008. год.

ČASOVNI POVOLJNO
Profesor NENAD
064/155-72-42

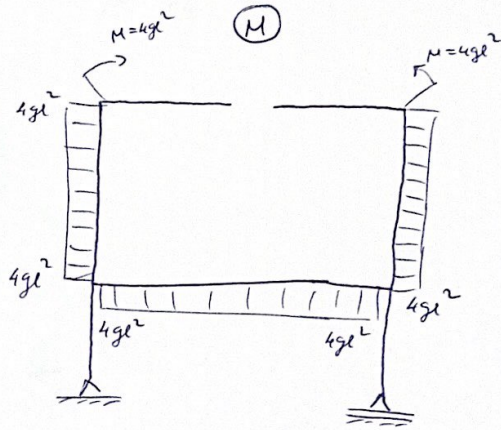
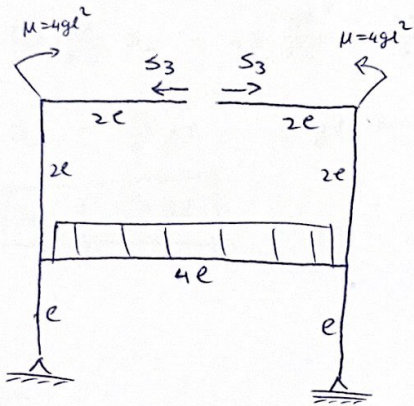
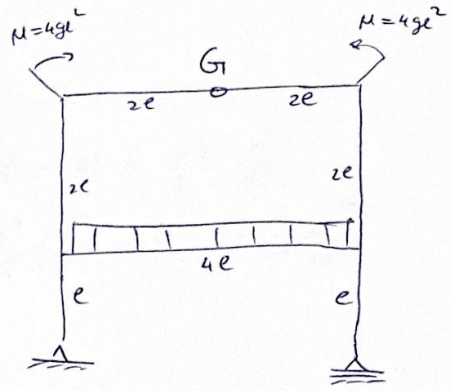


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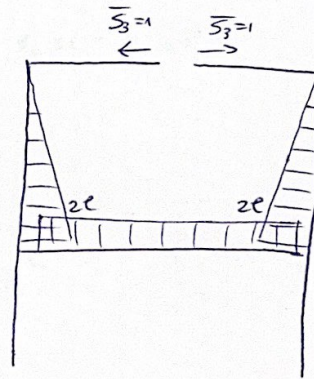
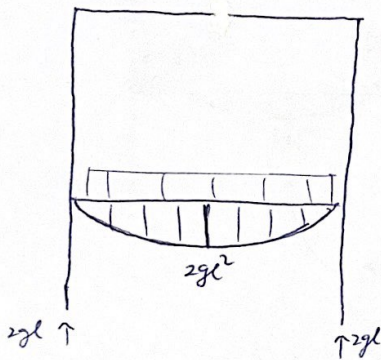
(4) $M = 4gl^2$, $EI = \text{const.}$

odrediti max. vrednost momenta M ,
 ako je $\sigma_{\text{dop}} = 8 \frac{\text{kJ}}{\text{cm}^2}$, I 14

sim. konst., sim. opt. $\Rightarrow S_2 = 0$



(2)



$$M = 2gl \cdot z - gz \cdot \frac{z}{2} = -\frac{1}{2}gz^2 + 2gl \cdot z$$

$$z=0 \Rightarrow M=0$$

$$z=2e \Rightarrow M = -\frac{1}{2}4gl^2 + 2gl \cdot 2e = 2gl^2$$

$$z=4e \Rightarrow M=0$$

$$\int_{G, rec}^{hor} = \alpha_{11} S_{\perp} + \Delta_1^H + \Delta_1^Q = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} 2e \cdot 2e \cdot \frac{2}{3} 2e \cdot 2 + 2e \cdot 4e \cdot 2e \right\} = \frac{e^3}{EI} \left(\frac{16}{3} + 16 \right) = \frac{64}{3} \frac{e^3}{EI}$$

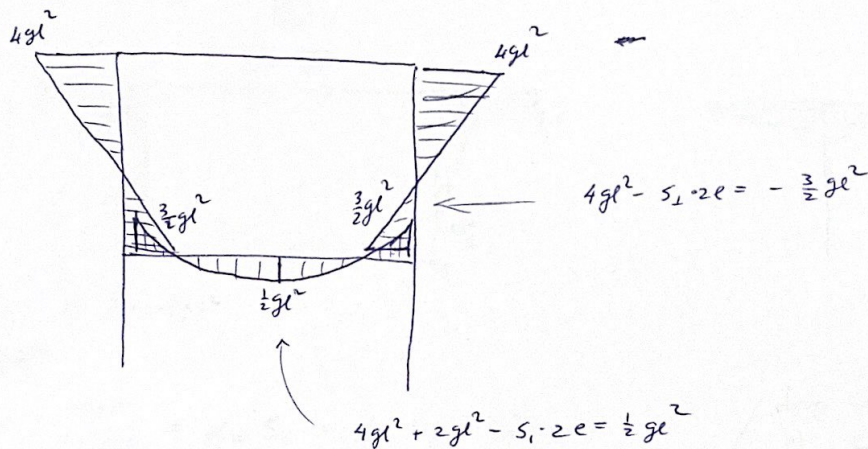
$$\Delta_1^H = \frac{1}{EI} \left\{ -4gl^2 \cdot 2e \cdot e \cdot 2 - 4gl^2 \cdot 4e \cdot 2e \right\} = \frac{gl^4}{EI} (-16 - 32) = -48 \frac{gl^4}{EI}$$

$$\Delta_1^Q = \frac{1}{EI} \left\{ -2 \cdot \frac{2}{3} \cdot 2gl^2 \cdot 2e \cdot 2e \right\} = -\frac{32}{3} \frac{gl^4}{EI}$$

$$\frac{64}{3} \frac{gl^3}{EI} S_{\perp} - 48 \frac{gl^4}{EI} - \frac{32}{3} \frac{gl^4}{EI} = 0 \quad / \cdot \frac{3}{64} \frac{EI}{e^3}$$

$$S_{\perp} - \frac{9}{4} gl - \frac{1}{2} gl = 0$$

$$\boxed{S_{\perp} = \frac{11}{4} gl}$$



$$\sigma_{max} = \frac{4gl^2}{W_x} = \frac{M}{W_x} \leq \sigma_{dop}$$

$$M \leq \sigma_{dop} \cdot W_x = 8 \frac{kN}{cm^2} \cdot 81,9 \text{ cm}^3 = 655,2 \text{ kNm} \Rightarrow \boxed{M_{max} = 655 \text{ kNm}}$$

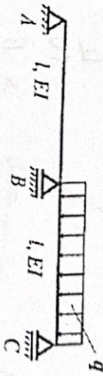
$$W_x = 81,9 \text{ cm}^3 \quad - \quad z = I 14$$

Писмени испит из ООК
-Октобар (допунски рок) 2011.-

1. Континуални гредни носач константне крутости оптерећен је према скици.
- Применом Методе растављивања решити носач и нацртати статичке дијаграме.
- Према вредности момента у пресеку напад ослона В извршити димензионисање носача, ако је пресек израђен од сталног легуре Г-профила, ЈОС С.В3.131. Потребни бројни подаци: $q=4\text{ kN/cm}$; $l=60\text{ cm}$.
 2. Задати статички неодређени равнотежни носач оптерећен је према скици. Одредити статички неопознате величине и нацртати дијаграме момента сила у функцији општих бројева. Задатак решити применом методе Вершилгиновић поступком, користећи особину симетрије.
 3. Просторни носач константног кружног попречног пресека оптерећен је према скици.
- Нацртати статичке дијаграме (од слаке силе појединачно).
- Еквивалентни напон у пресеку А сračунати применом Хипотезе дозвољени напон $\sigma_{\text{dov}}=10\text{ kN/cm}^2$ при чему је $F=1\text{ kN}$ и $l=60\text{ mm}$.
- Напомене:
- Испит traje три сата
- Дозвољено је само штампана литература – не записи!
- Резултати ће бити објављени најкасније до понедељка 14.11.2011. у 13 сати
- Улога у испитним радоме: понедељак 14.11.2011. год. у 13 сати – одб. 428.
- Ускени део испита одржаће се у уторак 15.11.2011. год. у 13 сати – одб. 428.
у Београду, 13.11.2011. год.

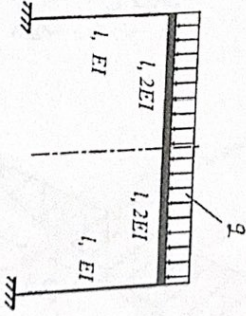
Из Кабинета

Група 1.



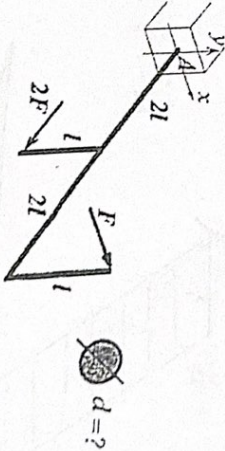
Задатак 1.

K-1



Задатак 2.

K-3



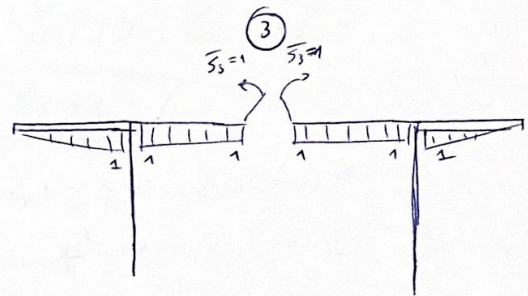
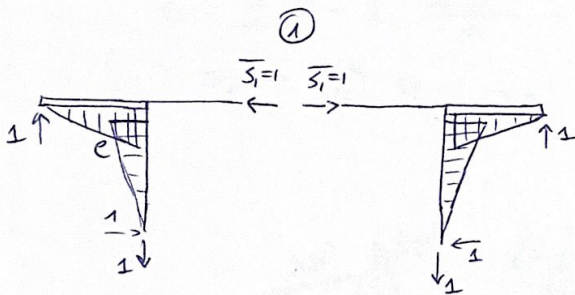
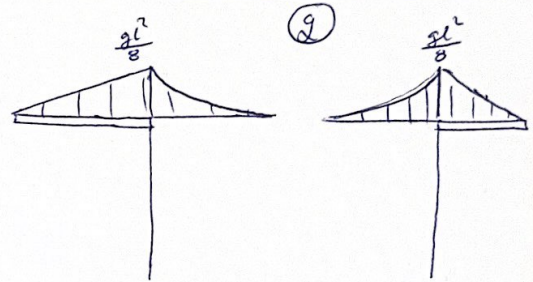
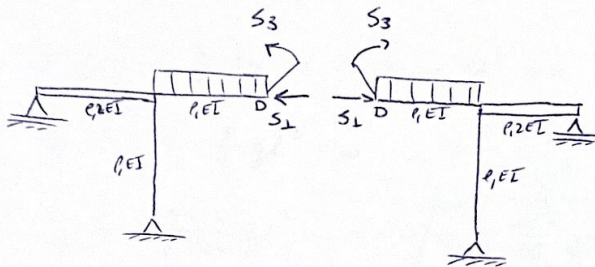
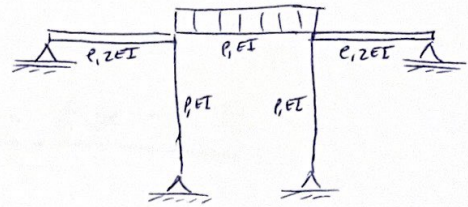
Задатак 3.

K-4

oktober 11

② naerdek zGini dijagram
womenta saujanya

sim. kaus, sim. opd. $\Rightarrow S_2 = 0$



$$M_f = -qz \cdot \frac{z}{2} = -\frac{1}{2} qz^2$$

$$(1) \quad f_{D,rec} = \alpha_{11} S_1 + \alpha_{13} S_3 + \Delta_1^q = 0$$

$$(2) \quad \varphi_{B,rec} = \alpha_{31} S_1 + \alpha_{33} S_3 + \Delta_3^q = 0$$

drugi uedu Ci Gio:
 (1) $f_{0,e}^{hor} = -f_{0,d}^{hor}$
 (2) $\varphi_{0,e} = -\varphi_{0,d}$
 Pa Ci anda posmetali kao posebne dijagrame (1) (2) (3) (4) (5) (6)

$$\alpha_{11} = \frac{1}{2EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \right\} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} c \right\} \cdot 2 = \frac{c^3}{EI} \left(\frac{1}{3} + \frac{2}{3} \right) = \frac{c^3}{EI}$$

$$\alpha_{13} = \frac{1}{2EI} \left\{ \frac{1}{2} c \cdot c \cdot \frac{2}{3} \cdot 1 \right\} \cdot 2 = \frac{1}{3} \frac{c^2}{EI}$$

$$\alpha_{33} = \frac{1}{2EI} \left\{ \frac{1}{2} \cdot 1 \cdot e \cdot \frac{2}{3} \cdot 1 \right\} \cdot 2 + \frac{1}{EI} \left\{ 1 \cdot \frac{c}{2} \cdot 1 \right\} \cdot 2 = \frac{c}{EI} \left(\frac{1}{3} + 1 \right) = \frac{4}{3} \frac{1}{EI}$$

$$\Delta_1^q = \frac{1}{2EI} \left\{ -\frac{1}{2} \frac{qe^2}{8} \cdot c \cdot \frac{2}{3} c \right\} \cdot 2 = -\frac{1}{24} \frac{qe^4}{EI}$$

$$\Delta_3^q = \frac{1}{2EI} \left\{ -\frac{1}{2} \frac{qe^2}{8} \cdot c \cdot \frac{2}{3} c \right\} \cdot 2 + \frac{1}{EI} \left\{ \frac{1}{3} \frac{qe^2}{8} \cdot \frac{1}{2} \cdot 1 \right\} \cdot 2 = \frac{qe^3}{EI} \left(-\frac{1}{24} - \frac{1}{24} \right) = -\frac{1}{12} \frac{qe^3}{EI}$$

$$(1) \quad \frac{e^3}{EI} s_{\perp} + \frac{1}{3} \frac{e^2}{EI} s_3 - \frac{1}{24} \frac{gl^4}{EI} = 0 \quad | \cdot \frac{EI}{e^3}$$

$$(2) \quad \frac{1}{3} \frac{e^2}{EI} s_{\perp} + \frac{4}{3} \frac{e}{EI} s_3 - \frac{1}{12} \frac{gl^3}{EI} = 0 \quad | \cdot 12 \frac{EI}{e}$$

$$(1) \quad s_{\perp} + \frac{1}{3} \frac{s_3}{e} - \frac{1}{24} gl = 0 \quad \Rightarrow \quad \underline{s_{\perp} = \frac{1}{24} gl - \frac{1}{3} \frac{s_3}{e}}$$

$$(2) \quad 4s_{\perp}e + 16s_3 - gl^2 = 0$$

$$4\left(\frac{1}{24} gl - \frac{1}{3} \frac{s_3}{e}\right)e + 16s_3 - gl^2 = 0$$

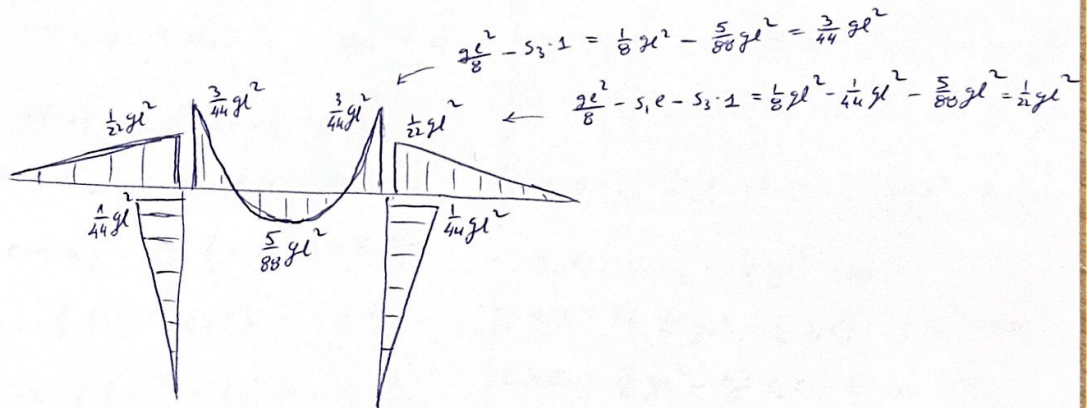
$$\frac{1}{6} gl^2 - \frac{4}{3} s_3 + 16s_3 - gl^2 = 0$$

$$\frac{44}{3} s_3 - \frac{5}{6} gl^2 = 0$$

$$s_3 = \frac{5 \cdot 3}{6 \cdot 44} gl^2 \quad \Rightarrow \quad \boxed{s_3 = \frac{5}{88} gl^2}$$

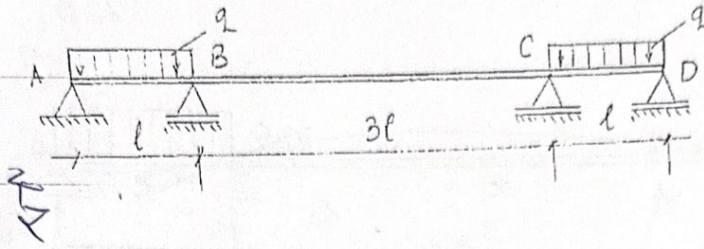
$$(1) \quad \Rightarrow \quad s_{\perp} = \frac{1}{24} gl - \frac{1}{3} \frac{\frac{5}{88} gl^2}{e} = gl \left(\frac{1}{24} - \frac{5}{264} \right) = gl \left(\frac{11}{264} - \frac{5}{264} \right) =$$

$$\Rightarrow \quad s_{\perp} = \frac{6}{264} gl \quad \Rightarrow \quad \boxed{s_{\perp} = \frac{1}{44} gl}$$



II ГРУПА

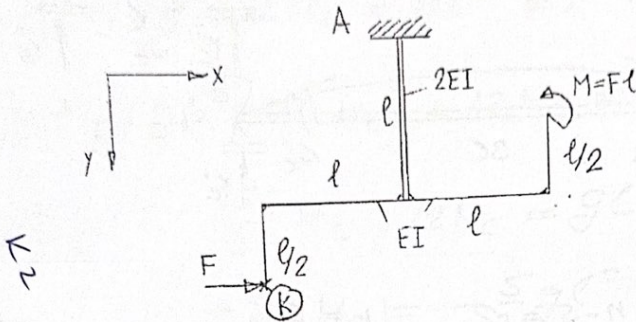
1.



$\sigma_{max} = ?$
 $q = 5 \text{ [kN/m]}$
 $l = 200 \text{ [cm]}$

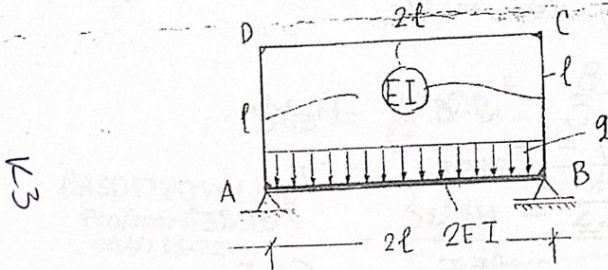
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2.

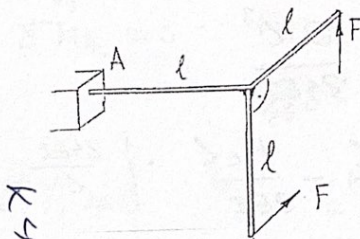


$f_k^y = ?$
 $f_k^x = ?$
 $\varphi_k = ?$

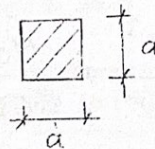
3.



4.

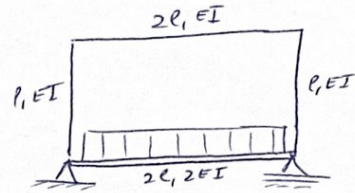


a) $\phi d = 5 \text{ [cm]}$
 b) $\phi d = 5 \text{ [cm]}$ } $F = 2 \text{ [kN]}$
 $l = 50 \text{ [cm]}$

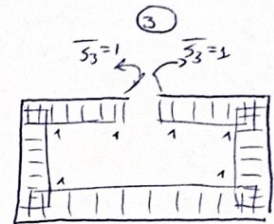
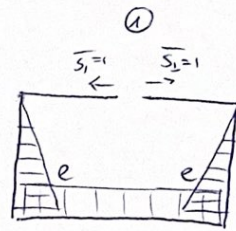
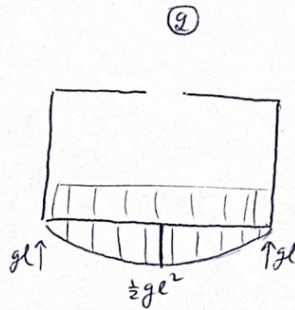
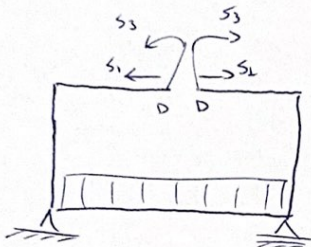


nepoznat rok

3) nacrtati zbirni dijagram momenta savijanja



slu. kon. , slu. opl. $\Rightarrow S_2 = 0$



$$M_x = qlx - qz \frac{z^2}{2} = -\frac{1}{2} qz^2 + ql \cdot z; \quad \begin{matrix} z=0 & \Rightarrow & M=0 \\ z=e & \Rightarrow & M = \frac{1}{2} ql^2 \\ z=2e & \Rightarrow & M=0 \end{matrix}$$

$$(1) f_{D,rec}^{hor} = \alpha_{11} S_1 + \alpha_{13} S_3 + \Delta_1^2 = 0$$

$$(2) f_{D,rec} = \alpha_{31} S_1 + \alpha_{33} S_3 + \Delta_3^2 = 0$$

$$\alpha_{11} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot \frac{2}{3} e \cdot 2 \right\} + \frac{1}{2EI} \{ e \cdot 2e \cdot e \} = \frac{5}{3} \frac{e^3}{EI}$$

$$\alpha_{13} = \frac{1}{EI} \left\{ \frac{1}{2} e \cdot e \cdot 1 \cdot 2 \right\} + \frac{1}{2EI} \{ e \cdot 2e \cdot 1 \} = 2 \frac{e^2}{EI}$$

$$\alpha_{33} = \frac{1}{EI} \{ 1 \cdot e \cdot 1 \cdot 4 \} + \frac{1}{2EI} \{ 1 \cdot 2e \cdot 1 \} = 5 \frac{e}{EI}$$

$$\Delta_1^2 = \frac{1}{2EI} \left\{ -2 \cdot \frac{2}{3} \frac{1}{2} ql^2 \cdot e \cdot e \right\} = -\frac{1}{3} \frac{ql^4}{EI}$$

$$\Delta_3^2 = \frac{1}{2EI} \left\{ -2 \cdot \frac{2}{3} \frac{1}{2} ql^2 \cdot e \cdot 1 \right\} = -\frac{1}{3} \frac{ql^3}{EI}$$

$$(1) \Rightarrow \frac{5}{3} \frac{e^3}{EI} S_1 + 2 \frac{e^2}{EI} S_3 - \frac{1}{3} \frac{ql^4}{EI} = 0 \quad | \cdot \frac{EI}{2e^2}$$

$$(2) \Rightarrow 2 \frac{e^2}{EI} S_1 + 5 \frac{e}{EI} S_3 - \frac{1}{3} \frac{ql^3}{EI} = 0 \quad | \cdot \frac{EI}{e}$$

$$\frac{5}{6} S_1 e + S_3 - \frac{1}{6} ql^2 = 0 \Rightarrow S_3 = \frac{1}{6} ql^2 - \frac{5}{6} S_1 e$$

$$2 S_1 e + 5 S_3 - \frac{1}{3} ql^2 = 0$$

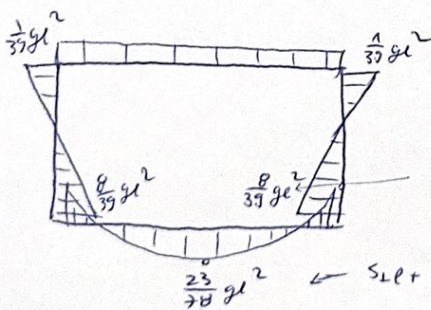
$$2 S_1 e + 5 \left(\frac{1}{6} ql^2 - \frac{5}{6} S_1 e \right) - \frac{1}{3} ql^2 = 0$$

$$2 S_1 e + \frac{5}{6} ql^2 - \frac{25}{6} S_1 e - \frac{1}{3} ql^2 = 0$$

$$-\frac{13}{6} S_1 e + \frac{1}{2} ql^2 = 0 \Rightarrow S_1 = \frac{3}{13} ql$$

$$S_3 = \frac{1}{6} ql^2 - \frac{5}{6} \cdot \frac{3}{13} ql^2 = \frac{1}{6} ql^2 - \frac{5}{26} ql^2 =$$

$$S_3 = \frac{26}{156} ql^2 - \frac{30}{156} ql^2 \Rightarrow S_3 = -\frac{1}{39} ql^2$$



$$S_1 e + S_3 e = \frac{3}{13} ql^2 - \frac{1}{39} ql^2 = \frac{8}{39} ql^2$$

$$S_1 e + S_3 e - \frac{1}{2} ql^2 = \frac{3}{13} ql^2 - \frac{1}{39} ql^2 - \frac{1}{2} ql^2 = \frac{23}{78} ql^2$$